«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
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ASYMPTOTIC LAWS IN RELATIVISTIC NUCLEAR PHYSICS AND THEIR EXPERIMENTAL VERIFICATION

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1. INTRODUCTION

Relativistic nuclear physics was born in Dubna and Berkeley in the early 1970s. In Dubna at the Synchrophasotron, deuterons and then more heavy nuclei up to the sulfur nuclei with an energy of 4.5 GeV/nucleon were first accelerated. At Berkeley beams of different relativistic nuclei, but with less energy were also obtained. An active research of nuclear interactions in GeV nuclear beams was then started.

Fig. 1. The Synchrophasotron and the Nuclotron of the Laboratory of High Energies of the Joint Institute for Nuclear Research

At the Joint Institute for Nuclear Research a specialized superconductive accelerator of relativistic nuclei — Nuclotron, able to accelerate practically all the nuclei at an energy of 6 GeV/nucleon [1] was built (Fig. 1).

In connection with N.N.Bogoliubov's anniversary we would like to remember how he understood the main problem of relativistic nuclear physics. In his talk at a general 1985 meeting of the USSR Academy of Sciences [2] he paid attention to the fact that over the past years the ideas of the theory of color quarks had started to penetrate more deeply in nuclear physics and the major problem is to explain the nature and the basic regularities of nuclear forces

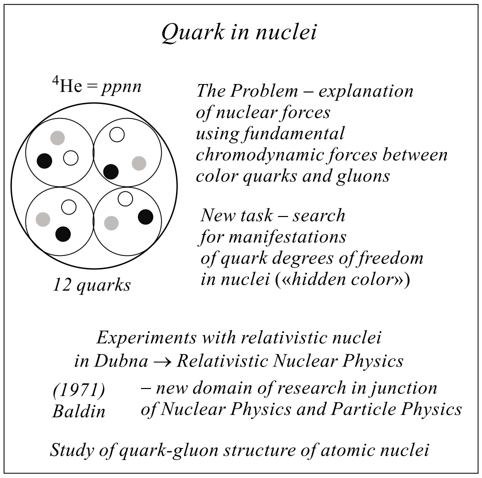

Fig. 2. A fragment from the talk of N.N.Bogoliubov [2]

proceeding from fundamental chromodynamic interactions of quarks and gluons. Figure 2 gives a fragment from Bogoliubov's talk. As is seen from this fragment, he considers that the main problem of relativistic nuclear physics is the search for manifestations of quark degrees of freedom in nuclei.

Being the Director of JINR, N.N.Bogoliubov gave constant support to the work on the Nuclotron. It is known that he was the author of the theory of superconductivity and he was interested in the quantum system with length of a quarter of kilometer. Figure 3 presents a photo on which Academicians N.N.Bogoliubov and A.M.Baldin are discussing the magnetic system of the Nuclotron.

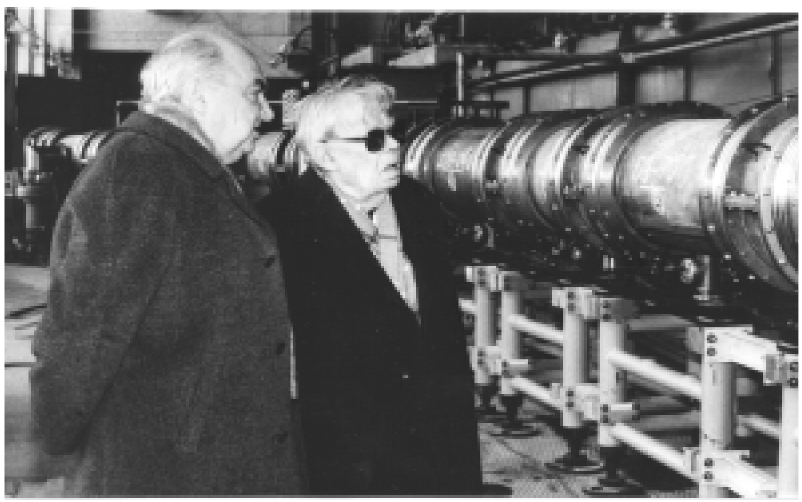

Fig. 3. Academicians N.N.Bogoliubov (right) and A.M.Baldin are discussing the magnetic system of the Nuclotron

2. CUMULATIVE EFFECT

First experiments with the deuteron beam of the Synchrophasotron carried out by the group of V.S.Stavinsky have resulted in the discovery of a nuclear cumulative effect [3] predicted earlier by A.M.Baldin [4].

The idea of this effect consists in the following: collisions of relativistic nuclei result in the production of particles in the region of energies, forbidden for one nucleon interactions. Otherwise, because the secondary particles have the momentum and energy, observed in experiment, it's necessary to suppose that several nucleons take part in the interaction, i.e., it's impossible to consider that the nucleons in the nucleus are quasi-free. Figure 4 gives a schematic view of the cumulative effect.

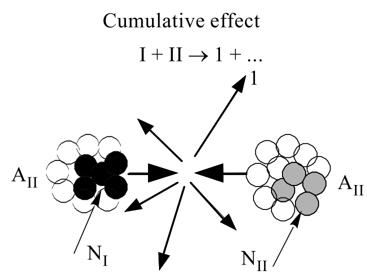

Fig. 4. A schematic view of the cumulative effect. Two nuclei with atomic numbers A_I and A_{II} interact between them and produce an inclusive particle 1. For the description of the kinematic parameters of inclusive particle 1 it is necessary to suppose that N_I nucleons from nucleus I participate with N_{II} from nucleus II

Later on the cumulative effect was investigated in detail in Dubna and in other scientific centres.

This research has resulted in the discovery of the quark-parton structure function of the nucleus analogous to the quark-parton structure function of the hadron. It was established that the experimental data for all nuclei from helium to uranium can be described by the following approximate equation for the cross sections

$$\sigma(A_I A_{II} \to h_1 + \dots) = k A_I^n A_{II}^{m(N_I)} \exp\left(-N_I/\langle N_I \rangle\right) \qquad (1)$$

at $0.5 \leq N_I \leq 3.3$ (cumulative region at $N_I > 1$),

$$\begin{array}{ll} m(N_I) = 2/3 + N_I/3 & (0.5 \leq N_I \leq 1) \\ m(N_I) \approx 1 & (N_I > 1). \end{array}$$

$N_I \approx 0.14$ characterises the sizes of a multi-quark system, from which cumulative particles are emitted. In this way, the nuclear quark-parton structure function can be taken as:

$$G(N_I) \sim \exp\left(-N_I/\langle N_I \rangle\right). \qquad (2)$$

In a more general case the cumulative effect can be realized in both nuclei A_I and A_{II} (double cumulative effect, Fig. 4), but with smaller probability.

3. DESCRIPTION OF INTERACTIONS OF RELATIVISTIC NUCLEI IN FOUR-VELOCITY SPACE

A relativistic invariant description of multiple particle processes in relative 4-velocity space was suggested by A.M.Baldin [6]. This approach turned out to be very fruitful and made it possible to obtain a number of new properties of relativistic nuclear interactions. The process of the interaction of two nuclei can be written as follows:

$$I + II \to 1 + 2 + \dots \tag{3}$$

where I and II are the interacting nuclei, and $1, 2, 3, \dots$ are the secondary particles. Following this approach relativistic invariant quantities:

$$b_{ik} = -(u_i - u_k)^2 \tag{4}$$

were introduced, where $u_i = p_i/m_i$, $u_k = p_k/m_k$ are 4-velocity particles i and k; $p_{i,k}$ and $m_{i,k}$ are their 4-momenta and masses. The distributions of the secondary particles as functions of b_{ik} have universal properties, which points to a common interaction mechanism on the quark-gluon level.

An important principle introduced in 4-velocity space by A.M.Baldin, is the correlation depletion principle (CDP) analogous to the Bogoliubov's CDP. CDP has been suggested by Bogoliubov in statistical physics as a universal property of the probability distributions for particle location in an ordinary space-time. The principle is based on an intuitive idea that the correlation between largely spaced parts of a macroscopic system practically vanishes and the distribution is factorized. From the mathematical point of view the principle means that probability distributions are desintegrated in independent factors (Fig. 5).

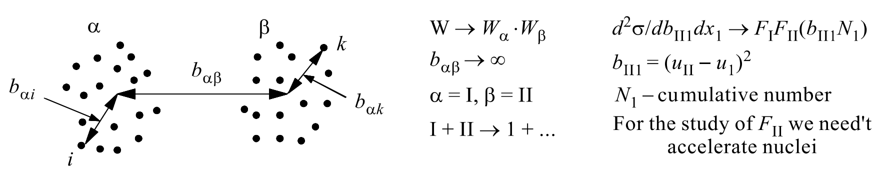

Fig. 5. Schematic view of the correlation depletion principle in four-velocity space. Correlation between largely spaced parts α and β of the particle system vanishes when the distance $b_{\alpha\beta}$ in four-velocity space between the centres of systems α and β tends to infinity. The probability distribution which characterizes the system is factorized $W \to W_\alpha \cdot W_\beta$ at $b_{\alpha\beta} \to \infty$

This principle makes it possible to study in detail cumulative processes as target nucleus fragmentation processes using intense proton beams as projectiles. In this case acceleration of relativistic nuclei is found to be unnecessary.

The second important property of relativistic nuclear interactions, which was used in four-velocities space, is the automodelity.

The introduction of a self-similarity parameter Π [7]:

$$\Pi = \min\,[\frac{1}{2}\sqrt{(u_I N_I + u_{II} N_{II})^2}] \tag{5}$$

leads to the description of the invariant production cross section for an inclusive particle in the form [8]:

$$E(d^3\sigma/d^3p) = C_1 \cdot A_I^{1/3+N_I/3} \cdot A^{1/3+N_{II}/3} \cdot \exp\,(-\Pi/C_2), \tag{6}$$

where $C_1 = 1.9 \cdot 10^4$ mb$\cdot$GeV$^{-2} \cdot$ c$^3\cdot$st^{-1}, $C_2 = 0.125 \pm 0.002$.

This equation describes a very large amount of experimental data in a wide region of change of the cross sections (by 10 orders of magnitude) and of the energy for different particles and interacting nuclei.

4. ASYMPTOTICS IN RELATIVISTIC NUCLEAR PHYSICS

Using the self-similarity parameter Π (5) an analytical expression was obtained for the inclusive invariant cross section of production of particles, nuclear fragments and antinuclei in relativistic nuclear collisions in the central rapidity region [9,10].

The quantities N_I and N_{II} become measurable if we take into account the law of conservation of four-momentum in the form

$$(N_I m_0 u_I + N_{II} m_0 u_{II} - m_1 u_1)^2 = (N_I m_0 + N_{II} m_0 + \Delta)^2, \tag{7}$$

neglecting the relative motion of the remaining not detected particles. Here m_0 is the nucleon mass, Δ is the mass of the particles providing conservation of the baryon number, strangeness and other quantum numbers. For antinuclei and K^- mesons (the case of antimatter formation) $\Delta = -m_1$. For particles produced without accompanying antiparticles (π mesons, jets and others) $\Delta = 0$.

Using condition (7) it is possible to find value (5) in the central rapidity region (here $N_I = N_{II} = N$):

$$\Pi = \frac{1}{2}\sqrt{2N^2 + 2N^2(u_I u_{II})} = \frac{N}{\sqrt{2}}\sqrt{1 + (u_I u_{II})} = N\cosh Y, \tag{8}$$

where Y is the rapidity of colliding nuclei in the c.m. system.

In the region of the rapidity of the inclusive particle $y = 0$ we have obtained

$$N = [1 + \sqrt{(\Phi_\delta/\Phi^2) + 1}] \cdot [\frac{m_T}{m_0}\cosh Y + \frac{\Delta}{m_0}] \cdot [1/(2\sinh^2 Y)], \tag{9}$$

where

$$\Phi = \frac{1}{m_0} \cdot [m_T \cosh Y + \Delta] \cdot (1/2 \sinh^2 Y),$$

$$\Phi_\delta = (\Delta^2 - m_1^2)/(4m_0^2 \sinh^2 Y), \tag{10}$$

m_1 — the inclusive particle mass, $m_T = \sqrt{m_1^2 + p_T^2}$ — the transverse mass.

Now we consider the asymptotic behaviour of the self-similarity parameter with increasing interaction energy:

$$s/(2m_I m_{II}) \approx (u_I u_{II}) = \cosh 2Y \to \infty.$$

In the collider energy region the self-similarity parameter Π assumes the finite value

$$\Pi_\infty = \frac{m_T}{2m_0}\left[1 + \sqrt{1 + (\Delta^2 - m_1^2)/m_T^2}\right]. \tag{11}$$

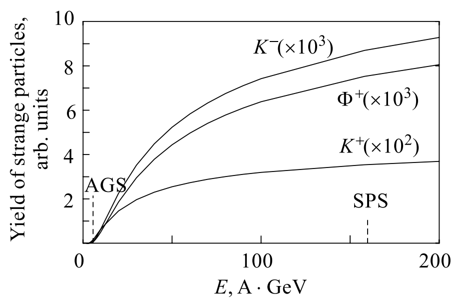

Fig. 6. Yield of strange particles in the central rapidity region (for $y = 0$) as a function of the collision energy

As is seen from Eq. (9), the effective number of the nucleons is involved in the reaction $N \to 0$ at $\cosh Y \to \infty$. In this connection, we may say with certainty that the hopes for obtaining dence and hot matter (in any case, for detecting it by fast inclusive particles) in ultrarelativistic nuclear collisions are not feasible.

The analytical representation for Π enables us to draw the following new conclusions:

1. There exists the limiting value of Π described by Eq.(11).

2. For $\Phi_\delta = 0$ the expression for Π is factorized and proportionality of it to the inclusive particle mass m_1 makes it possible to test in detail the self-similarity laws. From Eq.(9) it follows that the cross section (6) exponentially quickly decreases with increasing m_1. In particular, this implies that the probability of observing even light antinuclei and fragments in the region $y = 0$ is insignificantly small.

3. The yield of strange particles in the central rapidity region increases with increasing collision energy (Fig. 6).

4. The effective number of nucleons involved in the reaction decreases with increasing $\cosh Y$ (9).

5. A strong factorizable dependence of Π on m_T we have discovered explains the observed m_T scaling.

The results of our calculations for AGS and SPS energy are presented in the Table. Experimental results from Refs. 11–13 are also presented there.

Table

Ratios of the yields	$\bar{p}/p$	$\bar{d}/d$	K^-/K^+
Calculation (160 A · GeV)			
(the present paper) ($p_T = 0$)	0.16	0.027	0.25
NA52 (160 A · GeV)	≈ 0.1	≈ 0.01	$\approx 0{,}2$
NA44 (160 A · GeV)	≈ 0.08	-	≈ 0.4
Calculation (11 A · GeV)			
(the present paper) ($p_T = 0$)	0.00039	-	0.11
E866 (11 A · GeV)	≈ 0.0003	-	≈ 0.2

The results of our calculations are in satisfactory agreement with experiment.

Our predictions of the ratios of the production cross section for antiparticles to that for particles are presented in Fig. 7. The calculations were carried out for a fixed target and energy of incident nuclei in laboratory system.

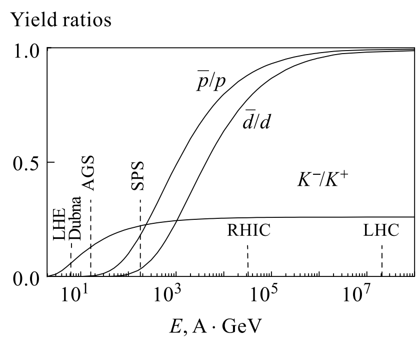

Fig. 7. Predictions of production cross section ratios for antiparticles to particles versus laboratory collision energy

5. CONCLUSIONS

For inclusive production cross sections for particles, nuclear fragments and antinuclei in relativistic nuclear collisions in the central rapidity region ($y = 0$)

- the analytical expression is obtained;
- the results of calculations are in agreement with available experimental data;
- the asymptotic behaviour as a function of increasing interaction energy is discovered;
- the predictions for RHIC and LHC energy are presented.

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