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NONLOCAL QUARK AND GLUON CONDENSATES WITHIN A CONSTRAINED INSTANTON MODEL

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We suggest a constrained instanton (CI) solution in the physical QCD vacuum which is described by large-scale vacuum field fluctuations. This solution decays exponentially at large distances. It is stable only if the interaction of the instanton with the background vacuum field is small and additional constraints are introduced. The CI solution is explicitly constructed in the ansatz form, and the two-point vacuum correlator of gluon field strengths is calculated in the framework of the effective instanton vacuum model. At small distances the results are qualitatively similar to the single instanton case, in particular, the form factor D_1 is small, which is in agreement with the lattice calculations.

The nonperturbative vacuum of QCD is densely populated by long-wave fluctuations of gluon and quark fields. The order parameters of this complicated state are characterized by the vacuum matrix elements of various singlet combinations of quark and gluon fields, condensates: $\langle : \bar{q}q : \rangle$, $\left\langle : F^a_{\mu\nu} F^a_{\mu\nu} : \right\rangle$, $\left\langle : \bar{q}(\sigma_{\mu\nu} F^a_{\mu\nu} \frac{\lambda^a}{2}) q : \right\rangle$, *etc*. The nonzero quark condensate $\langle : \bar{q}q : \rangle$ is responsible for the spontaneous breakdown of chiral symmetry, and its value was estimated a long time ago within the current algebra approach. The nonzero gluon condensate $\left\langle : F^a_{\mu\nu} F^a_{\mu\nu} : \right\rangle$ through trace anomaly provides the mass scale for hadrons, and its value was estimated within the QCD sum rule (SR) approach. The values of low-dimensional condensates were obtained phenomenologically from the QCD SR analysis of various hadron channels.

The nonlocal vacuum condensates or vacuum correlators [1, 2] describe the distribution of quarks and gluons in the nonperturbative vacuum. Physically, it means that vacuum quarks and gluons can flow through the vacuum with nonzero momentum. From this point of view the standard vacuum expectation values (VEVs) like $\langle : \bar{q}q : \rangle$, $\left\langle : \bar{q}D^2 q : \right\rangle$, $\left\langle : g^2 F^2 : \right\rangle$, $\ldots$ appear as expansion coefficients of the quark $M(x) = \left\langle : \bar{q}(0)\hat{E}(0,x)q(x) : \right\rangle$ and gluon $D^{\mu\nu,\rho\sigma}(x)$

correlators in a Taylor series in the variable $x^2/4$. The correlator $D^{\mu\nu,\rho\sigma}(x)$ of gluonic field strengths

$$D^{\mu\nu,\rho\sigma}(x-y) \equiv \left\langle : TrF^{\mu\nu}(x)\hat{E}(x,y)F^{\rho\sigma}(y)\hat{E}(y,x) : \right\rangle, \tag{1}$$

may be parameterized in the form consistent with general requirements of the gauge and Lorentz symmetries as

$$\begin{aligned} D^{\mu\nu,\rho\sigma}(x) \quad &\equiv \quad \frac{1}{24}\left\langle : F^2 : \right\rangle \Big\{ (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})[D(x^2) + D_1(x^2)] + \\ &+ \quad (x_\mu x_\rho \delta_{\nu\sigma} - x_\mu x_\sigma \delta_{\nu\rho} + x_\nu x_\sigma \delta_{\mu\rho} - x_\nu x_\rho \delta_{\mu\sigma}) \frac{\partial D_1(x^2)}{\partial x^2} \Big\}, \end{aligned} \tag{2}$$

where $\hat{E}(x,y) = P\exp\left(i\int_x^y A_\mu(z)dz^\mu\right)$ is the path-ordered Schwinger phase factor (the integration is performed along the *straight* line) required for gauge invariance and $A_\mu(z) = A_\mu^a(z)\dfrac{\lambda^a}{2}$, $F_{\mu\nu}(x) = F_{\mu\nu}^a(x)\dfrac{\lambda^a}{2}$, $F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + f^{abc}A_\mu^b(x)A_\nu^c(x)$. The P-exponential ensures the parallel transport of color from one point to another. In (2), $\left\langle : F^2 : \right\rangle = \left\langle : F_{\mu\nu}^a(0)F_{\mu\nu}^a(0) : \right\rangle$ is a gluon condensate, and $D(x^2)$ and $D_1(x^2)$ are invariant functions which characterize nonlocal properties of the condensate in different directions. The form factors are normalized at zero by the conditions $D(0) = \kappa$, $D_1(0) = 1 - \kappa$, that depend on the dynamics considered. For example, for the self-dual fields $\kappa = 1$, while in the Abelian theory without monopoles the Bianchi identity provides $\kappa = 0$.

In [3], one has shown that the instanton model of the QCD vacuum provides a way to construct nonlocal vacuum condensates. Within the effective single instanton (SI) approximation one has obtained the expressions for the nonlocal gluon $D_I^{\mu\nu,\rho\sigma}(x)$ and quark $M_I(x)$ condensates and derived the average virtualities of quarks λ_q^2 and gluons λ_g^2 in the QCD vacuum. The behavior of the correlation functions demonstrates that in the SI approximation the model of nonlocal condensates can well reproduce the behavior of the quark and gluon correlators at *short distances*. Really, the quark and gluon average virtualities, defined via the first derivatives of the nonlocal condensates $M_I(x^2)$, $D_I(x)$ at the origin,

$$\lambda_q^2 \equiv -\frac{8}{M_I(0)}\frac{dM_I(x^2)}{dx^2}\Bigg|_{x=0} = 2\frac{1}{\rho_c^2}\,, \qquad \lambda_g^2 \equiv -8\frac{dD_I(x^2)}{dx^2}\Bigg|_{x=0} = \frac{24}{5}\frac{1}{\rho_c^2}\,, \tag{3}$$

are connected with vacuum expectation values that parameterize the QCD SR,

$$\lambda_q^2 \equiv \frac{\left\langle : \bar{q}D^2q : \right\rangle}{\left\langle : \bar{q}q : \right\rangle}, \quad \lambda_g^2 \equiv \frac{\left\langle : F_{\mu\nu}^a\tilde{D}^2F_{\mu\nu}^a : \right\rangle}{\left\langle : F^2 : \right\rangle} = 2\frac{\left\langle : fF^3 : \right\rangle}{\left\langle : F^2 : \right\rangle} - 2\frac{\left\langle : g^4J^2 : \right\rangle}{\left\langle : F^2 : \right\rangle}, \tag{4}$$

where $\langle : fF^3 : \rangle = \langle : f^{abc} F^a_{\mu\nu} F^b_{\nu\rho} F^c_{\rho\mu} : \rangle$, $J^2 = J^a_\mu J^a_\mu$ and $J^a_\mu = \bar{q}(x) \dfrac{\lambda^a}{2} \gamma_\mu q(x)$. The value of $\lambda_q^2 \approx 0.5\ \mathrm{GeV}^2$ estimated in the QCD SR analysis [4] is reproduced at $\rho_c \approx 2\ \mathrm{GeV}^{-1}$. This number is close to the estimate from the phenomenology of the QCD vacuum in the instanton liquid model. Nevertheless, the SI approximation used evidently fails in the description of physically argued distributions at large distances.

In [5], it was suggested that the instanton $A^{CI}_\mu(x)$ is developed in the physical vacuum field $b_\mu(x)$ interpolating large-scale vacuum fluctuations. One has found that at small distances the instanton field dominates, and at large distances it decreases exponentially. This solution is called constraint instanton (CI). The long-wave vacuum field $b_\mu(x)$ is specified by the correlation function $\widetilde{B}(x^2)$ determined by its strength $\langle F_b^2 \rangle_b$ and the correlation length R. Within this model, by averaging over random color vector orientations of the background field with respect to the fixed instanton field orientation, one has found the equation

$$D^{ab}_\mu \left[A^{CI}\right] F^{CI,b}_{\mu\nu}(x) - \frac{N_c \langle F_b^2 \rangle_b}{24(N_c^2-1)} x^2 \Phi\left(x^2\right) A^{CI,a}_\mu(x) + \text{Constraint term} = 0, \tag{5}$$

governing the deformation of the instanton under the influence of the weak background vacuum field. The constraint term is added to stabilize the instanton against shrinking [6]. In (5)

$$\Phi\left(x^2\right) = 4 \int_0^1 d\alpha \int_0^1 d\beta \alpha\beta \widetilde{B}\left[(\alpha-\beta)^2 x^2\right], \qquad \Phi(0) = 1, \tag{6}$$

and N_c is the number of colors. The constraint independent asymptotics of the instanton solution at large distances is found as

$$A^{CI,a}_{\mu,\mathrm{asympt}}(x) = \overline{\eta}^a_{\nu\mu} \frac{2x_\nu}{x^2} a_{4/3} (\rho\eta_g)^2 K_{4/3}\left[\frac{2}{3}\left(\eta_g |x|\right)^{3/2}\right],$$

where

$$a_{4/3} = \frac{2}{\Gamma(1/3)\, 3^{1/3}} \tag{7}$$

is the normalization coefficient, $K_{4/3}(z)$ is modified Bessel function and $\Gamma(z)$ is the Gamma-function. This solution is exponentially suppressed at large distances $\sim \exp\left[-\dfrac{2}{3}\left(\eta_g |x|\right)^{3/2}\right]$ unlike the powerful decreasing SI. It is important to note that the form of this asymptotics is also independent of the model for the background field and the driven parameter $\eta_g \sim \left(\dfrac{N_c}{9\left(N_c^2-1\right)} R \langle F_b^2 \rangle_b\right)^{\frac{1}{3}}$

only weakly depends on it. Assuming that the external field is weak, the CI profile function is close to SI profile at distances smaller than ρ_c and it decreases exponentially at distances larger than η_g^{-1}. The knowledge of the constraint-independent parts of CI allowed to construct the solution in the ansatz form

$$A_\mu^{CI,a}(x) = \overline{\eta}^a_{\nu\mu} \frac{x_\nu}{x^2} \varphi_g\left(x^2\right), \qquad \varphi_g\left(x^2\right) = \frac{\overline{\rho}^2\left(x^2\right)}{x^2 + \overline{\rho}^2\left(x^2\right)}, \tag{8}$$

where the notation

$$\overline{\rho}^2\left(x^2\right) \;=\; a_{4/3}\eta_g^2 x^2 K_{4/3}\left[\frac{2}{3}(\eta_g x)^{3/2}\right], \qquad \overline{\rho}^2\left(0\right) = \rho^2$$

is introduced. By translational invariance the centre of CI can be shifted in (8) from the origin to an arbitrary position x_0: $x \to x - x_0$.

By averaging over the instanton orientations in the color space and taking the trace over color matrices the invariant functions $D\left(x^2\right)$ and $D_1\left(x^2\right)$ can be extracted. It is convenient to define the combinations of functions $D\left(x^2\right)$ and $D_1\left(x^2\right)$

$$A\left(x^2\right) = \delta_{\mu\rho}\delta_{\nu\sigma}\frac{D^{\mu\nu,\rho\sigma}(x)}{\left\langle 0\left|F_{\mu\nu}^2\right|0\right\rangle^{CI}} = D\left(x^2\right) + D_1\left(x^2\right) + \frac{1}{2}x^2\frac{\partial D_1\left(x^2\right)}{\partial x^2},$$

$$B\left(x^2\right) = 4\frac{x_\mu x_\rho}{x^2}\delta_{\nu\sigma}\frac{D^{\mu\nu,\rho\sigma}(x)}{\left\langle 0\left|F_{\mu\nu}^2\right|0\right\rangle^{CI}} = D\left(x^2\right) + D_1\left(x^2\right) + x^2\frac{\partial D_1\left(x^2\right)}{\partial x^2}, \tag{9}$$

taking the boundary condition, $D(0) + D_1(0) = 1$ and the asymptotic conditions $D(\infty) = D_1(\infty) = 0$. The final expressions for form factors A and B [5] are:

$$\begin{aligned}A(x^2) = \frac{8}{\pi}N_D\int_0^\infty drr^2\int_0^\infty dt\,\{&[\omega_1\left(z_+\right)\omega_1\left(z_-\right) + \omega_3\left(z_+\right)\omega_3\left(z_-\right)] \\ &\times\left(3 - 4\sin^2(\alpha_z)\right) - 2\omega_2\left(z_+\right)\omega_2\left(z_-\right) \\ &\times\left[r^2x^2\left(1 - 2\sin^2(\alpha_z)\right) - rx\left(z_+\cdot z_-\right)\sin(2\alpha_z)\right]\},\end{aligned} \tag{10}$$

$$\begin{aligned}B(x^2)=&\frac{16}{\pi}N_D\int_0^\infty drr^2\int_0^\infty dt\left\{\omega_1\left(z_+\right)\omega_1\left(z_-\right)\left(3 - 4\sin^2(\alpha_z)\right)\right. \\ &-\omega_1\left(z_+\right)\omega_2\left(z_-\right)\left[z_-^2 + 2t_-^2\left(1 - 2\sin^2(\alpha_z)\right) + 2rt_-\sin(2\alpha_z)\right] \\ &-\omega_2\left(z_+\right)\omega_1\left(z_-\right)\left[z_+^2 + 2t_+^2\left(1 - 2\sin^2(\alpha_z)\right) - 2rt_+\sin(2\alpha_z)\right] \\ &+\omega_2\left(z_+\right)\omega_2\left(z_-\right)\left[z_+^2z_-^2 + 2t_+t_-\left(z_+\cdot z_-\right)\left(1 - 2\sin^2(\alpha_z)\right)\right. \\ &\left.\left.+2rxt_+t_-\sin(2\alpha_z)\right]\right\},\end{aligned} \tag{11}$$

where

$$\omega_1(x) = x^2\varphi_g^2\left(x^2\right) - \varphi_g\left(x^2\right), \qquad \omega_2(x) = \varphi_g^2\left(x^2\right) + \frac{\partial\varphi_g\left(x^2\right)}{\partial x^2}, \tag{12}$$

$$x_\rho^2 = x^2 + \rho^2,$$

$z_\pm = (r, t_\pm)$, $t_\pm = t \pm \dfrac{x}{2}$, N_D is the normalization factor

$$N_D^{-1} = 6\int_0^\infty dyy^3\left(\omega_1^2(y) + \omega_3^2(y)\right), \tag{13}$$

and the phase factor

$$\alpha_z = r\int_{-\frac{x}{2}}^{\frac{x}{2}} d\tau\varphi_g\left(r^2 + (t+\tau)^2\right),$$

reflects the presence of the $\hat{E}$ exponent in the definition of the bilocal correlator. The form factors $D(x^2)$ and $D_1(x^2)$ are determined numerically by solving the equations (9) and plotted in the Figure in coordinate space. As it turns out, at a reasonable set of parameters, guaranteeing the smallness of the large-scale vacuum field fluctuations, the $D\left(x^2\right)$ structure is close to the SI induced function with the exponential asymptotics being developed at large distances. At the same time, the $D_1\left(x^2\right)$ structure is about two orders smaller than the $D\left(x^2\right)$ function at any reasonable choice of the parameter $\rho_c\eta_g$. The lattice data are in qualitative agreement with predictions of the constrained instanton model.

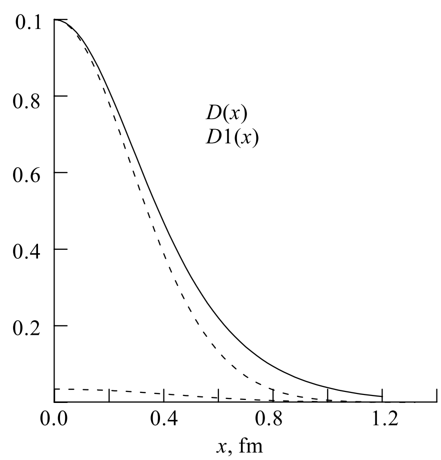

Fig. 1. The form factors D (top lines) and D_1 (bottom lines) (all normalized by $D(0)$) versus physical distance x, for the instanton size $\rho = 0.3$ fm and parameters $(\rho\eta_g)^2 = 0$ (solid lines) and $(\rho\eta_g)^2 = 1$ (dashed lines)

The nonperturbative part of the invariant functions $A(x^2)$ and $B(x^2)$ are the sum of short-range instanton induced contributions (10) and (11), multiplied by the weight factor $n_c 32\pi^2/\left\langle 0\left|F^2\right|0\right\rangle_{\text{total}}$, and the long-range contribution

$$\widetilde{B}\left(z^2\right) = \widetilde{D}\left(z^2\right) + \widetilde{D}_1\left(z^2\right) + z^2\partial\widetilde{D}_1\left(z^2\right)/\partial z^2 \tag{14}$$

modeled by exponentially decreasing function $\widetilde{B}_E\left(x^2\right) = \exp(-\left|x\right|/R)$, with the weight factor $\left\langle F_b^2\right\rangle_b / \left\langle 0\left|F^2\right|0\right\rangle_{\mathrm{total}}$. The constrained instanton model introduces two characteristic scales (correlation lengths). One is related to short distance behavior of the correlation functions and another with long range distance behavior. The first one, λ_g^{-1}, is predictable and expressed in terms of physical quantities.

The instanton model predicts the behaviour of nonpertirbative part of gluon correlation functions in the short and intermediate region assuming that it is dominated by instanton vacuum component, while the large-scale asymptotics is dominated by the background field.

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