УДК 539.14

DYNAMICAL EFFECTS PRIOR TO HEAVY ION FUSION

T. I. Mikhailova [a], *I. N. Mikhailov* [a], *I. V. Molodtsova* [a], *M. Di Toro* [b]

[a] Joint Institute for Nuclear Research, Dubna

[b] Laboratorio Nazionale del Sud, INFN, Catania, Italy

Dynamical effects in the initial phase of fusion reactions are studied following the evolution of two colliding ^{100}Mo ions. The role of elastic forces associated with the Fermi-surface deformation is shown by comparing the results obtained with and without taking the memory effects into account. The Bass barrier separating fused and scattered configurations and the lower bound for the extra push energy are estimated. Examples of cases are shown in which the excitation energy and deformation dependence of the friction parameter are fictitious and simulate the effects of collective motion related with the Fermi-surface deformations.

Изучаются динамические эффекты на начальной стадии реакции слияния на примере столкновения двух ионов ^{100}Мо. Сравнением результатов, полученных с учетом эффектов памяти и без их учета, показана роль упругих сил, связанных с деформацией поверхности Ферми. Дается оценка нижней границы энергии экстрапуша и барьера Басса, разделяющего конфигурации, приводящие к слиянию и к рассеянию. Приведены примеры, в которых зависимость параметра трения от энергии возбуждения и деформации является физически не обоснованной и подменяет собой эффекты коллективного движения, вызванные деформацией поверхности Ферми.

INTRODUCTION

The mutual influence of approaching ions determines the conditions for nuclear fusion. Of course, the Coulomb force slowing down the motion and the nuclear forces of a short range, changing strongly the height and the radial dependence of the fusion potential plays the major role in this process. Among the other factors of an utmost importance there is the strength of forces leading to the collective energy dissipation: forces accompanying the currents of nuclear matter and related with the departure from equilibrium in the distribution of nucleons in the momentum space. Here we focus our attention on dynamical effects produced by such forces. Examples of phenomena which are influenced by such dynamical effects are the «extrapush» characteristics and the competition of fusion with other processes leading to the reseparation of nuclei after the collision. We proceed in a vein of nuclear fusion model presented in Refs. 1–3 where the dynamical equations were formulated treating in an explicit way the Fermi-surface deformations. This approach is close to the one applied to the fission process developed in a series of publications of which we cite Ref. 4 (see also the review paper [5] and the references therein). The authors of these two approaches claim the adequacy of their models in treating various features of fission and fusion reactions and advocate for studies of memory or retardation effects in the action of the friction forces.

There are some new elements in the approach presented here. First of all, the matter distribution is described now by two collective coordinates instead of only one as in our previous

publications, and a more realistic than before potential is used. In our new formalism the retardation effects appear due to an additional extension of collective space: they are described by the new coordinate measuring the Fermi-surface deformations [3, 6]. The extension of the collective space for this particular coordinate marks the difference between our approach and the other well-known approaches of the fusion dynamics: the currently used adiabatic approach with the inclusion of dissipation via the friction force and the diabatic dynamics model (DDM) originated by Nörenberg [7, 8].

These novelties make it possible to treat consistently within the formalism presented here the diabatic effects considered in Ref. 9 and the loss of diabatic behavior due to the dissipation described in adiabatic approaches. Using the new techniques we show that the dynamical effects play an essential role during the heavy ions collision.

1. EQUATIONS OF MOTION

The results which will be presented below concern the head-on collisions of identical (spherical) ions. The dynamics of approaching ions is described by the evolution of the «elongation parameter» $L(t)$ defined as the distance between the centres of mass of colliding ions and by the intrinsic quadrupole moment of each of them:

$$q(t) = m \int_{V_1} d\mathbf{x}\, n(\mathbf{x}) \left[2z^2 - x^2 - y^2\right].$$

In this expression V_1 is the volume of one of the ions; $\mathbf{x}$ is the coordinate in the reference frame with the origin at the centre of mass of the ion.

Equations of motion are obtained on the basis of virial theorems [10] (see also [2] and [6]). The first of equations reads:

$$\mu \frac{d^2 L}{dt^2} = -\frac{\partial U(L,q)}{\partial L}, \tag{1}$$

where $\mu = mA/4$ is the reduced mass of the system of two ions having $A_1 \equiv A/2$ particles of mass m each, and

$$U(L,q) = \frac{1}{2} \int_V \int_V d\mathbf{x} d\mathbf{x}' n(\mathbf{x}) n(\mathbf{x}') V(\mid \mathbf{x} - \mathbf{x}' \mid) \tag{2}$$

is the potential energy. Here the integration goes over the volume of both ions, $V = V_1 + V_2$.

The second equation is

$$\frac{1}{2} \frac{d^2 q}{d\, t^2} = k(q, \dot{q}) - w_{2,0}(L,q) + \pi. \tag{3}$$

Here

$$w_{2,0}(L,q) = \int_{V_1} d\mathbf{x}\, n(\mathbf{x}) \left[2z \frac{\partial}{\partial z} - x \frac{\partial}{\partial x} - y \frac{\partial}{\partial y}\right] W(\mathbf{x}), \tag{4}$$

$W(\mathbf{x}) = \int_V d\mathbf{x}'\, n(\mathbf{x}')V(|\,\mathbf{x}-\mathbf{x}'\,|)$ being the mean potential, and $k(q,\dot{q})$ is defined as

$$k(q,\dot{q}) = m\int_{V_1} d\mathbf{x}\, n(\mathbf{x})\left[2u_z^2(\mathbf{x}) - u_x^2(\mathbf{x}) - u_y^2(\mathbf{x})\right], \tag{5}$$

where $\mathbf{u}(\mathbf{x})$ represents the collective velocity of matter with respect to the moving reference frame.

We suggest that the shapes of ions before the contact may be approximated by the family of spheroids. An ellipsoidal deformation of ions may be associated with the linear transformation of the liquid elements positions [10]. Let $\tilde{\mathbf{x}} = (\tilde{x},\, \tilde{y},\, \tilde{z})$ be the coordinates of a liquid element in the spherical body of radius R_1 ($R_1 = r_0 A_1^{1/3}$) filled by the homogeneously distributed incompressible matter with the density $n_0 = (4/3\,\pi r_0^3)^{-1}$. The transformation

$$x = f(t)\tilde{x}, \qquad y = f(t)\tilde{y}, \qquad z = \frac{1}{f(t)^2}\,\tilde{z} \tag{6}$$

defines the position of the same liquid element inside the spheroidal surface with the semiaxes $a_x = a_y \equiv a_\perp = R_1\, f,\ a_z \equiv a_\parallel = R_1/f^2$. The quadrupole moment of the ellipsoid is equal to

$$q = \frac{2}{5} m R_1^2 A_1 \left(\frac{1}{f^4} - f^2\right). \tag{7}$$

The velocity field $\mathbf{u}(\mathbf{x})$ is given by the time derivative of $\mathbf{x}$. Using Eqs. (6), (7) one finds

$$u_x = -2\dot{q}\, M(q)x, \quad u_y = -2\dot{q}\, M(q)y, \quad u_z = 4\dot{q}\, M(q)z, \tag{8}$$

where the effective mass function $M(q)$ is the same as in the expression for the kinetic energy: $T(q,\dot{q}) = M(q)\dot{q}^2/2$. This function and $\kappa(q) = k(q,\dot{q})/\dot{q}^2$ can be written as follows:

$$M(q) = \frac{5}{8\, m R_1^2 A_1}\ \frac{1}{f^2(1+2/f^6)}, \qquad \kappa(q) = -\frac{5}{8\, m A_1 R_1^2}\frac{1-4/f^6}{f^2(1+2/f^6)^2}. \tag{9}$$

One finds also the following important relations:

$$\kappa(q) = -\frac{1}{4M(q)}\frac{d\, M(q)}{d\, q}, \qquad w_{2,0}(L,q) = \frac{1}{4M(q)}\frac{\partial U(L,q)}{\partial q}. \tag{10}$$

The quantity $\pi(t)$ in equation (3) is the $(\lambda,\mu) = (2,0)$-component of the pressure tensor integrated over the volume of one ion:

$$\pi = \int_{V_1} d\mathbf{x}\,(2P_z - P_x - P_y), \tag{11}$$

where $P_i = P_{i,i}$ ($i = x,\, y,\, z$) are diagonal components of the pressure tensor.

On the basis of virial theorems we consider π as a collective variable satisfying the equation of motion

$$\frac{d\,\pi}{d\,t} + C\,\dot{q} = -\frac{\pi}{\tau}, \qquad \text{where} \qquad C\,\dot{q} = \int_{V_1} d\mathbf{x}(2P_z\partial_z u_z - P_x\partial_x u_x - P_y\partial_y u_y). \tag{12}$$

Using Eq. (8) one finds:

$$C = F_{\rm fs}^{(0)} M(q)/M(0), \quad \text{where} \quad F_{\rm fs}^{(0)} = \left[v_F/(r_0 A_1^{1/3}) \right]^2 \quad \text{and} \quad M(0) = 5/(24 m R_1^2 A_1).$$

The value of mean relaxation time parameter τ is fixed using the arguments of Ref. 11 where the giant multipole resonances, depicted as small-amplitude vibrations around the ground state configuration, were studied on the basis of virial theorems, i. e., on the same basis as in our approach to nuclear reactions. In the quoted paper it was found that the widths of two first isoscalar resonances with $\lambda = 2$ and $\lambda = 4$ are correctly reproduced when

$$\tau = \left(\frac{4}{3} \right)^2 \frac{r_0 {A_1}^{1/3}}{v_F} \qquad \text{and} \qquad \hbar/\tau = 24.9\, {A_1}^{-1/3}\ \text{MeV}.$$

We shall see later on that the deformation of ions remaines very small up to the moment of collision and that their heating is quite moderate. This choice of τ and the stability of parameters describing giant resonances exclude any noticeable influence on our calculations of the possible shape and temperature dependence of the relaxation time parameter. Consequently τ is considered to be constant.

2. COLLECTIVE ENERGY

One can see that Eqs. (1) and (3) are invariant under the time inversion. The dissipative element $(-\pi/\tau)$ is present only in Eq. (12). This property of equations of motion follows from their derivation in which it is assumed that the correlations between nucleons are of a short-range type and lead to the same invariance features of the collision integral as in the macroscopic bodies.

The term $-\pi/\tau$ may be ignored when the motion is fast in the time scale of τ. In this case the motion is quasi-elastic. In the opposite case of slow motion the term $\dot{\pi}$ may be ignored in Eq. (12). Then, the shape of ions changes with time as if they were «plastic». This double-faced nature of the considered system puts it in the class of «elastoplastic systems» introduced in the nuclear theory by W. Nörenberg [7]. It is convenient to study the differences between the elasoplastic and plastic dynamics giving the system a canonical form of classical mechanics.

2.1. Elastoplastic Rayleigh–Lagrange Dynamics. The procedure elaborated in Ref. 3 (see also Ref. 6), involving Eqs. (1), (3) and (12) multiplied by $\dot{L}$, $M(q)\dot{q}$ and π, respectively, and the kinematic relations (10), yields:

$$\frac{dE_c}{dt} \equiv \frac{d}{dt} \left\{ \frac{\mu}{2} \dot{L}^2 + M(q)\, \dot{q}^2 + U(L,q) + \frac{2M(0)}{F_{\rm fs}^{(0)}}\, \pi^2 \right\} = -2\mathcal{R}. \tag{13}$$

In Eq. (13) the quantity E_c is the collective energy. It contains the kinetic energy of translational motion of ions $\mu \dot{L}^2/2$, the sum of kinetic energies originated by the collective flow in two ions $M(q)\, \dot{q}^2$ and the potential energy $U(L,q)$. The term $2M(0)\, \pi^2/F_{\rm fs}^{(0)}$ may be associated with the kinetic energy of motion along some generalized coordinate Z, in respect to which π plays a role of generalized velocity: $\pi = \dot{Z}$. In the right-hand side of Eq. (13)

there appears the Rayleigh dissipation function $\mathcal{R} = (M(0)/\tau\, F_{\rm fs}^{(0)})\dot{Z}^2$. Collective energy remains constant when the dissipative terms are switched off, i. e., in the limit when $\tau \to \infty$. It satisfies the standard relation of classical mechanics [12] ($E_c = \sum_i \dot{Q}_i \partial\mathcal{L}_c/\partial\dot{Q}_i - \mathcal{L}_c$) with the Lagrangian function

$$\mathcal{L}_c = \frac{\mu}{2}\dot{L}^2 + M(q)\,\dot{q}^2 - U(L,q) + \frac{2M(0)}{F_{\rm fs}^{(0)}}\,\dot{Z}^2 + \mathcal{F}(q)\dot{Z}. \tag{14}$$

In the former equation the function $\mathcal{F}(q)$ is arbitrary. Choosing it as $\mathcal{F}(q) = 4\int dqM(q)$ one makes the Lagrange–Rayleigh equations of classical mechanics

$$d(\partial\mathcal{L}_c/\partial\dot{Q}_i)/dt - \partial\mathcal{L}_c/\partial Q_i = -\partial\mathcal{R}/\partial\dot{Q}_i$$

equivalent to the former equations of motion.

The Rayleigh dissipation function determines the work produced by the friction on the heat bath interacting with the system. The notion of a heat bath, understood as the multitude of intrinsic degrees of freedom interfering with the collective motion, seems to be well justified when the changes of collective energy are balanced by the statistical excitation (heating) of ions. From Eq. (13) it follows that $E_{\rm stat}(t) = E_c(t=-\infty) - E_c(t)$ is a monotonously rising function of time, as it should be in this case.

2.2. «Viscous» Limit. The contribution of π to the elastic forces is insignificant when the motion is slow in the time scale of the mean relaxation time parameter τ. In this case the time derivative of π in Eq. (12) may be neglected. Then the quantity π becomes a function of the state determined by L and q coordinates (and by $\dot{L}$ and $\dot{q}$) and the reaction dynamics is described by Eqs. (1), (3), where $\pi = -\tau\,(v_F/r_0A^{1/3})^2(M(q)/M(0))\dot{q}$. Such a dynamics, which we call «viscous», corresponds to the Lagrangian (14) from which the two last terms are removed. In this approximation the collective energy is given by an expression in the curly brackets in Eq. (13) without the last term, and the Rayleigh function is $\mathcal{R}_{\rm visc} = M(q)\beta\,\dot{q}^2$ with the friction coefficient

$$\beta_{\rm visc} = -\frac{\pi}{\dot{q}} = \tau\left(\frac{v_F}{r_0A^{1/3}}\right)^2\frac{M(q)}{M(0)}. \tag{15}$$

Description of fusion and fission reactions using the friction forces is typical of theoretical studies. However, both theoretical estimations of the friction parameter and the experimental data concerning it contain extremely large uncertainties [5]. Having this in mind, we include in the discussion of the deformation and «heating»of ions given in the following section the exposition of differences between the viscous and the elastoplastic scenarios.

3. DEFORMATION AND HEATING DURING THE APPROACH

Physical picture of processes, that take place just before a unification of heavy ions, was obtained analyzing within the model the head-on collision of two ^{100}Mo ions. The Coulomb potential ($U_{\rm Coul}(L,q)$) and the generalized surface potential of Refs. 13, 14 in the form of Yukawa-plus-exponential interaction ($U_{\rm fold}(L,q)$) are used to calculate the surface of potential energy $U(L,q) = U_{\rm Coul}(L,q) + U_{\rm fold}(L,q)$ and of its partial derivatives. A homogeneous

charge distribution inside the sharp surface is assumed, and the technique of Ref. 13 for conversion of volume integrals into surface integrals is used. Partial derivatives of potential energy are calculated using the same technique plus the Ostrogradsky–Gauss theorem. Applied to axially symmetrical figures this procedure transforms the sixfold double volume integrals into threefold integrals. In the following illustrations instead of the quadrupole moment q we use the parameter $\alpha = 1 - f \sim (5/4)(q/mR_1^2A_1)$. The value of L is given in the units of $2R_1$.

When the distance between the surfaces of two ions is large in comparison with the range of nucleon–nucleon interaction, the potential energy $U(L, \alpha)$ has a valley with a bottom in the region of oblate deformations ($\partial_q U = 0$ when $\alpha < 0$). The valley disappears at $L/2\,R_1 = 1.2$ giving way to a surface sloping towards the prolate shapes. For small values of $\mid \alpha \mid$ the potential energy ridge ($L_r(\alpha)$), where $\partial_L U = 0$, is situated at $L > 2R_1$. At $\alpha = 0$ $L_r(0)/2\,R_1 = 1.07$ and $U(L_r, 0) = 199.16$ MeV.

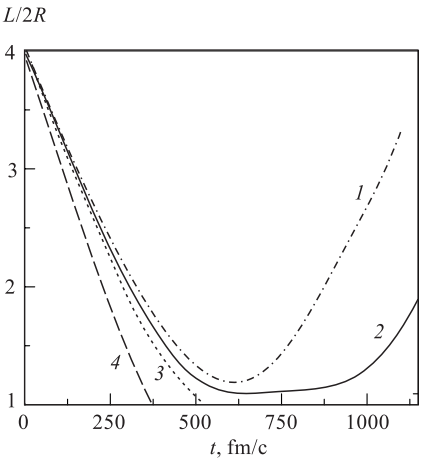

Fig. 1. Trajectories of the system ^{100}Mo+^{100}Mo in L-t plane for different values of incident energy: *1* — $E_{\rm in}$ = 3.8 MeV/nucl.; *2* — 4 MeV/nucl.; *3* — 4.4 MeV/nucl.; *4* — 6 MeV/nucl.

In Fig. 1 different types of $L(t)$ trajectories are presented. One can see that the nuclear interaction becomes important only when the incident energy ($E_{\rm in}$) is close to some «barrier» energy. For $E_{\rm in}$ = 3.8 MeV/nucl. (curve *1*) the time dependence of L is very close to that found in the elastic Coulomb scattering of nuclei. At slightly greater energies as given, e. g., by the curve *2* for $E_{\rm in} = 4$ MeV/nucl., the nuclear interaction influences the evolution of the system, which keeps, however, the character typical of scattering: colliding ions stop before coming to a contact (no fusion) and L increases after reaching a minimum. At still greater $E_{\rm in}$ (curves *3* and *4*, for $E_{\rm in} = 4.4$ and 6 MeV/nucl. respectively) the ions come into a contact and the formation of fused system becomes possible. Our calculations stop at the moment when the contact is established. This figure gives an impression of time-scales involved in the approach phase of the fusion reaction: typically, the ions pass less than 10^{-21} s within the range of nuclear interaction before hitting each other or being scattered.

Next figures allow one to see the memory effects in collisions: such effects are taken into account in the elastoplastic model, but are ignored in the «viscous » approximation. Trajectories drawn in full lines correspond to the elastoplastic dynamics; broken lines give the results of calculations in the viscous approximation.

Our parametrization of the relaxation time parameter τ makes $\beta_{\rm visc}$ close to the friction parameter $\beta_{\rm wall}$ corresponding to the «wall» formula ($\beta_{\rm wall} = 1/\tau_{\rm wall}$, $\tau_{\rm wall} = (3/4)\,v_F/R_1$) [15]. The relation between the values of friction parameters corresponding to the viscous approximation and to the wall formula is found comparing the rates of collective energy decrease as given in the two approaches:

$$\dot{E}_{\rm visc} = -2\beta_{\rm visc}\,M(q)\,\dot{q}^2, \qquad \dot{E}_{\rm wall} = -(3\rho\,v_F/4)\int_S ds\,u_n^2(s).$$

When the deformation of ions is small, one has

$$\frac{\beta_{\rm visc}}{\beta_{\rm wall}} \equiv \frac{\dot{E}_{\rm visc}}{\dot{E}_{\rm wall}} = (4/3)^3.$$

The «wall and window» friction[1] describes well the fast transformation of collective energy of colliding nuclei into the energy of statistical excitation. Used with the factor k in the interval $4 \leq k \leq 12$, it was successfully applied to the description of neutron multiplicity in «fission-like» heavy ions collisions [16]. On the other hand, our estimations in Refs. 2, 3 of the rate of heating during the fusion, made using the same value of τ as in this paper, are also in agreement with the experimental findings.

In Fig. 2 the trajectories of colliding ions are shown in the L-α plot for two different values of incident energy $E_{\rm in}$. Curves *1* and *2* correspond to $E_{\rm in}$ equal to 4 and 4.4 MeV per nucleon, respectively; the arrows beside the curves indicate the evolution of the shape in the time. Calculations show that the trajectories do not follow the valley in the potential energy landscape and that the nuclear shape remains very close to spherical: $\mid \alpha \mid$ never exceeds 0.04 in all considered cases. The small spheroidal deformation is however important, because it engenders the collective energy dissipation. The difference between the elastoplastic and viscous dynamics is evident signifying, in particular, the difference in estimations of the fast varying with the time component of the electromagnetic field accompanying the collision.

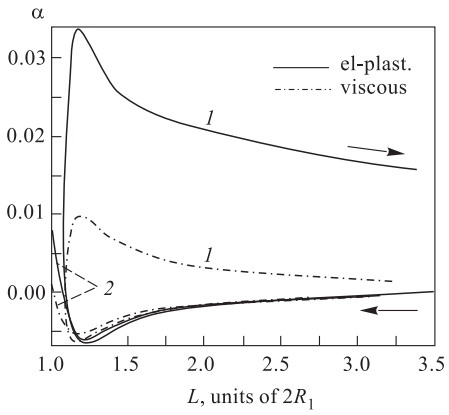

Fig. 2. Trajectories of the two-molibdenum system in α-L plane. Here and in the next figure the solid lines are obtained taking into account memory effects in friction forces, the broken lines correspond to the viscous approximation. Curves *1* and *2* correspond, respectively, to $E_{\rm in}$ = 4.0 and 4.4 MeV/nucl. Arrows beside the curves indicate the time evolution

Figure 3 shows the statistical excitation energy accumulated in the system as a function of incident energy. The results given for energies smaller than 4 MeV per nucleon correspond to cases when no contact between the nuclei is established. The statistical energy shown for these trajectories is calculated integrating Eq. (13) in the whole time interval of the scattering. It gives a rather schematized representation of what happens during the collision: in this case the mutual excitation of ions is given by the multiple Coulomb excitation of collective states. This must not lead us too far from the reality though: one may easily verify that the response of the elastoplastic system (parametrized as in our model) to the $l = 2$ wave of the time dependent electric field reproduces the general properties in the response of nuclei.

Calculations made for greater energies correspond to events at which the contact is established. Here the data are given for the statistical energy accumulated up to the moment of contact. The lowest energy at which the contact is established is equal to $E_{\rm in}$ = 4.013 MeV/nucl.

[1]Note, that during the approach there is no «window» between ions.

According to calculations, $E_{\rm stat}$ at this point is equal to 7 MeV. The broken line is slightly shifted to greater energies in respect to the solid line; the maximal value of statistical excitation energy accumulated before the contact is however not much different for two lines. Important differences between the two types of calculations are seen at energies greater than critical for the contact: retardation effects reduce the dissipation of collective energy as compared with the frictional mechanism of viscous approximation.

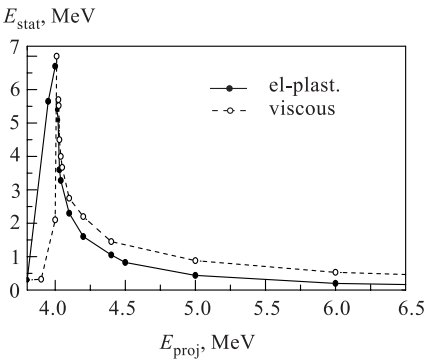

Fig. 3. Statistical excitation energy as a function of $E_{\rm in}$. Results given for energies not sufficient for the contact ($E_{\rm in} \leq$ 4.013 MeV/nucl.) show $E_{\rm stat}$ after the recoil of nuclei. Results for larger energies show $E_{\rm stat}$ accumulated before the contact

The fast rise of the dissipated collective energy with $E_{\rm in}$ at the energies lower than needed for establishing a contact does not call for a prolonged discussion: the greater the energy is in this domain, the closer is the approach of ions and the stronger is their mutual interaction. The transformation of the collective energy into the «heat» takes place almost entirely at the moment of the closest approach at which the rate of change in the configuration of the system is slow. The quality of the viscous approximation is expected to be the best in such conditions explaining the similarity of results obtained for the maximal dissipated energy in two types of calculations.

The minimal energy at which the fusion is possible corresponds to the trajectory separating events leading to the scattering of nuclei from those describing the formation of fused systems. In the considered case of two ^{100}Mo ions it is somewhat larger than the minimal energy at which the ions come into a contact. Hence, the minimal incident energy at which the contact is established gives the lower bound for the extra push energy. This lower bound is found to be close to 7.5 MeV. It is a little more than a half of the value found in Ref. 17 for the quantity $E_{\rm mean} - E_B = 12$ MeV, where $E_{\rm mean}$ is the energy at which the fusion probability is equal to 1/2. Our previous publication [2] shows that the elastoplastic nature of nuclei is responsible for a contribution to the extra push coming from the evolution after the moment when the contact between the nuclei is established. So, the calculations presented here are in a qualitative agreement with the earlier found results.

To explain the decrease of the dissipated energy with increasing $E_{\rm in}$ at the energies greater than 4 MeV/nucl. we ignore for a moment the differences between the results obtained within the elastoplastic and viscous dynamics. We suppose also that the forces $\mathcal{F}_L(L,q) = -\partial_L U(L,q)$, $\mathcal{F}_q(L,q) = -\partial U_q(L,q)$ do not depend in any essential way on q: $\mathcal{F}_{L,q}(L,q)/m = F_{L,q}(L)$. This approximation is reasonable when the deformations are small, and the energy is large. Then we find that $q(t)$ dependence, as determined in the simplest way by the viscous approximation, follows the equation $\ddot{q} + F(L(t)) + 2\beta\dot{q} = 0$, where $L(t)$ is the solution for the equation containing $\ddot{L}$. This equation is easily integrated giving

$$E_{\rm stat}(t_0) = \beta \int_{-\infty}^{t_0} dt \left\{ \int_{-\infty}^{t} dt' \exp\left[-2\beta(t-t')\right] F(L(t')) \right\}^2 .$$

When β is very large, and the exponential factor dominates in the integral taken in curly brackets, one has

$$E_{\rm stat}(L) \sim \frac{1}{4\beta}\left\{\int_{\infty}^{L}\frac{dL'}{[dL'/dt]}F(L')\right\}^2 . \qquad (16)$$

The time derivative dL'/dt increases with the incident energy making the integral to decrease. So, the ions colliding with the energies high above the barrier do not experience any noticeable loss of energy in contrast with the collisions at the energies close to the barrier. Neglecting this circumstance one may introduce some errors estimating the height of the potential barrier. This may affect the experimental estimations of the extra push energy and must be taken into account in the assessment of cold-fusion cross sections.

The minimal incident energy (E_i^0) at which the contact between ions is established and the energy of statistical excitation corresponding to collisions at this energy ($E_{\rm stat}^0$) determine an «effective» potential energy ridge (E_b) met by the system during its evolution ($E_b = E_{\rm in}^0/2 - E_{\rm stat}^0$). This potential energy ridge is found to be equal to 193.16 MeV which is in a remarkable agreement with the estimations of the Bass barrier for ^{100}Mo+^{100}Mo fusion given in Ref. 17 ($E_b = 194$ MeV).

We turn now to the excitation energy dependence of the friction parameter which is found to be a serious factor determining the fusion dynamics [18]. We want to show that the retardation effects in the considered model with the energy-independent relaxation time parameter produce phenomena explained elsewhere as coming from the energy dependence of the friction. To this end we consider the differences in the calculations of $E_{\rm stat}$ corresponding to the elastoplastic and viscous dynamics.

Let us suppose that the statistical energy estimations made within the elastoplastic model are exact, and let us try to reproduce the same estimations within the viscous approximation using a «renormalized», excitation energy dependent friction parameter $\beta_{\rm visc}(E_{\rm in})$. The calculation presented in Fig. 3 shows that the difference in the two estimations of $E_{\rm stat}(E_{\rm in})$ is small at $E_{\rm in} = 4$ MeV/nucl. and increases up to a factor of 3 when $E_{\rm in} = 7$ MeV/nucl. From Eq. (16) it follows that $E_{\rm stat}(E_{\rm in})$, as found in the viscous approximation, is roughly speaking in an inverse proportion to the friction parameter β. Then, to reconcile the viscous and elastoplastic pictures one must increase gradually $\beta_{\rm visc}(E_{\rm in})$ with increasing $E_{\rm in}$ by the same factor of 3 in this energy interval to compensate for this difference.

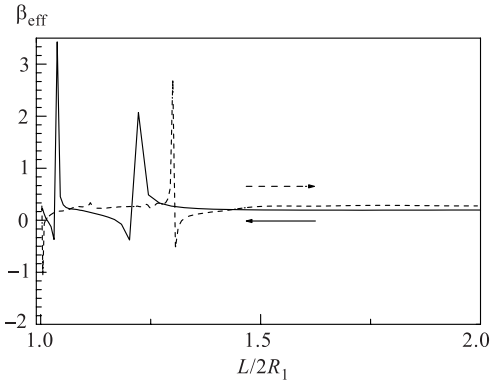

Fig. 4. Simulation of retardation effects by an «effective shape dependent friction parameter» $\beta_{\rm eff}(L) = -\pi/\dot{q}$ at $E_{\rm in} = 4$ MeV/nucl. The arrows indicate the time evolution. $\beta_{\rm eff}(L)$ is given in the units of $\beta_{\rm wall}$

For any arbitrarily chosen «trajectory» the «reconciliation» of elasoplastic and viscous dynamics can be done playing with the shape dependence of the friction parameter. Consider, e. g., the «L-dependent effective friction

parameter» $\beta_{\text{eff}}(L) = -\pi(L)/(dq/dt)_L$, where $\pi(L(t))$, $(dq/dt)_{L(t)}$, and $L(t)$ are parametrically defined by their dependence on the time following from the elastoplastic model. As is argued before (see Eq. (15)), this quantity represents the action of the friction force in the adiabatic regime.

Such an effective friction parameter for the trajectory corresponding to the scattering of two ^{100}Mo nuclei at 4 MeV per nucleon incident energy, is shown in Fig. 4. One sees that $\beta_{\text{eff}}(L)$ experiences variations of about 2 orders of magnitude showing that the dynamics at the moment of collision is not at all adiabatic. At some values of L it reaches much larger values than predicted by the wall formula and becomes negative at some other values of L passing through zero. It is different for the incoming and outgoing parts of the trajectory, reflecting the findings of Wilczinski [16]. The «strange» behavior of $\beta_{\text{eff}}(L)$ shown in this figure could be made in parallel with the very strange shape dependence of the friction parameter suggested by Gontchar et al. [19, 20].

4. CONCLUDING REMARKS

The discussion of dynamical phenomena encountered in nuclear reactions is limited in this paper to an approach phase in the head-on collisions of identical heavy ions. These limitations although narrowing the application field of the theory accentuate strongly the dynamical effects.

One may wonder why to limit oneself to the study of the approach phase of nuclear collisions? The answer is as follows: the dynamical effects which we are looking for, are the strongest at the moments of rapid and strong changes in the collective flow. The hitting of one heavy nucleus by another is one of such moments, and we want to learn first what happens at it.

These are the dynamical effects in the collective energy dissipation which interest us here mostly. Contrary to numerous other studies of such dynamical effects, the ones presented in this paper are done within the model which formally belongs to the «transport theory» approaches of nuclear dynamics. This is due to an extension of the collective space made including in it an «unusual» collective coordinate: a variable (Z) having as a conjugated momentum the quadrupole moment in the distribution of nucleons over momenta ($\pi(t)$). The latter describes the «Fermi-surface deformation». Our formulation of the theory has numerous advantages if compared with the others. We point out only one of them: we have no ambiguity in the estimation of the statistical excitation energy.

Comparing the calculations done considering π as a kinematically independent variable («elastoplastic regime») with calculations made in an approximation in which π is considered as a function of the state as determined by the distribution of the matter and of the currents in the coordinate space («viscous regime») we have found the following:

Only for slow collective motion, such as it is at the energies close to minimal needed for establishing a contact between the colliding nuclei, the viscous approximation is good. In this case one arrives at the description of fusion in terms of the usual friction force whose strength is comparable with the wall-formula estimation.

At larger energies «viscous» approximation becomes deficient. In some specific situations the dynamical effects may be treated introducing the excitation energy or/and shape dependence of the friction force. In the examples given here the «state» dependence of the friction

force is nothing but the simulation of more involved as is currently accepted collective dynamics. In the language of current theoretical approaches the effects discussed in the paper are called as the «retardation effects» in the action of the friction force. These examples show clearly that the name of a retarded friction is in a way misleading, standing behind a phenomenon combining the effects of elastic and dissipative forces.

The parametrization of our model is rather natural and economic in the number of parameters (the description of dissipation involves only one parameter treated as a constant: the mean relaxation time). Still without any fitting procedure we reproduce well the «Bass» barrier in $^{100}\mathrm{Mo}+^{100}\mathrm{Mo}$ collisions. The estimated lower bound of the extra push energy is equal to about a half of its experimental value. Having in mind the results of the previous studies in which an important contribution to the extra push is found from the «rebound» of unified ions leading to their reseparation, one may conclude that the model works not too badly to describe the beginning of fusion.

The basic condition for it is to reproduce the effects associated with the strong reduction of the friction force advocated by the studies of fusion–quasi-fission competition, neutron multiplicity, dispersion in the distribution of various parameters and so on. For example, the wall and window estimate for the friction force leads to a gross overestimation of the quasi-fission probability — to the drawback which is cured in [19] by introducing a «critical for fusion» configuration at which the friction must be strongly diminished. Microscopic calculations of the friction parameter involved in fission process also give much smaller values for friction than the wall formula [21, 22]. Our experience suggests that the elastoplastic model has good chances to explain such a reduction. It is confirmed by the study of competition between the fusion and quasi-fission [3] within the simplified version of elastoplastic model considered here.

It seems that much of the uncertainty in the understanding of nuclear dissipation lies in the fact that the friction, usually discussed in the literature, is a badly defined physical quantity. The calculations presented before hint strongly that these uncertainties may be eliminated or at least largely reduced by including into the collective space the states of multipole giant resonances. Physically it signifies the admition in the theory of the retardation effects in friction produced by the coupling between the geometrical and Fermi surfaces. Our paper shows that this can be done keeping intact the general strucure and methods of the currently used approaches to the nuclear reactions.

Acknowledgments. The report is based on the material obtained in the scientific collaboration of JINR with France and Italy. A part of the work was done in CSNSM-IN2P3 (Orsay, France) and in INFN-LNS (Catania, Italy). The scientific staff and the administration of these Laboratories are greatly acknowledged also.

REFERENCES

1. *Mikhailov I. N. et al.* // Nucl. Phys. A. 1994. V. 574. P. 593.

2. *Mikhailov I. N. et al.* // Nucl. Phys. A. 1996. V. 604. P. 358.

3. *Mikhailova T. I. et al.* // Nucl. Phys. A. 1998. V. 641. P. 64–94.

4. *Ayik S. et al.* // Zeit für Phys. A. 1990. V. 337. P. 413.

5. *Abe Y. et al.* // Phys. Rep. 1996. V. 257, No. 2, 3.
6. *Mikhailov I. N.* // Part. and Nucl. 2000. V. 31. P. 838.
7. *Nörenberg W.* // Phys. Lett. B. 1981. V. 104. P. 107.
8. *Rozmej P., Nörenberg W.* // Phys. Lett. B. 1986. V. 177. P. 278.
9. *Berdichevski D. et al.* // Nucl. Phys. A. 1989. V. 499. P. 609.
10. *Chandrasekhar S.* Ellipsoidal Figures of Equilibrium. Dover; N.Y., 1987.
11. *Balbutsev E. B., Mikhailov I. N.* Collective Nuclear Dynamics / Ed. by R. V. Jolos. L., 1990. P. 3.
12. *Goldstein H.* Classical Mechanics. Series of Physics. 2nd edition. Addison-Wesley Publ. Comp., 1959.
13. *Davies K. T. R., Nix J. R.* // Phys. Rev. C. 1976. V. 14. P. 1977.
14. *Krappe H. J., Nix J. R., Sierk A. J.* // Phys. Rev. C. 1979. V. 20. P. 992.
15. *Blocki J.* // Ann. of Phys. 1978. V. 113. P. 330.
16. *Wilczinski J., Sivek-Wilczinska K., Wilschut H. W.* // Phys. Rev. C. 1996. V. 54. P. 325.
17. *Quint A. B. et al.* // Zeit für Phys. A. 1993. V. 346. P. 119.
18. *Kolomietz V. M., Plujko V. A., Shlomo S.* // Phys. Rev. C. 1995. V. 52. P. 2480.
19. *Fröbrich P., Gontchar I. I.* // Phys. Rep. V. 292, No. 3, 4. P. 131.
20. *Gontchar I. I., Litnevski L. A.* // Zeit. für Phys. A. 1997. V. 359. P. 149.
21. *Hofmann H., Ivanyuk F. A., Yamaji Sh.* // Nucl. Phys. A. 1996. V. 598. P. 187.
22. *Ivanyuk F. A.* // Phys. Rev. C. 1997. V. 55. P. 1730.

Received on October 31, 2001.