60 years of Spontaneously Broken Symmetries in Quantum Theory

(From Bogoliubov's theory of Superfluidity to Standard Model)

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with participation of N. PLAKIDA and V. PRIEZZHEV

Motivation

"Phase transition in Quantum system, as a rule, is accompanied by the Spontaneous Symmetry Breaking".

XXth Century Folklore

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4. Superconductivity as a Superfluidity (Bogoliubov, 1958)
5. First QFT models with SSB up to the Higgs one (1960s)
6. Higgs model triumph in Electro-Weak theory (1980s)

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- Spontaneous Symmetry Breaking;
  From magnetics to Quantum Statistics
- Broken Symmetries in Quantum Field Theory

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- Constructivism against Reductionism
- The Struggle and the Unity of Opposites:
  Pragmatic (Phenomenological) and
  Fundamental (Microscopic) models

Reductionism and Constructivism

Physical EVENTS	⇒	LAWS of Nature	⇒	PRINCIPLES of Symmetry

WIGNER hierarchy: Events form basis for laws. Laws provide the raw material for principles.

"... the progression from events to laws of nature, and from laws of nature to symmetry or invariance principles, is what I meant by the hierarchy of our knowledge of the world around us ..." [Wigner 1964]

Reductionism, 2

"The supreme test of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction".

. [Einstein, 1918]

Reductionism, 3

"... not only to know how nature is organized (and how natural phenomena proceed), but possibly to achieve the goal which may be considered as utopian and daring – understand why nature is just the way it is".

[Einstein, 1929]

+ [Eddington, Heisenberg, ...]

Constructivism against Reductionism

"The ability to <u>reduce</u> everything to simple fundamental laws does not imply to start from these laws and <u>reconstruct</u> the Universe"

PRINCIPLES $\not\Rightarrow$ LAWS need ad hoc assumptions

$\not\Rightarrow$ EVENTS + approximation

Constructivism against Reductionism

"...the more the elementary-particle physicists tell us about the nature of the fundamental laws, the less relevance they seem to have to the very real problems of the rest of science."

[P. W. Anderson, 1972; NP-1977]

Logic of Modern Reductionism

PRINCIPLES

Symmetry, Quantum, ...

+Coupling Constants,
+ Representations

Dynamic EQUATIONS

+ Solution Symmetry= RG,
Path Integral, ...

G A P filled by phenomenology
(eff., approx. models)

Eqs. Solution =
= LAWS of Nature

+ Initial Conditions,
+ Integrals of Motion,
+ asymptotics

Physical EVENTS

Solution to Dyn. Eqs. give Laws [D.Sh '96]

Macroscopical vs Dynamic Eqs.

Задачей макроскопической теории является получение уравнений типа классических уравнений математической физики, которые отображали бы всю совокупность экспериментальных фактов, относящихся к изучаемым макроскопическим объектам.

[Bogoliubov 1958]

Microscopical vs Dynamic Eqs.

В микроскопической теории ставится более глубокая задача, – понять внутренний механизм явления, исходя из законов квантовой теории. ...

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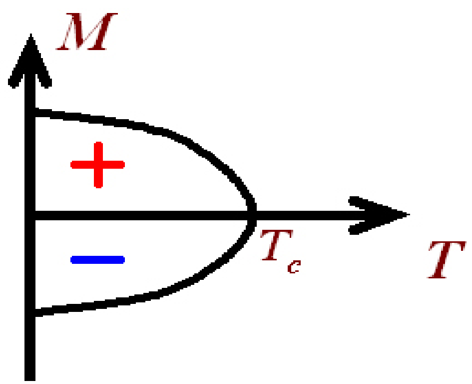
При этом ... надлежит получить также те связи между динамическими величинами, из которых вытекают уравнения макрос–копической теории. [Bogoliubov 1958]

Phenomenological vs Dynamic Eqs.

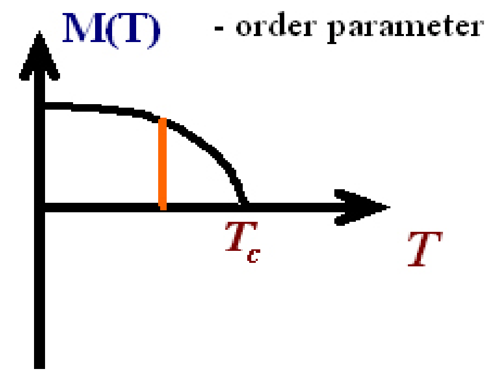
- From 4-fermion Fermi (1932) interaction to
  EW W, Z_0 Gauge Dynamics (1964) (with Higgs ...?)
- Landau SF (1940) phonon-roton model of He II
  vs Bogoliubov non-ideal Bose gas (1946)
- Ginzburg-Landau SC (1950) order parameter Ψ
  via Cooper pairs condensate ψ BSC- (1957);
  to Bogoliubov–SC by electron-phonon H_{Fr}
- Low-energy chiral models (Nambu,JL - 1961) via
  quark-meson model (Eguchi,Kikkawa 1976) ? vs ?
  QCD quark-gluon Gauge Dynamics
  <confinement, hadronization (2???)>

Phase transition with Symmetry Breaking

Order parameter [Landau 1937] in magnetics,



- add a weak field δH_+
- tend $V \to \infty$
- put $\delta H_+ \to 0$



Ferromagnetism in a finite volume V. In the termodynamic limit

Correlation function:

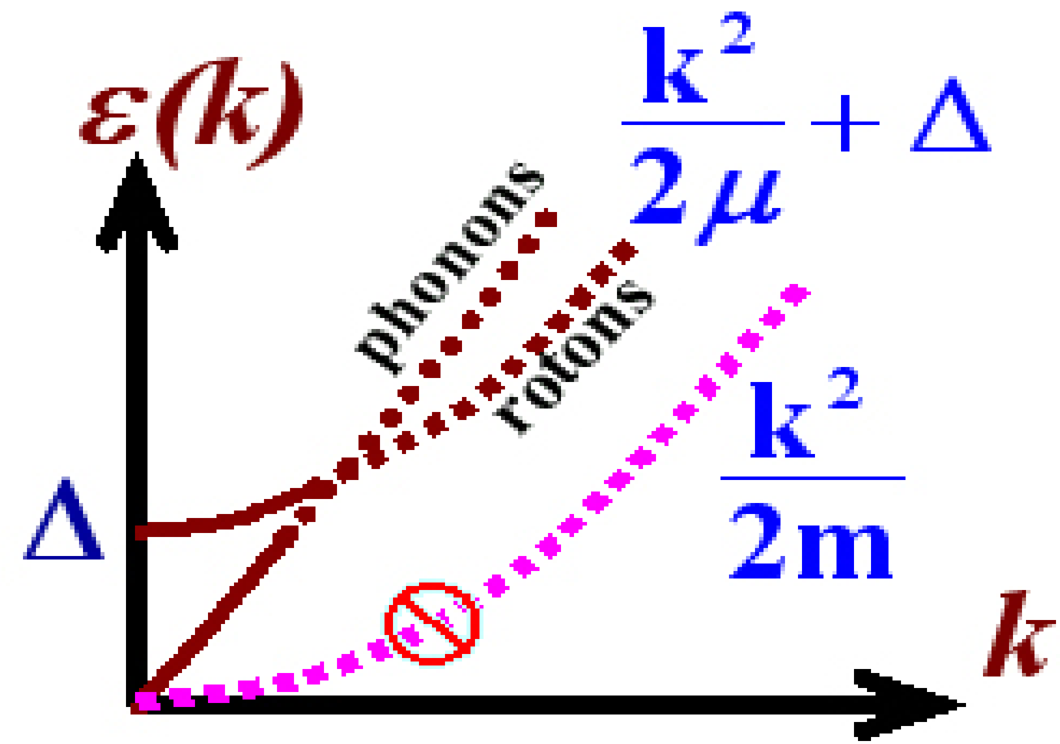
$$K_{\sigma\sigma}(\mathbf{r}) =< \sigma(\mathbf{0})\sigma(\mathbf{r}) > - < \sigma(\mathbf{0}) >< \sigma(\mathbf{r}) >$$

$$K_{\sigma\sigma}(\mathbf{r} \to \infty) = \begin{cases} 0, & T > T_c \\ M^2(T), & T < T_c \end{cases}$$

Phenomenology of Superfluidity

[Expt'l Discovery], Kapitsa (1937)

Landau 1941 phenomen. phonons-rotons theory



Energy loss at velocities $v < v_{crit}$ forbidden

Bogoliubov model for SuperFluid He II

Bogoliubov, Oct 1946 microscopic theory

$$H = \frac{\hbar^2}{2m} \int d\,x\,\Psi^*(x)\Delta\Psi(x) +$$

$$+ \int d\,x \int d\,y\,\Psi^*(x)\Psi(x)\,V(x-y)\,\Psi^*(y)\Psi(y).$$

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Shift by constant C, to single out the condensate

$$\Psi(x) = C + \phi(x) \qquad \Psi^*(x) = C + \phi^*(x) \tag{2}$$

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$$\Psi(x) = C + \phi(x) \qquad \Psi^*(x) = C + \phi^*(x) \qquad (3)$$

Transition to momentum p-picture

$$\Psi(x) = \frac{1}{\sqrt{V}} \sum_k a_k\, e^{\frac{i\,(q\,x)}{\hbar}},\ \ \phi(x) = \frac{1}{\sqrt{V}} \sum_{p \neq 0} b_p e^{\frac{i\,(p\,x)}{\hbar}}, \qquad C = \frac{1}{\sqrt{V}} a_0$$

yields $\quad a_k = a_0\, \delta_{k,0}\, \frac{C}{\sqrt{V}} + \left[1 - \delta_{k,0}\right]\, \delta_{k,p}\, b_p\,.$

Bogoliubov SuperFlu He II model, 2

$$H_{B1} = \sum_p T(p)\, a_p^+ a_p + \frac{1}{2V} \sum v(p_1 - p_2)\, a_{p_1}^+ a_{p_2}^+ a_{p_2} a_{p_1}; \quad T(p) = \frac{p^2}{2m}$$

non-ideal Bose gas with weak repulsion $v(0) > 0$.

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Corollary:

Condensate operators $a_0^+, a_0 \to \sqrt{N_0}$ big C–numbers.

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Corollary:
Condensate operators $a_0^+, a_0 \to \sqrt{N_0}$ big C–numbers.

Shift $\boxed{\psi(\mathbf{x}) = \psi_{\mathbf{0}} + \phi(\mathbf{x})}$ by constant $\psi_0 = \sqrt{N_0}$

or via binary operators: $b_p = \frac{a_0^+ \, a_p}{\sqrt{N_0}}\,; \quad b_p^+ = \frac{a_0 \, a_p^+}{\sqrt{N_0}}$

Bogoliubov SuperFlu model , 3

One gets in the leading order :

$$H_{B1} = E_0 + H_{B2}(b) + \ldots$$

(E_0 = condensate energy), H_2 – bilinear operator form

$$H_{B2}(b) = \frac{N_0}{2V}\sum_{p\neq 0} v(p)\,[b_p^+ b_{-p}^+ + b_p b_{-p}] + \sum_{p\neq 0}\left(T(p) + \frac{N_0}{V}v(p)\right) b_p^+ b_p, \qquad (1$$

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diagonalized by Bogoliubov (u, v) transformation

$$\xi_p = u_p b_p + v_p b_{-p}^+; \quad \xi_p^+ = u_p b_p^+ + v_p b_{-p}$$

with real coefficients $u_p^2 - v_p^2 = 1;\ u_{-p} = u_p;\ v_{-p} = v_p\,.$

Also by unitary transformation

$$\xi_p = U_\alpha^{-1} b_p\, U_\alpha = u_p b_p + v_p b_{-p}^+\,, \qquad U_\alpha = e^{\sum_p \alpha(p)\,[b_p^+ b_{-p}^+ - b_p b_{-p}]}\,.$$

Bogoliubov explanation for Landau spectrum

The (u,v) transformation $b_p \longrightarrow \xi_p$ correlates pairs of particles with opposite momenta. New Hamiltonian

$$H_{B2}(b) = H_{Bog3}(\xi)\,; \quad H_{Bog3} = \sum_{p\neq 0} E(p)\,\xi_p^+\,\xi_p\,,$$

$$E(p) = \sqrt{(T(p))^2 + T(p)\,v(p)}; \qquad T(p) = \frac{p^2}{2m}$$

describes new collective excitations.

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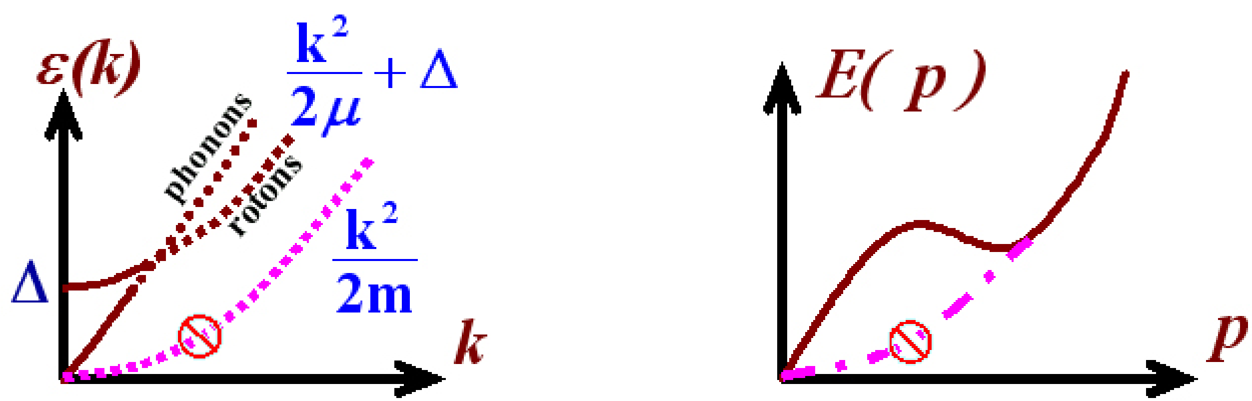

Рис. 2: (a) Phonon + roton spectra – Landau phenomenology; (b) Bogoliubov spectrum from non-ideal Bose-gas microscopical model.

Bogoliubov SF collective modes*

Diagonalization of Bogoliubov bilinear Hamiltonian

$$H_{B2} = \sum_p \epsilon(p)\, b_p^+ b_p + \sum_p v(p)[b_p^+ b_{-p}^+ + b_p b_{-p}] = \sum_p E(p)\, \xi_p^+ \xi_p$$

by unitary transformation $\xi_p = u_p b_p + v_p b_{-p}^+ = U_\alpha^{-1} b_p\, U_\alpha \,,$

$$U_\alpha = e^{\sum_p \alpha(p)\, [b_p^+ b_{-p}^+ - b_p b_{-p}]} \;; \qquad \alpha(p) = f[E(p), v(p)] \;.$$

New ground state $\Psi_0(\alpha) = U_\alpha^{-1}\, \Phi_0 \sim \boxed{\sim e^{\sum_p \alpha(p)\, [b_p^+ b_{-p}^+]}\, \Phi_0}$ is

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Bogoliubov (u, v) transformation and new ground state

$$\Psi_0(q) \sim e^{\sum_k c(k,q)\, b_k^+\, b_{q-k}^+}\, \Phi_0$$

of the same pair-correlated nature, is used now

in quantum optics to describe "squeezed states".

Gauge symmetry breaking in the SF state

Initial Hamiltonian $H_{B1}(a_p^+ \,, a_p)$ for normal states

$< a_p >= 0$ is invariant with respect to the

Phase (Gauge) transformation $a_p \rightarrow a_p \, e^{i\,\varphi}$ (GT)

related to conservation of particles number $< a_p^+ a_p >= n_p \,.$

The bilinear Bogoliubov model Hamiltonian $H_{B2}(b)$ is not compatible with GT, as well as the (u, v) canonical transformation and $H_{Bog3}(\xi)$.

Physically, this corresponds to non-conservation the number of particles with non-zero momenta

Ginzburg-Landau [1950] SuperConductivity

$\Psi(r) \sim$a system (of SC electrons) wave-function = 2-component order parameter for (SC) transition

$$\Psi(r) = |\Psi(r)| e^{i\Phi(r)}$$

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Free energy functional

$$F = F_n + \int \left(\frac{\hbar}{2m^*} |\vec{\nabla}\Psi(r)|^2 + a|\Psi(r)|^2 + b|\Psi(r)|^4 \right) dV$$

with $a \sim T - T_c\,,\quad b \approx$ const , m^* – effective mass

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SC current $j_\alpha = \frac{e^*\hbar}{m^*}\,|\Psi|^2\,\nabla_\alpha\Phi,$ $e*$ – effectve charge

Gor'kov (1959) : $m^* = 2m,\ e^* = 2e, \quad |\Psi|^2 = n_s/2\,.$

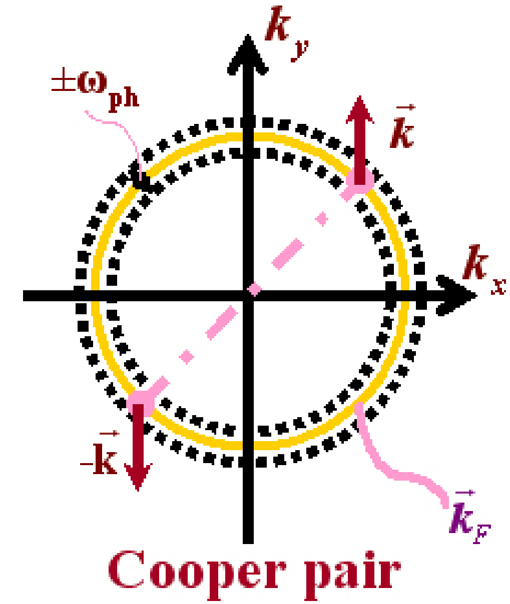
BSC SuperConductivity

BCS model:

$$H = \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}}\, c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{k},\vec{k}'} V_{\vec{k},\vec{k}'}\, c^{+}_{\vec{k}\uparrow} c^{+}_{-\vec{k}\downarrow} c_{-\vec{k}'\downarrow} c_{\vec{k}'\uparrow}\,,$$

- eff. Cooper pairs (antipodes) attraction

$\varepsilon_{\vec{k}} = \frac{\vec{k}^2}{2m} - \varepsilon_F$ - electron energy above ε_F



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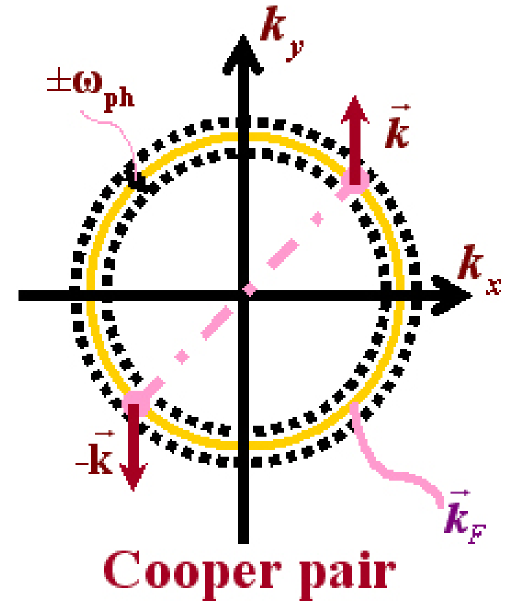
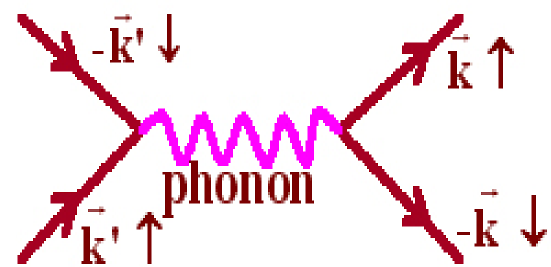


Effective electron-electron attraction in the vicinity of Fermi surface

$$V(\vec{k},\vec{k}') = \begin{cases} -V_C, & |\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}| < \omega_{ph} \\ 0, & |\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}| > \omega_{ph} \end{cases}$$



Semi-Phenomenological BSC theory, 2

Variational BCS wave function

$$|\Psi_{BCS}>= \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} c^{+}_{\vec{k}\uparrow} c^{+}_{-\vec{k}\downarrow})|0>; \quad c_{\vec{k}\sigma}|0>= 0.$$

New SC ground state: $\boxed{c_{\vec{k}\sigma}|\Psi_{BCS}>\neq 0}$

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- SC order parameter $=$ Cooper pair condensate:
  $< c^{+}_{\vec{k}\uparrow}c^{+}_{-\vec{k}\downarrow} >= \Psi(\vec{k}) = |\Psi(\vec{k})|exp[i\Phi(\vec{k})]$
- Phase symm breaking: $\widetilde{c}^{+}_{\vec{k}\sigma} = e^{i\phi}c^{+}_{\vec{k}\sigma} \Rightarrow \boxed{\widetilde{\Psi}(\vec{k}) = e^{2i\phi}\Psi(\vec{k})}$
- Energy gap: $\Psi(\vec{k}) = \frac{\triangle_{\vec{k}}}{2E_{\vec{k}}}$ $\quad \boxed{\triangle_0 \approx exp\left(-\frac{1}{\lambda}\right);}$ $\quad \lambda = N_0 V_C$
- SC temperature $T_c = 1.14\,\omega_{ph}\,exp\left(-\frac{1}{\lambda}\right); \quad 2\triangle_0 = 3.52T_c$

Bogoliubov SC theory

Fr$\ddot{o}$hlich electron-phonon model: $H_{Fr} =$

$$= \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}} c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{q}} \omega_{\vec{q}} b^{+}_{\vec{q}} b_{\vec{q}} + g_{Fr} \sum_{\vec{k},\vec{k}',\sigma} \sqrt{\frac{\omega(\vec{q})}{2V}} c^{+}_{\vec{k}\sigma} c_{\vec{k}'\sigma} (b^{+}_{\vec{q}} + b_{-\vec{q}})$$

Bogoliubov (u,v) transformation:

$$\alpha_{\vec{k}\uparrow} = u_{\vec{k}} c_{\vec{k}\uparrow} - v_{\vec{k}} c^{+}_{-\vec{k}\downarrow}; \qquad \alpha^{+}_{\vec{k}\uparrow} = u_{\vec{k}} c^{+}_{\vec{k}\uparrow} + v_{\vec{k}} c_{-\vec{k}\downarrow}$$

$$u^2_{\vec{k}} = 1 - v^2_{\vec{k}} = \frac{1}{2}\left(1 + \frac{\varepsilon_{\vec{k}}}{E_{\vec{k}}}\right)$$

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Gap solution :

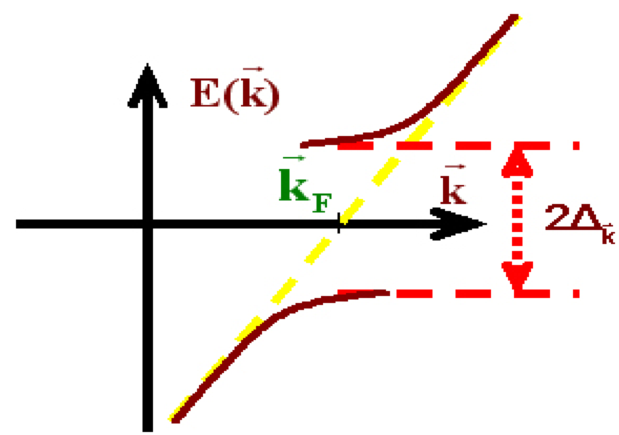
$$\triangle_B = \widetilde{\omega}\, exp\left(-\frac{1}{\rho_B}\right)$$

Microscopical $\rho_B = g^2_{Fr} N_0$ vs BCS phen $\lambda = V_{\rm C} N_0$

Excitation spectrum of quasiparticles ("Bogolons")

$$H_{Fr} \rightarrow H_B = \sum E_{\vec{k}}\, \alpha^{+}_{\vec{k},\sigma} \alpha_{\vec{k},\sigma}$$

$$E_{\vec{k}} = \sqrt{\varepsilon^2_{\vec{k}} + |\triangle_{\vec{k}}|^2}$$



Conclusion to Quantum Statistics (SF and SC)

1. The superfluid and the superconducting phase transitions are accompanied by the Spontaneous Symmetry Breaking

2. At these SSBs, the phase (gauge) symmetry, related to number of particles conservation, is broken

3. In the Symmetry Broken state both the amplitude and the phase of the order parameter are fixed

"$1/q^2$ theorem" $\sim$ Goldstone mode in QFT

Bogoliubov theorem on $1/q^2$ singularity (1961), (i.e., on long range forces) of Green function for systems with degenerate ground state was proven in context of "method of quasi-averages" for SSB.

Analog of this in QFT was proposed by Goldstone [1960]; massless excitations in QFT are known as the Goldstone (bosonic) modes.

The proof of Goldstone theorem analogous to Bogoliubov theorem, was given half a year later [1962]

SSB Transition to QFT; Early 60s

Spontaneous Breaking of Chiral (γ_5) Invariance

2-dim models with cutoff Λ

- Vaks + Larkin I, II [August 1960]
- Tavkhelidze [Aug 1960] {ref: Bogoliubov, Sept '60}
- Nambu [? 1960 Purdue Conf] {ref: Nobel Comm '08 doc }
- Nambu, Jona-Lasinio I [Oct 1960]

2-dim, + cutoff Λ

- Nambu, Jona-Lasinio II [May 1961]

2-dim without cutoff

- Arbuzov, Tavkhelidze, Faustov [Nov 1961]

Implication to QFT; Higgs field

Lagrangian for normal quantum scalar field with quartic self-interaction and stable ground state

$$L(\varphi, g) = \frac{1}{2}\,(\partial_\mu \varphi)^2 - V(\varphi), \qquad \boxed{V(\varphi) = \frac{m^2}{2}\,\varphi^2 + g\,\varphi^4;\ g > 0}$$

Since 60s, in QFT play with toy models à la Ginzburg-Landau with phantom scalar field $\mathbf{\Phi}(x)$, like the Higgs (1964) one

$$V_{\text{Higgs}}(\Phi^2) = \lambda\left(\mathbf{\Phi}(x)^2 - \Phi_0^2\right)^2; \quad \mathbf{\Phi}^2 = \Phi_1^2 + \Phi_2^2; \quad \Phi_0^2 = \text{const.}$$

with imaginary initial mass $\mu_{\text{H}}^2 = -4\lambda\,\Phi_0^2$ and the final one $m_{\text{Higgs}} = 2\sqrt{2\lambda}\,\Phi_0$ obtained after shift of field operator by constant

$$\Phi(x) \to \varphi(x) = \Phi(x) - \Phi_0\,,$$

like in Bogoliubov's SuperFluidity.

QFT; masses of fermions

In $\Phi(x) = \varphi(x) + \Phi_0$, constant $\Phi_0 \neq 0$ is the Vacuum Expectation Value of the Higgs field $< \Phi(x) >= \Phi_0$.

The main purpose of this trick is to attribute mass to particles of some other fields. Besides intermediate vector bosons W, Z_0, to leptons and quarks via Yukawa coupling

$$g_i\bar{\psi}\,\Phi(x)\,\psi \longrightarrow g_i\bar{\psi}\varphi(x)\,\psi + m_i\,\bar{\psi}\,\psi;\;\; m_i = g_i\,\Phi_0$$

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Hence, “Higgs mechanism” provides masses to fermions via Yukawa couplings along the rule

“One mass - one coupling constant”

In Standard Model, No of Yukawa couplings = 12
(besides the issue on neutrino masses !)

BS in QFT; Standard Model

In 60s the SSB mechanism in QFT models with degenerate vacuum formed cornerstone of Glashow-Weinberg-Salam gauge model of Weak and EM interaction with massive W and Z_0 vector mesons. Two Nobel Prizes :

- EW-theory; Glashow-Salam-Weinberg (NP-1979);
- W, Z_0-exp'tl; Rubbia+VanDer Meer (NP-1984)

Together with QCD, GWS-model forms Standard Model. Based on the principle "Dynamics from Symmetry", SM contains only 3 basic running couplings: $\bar{\alpha}_{i=1,2,3}(E)$ with the Renorm-group evolution.

BS in Standard Model; the Higgs issue

Up to now, so-called Higgs particle escaped of observation. Current window for it possible mass is

$$114\ \text{GeV} < M_{\text{Higgs}} \quad 154\ \text{GeV}\ .$$

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In such a case, the hopes of its direct observation on LHC look like illusive.

Message

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XXth Century Folklore

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2. Microscopic Superconductivity devised by BCS + Bogoliubov (1957) was shown to be a Superfluidity of Cooper pairs (Bogoliubov, 1958)
3. Higgs model in QFT is a replica of Ginzburg–Landau phenomenology for Superconductivity. Its physical base is still unknown

On some Nobel Prizes in Physics

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II

- EW-th renorm-tion t'Hooft+Veltman (NP 1999) Faddeev, Slavnov ?
- Nambu +Kobayashi,Maskawa (2008) - Goldstone,... experiment ?

Constuctivists vs Reductionists

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* * * *

Dedicated to the memory of great scientists

LANDAU	BOGOLIUBOV
born 1908	born 1909