«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
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STUDIES OF COLLECTIVE CURRENTS IN NUCLEI

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A search for fingerprints of «vortical currents» in nuclear structure data is presented. These currents affect little or not at all the changes of nuclear shape and volume. The paper starts with a classification of currents and with their parametrization applicable for studies of spherical and deformed nuclei. Here we generalize the concept of the time-dependent TAS transformation of liquid elements positions introduced by Riemanne's followers in their studies of currents in ellipsoidally shaped liquid bodies. We examine fingerprints of a dipole toroidal motion in the electromagnetic properties of rotational states built upon the collective octupole excitations in deformed nuclei and show one example where the experimental data give an indication of an important role played by such a mode. We study the conjecture according to which the vortical mode involved in the motion with a uniform circulation of nuclear matter may be regarded as an independent collective branch of excitation. This study allows us to interprete $\Delta I = 4$ staggering of the energies of superdeformed states. We discuss also another mechanism of producing regular perturbations in the spectrum of rotational bands: the installation of an unaxial octupole deformation with rapid rotation. We mention an analogy between our interpretation of unusual properties of superdeformed nuclear states and recent findings concerning the quantal apparatus produced using the modern technology (SQUID's). This analogy shows that the effects we are talking about may take place not only in atomic nuclei but also in other «mesoscopic» systems.

Описан поиск следов вихревого движения ядерной материи в данных о структуре ядра. Такое движение не влияет совсем или влияет слабо на изменения ядерной формы и объема. В обзоре дается классификация токов и их параметризация, удобная для изучения сферических и деформированных ядер. Делается обобщение концепции зависящего от времени «TAS»-преобразования координат элементов жидкости, предложенное последователями Римана в своих исследованиях потоков в жидких телах эллиптической формы. Изучаются следы дипольного тороидального движения в данных об электромагнитных свойствах состояний ротационных полос, выстроенных над коллективными октупольными возбуждениями, и приводится пример, указывающий на важную роль такого движения. Анализируется предположение, согласно которому вихревая мода движения с постоянной по объему циркуляцией скорости может рассматриваться как независимая ветвь возбуждений. Это исследование позволяет интерпретировать $\Delta I = 4$ искажения спектра супердеформированных (с.д.) полос. Обсуждается альтернативный механизм генерации периодических искажений спектра ротационных полос: возникновение неаксиальной октупольной деформации, вызванное быстрым вращением. Отмечается аналогия между нашей интерпретацией необычных свойств с.д. состояний атомных ядер и недавними открытиями, сделанными с помощью квантовых приборов, изготовленных на базе современной технологии (SQUID-ов). Аналогия говорит о том, что обсуждаемые нами эффекты могут иметь место не только в атомных ядрах, но и в других мезоскопических системах.

1. INTRODUCTION

It is well known by now that atomic nuclei reveal elastic properties when responding to perturbations creating collective flow of the matter inside them. The best studied branches of nuclear collective modes of motion are the ones generated by changes of the nuclear shape (and size). However, the scope of collective modes in nuclei is much larger than that which can be related with the changes of the nuclear geometry. We may mention here the giant magnetic resonance as one particular example of collective motion which is not directly related to the nuclear shape. An analysis of nuclear currents plays the leading role in the description of such a motion. In fact, such an analysis contributes much to the understanding of all kinds of nuclear collective motions. The currents reflect changes in the distribution of nucleons in the momentum space, and consequently they are related with the changes in the nuclear Fermi-surface determining the properties of quantal Fermi liquids.

It is the study of the properties of nuclear collective currents that unifies investigations reported here. The paper is organized as follows:

- In **Section 2** we give a classification of the types of collective currents in spherical and deformed nuclei. A practical method is suggested for the description of the motion in deformed nuclei introducing a «stretched» velocity field corresponding to a given type of motion in a spherical nucleus. Special attention is paid to the vortical motion. An important example is discussed concerning the vortical motion involved in the rotating Riemann S-ellipsoids.
- **Section 3** treats the quantization of the vortical motion in that case. The band structure issued from the coupling of the global rotation with the Kelvin circulation is examined and compared with the experimental energy spectra obtained for the yrast superdeformed states which show a characteristic staggering behaviour.

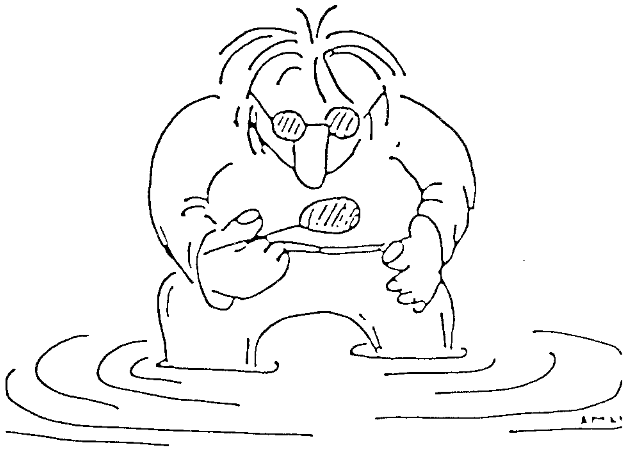


Fig.1. By examining the *flows* of the collective tide perhaps we might be able to learn something about its *ebbs* (elementary building blocks) [1]

- In **Section 4** we generalize the procedure described in the previous section to the case of nuclei having no axial symmetry. This allows us to show the relation of our description of the staggering in superdeformed bands with the description given in other publications.
- In **Section 5** we show that the most probable type of nonaxiality in superdeformed nuclei may be within the C_3-symmetry.
- **Section 6** is devoted to a more complicated type of vortical motion generating the dipole toroidal moment. The fingerprints of such a motion are presented.
- In **Section 7** we remark the parallels between the quantal descriptions of vortical motion in a liquid drop and of the motion of charged particles in an electromagnetic field (Aharonov-Bohm effects).

2. INTRINSIC VORTICITY AND OTHER NONTRIVIAL MODES OF NUCLEAR MOTION

In Refs. [2], [3] and [4] a scheme has been worked out to develop a velocity field parametrization for the study of the currents in deformed nuclei. This problem is of the same nature as the classical hydrodynamical problem of determining the flows of the matter in a homogeneous liquid filling a container whose shape and orientation in the space is changing with time [5]. This approach is currently applied in Ref.[6] to visualize the collective flow in some nontrivial cases.

The method suggested here is based on the knowledge of currents describing the motion in spherical nuclei. The parametrization of velocity fields for the latter can be easily done applying the techniques of spherical vector functions [7] $\mathbf{Y}^L_{l,M}(\mathbf{r}/r)$ by writing for the velocity field an expression

$$\mathbf{u}(\mathbf{r}) = \sum_{l,\,L,\,M} f_{l,L,M}(t)\,\mathbf{Y}^L_{l,M}(\mathbf{r}/r).$$

Expansion for currents in spherical vectors has much in common with the widely used in nuclear physics expansion for the distance of a point on the surface from the origin of the coordinate frame

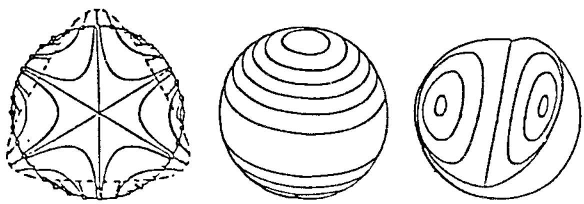
$$R(\mathbf{r}/r) = R_0 \sum_{l,\,m} \alpha_{l,m} Y_{l,m}(\mathbf{r}/r).$$

However, the former expansion provides possibilities of studying collective phenomena which are not related with the nuclear shape. Figure 2 allows one to see this feature in a clear way. Here some examples of velocity fields obtained using the spherical vectors expansion are presented. In the figure the lines of «solenoidal» (leaving unchanged the density) currents in spherical nuclei are shown.

Oscillations of the surface around the spherical shape of multipolarity 2, 3 and 4 can be described in terms of the shape parameters $\alpha_{l,m}$ with $l = 2, 3$ and 4 or in terms of the velocity fields $\mathbf{u}_{l,m}(\mathbf{r}) = f_{l,m} r^{l-1} \mathbf{Y}^{l-1}_{l,m}(\mathbf{r}/r)$. The pattern of velocity fields corresponding to the surface oscillations is shown in Fig.2 in the fragments marked as $L = 2b$ (for quadrupole vibrations), $L = 3$ (octupole case) and $L = 4$ (hexadecapole case).

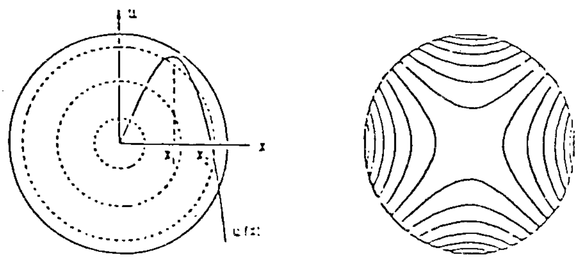
All the other fragments of Fig.2 describe the motion in the spherical body leaving undisturbed the surface. Among the modes of the motion shown in the picture there are two (isoscalar) $l = 1$ modes: the dipole toroidal mode ($L = 1a$) and the mode describing the distributed magnetism ($L = 1b$). In addition to these modes, Figure 2 shows the quadrupole magnetic mode ($L = 2a$) and the quadrupole toroidal mode ($L = 2c$).

The spherical vectors form a set of functions orthonormal on the sphere and may be used as a basis for numerical solution for dynamic equations of the motion in the normal and in the quantum fluids. However, to apply them, one must specify first of all the model of a quantum-fluid dynamics. These functions may be used differently as generic fields giving the possible motions in bodies (either classical or quantal) of an arbitrary shape. To do it, we suggest to generate velocity fields considering the displacements of liquid elements which appear after a series of transformations shown in Fig.3 and defined as:

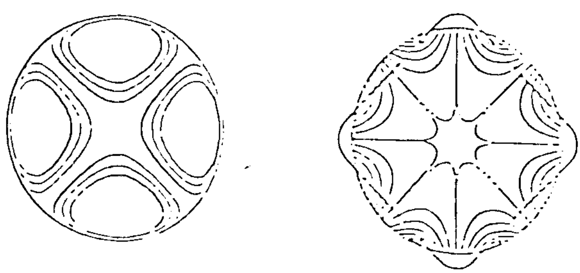


$L = 3$ $\qquad L = 2a$ $\qquad L = 1a$

$$f(r,t) = a(t) + b(t)r^2$$



$L = 1b$ $\qquad L = 2b$



$L = 2c$ $\qquad L = 4$

$$f(r,t) = a'(t)r + b'(t)r^3$$

Fig.2. Examples of «elementary» currents visualized by «the lines of currents». In these examples the function $f(r)$ in the expansions of the current in spherical vectors is given by polynoms in r whose power is < 4

$$\mathbf{r}(t) = \mathbf{TASA}_0^{-1}\mathbf{r}(0). \qquad (1)$$

Here $\mathbf{Ar}$ is the displacement of a given liquid element resulting from the transformation of the body's shape by the irrotational flow, $\mathbf{A}^{-1}\mathbf{r}$ describes the inverse displacement, $\mathbf{T}$ is a rotational matrix and finally $\mathbf{Sr}$ is a displacement resulting from one of the «elementary» modes of motion in a spherical nucleus.

The velocity field corresponding to this series of transformations is given by the following expression:

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{\Omega} \times \mathbf{r} + \mathbf{u}_A(\mathbf{T}^{-1}\mathbf{r}) + \mathbf{u}_S^{\text{str}}(\mathbf{T}^{-1}\mathbf{r}^{\text{str}}). \qquad (2)$$

The first term in eq.(2) describes the uniform rotation of the body ($\mathbf{\Omega}_i = \sum_{j,k} \varepsilon_{i,j,k}(\dot{\mathbf{T}}\mathbf{T}^{-1})_{j,k}$), while the second and the third ones are associated with the intrinsic motion. The second term corresponds to the irrotational flow transforming the body into a sphere ($\mathbf{u}_A = \dot{\mathbf{A}}\mathbf{A}^{-1}\mathbf{r}$). The last one gives the contribution from the chosen mode of «elementary vortical» flow «S» in the spherical nucleus ($\mathbf{u}_S = \dot{\mathbf{S}}\mathbf{S}^{-1}\mathbf{r}$)) «stretched» by the deformation so that

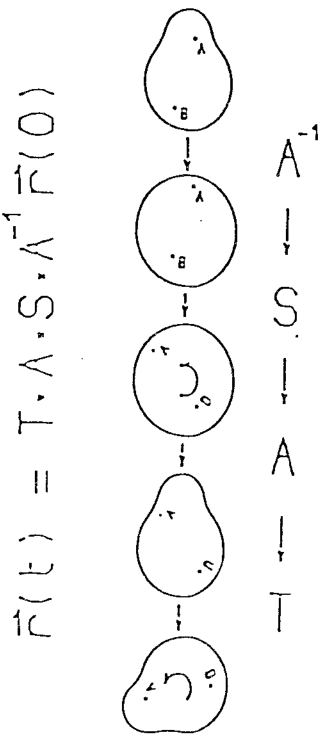


Fig.3. The transformations generating currents in deformed nuclei keeping their shapes

$$\mathbf{u}_S^{\text{str}} = \mathbf{A}\mathbf{u}_S, \quad \mathbf{r}^{\text{str}} = \mathbf{A}^{-1}\mathbf{r}. \qquad (3)$$

In the general case, the operator $\mathbf{A}$ must result from a variational calculation and thus demands numerical calculations. When the shape of the body is ellipsoidal, the operator $\mathbf{A}$ is a diagonal matrix whose matrix elements in the inertial frame are half the length (semiaxes) in the principal directions (a_x, a_y and a_z). When the «S» transformation is also a uniform rotation (around the z-axis of the ellipsoid), the corresponding collective velocity field $\mathbf{u}(\mathbf{r})$ in the laboratory frame has the following components on the principal axes of the nucleus:

$$u_x = -(\Omega + q\,\omega)y, \quad u_y = \left(\Omega + \frac{\omega}{q}\right)x, \quad u_z = 0 \quad \left(q = \frac{a_x}{a_y}\right)$$

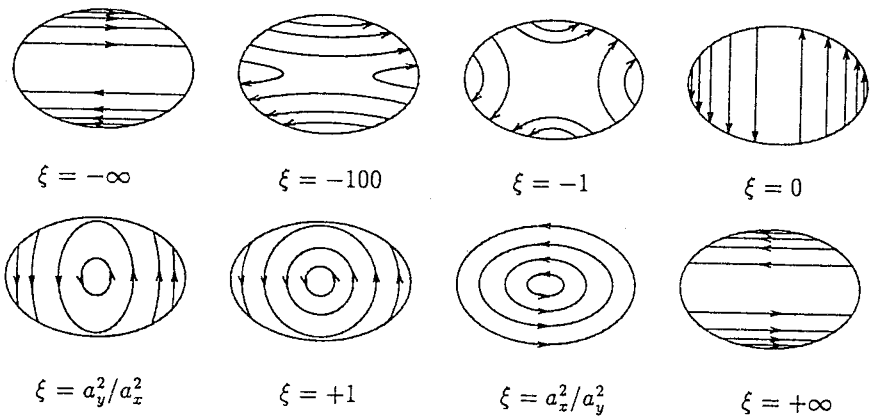


Fig.4. Flow patterns of Riemann's S-ellipsoids corresponding to various values of the dimensionless parameter ξ (see text). The picture shows the case when the global rotation frequency Ω and rot ($\mathbf{u}$) are parallel to one of the main axes of the inertia tensor (z-axis)

introducing the frequences (ω and Ω) of «S» and «T» rotations. The velocity field has a uniform circulation:

$$(\mathrm{rot}\ \mathrm{u})_z = 2\Omega\left(1 + \frac{\omega}{2\Omega}\left(q + \frac{1}{q}\right)\right).$$

This field is a particular solution of the well-known classical Dirichlet problem for the most general linear velocity field in the system bounded by an ellipsoidal surface as studied by Riemann [5], [8]. The pattern of the motion depends on the quantity

$$\xi = \frac{1 + q^{-1}\dfrac{\omega}{\Omega}}{1 + q\dfrac{\omega}{\Omega}}.$$

The corresponding parametrization of the velocity field comprises the uniform (rigid-body) rotation ($\xi = 1$), the motion of an ideal liquid without circulation within a nonspherical stationary rotating container ($\xi = -1$), various shear modes ($\xi = 0, \pm\infty$) and also the flow with an arbitrarily large angular momentum inside the container with an elliptic shape which does not change its orientation in the space ($\xi = a_x^2/a_y^2$). Figure 4 shows a variety of physically different types of flow which could be studied in this way (assuming $a_x < a_y \le a_x$).

In Refs. [2], [3], [4] we have generalized the usual Routhian or cranking approach for uniform rotations to allow the study of a variety of collective modes. It is done using a formal analogy between canonical transformations in Classical Mechanics and specific unitary transformations in Quantum Mechanics. The stationary motion in the presence of currents is described by a «cranked» Hamiltonian with the generalized cranking term [3]

$$\hat{H}_{\text{cranked}} = \hat{H} - \frac{1}{2\hbar} \sum_i (\mathbf{u} \cdot \hat{\mathbf{p}}_i + \hat{\mathbf{p}}_i \cdot \mathbf{u}), \qquad (4)$$

where **u** is the velocity field and the summation goes over all the particle's indices. In classical mechanics the cranking term represents the point-coordinates transformation to the coordinates attached to moving «liquid elements», whereas in quantum formulation it corresponds to the unitary transformation of the wave function

$$\Psi \to \Psi' = \exp(i\hat{S})\, \Psi \qquad (5)$$

with the $\hat{S}$-operator such that

$$\frac{d\hat{S}}{dt} \hat{S}^{-1} = \frac{1}{2\hbar} \sum_i (\mathbf{u} \cdot \hat{\mathbf{p}}_i + \hat{\mathbf{p}}_i \cdot \mathbf{u}). \qquad (6)$$

The presence of the cranking term in the Hamiltonian and the associated phase transformation of the wave function lead to changes in the distribution of nucleons in the momentum space and have numerous physical consequences [9].

Such an approach is applied to the coupling of a global rotation with a uniform intrinsic vortical motion in the aligned case discussed in the previous lines. Applying it, the nuclear inertia properties with respect to the motion were estimated in the simple case when the energy dependence on $\Omega = \dot{\Theta}$ and $\omega = \dot{\theta}$ was approximated by a quadratic form

$$E(\Omega, \omega) = \frac{A}{2} \Omega^2 + B\, \Omega\omega + \frac{C}{2} \omega^2. \qquad (7)$$

This was done using a truncated $\hbar$ expansion of the generalized cranking equation when the effective nucleon-nucleon interaction is a full-fledged Skyrme force [9]. The density was approximated as being constant within an ellipsoid whose semiaxes were given by

$$a_x = a_0 q^{2/3}, \quad a_y = a_z = a_0 q^{-1/3},$$

where a_0 is expressed in terms of the total particle number N and the usual size parameter r_0 by $a_0 = r_0 N^{1/3}$, whereas $q = a_x / a_y$ is the shape parameter. Introducing the parameter

$$\gamma = \frac{2}{5} m r_0^2 N^{5/3} q^{1/3},$$

which is equal to the rigid body moment of inertia times $q^{1/3}$, and defining the geometrical factor

$$R = \frac{1}{2}\left(q + \frac{1}{q} \right),$$

one obtains for the three inertia parameters in eq.(4) the following estimates:

$$A = \gamma R\left[1 - \left(\frac{D}{\gamma} \right) R \right] \quad B = \gamma\left[1 - \left(\frac{D}{\gamma} \right) R \right] \quad C = \gamma R\left[1 - \left(\frac{D}{\gamma R} \right) \right]. \qquad (8)$$

The semiclassical factor D/γ is given in terms of the real and effective masses in nuclear matter $(m^*/m)_{NM}$ by

$$\frac{D}{\gamma} = 10(9\pi)^{-2/3} (m^*/m)_{NM}^{-1} N^{-2/3} q^{-1/3}$$

and is small compared to one.

In this way one finds that the semiclassical approximation of the corresponding stationary solutions of the Hartree-Fock equations gives the classical Riemann results [8] at the Thomas-Fermi level with small corrections ($\sim N^{-2/3}$) at the order $\hbar^2$. The physical significance of the inertia parameters A, B and C will be studied in the next section.

3. QUANTAL ANALOGUE OF THE MOTION INVOLVED IN THE ROTATING RIEMANN *S*-ELLIPSOIDS AND STAGGERING IN SOME SUPERDEFORMED ROTATIONAL BANDS

The use of the quantal analogue of the motion involved in the rotating Riemann *S*-ellipsoids in nuclear physics had been already proposed by many authors, including D.R.Rowe [1], R.Y.Cusson [13], G.Rosensteel [10]—[12], P.Kramer [14] and one of the authors [15]. A simple quantization procedure of such a motion is suggested in the papers [2]-[4] and considered in more detail in Ref.[9]. To quantize the motion, one introduces the angular momentum $\hbar J$ and the «vortex» momentum $\hbar J$ as conjugate momenta (in classical mechanics terms) with respect to the angle variables (Θ and θ) of «*T*» and «*S*» rotations:

$$\hbar I = \frac{\partial E}{\partial \Omega}, \quad \hbar J = \frac{\partial E}{\partial \omega}. \qquad (9)$$

Then, the quantal Hamiltonian operator is introduced rewriting the energy in terms of I and J and substituting them by

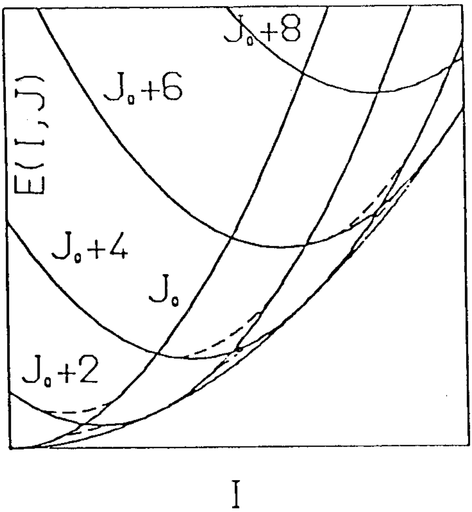


Fig.5. Schematic representation of the band pattern $E(I, J)$ in its dependence on the angular momentum (I) and «vorticity» (J) quantum numbers

$$\hat{I} = -i\frac{\partial}{\partial\Theta}, \quad \hat{J} = -i\frac{\partial}{\partial\theta}. \qquad (10)$$

The quantal states are determined from the Schroedinger equation for the wave function $\Psi_{I,J}(\Theta, \theta) = \pi^{-1/2}\exp(i(I\Theta + J\theta))$ imposing natural periodicity boundary conditions at the ends of the interval $0 \le \Theta,\ \theta \le \pi$. The precise formulation of the boundary conditions involves the intrinsic part of the wave function. Whatever its contribution, the periodicity conditions admit only a discrete set of quantum numbers I and J: $I = I_0 + 2k$, $J = J_0 + 2l$ with integer numbers k and l.

In the quadratic case one arrives at the following expression for the energy:

$$E(I, J) = \hbar^2\lambda\left(\frac{A}{2}I^2 - BIJ + \frac{C}{2}J^2\right) \qquad (11)$$

with

$$\lambda = (AC - B^2)^{-1}$$

(A, B, C being the inertia parameters defined in eq.(7)).

The corresponding quantal states form a set of rotational collective bands as illustrated in Fig.5. The collective band states belong to parabolas labelled by the vorticity number J. The parabola enveloping these parabolas is the classical yrast band with a moment of inertia equal to C. The yrast sequence is composed of states belonging to different collective bands. The passage from one band to another introduces a kink in the energy as a function of the angular momentum. The periodicity in I of such kinks corresponds to $\Delta I = 2B/C$. The maximal value of the kink (which does not change along the yrast line in the quadratic case) is equal to $\Delta E_\gamma = C/2(AC - B^2)$.

Recently, an energy staggering yielding a $\Delta I = 4$ periodic structure has been found experimentally for some rotational bands in superdeformed nuclear states. This structure whose amplitude corresponds merely to less than 0.1 per cent of the observed transition energies has been found in the presently available data

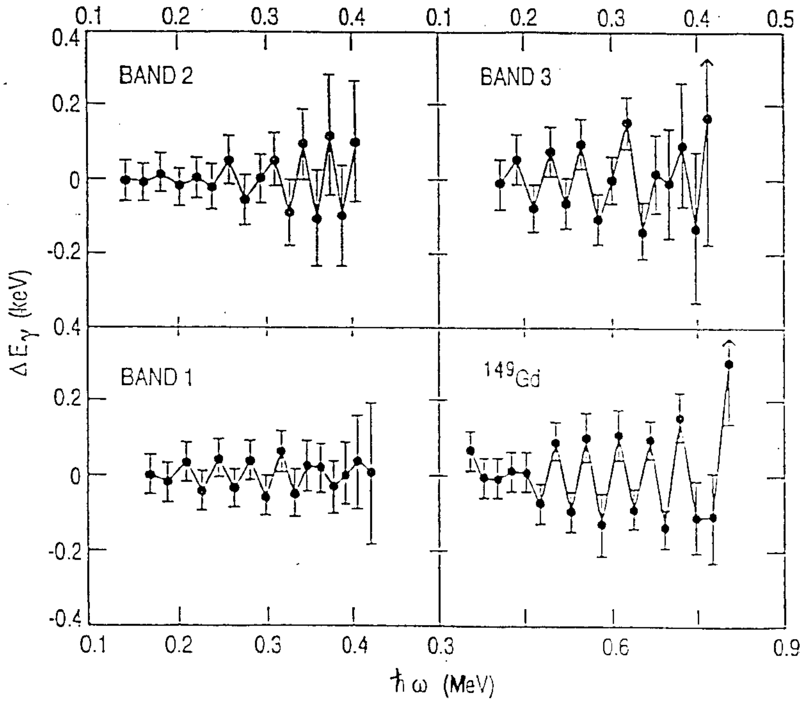


Fig.6. Experimental [17] staggering amplitude in ^{149}Gd (right lower corner) and in three superdeformed bands of ^{194}Hg (bands 1, 2, 3)

$$\Delta E_\gamma = \frac{3}{8}\left(E_\gamma(I) - \frac{1}{6}\,(4E_\gamma(I+2) + 4E_\gamma(I-2) - E_\gamma(I+4) - E_\gamma(I-4)\right).$$

only in ^{149}Gd for one and in ^{194}Hg for three bands [17] (see Fig.6). However a few other candidates are tentatively proposed in nuclei of the same superdeformation region or in the $A \simeq 130$ region [18].

Clearly, the coupling of the global rotation with the uniform vortical motion above considered may provide an explanation for such a staggering phenomenon. Indeed, within the described model when $C/B = 2$ the interval between the kinks is $\Delta I = 4$. In this case the kinks are absolutely regular. When C/B is close to two, the staggering becomes slightly modulated and may even change sign when increasing the angular momentum. All such features seem to appear in the experimental results as shown in Fig.6.

One finds also that the semiclassical estimates of the model parameters, if not exactly coincide with the values deduced from experimental data, are globally consistent with them [4].

No contradiction to the explanation of staggering in terms of coupling of two different «rotation-like» modes could be infered from the decay-time measurements of the states in staggered bands. In fact, the time of life measurements show within our model assumptions that the J-mixing does exist: othervise the $E2$ transitions along the yrast line would not be quenched but would exhibit a staggering which is not seen experimentally. It has been proved [9] that the level of quenching of $E2$ transitions resulting from our modelization of the yrast line is not inconsistent with the data, considering their currently available poor accuracy.

On the other hand, J-mixing leads to some quenching of the kink magnitude. The semiclassical theory prediction of the staggering amplitude which is done ignoring the mixing yields a value several times greater than experimentally found [9]. This difference may be attributed at least partly to the J-mixing of collective bands.

4. VORTICITY QUANTUM NUMBER CONSERVATION VERSUS AXIAL SYMMETRY VIOLATION

The $\Delta I = 4$ character of the staggering phenomenon has prompted some explanations involving a non-axial hexadecapole deformation, namely the $L, M = 4{,}4$ collective degree of freedom [16], [19], [20]. Does the suggested C_4-symmetry exclude the conservation of the vorticity quantum number? In other words, do the two lines of interpretation of the staggering phenomenon exclude each other? Apparently, it is not so. The rotational motion in high-spin superdeformed states must follow the rules of the quasiclassical theory. According to the presented model, in nuclei where the staggering phenomenon is found, the value of the quantum number in the states lying near the yrast line must be equal to $J \simeq (B/C)I \simeq I/2 >> 1$. Then to answer the question one may study the vortical motion in these nuclei using also semiclassical arguments.

The way of generating vortical currents suggested in **Section 2** is quite general and may be applied to the bodies of arbitrary shapes and to arbitrary currents going in three dimensions. Applying the technique similar to that which was used before (see, in particular, **Section 3**) one arrives at the Hamiltonian

$$\hat{H} = \hbar^2 \sum_{i=1}^{3} \lambda_i \left(\frac{A_i}{2} \hat{I}_i^2 - B_i \hat{I}_i \hat{J}_i + \frac{C_i}{2} \hat{J}_i^2 \right). \qquad (12)$$

Assuming that in the states lying near the yrast line, the global and the stretched rotations go predominantly around one of the principal axes of inertia tensors $A_{i,j}$, $B_{i,j}$, $C_{i,j}$, which we call as axis one, we rewrite the Hamiltonian in the following way:

$$\hat{H} = \hbar^2\lambda_1\left(\frac{A_1}{2}\mathbf{I}^2 - B_1\mathbf{IJ} + \frac{C_1}{2}\mathbf{J}_1^2\right) + \hbar^2\hat{h}'. \tag{13}$$

The second term in the r.h.s. of eq. (9) has the form:

$$\hat{h}' = \hat{h}'_A + \hat{h}'_B + \hat{h}'_C, \tag{14}$$

where

$$\begin{aligned} \hat{h}'_A &= \alpha_A(I_2^2 + I_3^2) + \beta_A(I_2^2 - I_3^2) \\ \hat{h}'_B &= -2\alpha_B(I_2J_2 + I_3J_3) + \beta_B(I_2J_2 - I_3J_3) \\ \hat{h}'_C &= \alpha_C(J_2^2 + J_3^2) + \beta_C(J_2^2 - J_3^2) \end{aligned} \tag{15}$$

and

$$\begin{aligned} \alpha_A &= \frac{1}{4}(\lambda_2A_2 + \lambda_3A_3 - 2\lambda_1A_1), \quad \beta_A = \frac{1}{4}(\lambda_2A_2 - \lambda_3A_3) \\ \alpha_B &= \frac{1}{4}(\lambda_2B_2 + \lambda_3B_3 - 2\lambda_1B_1), \quad \beta_B = \frac{1}{4}(\lambda_2B_2 - \lambda_3B_3) \\ \alpha_C &= \frac{1}{4}(\lambda_2C_2 + \lambda_3C_3 - 2\lambda_1C_1), \quad \beta_C = \frac{1}{4}(\lambda_2C_2 - \lambda_3C_3). \end{aligned} \tag{16}$$

When the two generalized momenta **I** and **J** are parallel and have the largest projection on the first axis in the intrinsic reference frame the following boson representation of **I** and **J** is appropriate (see Ref.[21] for the case of merely one set of angular momentum operators):

$$I_\pm = I_2 \pm iI_3$$

$$I_1 = I - c_I^\dagger c_I, \quad I_+ = \sqrt{2I}c_I^\dagger, \quad I_- = \sqrt{2I}c_I$$

$$J_1 = I - c_J^\dagger c_J, \quad J_+ = \sqrt{2J}c_J^\dagger, \quad J_- = \sqrt{2J}c_J.$$

$$[c_i, c_j^\dagger] = \delta_{i,j}$$

Using the boson operators one writes the first term on the r.h.s. of eq.(13) as

$$\hbar^2\lambda_1\left(\frac{A_1}{2}\mathbf{I}^2 - B_1\mathbf{IJ} + \frac{C_1}{2}\mathbf{J}_1^2\right) =$$

$$= E(I,J) + \hbar^2\lambda_1B_1(Ic_J^\dagger c_J + Jc_I^\dagger c_I - \sqrt{IJ}(c_I^\dagger c_J^\dagger + c_Jc_I)), \tag{17}$$

where

$$E(I, J) = \hbar^2\lambda_1 \left(\frac{A_1}{2} I(I+1) - B_1 IJ + \frac{C_1}{2} J(J+1) \right). \tag{18}$$

In this way one arrives at the following expression for the Hamiltonian :

$$\hat{H} = E(I, J) + \hbar^2 \hat{h}_{I,J}, \tag{19}$$

where $\hat{h}_{I,J}$ is a quadratic Hermitian polynomial:

$$\hbar^2 \hat{h}_{I,J} = \sum_{k,l} (\varepsilon_{k,l}(I, J) c_k^\dagger c_l + \Delta_{k,l}(I, J)(c_k^\dagger c_l^\dagger + c_k c_l)), \tag{20}$$

(the indexes k and l standing for I or J).

The boson operators

$$\hat{b}_k^\dagger = \sum_l (x_{k,l}(I, J) c_l^\dagger + y_{k,l}(I, J) c_l) \tag{21}$$

satisfying the commutation relation

$$[\hat{h}_{I,J}, b_k^\dagger] = \omega_k(I, J) b_k^\dagger \tag{22}$$

are the creation operators of the excited collective states; they determine also the yrast state wave function $\Psi_{\text{yrast}}(I, J)$ which is the vacuum with respect to the $\hat{b}$ operators. The contribution of the $\hat{h}_{I,J}$ operator to the yrast state energy $\Delta E(I, J)$ can be found when bringing this operator to the form

$$\hat{h}_{I,J} = \Delta E(I, J) + \hbar^2 \sum_{k=1}^{2} \omega_k(I, J) \hat{b}_k^\dagger \hat{b}_k. \tag{23}$$

The eigenfunctions of this Hamiltonian are given by the multiphonon configurations

$$\Psi_{n_I, n_J} = \frac{1}{\sqrt{n_I! n_J!}} (b_I^\dagger)^{n_I} (b_J^\dagger)^{n_J} \Psi_{\text{yast}}(I, J)$$

with the vacuum function $\Psi_{\text{yrast}}(I, J)$ such as $b_k \Psi_{\text{yrast}}(I, J) = 0$. The zero-point precession renormalizes the energy by

$$\Delta E(I, J) = \hbar^2 \langle \Psi_{\text{yrast}}(I, J) | \hat{h}_{I,J} | \Psi_{\text{yrast}}(I, J) \rangle$$

and may affect the staggering amplitude.

This renormalization has no evident relation with the one issuing from the effective Hamiltonians considered in Refs. [16], [19], [20] where the installation of the C_4 symmetry is advocated. In the quoted papers vortical currents are not considered. On the other hand, the Hamiltonians suggested in these papers contain, in addition to terms quadratic in the angular momentum operators I_i, also terms of the fourth power chosen so that the effective Hamiltonians possess the C_4-symmetry. In the language of this section such terms correspond to anharmonic corrections with respect to the precessional global rotation. The authors show that appropriate combinations of parameters in such Hamiltonians lead to the staggering pattern of the spectrum.

Admitting both the vortical motion and the anharmonic terms, one increases the number of parameters in such effective Hamiltonians and consequently increases the number of favorable combinations yielding staggering. Thus, the installation of a C_4-symmetry may, in principle, interfere (in constructive or destructive ways) with the quantized vortical motion in producing (or not) a persisting staggering in rotational superdeformed bands. One must have in mind, however, that whatever the chosen line of explanation for the staggering phenomenon is one must tune the parameters of the effective Hamiltonians in a very specific way: apparently the staggering appears as a result of an interplay of many tiny details of the nuclear structure and this happens only in very rare cases. There is no reason to think that various physically different ways of generating the staggering could be realised in the same nucleus.

We point out that the interplay of global rotation and of vortical motion may simulate the unharmonic effects in each of these modes even then no anharmonic terms are present in the Hamiltonian. Upon writing the one-phonon wave function ψ as

$$\psi = b^{\dagger}\Psi_{\mathrm{yrast}} = \begin{pmatrix} \varphi_I \\ \varphi_J \end{pmatrix}$$

with

$$\varphi_k = (x_k b_k^{\dagger} + y_k b_k)\,\Psi_{\mathrm{yrast}}$$

one obtains for the function φ_I an equation

$$\left(\hat{h}_{I,I} - \hat{h}_{I,J} \frac{1}{\hat{h}_{J,J} - \omega} \hat{h}_{J,I} - \omega \right) \varphi_I = 0.$$

The power expansion of the operator $\hat{h}_{I,J} \frac{1}{\hat{h}_{J,J} - \omega} \hat{h}_{J,I}$ yields an effective Hamiltonian with essentially the same general structure as that which could be obtained for the precessional motion starting from the effective Hamiltonians of the C_4-models. This shows merely that the effective Hamiltonians of this type may describe various physical phenomena not necessarily related with the symmetry of the nuclear mean field.

5. WHY NOT CONSIDER THE C_3-SYMMETRY INSTEAD OF C_4?

The model of two coupled collective modes is able, as we have seen, to explain the staggering in the yrast band with an arbitrary periodicity ΔI in the angular momentum scale depending on the ratio of inertia parameters. The C_4 symmetry model has an appealing feature of being seemingly related with the $\Delta I = 4$ nature of the staggering found experimentally. Here we show that this relation is not intimate at all, and that the type of the axial symmetry breaking is not directly related with the interval between the kinks.

The well-known differences between the even I and odd I states in deformed nuclei are indeed deeply related with the prevailing D_2-symmetry of the nuclear mean field (and thus with the C_2-symmetry contained in D_2-symmetry [22]). This difference is due to the nature of the angular variables determining the spacial orientation of the nucleus: these angles are associated with the orientation of the main axes of the nuclear inertia tensor. Indeed, the choice of the intrinsic reference frame associated with the inertia tensor is not unique. Correspondingly, the definition of intrinsic variables is not unique in this case: each of equivalent orientations of the principal axes of a quadrupoloid with respect to the reference frame corresponds to a particular set of intrinsic coordinates. These properties of systems with D_2-symmetry are incorporated in the unified nuclear model [21] and are expressed by the condition imposed on the wave function with respect to the coordinates transformation associated with a reference frame rotation by the angle π around each of the principal axes of the quadrupole tensor: the transformation of collective and intrinsic coordinates issuing from such an operation must leave the wave function unchanged. Such formulation of the theory lies in the origin of the differences in the description of states with positive and negative signatures and is deeply related with various «$\Delta I = 2$ staggering» phenomena.

One may imagine a physical system for which such a definition of collective angles is inappropriate: a system with the shape of a perfect cube. The inertia tensor of such systems possesses the spherical symmetry, and the choice, usual for nuclear physics, of angular coordinates is not adequate for them. Then one would be inclined to associate the collective angles with the hexadecapole moment instead of the quadrupole moment. The quantal collective motion in such systems must be quite different from that which is known in nuclear physics. Here one would expect various kinds of «$\Delta I = 4$ staggering».

Surely, the superdeformed nuclear states are not states to be understood with such a choice of the reference frame. Here we deal with a system having a large quadrupole moment. The image given for nuclei by the adepts of the C_4-symmetry reminds a cigar kept in a tightly packed rectangular box: looked from the thin side, it appears as an object of rectangular shape. The usual definition of collective angles must be perfectly justified in this case. Thus, no a priori limiting conditions additional to those existing in the unified model could be formulated and no a priory $\Delta I = 4$ staggering could be expected as a consequence of the $Y_{4,4}$ deformation. In corroboration with this statement, the effective Hamiltonians introduced in Ref.[19], [20] describe various kinds of spectra with or without staggering depending on the values of entering them parameters.

These arguments do not mean that the explanation of the staggering in terms of effective Hamiltonians in the above quoted papers is necessarily wrong. They indicate that the staggering phenomenon does not give a solid proof for the existence of a C_4-symmetry: these Hamiltonians may describe implicitly any kind of physics, and in particular the quantized vortical motion advocated in this paper.

The staggering effects and other irregularities of the rotational bands may be related, in principle, with other types of violation of the axial symmetry. In particular, the $Y_{3,3}$-deformation of the mean field has good reasons to be involved in the high-spin physics. This is because the octupole collective excitations carry an intrinsic angular momentum ($j_{\text{intr}} = 3\hbar$ for purely octupole one-phonon state) which has a natural tendency to align with the collective rotational angular momentum.

The alignment is reflected by a reduction of the energy interval between the yrast and the lowest $K^{\pi} = 0^{-}$ bands upon increasing the total angular momentum. It is found practically in all even-even deformed nuclei already at very moderate angular momenta. Figure 7 shows two examples of such a tendency [23]: one for a nucleus considered as beeing rigid with respect to octupole vibrations (^{232}Th) and the other for an octupolly soft nucleus (^{220}Ra). The Coriolis effects leading to the alignment may be studied within the model Hamiltonian [23]

$$\hat{H} = \hat{H}_{\text{rot}}(\hat{\mathbf{I}} - \hat{\mathbf{j}}) + \hat{H}_{\text{intr}}, \tag{24}$$

where $\hat{H}_{\text{rot}}$ represents a rotational part depending on the difference between the total angular momentum operator ($\hat{\mathbf{I}}$) and the angular momentum operator ($\hat{\mathbf{j}}$) associated with the octupole vibrations of the nuclear surface.

The second term in the above formula describes the excitation of vibrational degrees of freedom :

$$\hat{H}_{\text{intr}} = \sum_{K=-3}^{3} \omega_{|K|} b_K^{\dagger} b_K + h \left(\sum_{K=-3}^{3} b_K^{\dagger} b_K \right)^2 \tag{25}$$

containing thus an anharmonic correction in addition to the harmonic term.

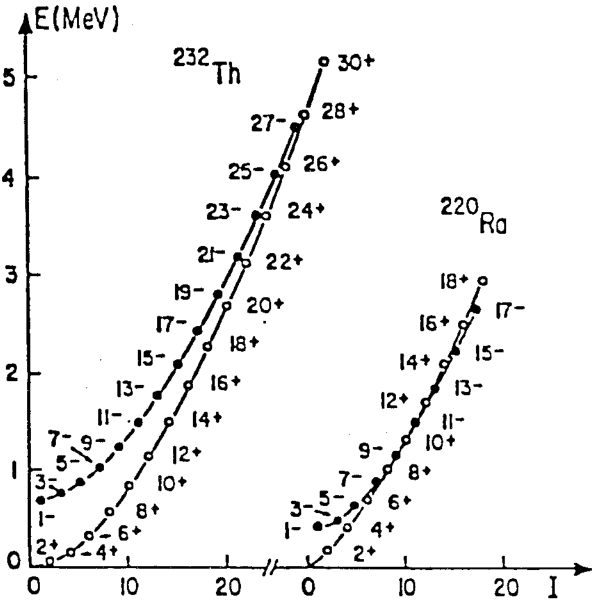


Fig.7. Energy of nuclear states as function of the spin for ^{232}Th [24] and ^{220}Ra [25] nuclei

The intrinsic angular momentum operator

$$\hat{\mathrm{j}} = \sum_{K,K'=-3}^{3} \hat{\mathrm{j}}_{K,K'}\, b_K^{\dagger} b_{K'} \tag{26}$$

establishes a coupling between rotational and vibrational motions and leads to the hybridization of the positive and negative collective bands.

The eigenfunction of the model Hamiltonian can be written as the product of multiphonon configuration states $|n_{K=-3},\ldots n_{K=3}\rangle$ with the spherical functions $D^I_{M,K}$ $(K = \sum K_i n_{K_i})$:

$$\Psi = \sum \psi_{[n_i]} D^I_{M,K} |n_{K=-3},\ldots n_{K=3}\rangle. \tag{27}$$

The amplitudes $\psi_{[n_i]}$ are found as solutions to the matrix equation

$$(\hat{H}_{\mathrm{intr}} - \Omega \hat{j}_x)[\psi] = \varepsilon[\psi] \tag{28}$$

with $\Omega = dE_{\mathrm{rot}}/dI$. The eigenenergies are then equal to

$$E = E_{\mathrm{rot}} + \varepsilon. \tag{29}$$

When Ω becomes large (i.e., when the spin increases) it is found that the lowest value of ε corresponding to an aligned superposition of one phonon states becomes negative. This means that at some spin value the one-phonon configurations become yrast states. At still larger values of spin the yrast states correspond to two-phonon configurations and so forth. Due to linear dependence on I of the energy of aligned multiphonon configurations which is weaker than the quadratic dependence on I of the energy of collective rotation, the intervals between the I-even and I-odd states decrease when the angular momentum increases.

However, the hybridization of I-even and I-odd yrast sequences does not proceed in a monotonic way. The quantized nature of the number of phonons and of the angular momentum results, as in the model discussed before, in a staggering of the yrast-states energies.

An analysis of the described system has been performed upon using coherent states $|\lambda\rangle$ such that $b|\lambda\rangle = \lambda|\lambda\rangle$ within a variational ansatz for finding the amplitudes ψ_{K_i} in the phonon operator $b = \sum \psi_{K_i} b_{K_i}$. It yields a smooth yrast line corresponding to a renormalized moment of inertia. The quantal staggering is lost within the variational treatment just like in the case considered in previous sections.

Coherent states are not eigenstates of the vibrational angular momentum $\mathbf{j}$ and may be considered as some deformed states. To find out the nature of the deformation which is involved here we associate the boson operators $b^{\dagger}_{K_i}$, b_{K_i} with the components of the octupole mass tensor and conjugated to it momentum $p_i \sim i\hbar[T, q_i]$ (T being the kinetic energy operator). Then we find that the optimal shape of the nucleus at high spins involves an octupole $Y_{3,3}$ deformation.

These results published first in Ref.[23] led the authors to conclude that the installation of such a deformation is a potentially general property of nuclei at high spins*.

One cannot expect that such a schematic model may go far in explanation of the high-spin states in atomic nuclei. Anyway, it allowed one to predict an unusual pattern of the spectrum in the nucleus ^{220}Ra soft in respect to the octupole vibrations. In Fig.8 we reproduce the comparison of the experimentally found values of the ratio

$$R = \frac{2E_{\gamma}((I+1)^{-} \to I^{+})}{E_{\gamma}((I+2)^{+} \to I^{+})}$$

*That is the property expected at sufficiently high spins which may turn out however to be higher than critical for the fission in some nuclei.

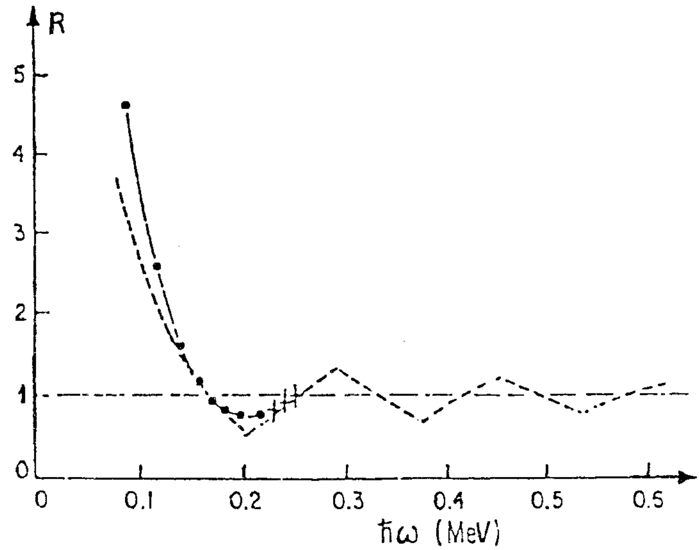


Fig.8. The calculated and experimental transition energy ratios $R = 2E_\gamma((I+1)^- \to I^+)/E_\gamma((I+2)^+ \to I^+)$ in ^{220}Ra. The symbols in the picture stand for: – – • – – experiment [25], – – + – – experiment [26], – – – – – theory [23],

with the values corresponding to the presented above model. The approach of R to one is taken in the literature as an indication for the break-up of the left-right symmetry of the nuclear mean field. Both the experimental data (the high-spin part of which was obtained about five years after publication of Ref.[23]) and the theory show that the installation of the octupole symmetry is accompanied by some staggering effects with the periodicity having nothing in common with the type of the symmetry involved.

The high-spin superdeformed states are excellent candidates for the search for the deformation of this type, although the model discussed before cannot yield a quantitative prediction here. One knows that an appreciable fraction of the octupole strength is concentrated around the excitation energy [27] of the «low-lying octupole resonance» (LEOR)

$$E_{\rm LEOR} = \frac{31.4}{A^{1/2}} \sqrt{7/4 - X}\ ({\rm MeV})$$

throughout the periodic table (in the above formula $X = Z^2/49A$ is the fissility parameter). In rare earth nuclei this excitation energy becomes equal to the magnitude of the Coriolis coupling between octupole states at the rotational frequency of about 1 MeV which is not too far from the frequencies found in the superdeformed bands in this region of nuclei.

6. HIGHER-ORDER EFFECTS IN THE ELECTROMAGNETIC RADIATION AS A PROBE OF VORTICAL MOTION

The uniform circulation may manifest itself only in systems with an appreciable quadrupole deformation. Indeed, the «stretched» rotation becomes indistinguishable from the uniform rotation of the matter when the container has a spherical shape. Strictly speaking, it is expected only in ellipsoidally shaped bodies. However, other types of vortical motion may exist also in spherical or slightly deformed nuclei with arbitrary shapes. For example, in spherical nuclei the dipole toroidal motion produces the currents with the velocity field

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{a}(t)\left(1 - 2\frac{r^2}{R^2}\right) + (\mathbf{a}(t)\cdot\mathbf{r})\frac{\mathbf{r}}{\mathbf{R}^2}, \tag{30}$$

where $\mathbf{r}$ is a position of a moving liquid element and R is the radius of a sphere representing the nuclear surface. This type of motion is known as the Hill's vortex [5]. The vector $\mathbf{a}(t)$ determines the amplitude and the direction of the Hill's vortex. The lines of current of such a velocity field are shown in the upper right corner in Fig.2. The procedure described in **Section 2** gives for the stretched toroidal current in ellipsoidally deformed nuclei the following expression [6]:

$$\mathbf{u} = u_\rho \mathbf{e}_\rho + u_z \mathbf{e}_z \tag{31}$$

with

$$u_\rho = Az\rho, \quad u_z = A\left(b^2 - z^2 - 2\frac{b^2}{a^2}\rho^2\right). \tag{32}$$

Here, A is the amplitude of the vortex aligned with the z-axis, a and b are the semiaxes of the ellipsoid determining the surface. The corresponding lines of the current in ellipsoidally deformed nuclei are shown in Fig.9.

Fingerprints of one particular mode of such a motion (toroidal mode) have been already looked for in the data on the nuclear structure for some years already.

The toroidal currents in an electrically charged liquid interact with the transverse electromagnetic field and consequently may influence the dipole-electric transitions between nuclear eigenstates [28]. Strong deviations from predictions of the adiabatic theory for the absolute values of $E1$-transitions found recently in the multiple Coulomb excitation of ^{226}Ra have been analysed

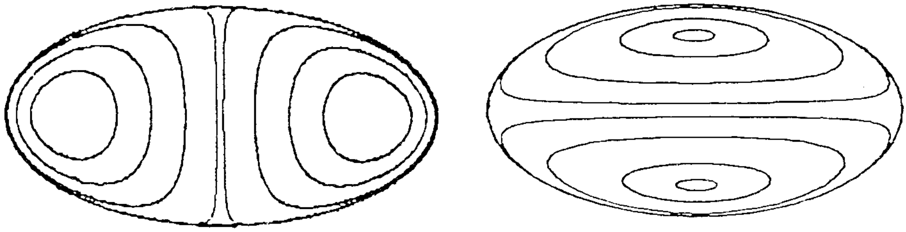


Fig.9. The lines of the current corresponding to the toroidal dipole modes in deformed nuclei: right — the Hill's vortex is aligned with the short axis, left — the Hill's vortex is aligned with the long axis

in order to estimate the contribution of the dipole toroidal moment to the $E1$ transition probabilities [29]. The analysis is performed taking into account the effects of Coriolis coupling between the negative-parity bands and including in the $E1$ transition operator the terms describing the interference of the «pure» electric dipole and toroidal moments. This is done writing for the intrinsic part of the dipole electric operator an expression:

$$M_K \propto \mathbf{d}_K + E_\gamma \mathbf{t}_K .$$

Here, $\mathbf{d}$ is the standard dipole electric moment and t is the dipole toroidal moment of the nucleus; E_γ is the γ-ray energy.

In Fig.10 we show the best fit of experimental results obtained in this way in comparison with the best fit obtained assuming that the toroidal contribution is absent ($\mathbf{t} = 0$). Here, the «effective electric dipole moment» is given:

$$Q^{\mp} = \sqrt{\frac{4\pi}{3} \frac{(2I+1)}{((I+1/2) \mp 1/2)} B(E1; I^- \to (I \mp 1)^+_{\rm gr})}$$

extracted in Ref.[30] from experimental data and calculated from the model. It is seen that the admission of toroidal currents allows one to improve essentially the reproduction of experimental results.

In Ref.[29] the energy weighted sum of matrix elements squared for the toroidal dipole operator is estimated ($S_{\rm tor}$). A phenomenological treament shows that the strength of toroidal transitions between the ground and the lowest negative-parity band is rather large: $|\langle 1^- | \mathbf{t} | gr \rangle|^2 (E^{1-} - E^{gr}) \simeq 0.13 S_{\rm tor}$. This allows one to think that the collective toroidal current plays an important role in the structure of the lowest negative-parity band in ^{226}Ra. Such a conclusion

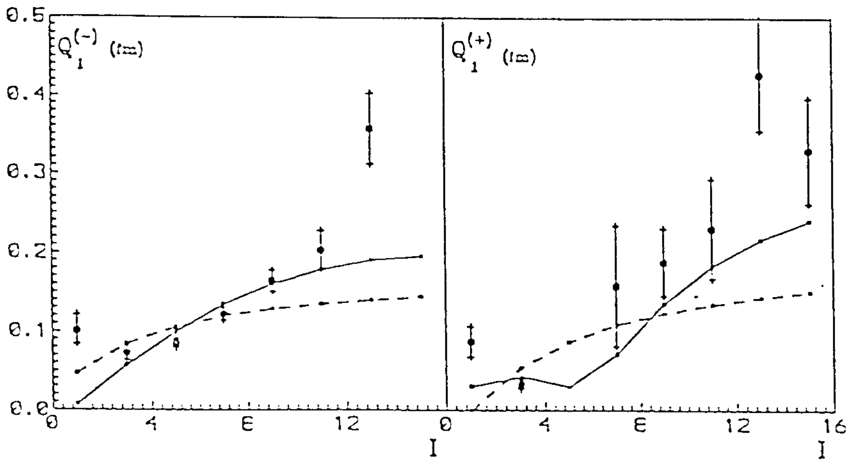


Fig.10. Comparison of the «effective» dipole moments $Q^{\pm}$ extracted from the experimental data and calculated admitting toroidal contributions to the transition matrix element (full line) and without such contributions (broken line)

calls for verifications which could be done by analysing both theoretically and experimentally the internal conversion process accompanying the nuclear transitions studied in this paper. Indeed, the internal convertion coefficients give an independent test of the presence of such currents.

7. ANALOGY WITH THE AHARONOV-BOHM EFFECT

We want to give here an image of the phenomena discussed before somewhat different from previously presented. The emphasis of our communication is made on collective motions which do not change the distribution of the matter in space. The existence of such motions in macrophysics does not need any proof: with a very tiny idealization of the reality, one may think that the electric current in an ordinary conductor is not related with any changes in the electric circuit. The amplitude of the current may be regulated at one's desire and may be considered as one of the parameters determining the physical state of the circuit. Certainly, intrinsic vortical currents exist in microscopic systems like atomic nuclei. The value of the moment of inertia of deformed nuclei indicates that the motion involved in the collective rotation is different from the global (rigid body) rotation. Magnetic properties of spherical nuclei reflect the presence of currents yielding a circulation of the nuclear matter. The previous section has also given an evidence for the dipole toroidal currents in heavy nuclei.

Still a rather fundamental question concerning the vortical currents in microscopic systems may be raised: may such currents represent, or not, degrees of freedom independent from those which are associated with the distribution of the matter in the space? Such a question is prompted by the following simplistic consideration. In quantal systems consisting of identical particles one could think that their (infinitesimally slow) displacements would affect the physical state only inasmuch as by changing the distribution of the matter. Arguing like this, one may say that the second of the transformations considered in **Section 2** (the **S**-transformation in eq.(1)) must be considered as unphysical in application to atomic nuclei, and all theoretical constructions based on it are erroneous. However, this argument fails in the case of the time-dependent transformations, because the latter reflect the changes of the particle distribution in the momentum space. The state Ψ' satisfying the generalized cranking equation (4) is quite different from the one corresponding to the same distribution of the matter in space but which is obtained using a variational approach involving only time-even constraints.

The wave function describing the stationary motion contains a time-dependent phase factor (see eq.(5)). It is precisely the place, where the motion generating parameters appear. Representing no measurable quantities in the sense of classical picture, such parameters play an important role in the quantal description of the motion. These aspects of the theory have been shown in this paper for the case of the motion involving the Kelvin circulation. In fact, such aspects of the theory are well known in the literature : they play the central role in the so-called Aharonov-Bohm effect [31,32,33]. The latter concerns the reaction of the quantal system on the electromagnetic field in special circumstances when the electromagnetic field is localized in the region inattainable to a charged particle but affects the phase of its wave function through the electromagnetic potentials (the scalar potential and especially the vector potential **A**). When the charged particle passes the region of a nonvanishing value of **A** the presence of a remote electromagnetic field manifests itself by a variety of interference effects.

The Aharonov-Bohm effect stems from the accentuation of the role of the phase of the wave-function: the changes in physical arrangements responsible for the changes of the phase of the wave function may lead to measurable effects even in cases when such effects are not expected for classical systems. The model suggested in this paper for the explanation of the energy staggering in superdeformed bands gives a new example of such a situation.

Since the publication of the paper by Y.Aharonov and D.Bohm, a number of possible experiments has been discussed in which the phase of the wave function plays an unexpected role. In particular, there was considered a system consisting of an electron in a circular circuit placed in the static magnetic field. If the circuit is infinitely thin and the electron-ion interaction is absent the quantum state of an electron is given by the Schroedinger equation with the Hamiltonian

$$\hat{H}_e = \frac{1}{2m}\left(\hat{p}_\theta - \frac{eA}{c}\right)^2 \qquad (33)$$

with

$$\hat{p}_\theta = \frac{\hbar}{iR}\frac{\partial}{\partial\theta},$$

where R is the radius of the circuit. Naturally, the solutions of the Schroedinger equation are

$$\psi_n(\theta) = \sqrt{1/2\pi}\,\exp\,[i(Rk_n\,\theta - \frac{e}{\hbar c}\int_0^\theta d\,\mathbf{l}(\theta')\mathbf{A}(\theta'))]$$

with k_n quantum numbers determined by the boundary periodicity conditions:

$$2\pi Rk_n - \frac{e}{\hbar c}\Phi = 2\pi n, \qquad (34)$$

where, $n = 0, \pm1, \pm2, \ldots$, and

$$\Phi = \int_c d\,\mathbf{l}\cdot\mathbf{A} = \int_s d\,\mathbf{s}\cdot\mathbf{H} \qquad (35)$$

is the magnetic flux passing through the circuit.

The energy of the electron in the quantum state n

$$E = \bar{\hat{H}}_e = \frac{(\hbar k_n)^2}{2m}$$

does not depend in an explicit way on the magnetic field. However, the allowed values of k_n depend on the flux according to eq. (34). In particular, the ground state energy is not necessarily equal to zero whenever their is a nonvanishing magnetic flux through the circuit. Indeed the lowest $|k_n|$ value is either

$$k_0 = \frac{1}{R}\left(\left[\frac{e}{hc}\Phi\right] - \frac{e}{hc}\Phi\right)$$

or $k_0 - 1$ (here $[x]$ means the integer part of x and it is assumed that $\Phi \geq 0$). Thus, the energy of the lowest state is a «staggering» function of the flux.

The energy staggering in the electric circuit is accompanied by the presence of an electric current whose magnitude depends non-monotonically on the magnetic flux Φ. The magnitude of the current is equal to

$$(j_{el})_n = \frac{\hbar e}{m} k_n .$$

From the above discussion it follows that whenever $\Phi/(e/hc)$ is not an integer, one expects to register the permanent electric current in the circuit (even when it is in its ground state). Very fine recent experiments on Superconducting Quantum Interference Devices (SQUID) representing electric «mesoscopic» chains with the size of the order of $3\mu m$ show indeed the presence of such currents [34], [35].

It is easy to establish parallels between the model of **Section 3** describing the irregularities in superdeformed bands with the system discussed in the previous paragraphs. To do it, we examine in a slightly different way the Hamiltonian in eq.(13) to which we add a term describing the coupling of the rotational degrees of freedom with the degrees of freedom of the intrinsic motion:

$$\hat{H} = \hbar^2\lambda_1 \left(\frac{A_1}{2}\hat{\mathbf{I}}^2 - B_1\hat{\mathbf{I}}\hat{\mathbf{J}} + \frac{C_1}{2}\hat{\mathbf{J}}^2 \right) + \hbar^2\hat{h}' + V_{\text{coupl}} \tag{36}$$

assuming that $[\hat{\mathbf{I}}, V_{\text{coupl}}] = 0$. The eigenfunctions have exact angular momentum quantum numbers (I and M) and may be written in the usual way as superpositions of states with definite projection of the angular momentum on a quantization axis. Choosing the latter as the axis of rotation yields the expansion of the wave function in the spherical harmonics $D^I_{M,K}(\Theta)$ depending on the orientation of the system in space :

$$\Psi_{I,M} = \sum_n D^I_{M,I-n}(\Theta)\Psi_n . \tag{37}$$

We represent the eigenfunction $\Psi_{I,M}$ as a column vector (dropping the I and M indices)

$$\Psi_{I,M} \to \Psi = \begin{pmatrix} \Psi_0 \\ \Psi_1 \\ \Psi_2 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \tag{38}$$

and write the Schroedinger equation in a matrix form

$$\hat{H}_I \Psi = E_I \Psi. \tag{39}$$

The state Ψ having a fixed angular momentum quantum number I, the projected Hamiltonian $\hat{H}_I$ is

$$\hat{H}_I = \hbar^2 \left(\frac{A_1}{2} I(I+1) - B_1(I - \hat{n})\hat{J}_1 + \frac{C_1}{2} \hat{J}^2 + \hat{h}_I \right) + V_{\text{coupl}}. \tag{40}$$

As a starting point we make the same approximation as in **Section 4**. We neglect the terms $\hat{h}_1'$ and V_{coupl}. The first of them contributes to the precessional motion while the second describes the J-mixing. Then the I and K quantum numbers are conserved:

$$\hat{I}^2 \Psi = I(I+1)\Psi, \quad \hat{n}\Psi = n\Psi, \quad K = I - n.$$

In this case the dynamical equation for Ψ becomes:

$$(\hat{H}_{\text{stag}} + \hbar^2 B_1 (I - n_I)\hat{n}_J)\Psi = \Delta E(I)_I \Psi, \tag{41}$$

where

$$\Delta E_I = E(I) - \overline{E(I)}$$

$$\overline{E(I)} = \frac{\hbar^2}{2} \left(A_1 I(I+1) - \frac{B_1^2}{C_1} (I - n_I)^2 \right) \tag{42}$$

and also

$$\hat{H}_{\text{stag}} = \frac{\hbar^2 C_1}{2} \left(\hat{J} - \frac{B_1}{C_1} (I - n_I) \right)^2. \tag{43}$$

The solution for $\Delta E(I)$ is

$$\Delta E(I) = \frac{\hbar^2 C_1}{2} \left(J - \frac{B_1}{C_1} (I - n_I) \right)^2 + \hbar^2 B_1 (I - n_I) n_J. \tag{44}$$

When $n_I = n_J = 0$ one easily recognises in the last expressions the results presented in **Section 3** and **4** for the smooth and staggering parts of the energy of the system in which the quantized Kelvin circulation is coupled with the global rotation.

The Hamiltonians in eq. (33) and eq. (43) have an important property in common: they represent positive definite operators each depending on the

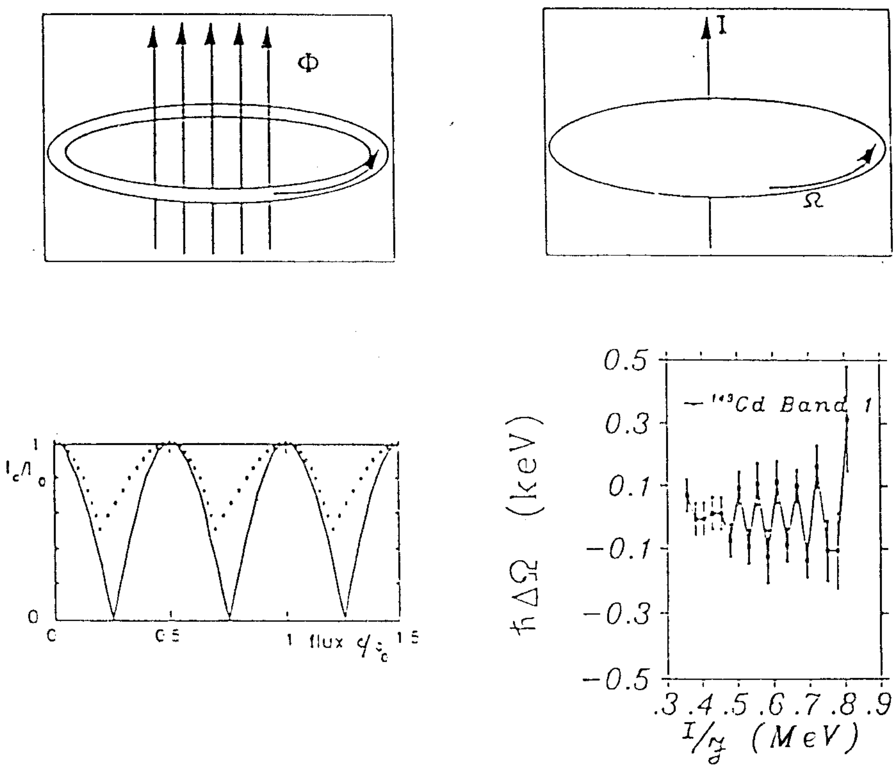


Fig.11. Parallels in the properties of mesoscopic conducting rings (left side) and the rotational motion in the nuclear model with quantized Kelvin circulation (right side). See explanation in the text

difference of a quantized quantity ($\hat{J}$ and $\hat{p}_\theta$ correspondingly) and an «external» parameter (I and A). For these reasons the «ground state energy» as a function of the corresponding «external» parameter has repetitive kinks following the quantal changes in the ground state wave function. The possible relation of such kinks with the observed $\Delta I = 4$ energy staggering has been already discussed in this paper. The experimental proof for the existence of such kinks in the electric circuit has also been given, although the direct measurements of the energy of a circuit with the precision necessary to register the tiny quantal effects turn out to be impossible. The proof of the considered phenomenon has been done measuring the accompanying electric current.

The parallels between the energy staggering in the superdeformed bands and the permanent electric currents are illustrated in Fig.11. The upper part of the figure shows the two quantal systems considered before (the «mesoscopic» electric chain in the magnetic field at the left and the rotating nucleus with vortical currents at the right). The lower part of the figure presents the quantal phenomena establishing the similarity of the two systems. The permanent electric currents in SQUIDs (solid line for an ideal «symmetric» SQUID and dotted line for the real SQUID with finite inductivity) are shown on the left side of the picture. At the right we reproduce once more the data concerning the energy staggering in the superdeformed band in ^{149}Gd explained in **Section 3** within the model of a vortical motion coupled with the global rotation.

One may go much further in studying parallels in the properties of these two kinds of systems. One may establish a similarity of effects produced by the finite width of an electric circuit and of the precessional motion in the nuclear model presented in **Section 3**. Both factors are related with the activation of an additional degree of freedom. If the possibility of a motion in an additional direction is open then the closed orbitals may be destroyed and thus the kinks may be deminished. One maý see the similarity between the «J» and «k_n» mixing in the corresponding models. The mixing of these quantum numbers diminishes the role of the limiting conditions and also diminishes the amplitude of the kinks.

8. CONCLUDING REMARKS

To conclude we may refer once more to Figure 1 picturing an oldish man looking for the treasure in a pool of water. The particular pool in which we have dwelt here may look to some readers rather shallow and its waters much disturbed by the previous passage of so many gold searchers. It is our prejudice however that whatever the fate of some tentative explanations proposed here might ultimately be, taking into account in a systematic way of the dynamics associated with shape conserving collective currents is a rich field of investigations deserving a particular attention.

As a matter of fact we have considered here mostly but not exclusively a particular type of such motions namely the uniform intrinsic vortical modes whose classical analogues are the S-type Riemann ellipsoids. Their quantization has yielded such interesting spectroscopic features as the regular presence of kinks in the energies of the yrast states. Even though our proposition of such a phenomenon to explain the observed $\Delta I = 4$ staggering still remains to be confirmed, it is not without significance that it may be ascribed as an analogue of the Aharonov-Bohm effect in rotating nuclei.

The influence of the breaking of axial symmetry as well as intrinsic parity symmetry has also been discussed. Under some reasonable model assumptions, breaking these symmetries one retrieves, roughly speaking, the same coupling scheme of collective modes and therefore the same type of spectroscopic properties as in axially and reflection invariant nuclei.

We have sketched also some directions of further work which go beyond the uniform intrinsic vortical motion either by considering more complicated intrinsic vortical modes or by explicitly taking into account the mixing of the Kelvin circulation quantum number.

Altogether most of our ideas brought up in the present review call for an assessment through specific microscopic calculations. In particular the crucial role of pairing correlations should be studied in a detailed fashion. Such studies will without any doubt illustrate new facets of the rich dynamical behavior of the atomic nuclei considered as small quantal fermionic droplets.

This paper is dedicated to Professor Vadim G. Soloviev not only for many illuminating scientific discussions on various aspects of nuclear collective motion but also for his constant example of positive, enthusiastic and deep attitude in front of the challenging and sometimes overwhelming body of nuclear spectroscopic data.

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