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STRENGTHENING AND DAMPING OF SYNCHROTRON OSCILLATIONS

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Resonance strengthening and damping of synchrotron oscillations of collider bunch halo particles was studied by simulation. It was shown that the strengthening of particle synchrotron oscillations can be high efficient with using a resonance pulse sequence. The resonance damping of particle synchrotron oscillations is only possible when the inverse population of the accelerated bunch halo is realized.

Resonance method of synchrotron oscillation strengthening can be used for the extraction of beam halo particles with a bent crystal to improve the background conditions for colliding beam experiments and to fulfill simultaneously some fixed target experiments.

Резонансное усиление и демпфирование синхротронных колебаний частиц гало пучка коллайдера исследовано моделированием. Показано, что может быть достигнута высокая эффективность усиления колебаний с использованием резонансной импульсной последовательности. Демпфирование колебаний возможно только в случае инверсной заселенности гало ускоренного сгустка.

Резонансный метод усиления синхротронных колебаний может быть использован для обеспечения вывода частиц гало пучка коллайдера с помощью изогнутого кристалла с целью улучшения фоновых условий коллайдерных экспериментов и проведения одновременно экспериментов на фиксированной мишени.

INTRODUCTION

Particles accelerated by radio frequency (RF) fields form bunches and circulate in the accelerator performing longitudinal oscillations about the bunch center. The bunch center positions correspond to synchronous particles with the energy E_s the revolution period of which T_0 is strictly adjusted with the RF field period, $T_0 = hT_{\mathrm{rf}}$, where h is an integer called harmonic number. The synchronous particles always pass through the accelerating gap having the same synchronous phase φ_s with respect to the RF voltage. The longitudinal oscillations of bunch particles occur simultaneously with the oscillations of particle energy and phase relative to their synchronous values.

The method of the strengthening of synchrotron oscillations of beam halo particles to extract them from the collider by a bent crystal for fixed target experiments has been suggested in [1]. For this purpose, the crystal is located at the azimuth with a high dispersion where the energy deviation of particles from the synchronous energy can be translated into some transverse displacement. The growth of synchrotron oscillation amplitudes of particles is achieved with using longitudinal voltage pulses in resonance with the oscillations. The accelerated or decelerated pulses can be realized with using a longitudinal kicker. The pulse position into the bunch halo region is fixed due to synchronization with a certain phase of the RF accelerating voltage.

This method for the particle ejection from the beam halo without perturbing the bunch core can be used with success to bring the particles onto a bent crystal for extraction of them from the existing and now constructed colliders. With using the method, one can improve the background conditions for the colliding-beam experiments [2] and receive a possibility of performing simultaneously the fixed target experiments.

The method of the resonance strengthening of synchrotron oscillations is investigated here in detail. The comparison of the resonance pulse action with a periodic pulse action at synchrotron oscillation frequencies is given. The possibility for the resonance damping of synchrotron oscillations of bunch halo particles is considered.

1. RESONANCE PULSE SEQUENCE

The time variation of the phase and energy deviations $\delta\varphi$, δE from the synchronous values φ_s, E_s is described by the system of equations (here for ultra-relativistic case)

$$\frac{d\delta}{dt} = \frac{1}{T_0}\frac{eV_0}{E_s}(\sin\varphi - \sin\varphi_s), \tag{1}$$

$$\frac{d}{dt}(\delta\varphi) = h\omega_0\alpha\delta, \tag{2}$$

where $\delta = \delta E/E_s$, V_0 is the amplitude of the accelerating voltage, $V = V_0 \sin\varphi$, ω_0 is the revolution frequency, α is the momentum compaction factor. The equations have the Hamilton form. For the storage rings or colliders without accounting necessity to compensate particle energy losses, that is, when $\varphi_s = \pi$, the Hamiltonian can be presented in the form [3]

$$H = k\delta^2 + \cos\varphi, \quad k = \pi h\alpha\frac{E_s}{eV_0}. \tag{3}$$

The particle synchrotron oscillations occur in the effective potential $U(\varphi) = \cos\varphi$, therefore they are strongly nonlinear. The oscillation period T_s grows with the oscillation amplitude. So, the longitudinal kicks for the resonance strengthening or damping of particle synchrotron oscillations have to follow with increasing or decreasing time intervals, that is, the resonance pulse sequence is aperiodic.

Let us determine the resonance pulse sequence. The calculations will be performed for the Tevatron Collider parameters (see the table). Here L_0 is the accelerator circumference, $l_w = L_0/2h$ is the bucket half-width, $\delta_h = (2eV_0/\pi\alpha h E_s)^{1/2}$ is the bucket half-height.

The finite-difference equations were used to calculate particle trajectories in the φ-δ phase space:

$$\delta_{n+1} = \delta_n + \frac{eV_0}{E_s}\sin\varphi_n, \tag{4}$$

$$\varphi_{n+1} = \varphi_n + 2\pi h\alpha\delta_{n+1}. \tag{5}$$

The kicker action onto a particle changes the particle energy by the value eV_p, where V_p is the pulse voltage. This action occurs when the particle is at a distance l_p from the bunch center (at the points *1* or *2* at the phase trajectory of the particle, see Fig. 1,*a*). The pulse localization relative to the bunch center is realized by means of the pulse tie with the phase φ_p of the accelerating voltage, which is removed at $\psi_p = \pi l_p / l_w$ from the synchronous phase.

Table. **Some relevant accelerator parameters**

Parameters	Value	Parameters	Value
$E_{\rm S}$, GeV	1000	h	1113
V_0, MV	1.1	$T_{\rm rf}$, ns	18.83
L_0, m	6283.1923	l_w, m	2.823
α	$2.90332 \cdot 10^{-3}$	δ_h	$0.4655 \cdot 10^{-3}$

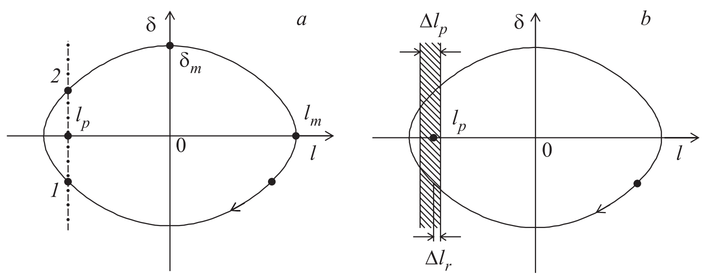

Fig. 1. Schematic representation of a particle trajectory in the l-δ phase space. *a*) The details for the resonance sequence calculation. Here l_p is the distance from the bunch center where a particle undergoes the strengthening or damping action. *1* and *2* are the corresponding points of the pulse actions at the phase trajectory. l_m and δ_m are the amplitudes of the displacement and relative energy deviation of a particle. *b*) The details of the pulse sequence actions on a halo particle. Here Δl_p is a pulse length, the pulse region is dashed. Δl_r is one turn displacement of a particle, $\Delta l_p > \Delta l_r$. The arrow shows the direction of a particle motion

As a result of the pulse action the particle jumps onto an orbit with a bigger or smaller amplitude. It depends on the sign of both the pulse and the energy deviation of the particle. Then the phase trajectory of the particle in this perturbed state is calculated up to the next pulse action at the phase φ_p and so on. Thus, the whole process of the resonance strengthening or damping is followed. The turn number $N_p(k)$ and the energy deviation amplitude of the particle are registered at every pulse action on the particle, where k is the action number. The sequence of the pulses which appear at the turn numbers $N_p(k)$ and at the phase φ_p of the RF voltage is *the resonance pulse sequence*.

The oscillation strengthening can be done by using the accelerating pulse, $V_p > 0$, when a particle energy deviation $\delta > 0$ (at the point *2* of the phase trajectory, Fig. 1,*a*), or by the decelerating pulse when $\delta < 0$ (at the point *1*). Both cases can be also used together but we will not consider such pulse sequences here. There are the same possibilities for the oscillation damping, but the signs of V_p and δ have to be opposite.

The increase of the particle oscillation amplitude due to the pulse action depends on the action phase φ_p. Therefore, the resonance pulse sequences are different for the different φ_p. The growth or decrease of the oscillation amplitude are most fast when $\varphi_p = \pi$. The action asymmetry for $\varphi_p \neq \pi$ leads to a significant difference in the use of accelerating and decelerating pulses. We will consider here, as in [1], the pulse actions, which only affect the bunch halo.

Figure 2,*a* shows the growth of the oscillation amplitude δ_m of a halo particle driven by a resonance strengthening sequence. The start amplitude of the sequence $\delta_{ms} = 0.35 \cdot 10^{-3}$, the action phase φ_p corresponds to the distance from the bunch center $l_p = -1.5$ m. Two cases with the pulse voltage $V_p = 0.5$ and 1 MV were considered. These values are close to the accelerating voltage. This was done to reduce the CPU time because the strong pulses give a faster increase of the oscillation amplitude. The considered pulse displacement l_p from the bunch center is bigger than one half of a maximum one l_w. This is a periphery of the real particle distribution in the Tevatron bunch [4]. Figure 2,*b* shows the decrease of the oscillation amplitude of a particle driven by a resonance damping sequence, the start amplitude of the sequence $\delta_{ms} = 0.46 \cdot 10^{-3}$.

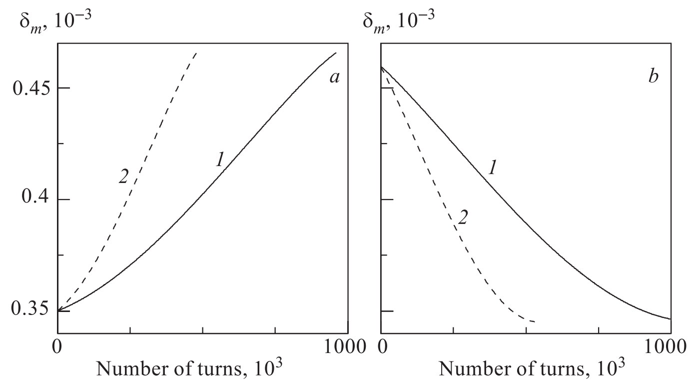

Fig. 2. The oscillation amplitude δ_m as a function of turn number for a halo particle driven by a single resonance sequence. *a*) For the strengthening sequence, $\delta_{mo} = 0.35 \cdot 10^{-3}$. *b*) For the damping sequence, $\delta_{mo} = 0.46 \cdot 10^{-3}$. The curves *1* and *2* are for the pulse voltage $V_p = 0.5$ MV and 1 MV correspondingly

The pulse duration did not figure as a parameter at the calculation of the resonance sequence. The only parameter was a pulse voltage. However, for a selective action onto the bunch halo the pulse duration has to be considerably shorter than the period of the accelerating voltage, $\Delta t_p \ll T_{\rm rf}$. Besides, a high precision of the pulse synchronization with the accelerating voltage is necessary.

The pulse actions of the calculated sequence are synchronous with longitudinal oscillations of a very small fraction of particles which were near the start point (φ_p, δ_p) of the phase space at the initial moment. The particles will be considered as captured by the sequence if their synchrotron oscillations are increased or decreased under the sequence action. The number of captured particles can be increased a little if the pulses are lengthened.

To capture considerably more of the halo particles the sequence has to be repeated for a number of subsequent turns N_{on} [1]. The repetition number $N_{on} < N_s = T_s(\delta_{mo})/T_o$, but it can be bigger than $N_s/2$. This allows achieving a high efficiency of the oscillation strengthening and damping. The pulse sequence received by the repetition of the single sequence will be called *a multiple resonance sequence*. The repetition number N_{on} determines the number of turns during which the pulses occur for one period of synchrotron oscillations. Therefore, N_{on} will be also called *a sequence action interval*.

Let us mark in as N_{12} the number of turns, which a particle makes moving along the phase trajectory from the point *1* to the point *2*, and as N_{21} the additional number of turns, $N_{21} = N_s - N_{12}$ (Fig. 1,*a*). Figure 1,*b* shows *the pulse region* (dashed) at the $l-\delta$ phase space where a particle can get the increase of energy from the kicker if the particle is there at the turns determined by the sequence action interval. The region width Δl_p equals the pulse length that is for the relativistic case considered $\Delta l_p = c\Delta t_p$. The pulse boundaries are symmetric relative to the action phase in the case of short pulses. For one turn a particle shifts at the distance $\Delta l_r = \alpha L_0 \delta$ moving clockwise along the phase trajectory. The step size Δl_r is different for the halo particles crossing the pulse region with different amplitudes, $\Delta l_r = (1-5.5)$ mm. Therefore, it is impossible to have the pulse duration at which a particle undergoes the pulse action and besides only one at every synchrotron oscillation. Such situation should give a resonance growth of the amplitude.

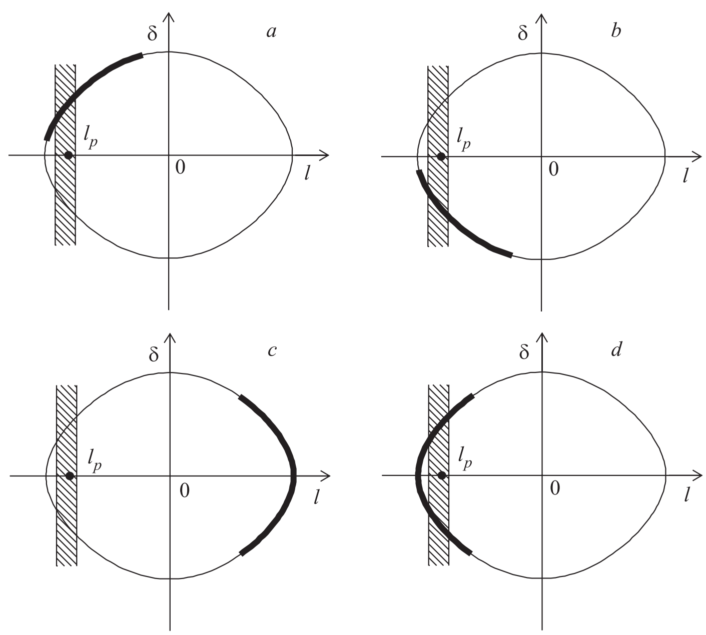

Fig. 3. Schematic representation of possible positions of a sequence action interval (thick lines) at the phase trajectory of a halo particle, the pulse region is dashed. A particle undergoes the pulse actions near the point *2* (*a*) or near the point *1* (*b*). It does not undergo any pulse action (*c*) or undergoes both the actions (*d*)

The particle changes the amplitude and the step size at every turn in the pulse region during the action interval. The number of actions experienced by a particle for one passage through the pulse region can achieve a value $\Delta l_p/\langle \Delta l_r \rangle$, where $\langle \Delta l_r \rangle$ is the average step size.

The possible positions of the sequence action interval at the phase trajectory of a particle are shown in Fig. 3. These positions correspond to the situations when a particle undergoes only the strengthening pulse actions (*a*) or only the damping actions (*b*) during one synchrotron oscillation. Besides, a particle can undergo no pulse action (*c*) or undergo both actions (*d*).

2. RESONANCE STRENGTHENING OF SYNCHROTRON OSCILLATIONS

To study the strengthening and damping of synchrotron oscillations of bunch halo particles under the action of the multiple resonance sequence the phase trajectories of particles are again calculated using the equations (4), (5). The pulses of the sequence would act on a particle when its phase is in the pulse region near φ_p and when the turn number is in the sequence action interval between $N_p(k)$ and $N_p(k) + N_{on}$ (the sequence action interval covers the pulse region).

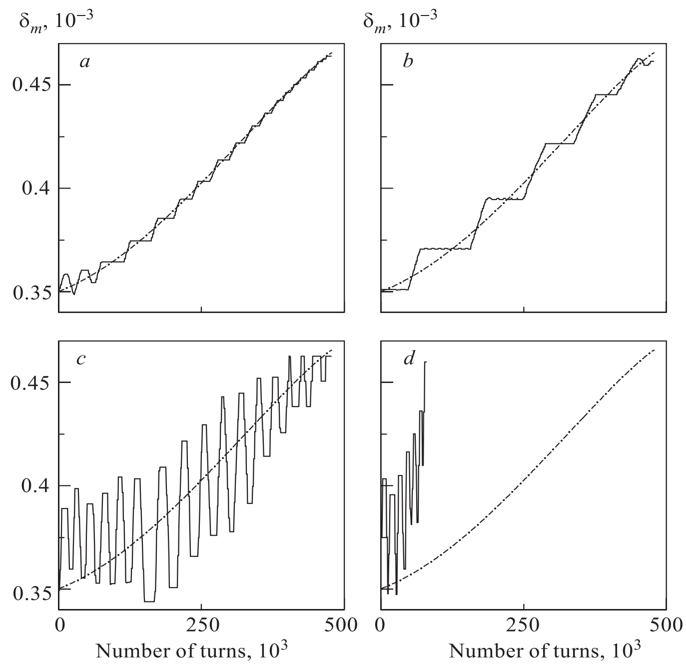

Fig. 4. The oscillation amplitude δ_m as a function of turn number for a halo particle driven by a resonance strengthening sequence with different pulse length Δl_p and repetition number N_{on}. With short pulses $\Delta l_p = 1$ cm, $N_{on} = 100$ (*a*), 1500 (*b*). With long pulses $\Delta l_p = 10$ cm (*c*) and 20 cm (*d*), $N_{on} = 640$. The pulse voltage $V_p = 1$ MV. The dash-dotted line is for a single resonance sequence

Figure 4 shows the growth of the energy deviation amplitude of a particle under the action of the resonance strengthening sequence with different pulse duration and repetition number. The amplitude growth is not monotonous but its average rate is close to the resonance one. There are flat and lowering intervals, which form steps and oscillations of the amplitude. Here it is of importance the ratio between the repetition number N_{on} and the number of turns N_{12} or N_{21} completed by a particle with a given amplitude moving along the phase trajectory.

The flat parts of the curves correspond to the situation when a particle does not undergo any pulse actions (Fig. 4, *a*) or undergoes both the strengthening and damping actions during one synchrotron oscillation, which compensate each other (Fig. 4, *b*). The intervals of amplitude growth or decrease correspond to the situations when a particle undergoes successive strengthening or damping pulse actions.

The intervals of amplitude growth and constancy decrease from the beginning to the end of the sequence action. This occurs because the sequence action interval faster moves along the phase trajectory of a particle when its oscillation amplitude is high. The lengths of the intervals of amplitude growth and constancy increase with increasing the repetition number N_{on} of the resonance pulse sequence (compare (*a*) and (*b*) in Fig. 4). The amplitude oscillations increase with increasing the pulse duration at the same N_{on}. Therefore, a particle can be ejected from the bucket before the completion of the sequence action (Fig. 4, *c*).

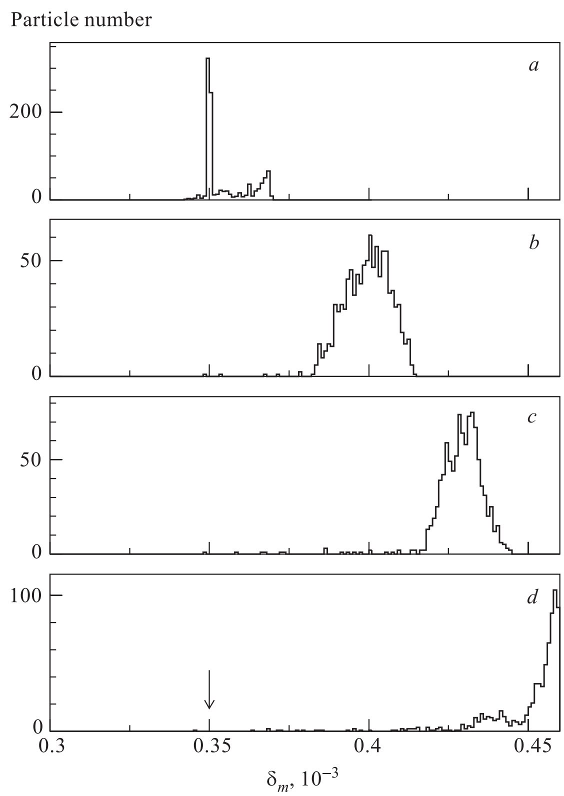

Fig. 5. The distributions of halo particles in synchrotron oscillation amplitudes δ_m under the action of the resonance strengthening sequence at different turn numbers: a — $25 \cdot 10^3$; b — $250 \cdot 10^3$; c — $350 \cdot 10^3$; d — $450 \cdot 10^3$. The pulse duration $\Delta l_p = 2$ cm, $N_{on} = 640$. The arrow shows the initial amplitude of particles

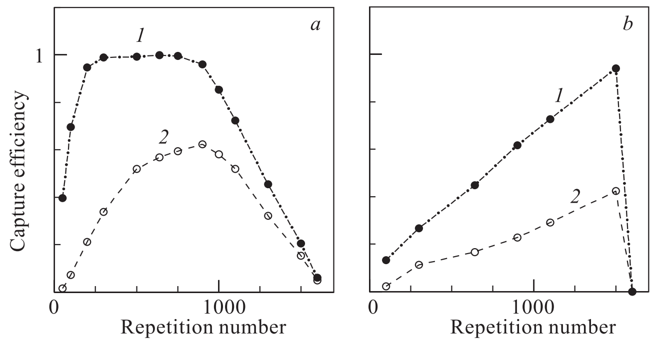

Fig. 6. The dependence of the capture efficiency (*1*) of halo particles by the resonance strengthening sequence and their ejection efficiency from the bucket (*2*) on the repetition number with using accelerating (*a*) or decelerating (*b*) pulses. The initial amplitude $\delta_{mo} = 0.35 \cdot 10^{-3}$, the pulse duration $\Delta l_p = 2$ cm

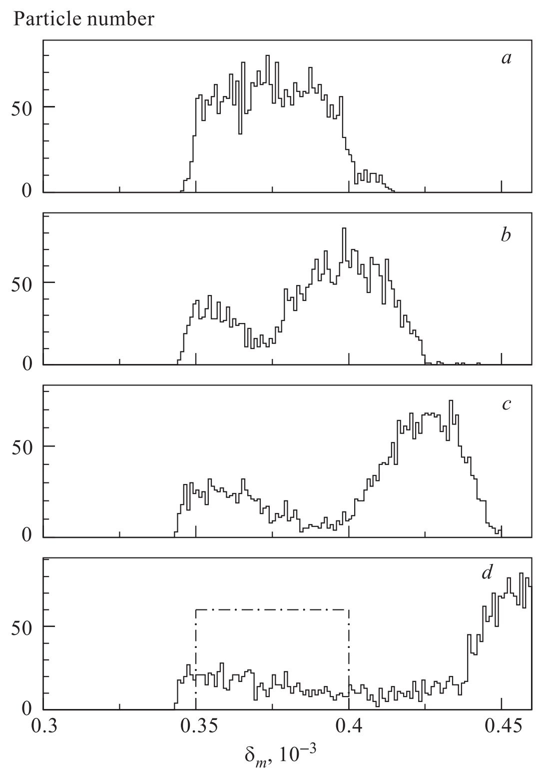

Fig. 7. The distributions of halo particles in amplitudes δ_m under the action of the resonance strengthening sequence at different turn numbers: $25 \cdot 10^3$ (*a*), $225 \cdot 10^3$ (*b*), $325 \cdot 10^3$ (*c*), $425 \cdot 10^3$ (*d*). The initial distribution with $\delta_{mo} = (0.35{-}0.40) \cdot 10^{-3}$ is uniform (dashed line). The pulse length $\Delta l_p = 5$ cm, $N_{on} = 1500$

Figure 5 shows the evolution of the particle distributions in synchrotron oscillation amplitudes under the action of the multiple resonance sequence with short pulses $\Delta l_p = 2$ cm, $N_{on} = 640$. The initial amplitude was $\delta_{mo} = 0.35 \cdot 10^{-3}$ (it is shown by the arrow in Fig. 5, *d*). The distribution shifts as a whole to a maximum value δ_h acquiring some width. The distribution width is caused by the nonmonotonous change of the oscillation amplitude of particles about the resonance growth curve. The case presented in the figure corresponds to the optimum conditions.

Figure 6 shows the dependences of the capture efficiency of particles by the resonance sequence (*1*) and their ejection efficiency from the bucket (*2*) on the repetition number when accelerating (*a*) or decelerating (*b*) pulses were used. The dependences were calculated with using short pulses $\Delta l_p = 2$ cm for particles with the initial amplitude $\delta_{mo} = 0.35 \cdot 10^{-3}$ which is equal to the start amplitude of the sequence δ_{ms}. The capture event was fixed when a particle amplitude became bigger than $\delta_m = 0.4 \cdot 10^{-3}$. The difference of the dependence (*a*) and (*b*) is caused by the different number of turns to appear the pulses, which compensate the oscillation strengthening. The compensating pulses appear through N_{12} turns ($N_{12} = 100{-}1000$) for (*a*) and through N_{21} turns ($N_{21} = 1500{-}1800$) for (*b*). The decelerating short pulses prove to be some more efficient for the oscillation strengthening than the accelerating ones.

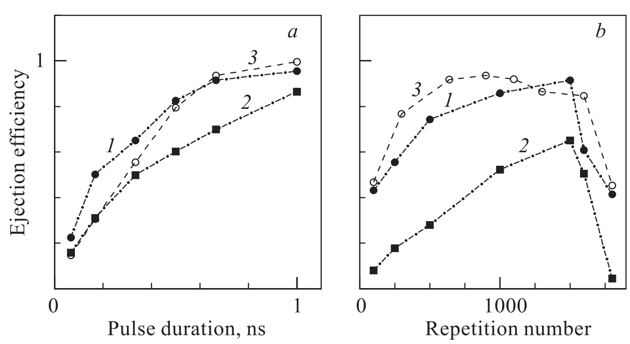

Fig. 8. The dependence of the ejection efficiency of halo particles from the bucket on the pulse duration (*a*) and the sequence repetition number (*b*). In (*a*) with accelerating pulses $V_p = 1$ MV (*1*) and 0.5 MV (*2*), $N_{on} = 1500$, and with decelerating pulses $V_p = -1$ MV, $N_{on} = 900$ (*3*). In (*b*) with accelerating pulses $V_p = 1$ MV, $\Delta l_p = 20$ cm (*1*), 10 cm (*2*), and with decelerating pulses $V_p = -1$ MV, $\Delta l_p = 20$ cm (*3*)

When the initial amplitude of particles is bigger than the start amplitude of the sequence the particles find themselves under the damping action of pulses before the sequence come to the amplitude δ_{mo}. Their amplitudes fast reduce. This is an obstacle to capture these particles by the sequence. However, it is possible to increase the capture efficiency for particles with $\delta_{mo} > \delta_{ms}$ by means of increasing the pulse duration.

Figure 7 shows the evolution of the halo particle distributions in synchrotron oscillation amplitudes under the action of the resonance strengthening sequence with the pulse duration $\delta_p = 5$ cm. The initial distribution of halo particles with $\delta_{mo} = (0.35{-}0.40) \cdot 10^{-3}$ is uniform in the amplitudes and along the phase trajectories. The used pulse duration is already sufficient to shift successively a bigger fraction of halo particles towards the separatrix. The efficiency of particle ejection from the bucket is 50 %.

The dependence of the ejection efficiency of particles from the bucket on the pulse duration (*a*) and the repetition number of the sequence (*b*) are shown in Fig. 8. The efficiency approaches to 100 % with long pulses when the particle ejection often occurs before the completion of the sequence action due to big oscillations of particle amplitudes about the resonance growth curve (see Fig. 4, *d*).

3. OSCILLATION STRENGTHENING WITH PERIODIC PULSE SEQUENCE

Let us compare the resonance sequence action onto the bunch halo with the action of the periodic sequence of pulses at synchrotron oscillation frequencies. At first, let us note the peculiarity of selective actions of short pulses on the bunch.

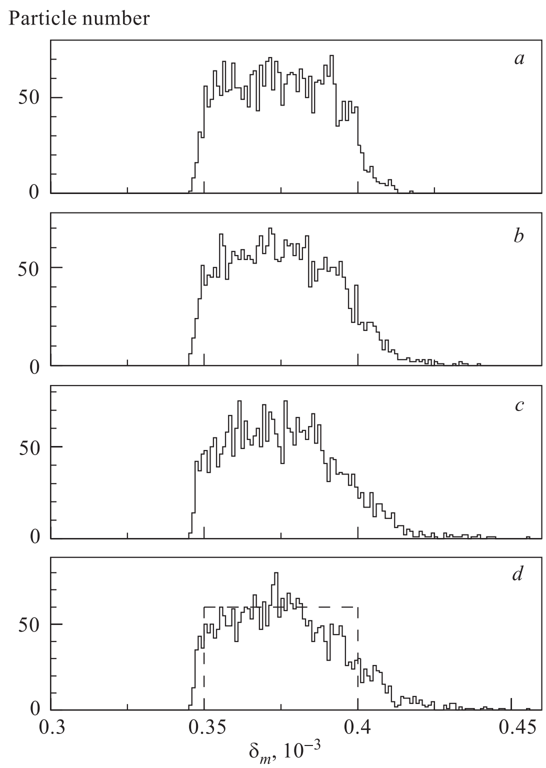

Fig. 9. The distributions of halo particles in oscillation amplitudes δ_m under the action of the periodic sequence at different turn numbers: $25 \cdot 10^3$ (*a*), $225 \cdot 10^3$ (*b*), $325 \cdot 10^3$ (*c*), $425 \cdot 10^3$ (*d*). The pulse length $\Delta l_p = 5$ cm, the period $T_p/T_0 = 1620$, $N_{on} = 1500$. The initial distribution is shown by dashed line

When the pulse length is much smaller than the bunch length, not every pulse of the sequence acts on a bunch particle. The selective action also occurs because the pulses act at a certain phase of the accelerating voltage. Only a small particle fraction with synchrotron oscillation frequencies being close to the pulse frequency perceives the pulses as a periodic sequence. However, this continues only a short time until the frequencies are close.

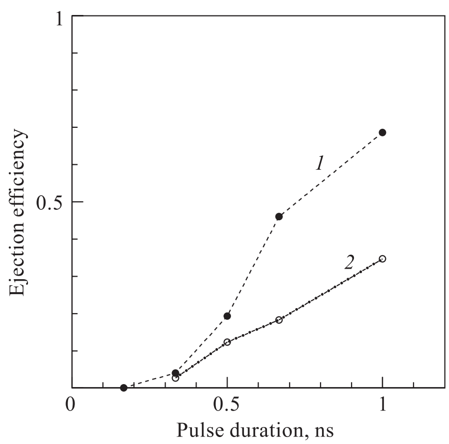

Fig. 10. The dependence of the ejection efficiency of halo particles from the bucket on the pulse duration of the periodic sequence. The sequence period $T_p/T_0 = 3000$ (*1*), 1500 (*2*), $N_{on} = 200$

The action of the sequence of pulses following with a period $T_p \sim T_s$ on the bunch halo particles is weak. Let us consider repeating a periodic pulse sequence for a number N_{on} of subsequent turns as it was done for the resonance sequence. Figure 9 shows the halo particle distributions in synchrotron oscillation amplitudes at different turns under the action of the periodic sequence with the same pulse duration as for the case in Fig. 7. The pulse period $T_p/T_0 = 1620$ (this is a period for particles with the amplitude $\delta_m = 0.35 \cdot 10^{-3}$) and the repetition number $N_{on} = 1500$. Unlike the action of the resonance sequence (see Fig. 7), a weak diffusion of halo particles towards the separatrix is only observed.

Figure 10 shows the dependence of the ejection efficiency of particles from the bucket on the pulse duration of the periodic sequence. The sequence period $T_p/T_0 = 3000$ (*1*), 1500 (*2*) (these are the synchrotron oscillation periods for particles with $\delta_m \approx \delta_h$ and $\delta_m \approx 0.29 \cdot 10^{-3}$) and the repetition number $N_{on} = 200$. The ejection efficiency becomes high for the pulse duration about 1 ns, but it is still smaller than with using the resonance sequence.

4. RESONANCE DAMPING OF SYNCHROTRON OSCILLATIONS

Figure 11 shows the decrease of the oscillation amplitude of a bunch halo particle under the action of the resonance damping sequence with different pulse duration and the sequence repetition number. The amplitude decrease is not monotonous. However, it follows the resonance-damping curve (Fig. 2, *b*) in average though there are considerable amplitude oscillations with the intervals of the amplitude growth. The amplitude oscillations become bigger with increasing the pulse duration. This leads to the ejection of particles from the bucket at the beginning of the sequence action. In the case of long pulses, ejecting jumps of the amplitudes occur for most of the halo top particles because their initial amplitudes are close to the maximum value. So, the oscillation damping becomes impossible with long pulses.

Figure 12 shows the evolution of the particle distribution in oscillation amplitudes under the action of the resonance damping sequence with short pulses $\Delta l_p = 2$ cm, $N_{on} = 640$. The initial amplitude $\delta_{mo} = 0.46 \cdot 10^{-3}$ (the position is shown by arrow in Fig. 12, *d*). Most of the particles are captured by the sequence. They successively shift deep into the bunch halo.

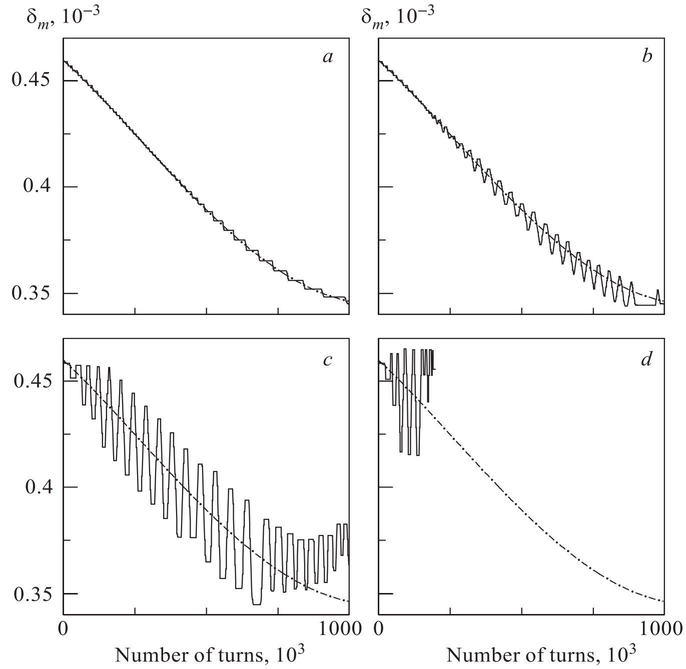

Fig. 11. The oscillation amplitude δ_m as a function of turn number for a halo particle driven by a resonance damping sequence with different pulse length Δl_p and repetition number N_{on}. With short pulses $\Delta l_p = 2$ cm, $N_{on} = 100$ (*a*), 1500 (*b*). With long pulses $\Delta l_p = 10$ cm (*c*) and 20 cm (*d*), $N_{on} = 640$. The pulse voltage $V_p = 0.5$ MV

The dependence of the capture efficiency by the damping sequence on the repetition number for the particles with $\delta_{mo} = \delta_{ms}$ is shown in Fig. 13. The dependence is different with using decelerating (*1*) or accelerating (*2*) pulses. In the case of using decelerating pulses (applied at point *2*, see Fig. 1,*a*), the dependence maximum is at the repetition number $N_{on} = 640$. This number is equal to the turn number N_{12} completed by a particle with the amplitude δ_{mo} moving between the action phases along the phase trajectory.

In the other case of using accelerating pulses (applied at point *1*, see Fig. 1,*a*) the capture grows with the repetition number up to $N_{on} = 1500$, which approximately equals the number of turns N_{21} for particles with the considered amplitude. In both cases a further increase of the repetition number leads to the compensation of the damping pulse actions because particles begin also undergo the strengthening actions at the same oscillation period. The maximum capture is about 75 % while it was about 100 % for the strengthening sequence.

However, in the case of short pulses the maximum capture is not bigger than 15 % for particles the initial amplitudes of which are smaller than the start amplitude of the sequence, $\delta_{mo} < \delta_{ms}$. These particles undergo the actions of the strengthening pulses before the sequence achieves their initial amplitude δ_{mo} therefore they have a small probability for the capture. Contrary to the situation with the oscillation strengthening, the increase of the pulse duration for the damping sequence does not lead to a considerable growth of

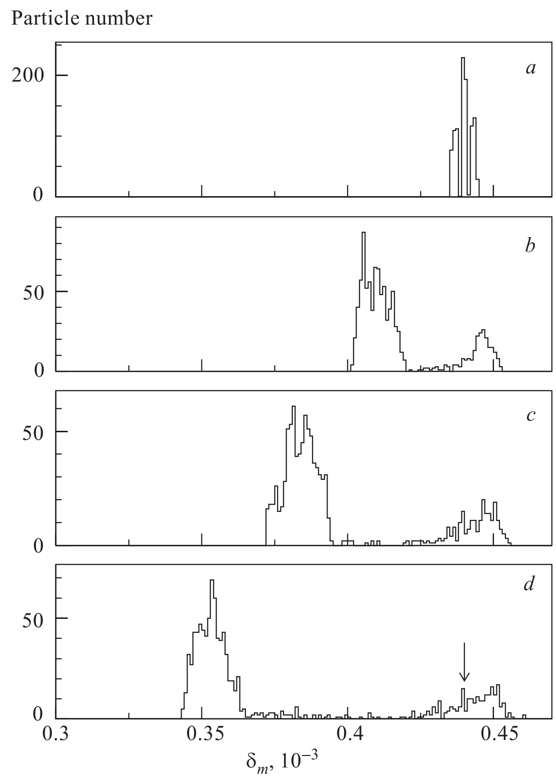

Fig. 12. The distributions of halo particles in amplitudes δ_m under the action of the resonance damping sequence at different turn numbers: $50 \cdot 10^3$ (*a*), $250 \cdot 10^3$ (*b*), $450 \cdot 10^3$ (*c*), $750 \cdot 10^3$ (*d*). The pulse duration $\Delta l_p = 2$ cm, $V_p = 0.5$ MV, $N_{on} = 640$. The arrow shows the initial amplitude of particles

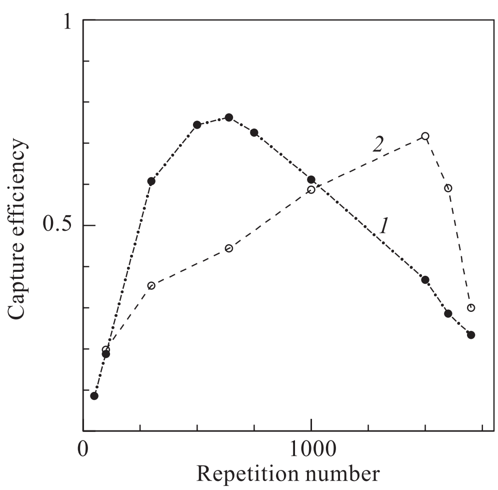

Fig. 13. The dependence of the capture efficiency of halo particles by the damping sequence on the repetition number. The initial amplitude $\delta_{mo} = \delta_{ms}$, where $\delta_{ms} = 0.44 \cdot 10^{-3}$ is the start amplitude of the sequence. The pulse length $\Delta l_p = 2$ cm, $V_p = 0.5$ MV. The curve *1* and *2* are for decelerating and accelerating pulses correspondingly

the capture for particles with $\delta_{mo} < \delta_{ms}$. This occurs because the amplitude oscillations with using long pulses eject particles from the bucket with a high probability (Fig. 11, *d*). Besides, the action of the damping sequence leads to a considerable increase of the oscillation amplitudes of particles, which are near the halo bottom.

Figure 14 shows the halo particle distributions in oscillation amplitudes as a result of the action of the damping sequence with $\Delta l_p = 2$ cm, $N_{on} = 200$. The initial distributions of the halo particles with $\delta_{mo} = (0.34{-}0.44) \cdot 10^{-3}$ in the oscillation amplitudes and along the phase trajectories were uniform. The distributions for the lower and upper parts of the considered bunch halo are shown separately. Dashed lines show the corresponding initial distributions. About 15 % of particles from the upper part of the halo which are close to the start amplitude of the sequence (here $\delta_{mo} = 0.44 \cdot 10^{-3}$) cool down to the halo bottom (Fig. 14, *b*). However, the particles of the halo bottom increase their oscillation amplitudes (Fig. 14, *a*). Therefore, at the consideration of the whole halo the oscillation damping is not observed under the action of the damping sequence (Fig. 14, *c*).

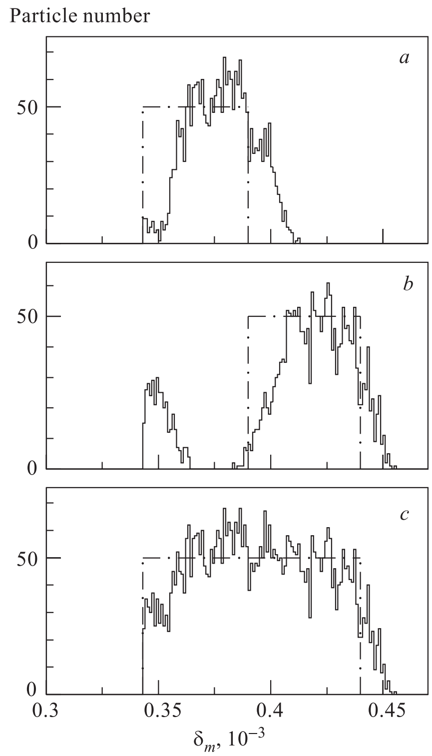

Fig. 14. The distributions of halo particles in oscillation amplitudes δ_m as a result of the action of the damping sequence with $\Delta l_p = 2$ cm, $N_{on} = 200$. *a*) For the lower part of the halo with $\delta_{mo} = (0.34{-}0.39) \cdot 10^{-3}$; *b*) for the upper part with $\delta_{mo} = (0.39{-}0.44) \cdot 10^{-3}$; *c*) for the whole halo with $\delta_{mo} = (0.34{-}0.44) \cdot 10^{-3}$. Dashed lines show the initial distributions

CONCLUSION

The simulation results presented show that the damping of synchrotron oscillations of all bunch halo particles, which undergo the action of the resonance pulse sequence, is not realized. Cooling one part of the halo, we are heating the other. This is a particular illustration of the realization of the Liouville's theorem.

The other possibilities for construction of the resonance damping sequence of pulses to reduce the perturbation of a lower halo fraction were also considered, including the fast sequence when the damping pulses do not fix to the same phase of the accelerating voltage but move along the bunch from turn to turn following to one of the halo particles along its phase trajectory. The sequence like this can be effective to mix (exchange) the neighbouring regions of the bunch halo.

The cooling of the bunch halo with using the resonance pulse sequence is nevertheless possible when the inverse population of the halo is realized. The cooling can be high efficient in the ideal case when there are only particles with a certain oscillation amplitude (or with a narrow amplitude distribution) and the other phase space of the halo is empty.

On the other hand, the strengthening of synchrotron oscillations of bunch halo particles with using the resonance pulse sequence is highly efficient. This method without perturbing the bunch core can be successfully applied to deliver halo particles to a bent crystal for their extraction from the existed and constructed colliders. So, this is additional possibility to control beam losses which can give improving the background conditions for colliding beam experiments and a possibility to perform simultaneously some fixed target experiments.

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