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TOPOLOGICAL AND NONTOPOLOGICAL SOLUTIONS FOR THE CHIRAL BAG MODEL WITH CONSTITUENT QUARKS

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The three-phase version of the hybrid chiral bag model, containing the phase of asymptotic freedom, the hadronization phase as well as the intermediate phase of constituent quarks, is proposed. For this model the self-consistent solutions of different topology are found in $(1+1)$D with due regard for fermion vacuum polarization effects. The renormalized total energy of the bag is studied as a function of its geometry and topological charge. It is shown that in the case of nonzero topological charge there exists a set of configurations being the local minima of the total energy of the bag and containing all the three phases, while in the nontopological case the minimum of the total energy of the bag corresponds to vanishing size of the phase of asymptotic freedom.

В данной работе предложен трехфазовый вариант гибридной киральной модели мешка, содержащий наряду с фазами асимптотической свободы и адронизации промежуточную фазу конституентных кварков. Для этой модели в $(1+1)$-мерном случае с учетом эффектов поляризации фермионного вакуума найдены самосогласованные решения с различной топологией. Изучена зависимость перенормированной полной энергии мешка от параметров, характеризующих его геометрию, и от его топологического заряда. Показано, что в случае ненулевого топологического заряда существует множество конфигураций, являющихся локальными минимумами полной энергии мешка и содержащих все три фазы, в то время как в нетопологическом случае минимум полной энергии мешка соответствует нулевому размеру фазы асимптотической свободы.

INTRODUCTION

At present the models of quark bags [1–4] turn out to be one of the most perspective approaches to the study of the low-energy structure of baryons. The most promising results have been obtained within so-called hybrid chiral bag models (HCBM) [5–7], where asymptotically free massless quarks and gluons are confined in a chirally invariant way in a spatial volume, surrounded by the colorless purely mesonic phase, described by some nonlinear theory like the Skyrme model [8]. However, in such two-phase HCBM there is no place for massive constituent quarks, whose concept is one of the cornerstones in the hadronic spectroscopy [9]. From the last point of view the most attractive situation should be that in which first the initially free, almost massless current quarks transmute into «dressed», via interaction, massive constituent quarks carrying the same quantum numbers of color, flavor

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and spin, and only afterwards there emerges the purely mesonic colorless phase. The first step towards such a version of the bag is made in the three-phase chiral model, wherein the additional intermediate phase of interacting quarks and mesons with nonzero radial size is introduced [10, 11]. This model allows one to take self-consistently into account: i) the phase of asymptotic freedom with free massless quarks; ii) the phase of constituent quarks, which acquire an effective mass due to the chirally invariant interaction with the meson fields in the intermediate domain of finite size; iii) the hadronization phase, where the quark degrees of freedom are completely suppressed, while the nonlinear dynamics of meson fields leads to the appearance of the c-number boson condensate in the form of a classical soliton solution, which keeps up the topological nature of the model as well as the relevant quantum numbers.

It should be mentioned that the direct quark-meson interaction is also considered in a number of other approaches to the description of low-energy hadron structure, in particular, in the cloudy bag models [12–14], as well as in various versions of the chiral quark-soliton models [15–19]. However, the role of this interaction in each of these approaches is substantionally different. In the cloudy bag models such $\pi\bar{q}q$ coupling is treated only perturbatively, while in quark-soliton models it is considered as the main source for nonlinear dynamical generation of the quark bag structure in the whole space. In the case under consideration an intermediate variant takes place, where the contribution of the direct chiral quark-meson coupling to the properties of the system is nonperturbative, while the confinement of quarks is ensured by appropriate boundary conditions. Such an approach allows one to realize the nonlinear mechanism of dynamical mass generation in the intermediate domain, but, unlike the quark-soliton models, preserves the total confinement.

In the present paper a toy $(1+1)$D model of such kind is considered, in which in the intermediate domain the one-flavor fermion field is coupled in a chirally invariant way to the real scalar field, which possesses a nonlinear soliton solution in the exterior region. For this model the self-consistent solutions with different values of topological charge, namely 1, 2, and 0, are found with due regard for the fermion vacuum polarization effects. For these solutions the renormalized total energy of the bag is studied as a function of its geometry and topological charge. It is shown that for nonzero topological charge there exists a set of configurations being the local minima of the total energy of the bag and containing all the three phases, while in the nontopological case the minimum of the bag's energy corresponds to vanishing size of the phase of asymptotic freedom.

1. LAGRANGIAN AND EQUATIONS OF MOTION

The division of space into separate bag's phases is performed by means of a set of subsidiary fields $\theta(x)$, whose self-interaction is supposed to be strong enough to neglect the influence of the matter fields ϕ on the dynamics of θ to the leading order, and thereafter to use θ as background fields for the dynamics of ϕ's [10, 20]. One can obviously introduce as many fields $\theta(x)$ as needed with the appropriate self-interaction, which will determine an (almost) rectangular division of space into domains, corresponding to different phases. Note that in this approach the Lorentz covariance will be broken only spontaneously, on the level of solutions of equations of motion, so, in order to restore the broken Lorentz symmetry, one can freely use the framework of covariant group variables [21].

Within this framework, the model we consider is described by the following Lagrangian [10]:

$$\mathcal{L} = \bar{\psi} i\hat{\partial}\psi + \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m_0^2\varphi^2\theta_{\mathrm{I}} - \frac{M}{2}\left[\bar{\psi}, \mathrm{e}^{ig\gamma_5\varphi}\psi\right]_- \theta_{\mathrm{II}} - \left(\frac{M_0}{2}\left[\bar{\psi}, \mathrm{e}^{ig\gamma_5\varphi}\psi\right]_- + V(\varphi)\right)\theta_{\mathrm{III}}, \tag{1.1}$$

with $\theta_{\mathrm{I}} = \theta(|x| < x_1),\ \theta_{\mathrm{II}} = \theta(x_1 \le |x| \le x_2),\ \theta_{\mathrm{III}} = \theta(|x| > x_2)$ being the step functions, which pick out the inner, intermediate and exterior bag's domains correspondingly, combined with the rule that upon the field variation all the surface terms, which should appear on the boundaries between domains, must be dropped. In (1.1) the vacuum pressure term B is absent, although it is physically quite reasonable as taking account for the gluonic input to the bag structure. The reason is that in this model due to the existence of the intermediate phase the Dirac sea polarization behaves very specifically and itself produces the required «inward pressure», what is the main role of the B-term in the two-phase HCBM. Therefore we can drop it without serious loss of physical content, focusing attention mostly on fermion vacuum polarization effects.

To form the bag, we suppose M_0 to be very large, which leads to the dynamical suppression of fermions in the exterior domain III, and simultaneously take $m_0 \to \infty$, so the boson field vanishes in domain I. According to the general approach accepted in HCBM, the boson field is treated in the mean-field approximation, i. e., it is assumed to be a c-number field. Henceforth we shall consider the rest frame of the bag, where $\varphi(x)$ becomes a stationary classical background for fermions. In domain I we have $\varphi(x) = 0$, while in the bag's exterior $\varphi(x)$ decouples from fermions due to the infinite effective mass of the latters and is formed uniquely by the self-interaction $V(\varphi)$. We shall suppose that the self-interaction $V(\varphi)$ leads to soliton-like solutions of equations of motion and is an even function. Then the boson field could be either odd (the topological charge is nonzero) or even (the topological charge vanishes) function.

The equations of motion, following from (1.1), read:
in domain I

$$i\hat{\partial}\psi = 0, \tag{1.2a}$$

$$\varphi = 0, \tag{1.2b}$$

in domain II

$$\left(i\hat{\partial} - M\,\mathrm{e}^{ig\gamma_5\varphi}\right)\psi = 0, \tag{1.3a}$$

$$\varphi'' = ig\frac{M}{2}\langle\left[\bar{\psi}, \gamma_5\,\mathrm{e}^{ig\gamma_5\varphi}\psi\right]_-\rangle, \tag{1.3b}$$

and in domain III

$$\left(i\hat{\partial} - M_0\,\mathrm{e}^{ig\gamma_5\varphi}\right)\psi = 0, \tag{1.4a}$$

$$-\varphi'' + V'(\varphi) = 0, \tag{1.4b}$$

where $\langle\quad\rangle$ in Eq. (1.3b) stands for the expectation value with respect to the fermionic state of the bag. To simplify calculations, we put further $g = 1$, because the dependence on it can be easily restored by means of the substitution $\varphi \to \varphi/g$. Then the spectral problem for fermionic wavefunctions ψ_ω with definite energy ω takes the form

$$\omega\psi_\omega = -i\alpha\psi'_\omega + \beta\,\mathrm{e}^{i\gamma_5\varphi}\left[M\theta_{\mathrm{II}} + M_0\theta_{\mathrm{III}}\right]\psi_\omega. \tag{1.5}$$

Upon taking $M_0 \to \infty$, we get that $\psi_\omega \to 0$ in domain III in such a way that the term $M_0\psi_\omega$ in Eq. (1.5) vanishes, and the chiral boundary conditions [3, 5–7, 22] at the points $\pm x_2$ appear instead:

$$\pm i\gamma^1\psi_\omega(\pm x_2) + \mathrm{e}^{i\gamma_5\varphi(\pm x_2)}\psi_\omega(\pm x_2) = 0. \tag{1.6}$$

In domain I, Eq. (1.5) is the equation for free massless fermions:

$$\omega\psi_{\mathrm{I}} = -i\alpha\psi_{\mathrm{I}}', \tag{1.7}$$

while in the intermediate domain II one has

$$\omega\psi_{\mathrm{II}} = -i\alpha\psi_{\mathrm{II}}' + \beta M\,\mathrm{e}^{i\gamma_5\varphi}\psi_{\mathrm{II}}. \tag{1.8}$$

The wavefunction's continuity on the boundary between domains I and II gives

$$\psi_{\mathrm{I}}(\pm x_1) = \psi_{\mathrm{II}}(\pm x_1), \tag{1.9}$$

while $\psi_{\mathrm{II}}(\pm x_2)$ are subject of the boundary conditions (1.6). At the same time, the field φ in Eq. (1.8) has to be determined self-consistently from Eq. (1.3b) with corresponding continuity conditions at points $|x| = x_{1,2}$.

2. SOLUTIONS WITH NONZERO TOPOLOGICAL CHARGE

The essential feature of this model is that the coupled equations (1.3) in the closed intermediate domain II of finite size $d = x_2 - x_1$ possess simple and physically meaningful solution, which would be unacceptable if these equations were considered in the infinite space. In order to obtain this solution in the most consistent way, we perform first in domain II the chiral Skyrme rotation

$$\psi_\omega = \exp\left(-i\gamma_5\varphi/2\right)\chi_\omega, \tag{2.1}$$

by virtue of which Eq. (1.8) and the boundary conditions (1.6) transform correspondingly into

$$\left(\omega - \frac{1}{2}\varphi'\right)\chi_\omega = -i\alpha\chi_\omega' + \beta M\chi_\omega, \tag{2.2}$$

$$\pm i\gamma^1\chi_\omega(\pm x_2) + \chi_\omega(\pm x_2) = 0. \tag{2.3}$$

It follows from Eq. (2.2) that, if we assume the linear behavior for the field $\varphi(x)$ in domain II, namely,

$$\varphi' = \mathrm{const} = 2\lambda, \tag{2.4}$$

then it becomes the equation for free massive fermions:

$$\nu\chi = -i\alpha\chi' + \beta M\chi, \tag{2.5}$$

with eigenvalues $\nu = \omega - \lambda$. So the fermions being massless in domain I acquire the mass M in domain II due to the coupling to the field φ, whence the intermediate phase emerges describing massive quasifree «constituent quarks».

The most important feature of Eq. (2.5) is that it reveals the sign symmetry $\nu \to -\nu$, which corresponds to the unitary transformation of the fermionic wavefunction

$$\chi \to \tilde{\chi} = i\gamma_1 \chi, \tag{2.6}$$

while the chiral currents

$$j_5 = i\bar{\psi}\gamma_5 \, \mathrm{e}^{i\gamma_5 \phi}\psi = i\chi^+ \gamma_1 \chi \tag{2.7}$$

coincide for these sign-symmetric states:

$$j_5 = i\chi^+\gamma_1\chi = i\tilde{\chi}^+\gamma_1\tilde{\chi} = \tilde{j}_5. \tag{2.8}$$

However, the sign symmetry of Eq. (2.5) itself cannot ensure the corresponding one for the fermion spectrum, since it takes place in domain II only, while the latter has to be determined from the Dirac equation on the unification $\mathrm{I} \cup \mathrm{II}$. Meanwhile in domain I one has Eq. (1.8), which possesses another symmetry, namely, $\omega \leftrightarrow -\omega$. That means that the sign symmetry $\nu \leftrightarrow -\nu$ of the fermionic spectrum could hold only for some discrete values of the derivative φ' in domain II. These values are determined from the algebraic equation for fermionic energy levels, which is obtained from the straightforward solution of Eqs. (1.6)–(1.9) and reads

$$\exp\left(4i\omega x_1\right) = \frac{1 - \mathrm{e}^{-2ikd}\dfrac{M - i(\nu + k)}{M - i(\nu - k)}}{1 - \mathrm{e}^{-2ikd}\dfrac{M + i(\nu - k)}{M + i(\nu + k)}} \; \frac{1 - \mathrm{e}^{2ikd}\dfrac{M - i(\nu - k)}{M - i(\nu + k)}}{1 - \mathrm{e}^{2ikd}\dfrac{M + i(\nu + k)}{M + i(\nu - k)}}, \tag{2.9}$$

where $\nu^2 = k^2 + M^2$. It is easy to find from (2.9) that the fermionic spectrum reveals the symmetry $\nu \leftrightarrow -\nu$, if

$$4\lambda x_1 = \pi s, \tag{2.10}$$

where s is integer, since for such values of $\varphi'(x)$ in domain II the left-hand side (l. h. s.) of Eq. (2.9) reduces to $(-1)^s \, \exp\left(4i\nu x_1\right)$.

When the condition (2.10) is fulfilled, the following consequence of arguments becomes reasonable. In the right-hand side (r. h. s.) of Eq. (1.3b), which determines $\varphi''(x)$ in domain II, we have the vacuum expectation value (v. e. v.) of the C-odd chiral current

$$J_5 = \frac{1}{2}\left[\bar{\psi}, i\gamma_5 \, \mathrm{e}^{i\gamma_5\phi}\psi\right]_- = \frac{1}{2}\left[\chi^+, i\gamma_1\chi\right]_-, \tag{2.11}$$

with χ being now the secondary quantized fermion field in the chiral representation (2.1)

$$\chi(x,t) = \sum_n b_n \chi_n(x) \, \mathrm{e}^{-i\omega_n t}, \tag{2.12}$$

where $\chi_n(x)$ are the normalized solutions of the corresponding Dirac equation, while b_n, b_n^+ are the fermionic creation-annihilation operators, which obey the canonical anticommutation relations

$$\{b_n, b_{n'}^+\}_+ = \delta_{nn'}, \quad \{b_n, b_{n'}\}_+ = 0. \tag{2.13}$$

The average over the given bag's state includes, by definition, the average over the filled sea of negative energy states $\omega_n < 0$ plus possible occupied valence fermion states with $\omega_n > 0$, which are dropped for a moment because their status is discussed specially below. Finally,

$$\langle J_5 \rangle = \langle J_5 \rangle_{\text{sea}} = \frac{1}{2} \left(\sum_{\omega_n < 0} - \sum_{\omega_n > 0} \right) \chi_n^+ i \gamma_1 \chi_n. \tag{2.14}$$

It should be emphasized that in Eq. (2.14) the division of fermions into sea and valence ones is made in correspondence with the sign of their eigenfrequencies ω_n, which differ from sign-symmetric ν_n by the shift in λ:

$$\omega_n = \nu_n + \lambda, \tag{2.15}$$

and so do not possess the sign symmetry $\omega \leftrightarrow -\omega$. However, if we suppose additionally that ν_n and λ are such that for all n the signs of ν_n and ω_n coincide, i. e., after shifting by λ none of ν_n's changes its sign, then the condition $\omega_n \gtrless 0$ in Eq. (2.14) will be equivalent to the condition $\nu_n \gtrless 0$. Thence

$$\langle J_5 \rangle_{\text{sea}} = \frac{1}{2} \left(\sum_{\nu_n < 0} - \sum_{\nu_n > 0} \right) \chi_n^+ i \gamma_1 \chi_n = 0 \tag{2.16}$$

by virtue of the relation (2.8). In turn, it means that Eq. (1.3b) in domain II reduces to $\varphi'' = 0$, which is in complete agreement with our initial assumption that $\varphi'(x) = \text{const}$ in domain II. In other words, we obtain the solution of the coupled Eqs. (1.3) in domain II in the form

$$\varphi(x) = \begin{cases} 2\lambda(x - x_1), & x_1 \le x \le x_2, \\ 2\lambda(x + x_1), & -x_2 \le x \le -x_1, \end{cases} \tag{2.17}$$

where λ takes discrete values from (2.10), while the fermionic spectrum is determined from the relation (2.15) with ν_n being defined from Eq. (2.9) after replacing the l. h. s. to $(-1)^s \, \exp(4 i \nu x_1)$.

There are the following keypoints that make this solution meaningful. The first is the finiteness of the intermediate domain size d, because for an infinite domain II the solution (2.17) would be unacceptable. In our case, however, the size of the intermediate domain is always finite by construction, while the boson field $\varphi(x)$ acquires the solitonic behavior in domain III due to self-interaction $V(\varphi)$. Here the following circumstance manifests itself again: in $(1+1)$D the chiral coupling $\bar{\psi}\, \mathrm{e}^{i\gamma_5 \varphi} \psi$ itself cannot cause the solitonic behavior of the scalar field by virtue of the effects of fermion-vacuum polarization only, i. e., without some additional self-interaction of bosons [23]. The second point is the discreteness and the $\nu \leftrightarrow -\nu$ symmetry of the fermionic spectrum, which leads in turn to a reasonable method of calculation for the average of the chiral current J_5 over the filled Dirac sea (2.16), as well as for other C-odd observables like the total fermion number. After all, in the case we consider the boson field is continuous everywhere and so is topologically equivalent to the odd soliton that would take place in absence of fermions due to the self-interaction $V(\varphi)$ only. So the topological number of the boson field does not depend on the existence and sizes of the spatial domains containing fermions ($\mathrm{I} \cup \mathrm{II}$). On the other hand, the baryon number of the hybrid bag is, by definition, the sum of the topological charge of the boson soliton and the fermion number of the bag interior. In our case the latter is zero (for the ground state), hence the baryon number of the bag is determined by the topological charge of the boson field only and does not depend on the sizes of domains I and II containing fermions, which meets the general requirements for hybrid models. Some more details concerning the status of this solution of Eqs. (1.3) can be found in Ref. [10].

It should also be mentioned that, although the (topological) quantum numbers of such a bag are determined by its solitonic component, it does not mean that the filled fermion levels with positive energy could not exist at all. This would take place for small enough values of the parameter λ only. If λ increases, the negative levels $\omega_n = -|\nu_n| + \lambda$ should move into the positive part of the spectrum. The change of sign of each such level will decrease $\langle Q \rangle_{\rm sea}$ by one unit of charge, but if we fill the emerging positive level with the valence fermion, then the sum $Q_{\rm val} + Q_{\rm sea}$ remains unchanged. Analogously, the total axial current will be equal to $J_{\rm val} + J_{\rm sea}$ and will not change either, which ensures the vanishing r. h. s. of Eq. (1.3b) and so preserves the status of linear function (2.17) as the self-consistent solution of the field equations. Therefore, the existence or absence of valence fermions in such construction of the ground state of the bag depends actually on the relation between λ and $|\nu|_{\rm min}$ and so appears to be a dynamical quantity like the other bag's parameters (the size and mass), which are determined from the total energy minimization procedure.

Another essential feature of this bag configuration is that (2.17) ensures the self-consistent solution of Eqs. (1.3) for even values $s = 2r$ in (2.10) only. The reason is that for odd values $s = 2r + 1$ the fermionic spectrum obtained from the solution of Eqs. (1.6)–(1.9) under the condition $\varphi' = 2\lambda$ will always contain the nondegenerate energy level $\chi_0(x)$ with zero frequency $\nu_0 = 0$, whereas for even values $s = 2r$ one has $\nu_n \neq 0,\ \forall n$. According to the general theory [24], such a zero mode causes fractionalization, which means that its contribution to all C-odd observables will be given by the operator $(1/2)(b_0^+ b_0 - b_0 b_0^+)$ with the eigenvalues $\pm 1/2$ and the numeric coefficient determined by $\chi_0(x)$. On the other hand, in Eq. (1.3b) the chiral current should be averaged over its eigenvector in order to keep up the vanishing dispersion of the r. h. s., otherwise the system of equations (1.3) would be ill-defined. So the operator part of the zero mode contribution to the r. h. s. of (1.3b) reduces to the factor $\pm 1/2$, while $\chi_0(x)$ appears to be such that the corresponding chiral current in domain II does not vanish (it is proportional to $\exp(-2M|x|)$). Hence for odd values $s = 2r + 1$ the r. h. s. of Eq. (1.3b) does not vanish, and the function (2.17) is no longer the self-consistent solution of Eqs. (1.3). However, it is easy to find the way of constructing analogous bags, where the odd values $s = 2r + 1$ are allowed instead of the even ones, utilizing the specific, for such $(1 + 1)$D bag models, possibility to choose in the model Lagrangian the signs of the chiral fermionic masses M, M_0, independently to the right and to the left of the central domain of asymptotic freedom. This question is worked out more explicitly in Ref. [25].

3. THE TOTAL ENERGY OF THE BAG FOR THE NONZERO TOPOLOGICAL CHARGE

As a result, for the bag with the topological charge 1 the boson field is zero in domain I, in domain II it is the linear function (2.17) with $\lambda = \pi r/2x_1$, which after restoring the g-dependence is sewn together with the odd soliton solution of Eq. (1.4b) in the bag's exterior. The typical behavior of $\varphi(x)$ is presented in Fig. 1. For simplicity we shall suppose that in domain III the asymptotic expansion of the soliton solution of Eq. (1.4b) for large $|x|$ can be used, namely,

$$\varphi_{\rm sol}(x) = \frac{\pi}{g}\left(1 - A\,{\rm e}^{-mx}\right), \quad x > x_2, \quad (3.1)$$

with m being the meson mass in the bag's exterior (for $x < -x_2$, $\varphi_{\rm sol}(x)$ is determined via oddness).

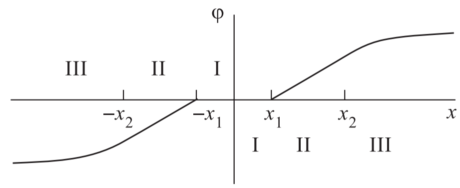

Fig. 1. The configuration of the boson field for a single bag with the topological charge 1

The factor π/g means that we deal actually with the phase soliton with the total amplitude being multiple of $2\pi/g$, since it is the period of the initial chiral interaction $\bar{\psi}\exp{(i\gamma_5 g\varphi)}\psi$. The constant A is determined from the continuity conditions for boson field at points $x = \pm x_2$, which gives

$$x_1 = \frac{r}{r+1}(x_2 + 1/m), \quad d = \frac{x_2 - r/m}{r+1}. \quad (3.2)$$

The condition $d \geq 0$ gives rise then to an additional restriction for the size of the confinement domain

$$mx_2 \geq r, \quad (3.3)$$

which shows that r could be naturally interpreted as the index enumerating the excited states of the bag, whose sizes increase with r.

For the total energy of the boson field, one finds

$$E_\varphi = m\frac{\pi^2}{g^2}\frac{r+1}{mx_2+1}. \quad (3.4)$$

The total energy of the bag is the sum of E_φ and the fermionic contribution E_ψ:

$$E_{\rm bag} = E_\varphi + E_\psi. \quad (3.5)$$

As it follows from (3.4), the boson field energy decreases smoothly for increasing x_2, so all the nontrivial dependence of the total bag energy $E_{\rm bag}$ on the model parameters originates from the fermionic contribution E_ψ, which is the sum of the filled Dirac sea of negative energy states and positive energy valence fermions:

$$E_\psi = E_{\rm val} + E_{\rm sea}. \quad (3.6)$$

Bearing in mind that the charge conjugation symmetry dictates the following definition of the Dirac sea energy [23, 26]:

$$E_{\rm sea} = \frac{1}{2}\sum_{\omega_n<0}\omega_n - \frac{1}{2}\sum_{\omega_n>0}\omega_n, \quad (3.7)$$

for the ground state of the bag described above, the sum (3.6) can be reduced to a single universal expression. If the transformation from ω_n to ν_n is sign-preserving for all n and so there are no valence fermions in the ground state of the bag, one finds from (3.7)

$$E_\psi = E_{\rm sea} = \frac{1}{2}\sum_{\nu_n>0}(-\nu_n + \lambda) - \frac{1}{2}\sum_{\nu_n>0}(\nu_n + \lambda) = -\sum_{\nu_n>0}\nu_n. \quad (3.8)$$

If the parameter λ appears to be large enough, the initially negative level $\omega_n = -|\nu_n| + \lambda$ changes its sign and turns into the occupied valence state. In this case it is convenient to calculate E_ψ in two steps. At first, we consider the contribution from all states with $|\nu_n| > \lambda$ to $E_{\rm sea}$, which in analogy to (3.8) reads

$$E'_{\rm sea} = -\sum_{\nu_m > \lambda} \nu_m. \tag{3.9}$$

To this expression the energy of emerging valence fermion $E_{\rm val} = -|\nu_n| + \lambda$ and the contribution of the positive levels with $\omega_n = \pm|\nu_n| + \lambda$ to the Dirac sea energy should be added, which gives

$$E_\psi = -|\nu_n| + \lambda - \frac{1}{2}[(-|\nu_n| + \lambda) + (|\nu_n| + \lambda)] + E'_{\rm sea} = -\sum_{\nu_n > 0} \nu_n, \tag{3.10}$$

i. e., the same expression (3.8) as we have got for the energy of fermions without filled valence states.

For what follows it is convenient to introduce a set of new parameters, in terms of which the total energy of the bag takes the most appropriate form. First, we introduce the dimensionless quantities

$$\alpha = 2Mx_1, \quad \beta = 2Md, \quad \rho = 2Mx_2, \tag{3.11}$$

and consider in more detail Eq. (2.9), which determines the energy levels ν_n. This equation has two branches of roots. The first one corresponds to real k and in terms of α and β can be transformed into the following form:

$$\tan\left(\alpha\sqrt{1+x^2}\right) = \frac{x}{\sqrt{x^2+1}} \, \frac{x\cos\beta x + \sin\beta x}{1 - \cos\beta x + x\sin\beta x}, \tag{3.12}$$

where the unknown quantity is the dimensionless x defined through $k = Mx$, so that $\nu = M\sqrt{1+x^2}$. The real roots x_n of (3.12) belong to the semiaxis $0 \le x_n < \infty$, since the fermionic wavefunctions are actually the standing waves in a finite spatial box with degeneracy in the sign of k, while the corresponding frequencies ν_n lie in the interval $M \le \nu_n < \infty$. The second branch corresponds to imaginary $k = iMx,\ \nu = M\sqrt{1-x^2},\ 0 \le x \le 1$ and can be derived from (3.12) through the analytical continuation:

$$\tan\left(\alpha\sqrt{1-x^2}\right) = \frac{x}{\sqrt{1-x^2}} \frac{x\cosh\beta x + \sinh\beta x}{\cosh\beta x + x\sinh\beta x - 1}\ . \tag{3.13}$$

For this branch one has $0 < \nu_n \le M$.

Thus, ν_n and so E_ψ appear to be functions of two dimensionless parameters α and β, whose sum is the dimensionless total size of the confinement domain ρ:

$$\alpha + \beta = \rho. \tag{3.14}$$

Proceeding further, it is convenient to extract the mass of the «constituent quark» M from the sea energy and fermionic frequencies as a dimensional factor:

$$\varepsilon_n = \nu_n/M = \sqrt{1+x_n^2}, \tag{3.15}$$

hence $E_\psi = -M \sum_n \varepsilon_n$. Upon introducing the dimensionless ratio of the two mass parameters of the model

$$\mu = m/2M, \tag{3.16}$$

the dimensionless energy of fermions $\mathcal{E}_\psi = E_\psi/M$ and analogously the dimensionless total energy $\mathcal{E}_{\rm bag} = E_{\rm bag}/M$, for the latter one finds

$$\mathcal{E}_{\rm bag} = \mathcal{E}_\psi(\alpha, \beta) + 2\mu \frac{\pi^2}{g^2} \frac{r+1}{\mu\rho + 1}, \tag{3.17}$$

where the dimensionless parameters $\alpha,\ \beta$ are determined through μ and ρ as

$$\alpha = \frac{r}{r+1}\ (\rho + 1/\mu), \quad \beta = \frac{\rho - r/\mu}{r+1}. \tag{3.18}$$

So the total energy of the bag depends ultimately on two dimensionless parameters, μ and ρ, where the parameter μ is fixed by the ratio of the masses m and M, while the optimal value of the bag's size should be found from the minimum of the total energy $\mathcal{E}_{\rm bag}(\rho)$ for given μ.

To study the behavior of $\mathcal{E}_{\rm bag}(\rho)$, first of all we have to renormalize the fermion sea energy $\mathcal{E}_\psi$, which is obviously UV-divergent. Let us start with the asymptotics of roots of Eq. (3.12) for $x_n \gg 1$. Representing Eq. (3.12) as

$$\sin\left(\alpha\sqrt{1+x^2}\right) = \frac{1}{2}\left(\sqrt{1+x^2} + x\right)\sin\left(\alpha\sqrt{1+x^2} + \beta x + \delta\right) + \\ + \frac{1}{2}\left(\sqrt{1+x^2} - x\right)\sin\left(\alpha\sqrt{1+x^2} - \beta x - \delta\right), \tag{3.19}$$

where $\delta = \arctan x$, one finds

$$\varepsilon_n(\alpha, \beta) = \frac{\pi/2 + \pi n}{\rho} + \frac{(-1)^{n+1}\sin\left[(\pi/2 + \pi n)\alpha/\rho\right] + 1 + \beta/2}{\pi/2 + \pi n} + O(1/n^2). \tag{3.20}$$

In the expression (3.20) the first term yields the quadratic and linear divergences in $\sum_n \varepsilon_n$, the second one produces the logarithmic one, while the term with the sine does not cause any divergence at all. To compensate the contribution of the first term, the energy of the sea of free fermions contained in the same «volume» ρ should be subtracted, while the logarithmic divergence, proportional to $\beta/2$, is cancelled by the relevant one-loop counterterm of the boson self-energy [10]. The remaining logarithmic divergence, associated with the term $1/(\pi/2 + \pi n)$, does not depend on the bag parameters and originates from the fermion confinement inside the bag, rather than from some local interaction. Actually, it is the divergent part of the energy of interaction between fermions and the confining potential (bag boundaries). The appearance of such diverging boundary energy in $\mathcal{E}_\psi$ is a specific feature of fermion vacuum polarization in all the bag models [6, 7, 27–32].

In the considered three-phase model this effect acquires some additional features. First, it takes place for nonzero size $d \neq 0$ of the intermediate phase only, while the corresponding boundary energy is negative and diverges as $(-\sum_n 1/(\pi/2 + \pi n))$. More particularly, if $\alpha \to \rho$, then $(-1)^{n+1}\sin\left[(\pi/2 + \pi n)\alpha/\rho\right] \to -1$, hence there remains only the logarithmic term $\beta/(\pi + 2\pi n)$ in the asymptotics (3.20). Therefore in this limit $\mathcal{E}_\psi$ becomes finite just after subtraction of the energy of perturbative vacuum and addition of the one-loop counterterm. On the other hand, the limit $\alpha \to \rho$ is equivalent to $\beta/\alpha \to 0$, and so the infinite energy of the interaction between fermions and bag boundaries takes place only for $d \neq 0$ and the finite size of the central domain (the phase of asymptotic freedom) of the bag.

So the considered three-phase bag model does not actually reveal the ability for the smooth transition into a two-phase configuration for $d \to 0$, although such an opportunity exists formally on the level of the initial Lagrangian (1.1). In fact, in the case of the two-phase bag ($d \equiv 0$) the exact fermion levels are $\varepsilon_n = (\pi/2 + \pi n)/\rho$, hence the single subtraction of the perturbative vacuum energy suffices for renormalization of $\mathcal{E}_\psi$. Therefore the transition between two- and three-phase bag configurations requires an infinite amount of energy, which is a specific feature of such many-phase systems. Note also that in the case of the two-phase bag ($d \equiv 0$) massless fermions are reflected directly from the bag boundaries. So in the three-phase model the infinite boundary energy of the bag is intimately bound up with the circumstance that for $d \neq 0$ the boundaries of the bag reflect massive fermions.

Within the three-phase models we have an opportunity to demonstrate this effect in an even more apparent way. For these purposes let us consider the $(1+1)$-dimensional analog of a «dibaryon», i. e., the configuration with the topological charge 2. Such an object consists of two identical topological bags of the type described above, which are placed so close to each other that their neighboring intermediate domains overlap. The corresponding Lagrangian takes the form

$$\mathcal{L} = \bar{\psi} i\hat{\partial}\psi + \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{M}{2}\left[\bar{\psi}, \mathrm{e}^{ig\gamma_5\varphi}\psi\right]_- \theta_{\mathrm{I}} -$$
$$-\frac{1}{2}m_0^2\left((\varphi - \pi/g)^2\theta_{\mathrm{II}}^{(+)} + (\varphi + \pi/g)^2\theta_{\mathrm{II}}^{(-)}\right) - \left(\frac{M_0}{2}\left[\bar{\psi}, \mathrm{e}^{ig\gamma_5\varphi}\psi\right]_- + V(\varphi)\right)\theta_{\mathrm{III}}, \quad (3.21)$$

where $\theta_{\mathrm{I}} = \theta((|x| \leq x_0) \cup (x_1 \leq |x| \leq x_2))$, $\theta_{\mathrm{II}}^{(\pm)} = \theta(x_0 < \pm x < x_1)$, $\theta_{\mathrm{III}} = \theta(|x| > x_2)$, with the same rule concerning field variations as for (1.1).

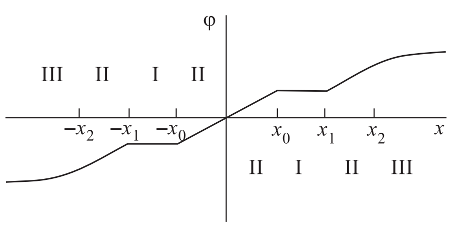

Fig. 2. The boson field profile for the «dibaryon»

The self-consistent solution of the model (3.21) corresponding to such a «dibaryon» configuration is again constructed assuming the linear behavior (2.4) for the boson field in the intermediate domains and taking account of the sign symmetry $\nu \leftrightarrow -\nu$ as well as of the conservation of the chiral current $j_5 = \tilde{j}_5$ for the transformations $\chi \to \tilde{\chi} = \sigma_2\chi$. Omitting some straightforward, but lengthy calculations, let us present the main results.

The profile of the boson field, corresponding to the dibaryon configuration, is shown on Fig. 2. For the intermediate domains of this configuration, one obtains $\varphi' = \mathrm{const} = 2\lambda$, where λ satisfies the condition

$$2\lambda a = \pi s, \quad a = x_1 - x_0. \quad (3.22)$$

The latter is quite analogous to Eq. (2.10) for a single isolated bag, since the parameter $2x_1$ in (2.10), as well as a in (3.22), is the size of the domain of asymptotic freedom for a single bag. However, in the case of the dibaryon there are no zero modes in the fermionic spectrum for any values of s, hence no additional restrictions imposed on the integer s in Eq. (3.22).

It is obvious that the $(1+1)$-dimensional model (3.21) cannot be considered as a realistic model of the dibaryon to any extent. However, being simple and nontrivial simultaneously, it turns out to be a very fruitful illustration for the study of the origin of additional logarithmically divergent terms $1/(\pi/2+\pi n)$ in the UV asymptotics of the fermionic spectrum in such three-phase bag models. The latter is again obtained from the corresponding transcendent equation for fermion levels, which in the trigonometric form reads

$$\sin\left(2\alpha\sqrt{1+x^2}\right)\left(x\sqrt{1+x^2}\cos\left((\beta+\gamma)x+\delta\right)-x\cos\gamma x\right)+$$

$$+\cos\left(2\alpha\sqrt{1+x^2}\right)\left(x^2\sin\left((\beta+\gamma)x+\delta\right)-\sqrt{1+x^2}\sin\gamma x+\sin\gamma x\sin\left(\beta x+\gamma\right)\right)+$$

$$+\left(\sqrt{1+x^2}-\cos\left(\beta x+\gamma\right)\right)\sin\gamma x=0, \tag{3.23}$$

where $\alpha = Ma$, $\beta = 2Md$, $d = x_2 - x_1$, $\gamma = 2Mx_0$, $\delta = \arctan x$. The parameter $d = x_2 - x_1$ is the size of the outward intermediate domains for each of the single bags forming the dibaryon, while $2x_0$ is the size of their common internal intermediate domain, i. e., the domain of their mutual interaction. The UV asymptotics of ε_n's in this case has the following form:

$$\varepsilon_n(\alpha,\beta,\gamma)=\frac{\pi/2+\pi n}{\rho}+$$

$$+\frac{(-1)^{n+1}\left(\sin\left[(\pi/2+\pi n)(2\alpha+\gamma)/\rho\right]-\sin\left[(\pi/2+\pi n)\gamma/\rho\right]\right)+1+(\beta+\gamma)/2}{\pi/2+\pi n}+O(1/n^2), \tag{3.24}$$

where

$$\rho=2\alpha+\beta+\gamma=M(2a+2d+2x_0)=2Mx_2 \tag{3.25}$$

is the total dimensionless bag's size. As in the case of a single isolated bag, the main divergent term in the asymptotics (3.24) corresponds to the sea energy of free fermions in the «volume» ρ, while the logarithmic term, proportional to $(\beta+\gamma)/2$, is exactly compensated by one-loop self-energy counterterm. The change of the coefficient in this term compared to (3.20) is caused by the fact that in the considered case the interaction between fermions and boson field takes place in the domain of the size $2d + 2x_0$. Besides this, there remains again a logarithmically divergent term $1/(\pi/2 + \pi n)$, which corresponds to the (infinite) energy of the interaction between fermions and the confining potential (bag boundaries). It follows from Eq. (3.24) that on the level of divergent terms the boundary energy of the dibaryon coincides exactly with that of a single isolated bag. So we are led to an unambiguous conclusion that it is indeed the effect of fermion confinement in a simply connected domain, which gives rise to the term $1/(\pi/2 + \pi n)$ in Eqs. (3.20) and (3.24), since in the dibaryon configuration the number of boundary points is just the same as in the case of one isolated bag. Note also that the direct consequence of this circumstance is that in $(1 + 1)$D the dibaryon configuration cannot be obtained as a result of continuous fusion of two isolated bags, since when they are separated enough from each other, the sum of their boundary energies is twice larger than that of the dibaryon. In other words, in $(1 + 1)$D the reconstruction of the bag's boundary in the fusion-fission processes requires an infinite amount of energy.

After all, it follows from (3.24) that for $\beta \to 0$, i. e., for vanishing outward intermediate domains of the dibaryon, one gets $(-1)^{n+1} \sin\left[(\pi/2+\pi n)(2\alpha+\gamma)/\rho\right] \to -1$, which compensates the term $1/(\pi/2+\pi n)$, and the infinite interaction energy between fermions and bag boundaries disappears. This circumstance provides with one more argument the assertion, made for a single bag by analysis of the asymptotics (3.20), that the infinite boundary energy appears only when fermions pass through the intermediate phase just before reflection from the bag boundaries.

As a result, for a three-phase bag with $d \neq 0$ the extraction of the finite part from $\mathcal{E}_\psi$ consists actually of two separate procedures. The first one is the standard renormalization onto perturbative vacuum with account of the one-loop counterterm, caused by virtual fermion pairs [10]. The second one is the compensation of the boundary energy by means of an appropriate subtraction, and both procedures suffer from an ambiguity in the choice of subtraction point. In the «classical» renormalization scheme, this uncertainty is resolved by fixing the physical values for a corresponding number of parameters. For obvious reasons, we avoid doing that in our «toy» $(1+1)$D model, but consider instead the most straightforward approach to the compensation of divergences in the sum (3.8), which keeps up the continuous dependence of the result of substraction on the model parameters. The essence of this approach is that we subtract from $\sum_n \varepsilon_n$ another sum with the same summation index n, whose common term coincides exactly with the divergent part of asymptotics (3.20). The result is the finite quantity

$$\tilde{\mathcal{E}}_\psi = -\sum_n \left[\varepsilon_n - \left(\frac{\pi/2+\pi n}{\rho} + \frac{1+\beta/2}{\pi/2+\pi n}\right)\right]. \tag{3.26}$$

This method requires no additional counterterms, because all the divergences are already cancelled by the subtracted sum. Of course, to some extent the physical meaning of such a procedure is lost. It should be emphasized, however, that it is only the $(1+1)$D case when the theory with coupling $\mathcal{L}_{\rm I} = G\bar{\psi}(\sigma + i\gamma_5\pi)\psi$ is (super)renormalizable and any counterterm has explicit physical meaning. For higher space dimensions this is already not true and so the procedure of compensation of divergences in the energy based on (3.26) should not be considered as having no motivation. For more detailed discussion on the extraction of the finite part from the divergent Dirac sea energy in $(3+1)$D HCBM see Refs. [30–32].

Now — having dealt with the renormalization of $\mathcal{E}_\psi$ in this way — let us turn to the study of the total bag energy

$$\mathcal{E}_{\rm bag} = \tilde{\mathcal{E}}_\psi(\alpha,\beta) + 2\mu\frac{\pi^2}{g^2}\,\frac{r+1}{\mu\rho+1} \tag{3.27}$$

as a function of the parameters μ and ρ. The convergent logarithmic sine-term in the asymptotic expression (3.20) gives rise to the first feature of $\mathcal{E}_{\rm bag}$. For these purposes we transform this term to the form

$$\left(\tilde{\mathcal{E}}_\psi\right)_{\log}(\alpha,\beta) = \frac{1}{\pi}\sum_{n\gg 1}(-1)^n\frac{\sin\left[(\pi\alpha/\rho)(n+1/2)\right]}{n+1/2} \tag{3.28}$$

and then use the well-known relation

$$\sum_{n=0}^{\infty}(-1)^n\frac{\sin\left[z(n+1/2)\right]}{n+1/2} = \ln\tan(\pi/4+z/4), \quad |z|<\pi. \tag{3.29}$$

The sums (3.28) and (3.29) possess the similar common term, while the sum (3.28) diverges as $(-\ln(\pi - z))$ when $z \to \pi$. So for $\pi\alpha/\rho \to \pi$, which implies $\beta \to 0$, the sum (3.29) will show the similar behavior, namely,

$$\left(\tilde{\mathcal{E}}_\psi\right)_{\log}(\alpha, \beta) \to -(1/\pi)\ln\beta, \quad \beta \to 0. \tag{3.30}$$

Therefore, both the renormalized fermion energy (3.26) and the total bag's energy (3.27) reveal the logarithmic singularity for $\beta \to 0$, i. e., for $\rho \to r/\mu$, which confirms the qualitative analysis of the vacuum polarization effects in the three-phase bag, performed above. More precisely, after the subtraction (3.26) the renormalized $\tilde{\mathcal{E}}_\psi$ includes $\sum_n 1/(\pi/2 + \pi n)$ as a counterterm for the divergent part of the boundary energy, but the latter disappears for $\beta \to 0$.

$\mathcal{E}_{\rm bag}$ will also grow for $\rho \to \infty$. In this case $\alpha/\rho \to r/(r+1)$, so the logarithmic term (3.28) remains finite, which means that we have to deal now with the whole sum (3.26). However, the leading order behavior of $\tilde{\mathcal{E}}_\psi$ can be evaluated from (3.26) quite effectively by virtue of the fact that for $\rho \to \infty$ the fermionic spectrum becomes quasicontinuous, which allows one to transform the sum over x_n into integral over dx. The analysis of distribution of the roots of Eq. (3.12) shows that in this limit $\sum_n \varepsilon_n$ can be estimated by the following (divergent) integral:

$$\sum_n \varepsilon_n \to \frac{1}{\pi} \int dx\ \sqrt{1+x^2}\ \left[\beta + \frac{1}{1+x^2} + \alpha \frac{x^2}{x^2 + \sin^2\left(\alpha\sqrt{1+x^2}\right)} - \right.$$
$$\left. - \frac{\sin\left(\alpha\sqrt{1+x^2}\right)\cos\left(\alpha\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(x^2 + \sin^2\left(\alpha\sqrt{1+x^2}\right)\right)}\right]. \tag{3.31}$$

For the subtracted sum in (3.26) one finds

$$\sum_n \left(\frac{\pi/2 + \pi n}{\rho} + \frac{1+\beta/2}{\pi/2 + \pi n}\right) \to \frac{\rho}{\pi} \int dx\ \left(x + \frac{1+\beta/2}{\rho x}\right). \tag{3.32}$$

The integrals (3.31) and (3.32) have obviously the same divergent part $(1/\pi)\ \int dx(\rho x + 1/x + \beta/2x)$, and so their difference yields a converging integral, in agreement with the subtraction procedure. The leading term of the integrand in this difference, taken with the (correct) inverse sign, is $\beta/8\pi x^3$. Since $\beta \to \rho/(r+1)$ for $\rho \to \infty$, this finally leads to the emergence of the positive, proportional to ρ, contribution to $\tilde{\mathcal{E}}_\psi$ and correspondingly to $\mathcal{E}_{\rm bag}$.

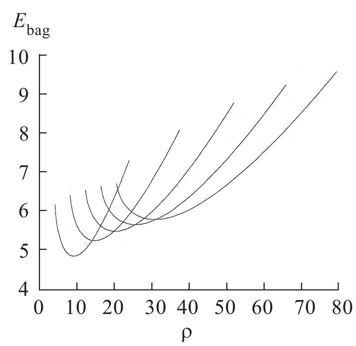

Fig. 3. The dependence of the topological bag's energy on its size

The numerical calculations confirm completely such qualitative predictions for the behavior of $\mathcal{E}_\psi(\rho)$ and $\mathcal{E}_{\rm bag}(\rho)$. The values of free parameters μ and g are chosen as $\mu = 0.25$, which corresponds approximately to the ratio $m_\pi/2M_Q$, and $g = 1$, because the energy of the boson soliton does not have any significant influence on the main properties of $\mathcal{E}_{\rm bag}(\rho)$. The results of $\mathcal{E}_{\rm bag}(\rho)$ calculation for $r = 1, 2, 3, 4, 5$ are depicted on Fig. 3 and show that the size and energy of the solution, determined from the minimum of $\mathcal{E}_{\rm bag}(\rho)$, grow continuously for increasing r, whereas the curvature of $\mathcal{E}_{\rm bag}(\rho)$ in the minimum decreases, which supports the interpretation of configurations with $r > 1$ as excited states of the bag.

4. BAGS WITH ZERO TOPOLOGICAL CHARGE

For bags with vanishing topological charge the relevant configuration of the boson field should be an even one $\varphi(x) = \varphi(-x)$. The principal difference between this case and the previous one is that for even $\varphi(x)$ the sign symmetry $\omega \leftrightarrow -\omega$ is a characteristic feature of the spectral problem for fermions (1.6)–(1.9), which can be easily justified by means of the following transformation of fermionic wavefunctions:

$$\psi_\omega(x) \to \psi_{-\omega}(x) = \pm\gamma_5\psi_\omega(-x). \tag{4.1}$$

However, the corresponding chiral currents are related now in the following way:

$$j^5_{-\omega}(x) = -j^5_\omega(-x), \tag{4.2}$$

so there is no automatic compensation between positive- and negative-frequency terms in the v. e. v. of $J_5(x)$. From (4.2) one can derive only the relation

$$\langle J_5(x)\rangle_{\text{sea}} = \langle J_5(-x)\rangle_{\text{sea}}, \tag{4.3}$$

which guarantees the consistence of Eq. (1.3b) with respect to parity. The direct consequence of such fermion properties is that the even configuration of the boson field, similar to (2.17),

$$\varphi(x) = \begin{cases} +2\lambda(x - x_1), & x_1 \le x \le x_2, \\ -2\lambda(x + x_1), & -x_2 \le x \le -x_1, \end{cases} \tag{4.4}$$

is not an exact solution of Eqs. (1.3), since in this case $\langle J_5(x)\rangle_{\text{sea}} \not\equiv 0$ in domain II.

Nevertheless, the configuration (4.4) plays an important role in the study of the nontopological case. First of all, for $g \ll 1$ it turns out to be a rather good approximation to the precise solution. To argue this statement, let us note firstly that the replacement $\varphi = \tilde{\varphi}/g$ removes g from Eq. (1.3a), while Eq. (1.3b) will contain g only as a coefficient in the r. h. s., namely,

$$\tilde{\varphi}'' = ig^2\frac{M}{2}\langle\big[\bar{\psi}, \gamma_5\,\mathrm{e}^{i\gamma_5\tilde{\varphi}}\psi\big]_-\rangle. \tag{4.5}$$

Assuming further that the potential $V(\varphi)$ depends on g as

$$V(\varphi) = W(g\varphi)/g^2, \tag{4.6}$$

where $W(f)$ should be an even polynom to maintain the (anti)symmetry of soliton solutions, for small g there appears a quite natural expansion in powers of g^2 in the problem. Within this expansion, the zero-order approximation for the rescaled boson field $\tilde{\varphi}(x)$ is the configuration (4.4) in domain II, $\tilde{\varphi}(x) \equiv 0$ in domain I, while in domain III it is given by the even soliton solution of Eq. (1.4b). As in the topological case, to simplify calculations we retain only the asymptotics of this solution, which means

$$\tilde{\varphi}_{\text{sol}}(x) = \pi\left(1 - A\,\mathrm{e}^{-m|x|}\right), \quad |x| > x_2. \tag{4.7}$$

Merging (4.4) and (4.7) via continuity of φ and φ' gives rise to the following relation:

$$2\lambda = \frac{\pi m}{md + 1}, \tag{4.8}$$

whence for the energy of the boson field one finds

$$E_{\tilde{\varphi}} = \frac{\pi^2 m}{md+1}. \tag{4.9}$$

(By returning to the initial φ the dependence on g in E_φ is restored by adding the coefficient $1/g^2$.)

Now let us show that the first-order $O(g^2)$ correction to the energy of the boson field (4.9) vanishes exactly for any current in the r. h. s. of Eq. (4.5), provided the asymptotics (4.7) for the boson field in domain III remains valid beyond the perturbation expansion in g^2, which implies that the corrections caused by the r. h. s. of (4.5) could disturb solely the value of the parameter A. Further, we shall consider only the positive semiaxis. The contribution of the negative one is exactly the same.

From the relation (4.7) we derive

$$m\tilde{\varphi}(x_2) + \tilde{\varphi}'(x_2) = \pi m, \tag{4.10}$$

while in domain III

$$\tilde{\varphi}'_{\mathrm{III}}(x) = \tilde{\varphi}'(x_2)\, \mathrm{e}^{-m(x-x_2)}, \tag{4.11}$$

which is valid beyond the g^2-expansion as well. Using the virial theorem, which is also relevant beyond this expansion, we obtain the following general expression for the contribution of domain III to $E_{\tilde{\varphi}}$:

$$E_{\tilde{\varphi}\mathrm{III}} = \int\limits_{\mathrm{III}} dx\ \tilde{\varphi}'^2 = \frac{\left(\tilde{\varphi}'(x_2)\right)^2}{2m}. \tag{4.12}$$

Proceeding further, on account of the first-order correction from the nonvanishing $\langle J_5(x)\rangle$ one obtains for the boson field in domain II

$$\tilde{\varphi}(x) = 2\lambda(x - x_1) + g^2\tilde{\varphi}_1(x). \tag{4.13}$$

At the same time, it follows from the condition $\tilde{\varphi}_{\mathrm{I}}(x) \equiv 0$ and the boundary conditions (4.10) that

$$\tilde{\varphi}_1(x_1) = 0, \quad m\tilde{\varphi}_1(x_2) + \tilde{\varphi}'_1(x_2) = 0. \tag{4.14}$$

Then for the boson field energy in domain II with the first $O(g^2)$ correction, one finds

$$E_{\tilde{\varphi}\mathrm{II}} = \frac{1}{2}\int\limits_{\mathrm{II}} dx\ \tilde{\varphi}'^2 = 2\lambda\left(\lambda d + g^2\tilde{\varphi}_1(x_2)\right). \tag{4.15}$$

On the other hand, it follows in the same approximation from (4.12) and (4.13) that

$$E_{\tilde{\varphi}\mathrm{III}} = \frac{2\lambda}{m}\left(\lambda + g^2\tilde{\varphi}'_1(x_2)\right). \tag{4.16}$$

Returning to Eq. (4.14), one finds that in the sum $E_{\tilde{\varphi}\mathrm{II}} + E_{\tilde{\varphi}\mathrm{III}}$ the contribution of $\tilde{\varphi}_1$ vanishes. In other words, within the g^2-expansion the corrections to the leading approximation (4.9) in $E_{\tilde{\varphi}}$, caused by the nonvanishing $\langle J_5(x)\rangle$, start from the second-order $O(g^4)$ only.

At the same time, for fermions the leading order of g^2-expansion is $O(g^0)$. In this approximation the spectral problem (1.6)–(1.9) leads to the following equation for the fermionic spectrum:

$$\exp(4i\omega x_1) = \left[\frac{\nu_- + k_-}{\nu_+ + k_+}\right] \frac{1 - \mathrm{e}^{-2ik_+ d}\dfrac{M - i(\nu_+ + k_+)}{M - i(\nu_+ - k_+)}}{1 - \mathrm{e}^{-2ik_+ d}\dfrac{M + i(\nu_+ - k_+)}{M + i(\nu_+ + k_+)}} \frac{1 - \mathrm{e}^{2ik_- d}\dfrac{M - i(\nu_- - k_-)}{M - i(\nu_- + k_-)}}{1 - \mathrm{e}^{2ik_- d}\dfrac{M + i(\nu_- + k_-)}{M + i(\nu_- - k_-)}}, \tag{4.17}$$

where $\nu_\pm = \omega \pm \lambda, \quad \nu_\pm^2 = k_\pm^2 + M^2$. The total energy of the bag is still given by the sum (3.5), where the fermion energy has the form

$$E_\psi = -\sum_{\omega_n < 0} \omega_n. \tag{4.18}$$

Like in (3.8), in (4.18) the inequality $\omega_n < 0$ is strict, because for the configuration (4.4) there are no levels with $\omega_n = 0$ for any values of x_1, x_2.

Finally, after restoring the dependence on g^2 in E_φ we obtain the following expression for the total energy of the bag:

$$E_{\mathrm{bag}} = \frac{\pi^2}{g^2}\,\frac{m}{md+1} + E_\psi + O(g^2), \tag{4.19}$$

where the two first leading terms in E_{bag} — the bosonic $O(1/g^2)$ and fermionic $O(g^0)$ — are determined by the zero-order approximation for the boson field (4.4), (4.7) only, while the corrections start with $O(g^2)$ terms, at once in the bosonic and fermionic parts of the total energy. Moreover, the considerable simplicity of Eq. (4.17) makes it possible to analyze the fermionic spectrum in a semianalytical way, which in turn allows one to use the configuration (4.4), (4.7) as a trial one for a qualitative study of the nontopological bag properties for even larger values $g \simeq 1$.

Thus, in further analysis of the main properties of the nontopological bag we shall use the first two terms in the total energy (4.19), which can be found directly from the configuration (4.4), (4.7). Recalculating E_φ to dimensionless variables, introduced in (3.11), (3.14)–(3.16), one obtains

$$\mathcal{E}_{\mathrm{bag}}(\alpha, \beta) = \mathcal{E}_\psi(\alpha, \beta) + \frac{\pi^2}{g^2}\frac{2\mu}{\mu\beta + 1}, \tag{4.20}$$

where $\alpha,\ \beta$ are now independent parameters. So the study of the bag's energy as a function of its geometry becomes a qualitatively different problem of finding the two-dimensional surface $\mathcal{E}_{\mathrm{bag}}(\alpha, \beta)$.

The extraction of the finite part from $\mathcal{E}_\psi(\alpha, \beta)$ undergoes the same main stages as in the topological case, but reveals some peculiar features, caused by the independence of $\alpha,\ \beta$. After some algebra the UV asymptotics of the energy levels can be presented in the form

$$\varepsilon_n(\alpha, \beta) = \frac{\pi/2 + \pi n}{\rho} + \frac{(-1)^{n+1}\cos(2\lambda d)\sin[(\pi/2 + \pi n)\alpha/\rho] + 1 + \beta/2}{\pi/2 + \pi n} + O(1/n^2). \tag{4.21}$$

From the structure of the logarithmic term in (4.21) we immediately deduce that, as for the topological case, the renormalized via asymptotics $\tilde{\mathcal{E}}_\psi$, and so $\mathcal{E}_{\rm bag}$, acquire the logarithmic divergence $(-\ln \beta/\pi)$ for $\beta \to 0$. Besides this, $\tilde{\mathcal{E}}_\psi$ and $\mathcal{E}_{\rm bag}$ increase for $\beta \to \infty$ and finite α. Since ρ grows together with β, this effect turns out to be quite similar to the increase of $\tilde{\mathcal{E}}_\psi$ and $\mathcal{E}_{\rm bag}$ for $\rho \to \infty$ in the topological case: in the UV domain the difference between ν_+ and ν_- vanishes and Eq. (4.17) turns into (2.9). Thereon to analyze the renormalized $\tilde{\mathcal{E}}_\psi$ one may use the integral approximation (3.31), (3.32), in which for $\beta \to \infty$ the main term in the integrand of $\tilde{\mathcal{E}}_\psi$ is positive and proportional to β.

The behavior of $\mathcal{E}_\psi$ and $\mathcal{E}_{\rm bag}$ for $\alpha \to \infty$ and finite β requires special consideration, since in this case the logarithmic sine-term in asymptotics (4.21) becomes significant again, but, unlike the case of $\beta \to 0$, there appears now an additional factor $\cos(2\lambda d)$. Since $2\lambda d = \pi\mu\beta/(\mu\beta+1)$, the sign of this multiplier can be either positive or negative depending on the current value of β. However, in this limit another effect comes into play, namely, the proportional to α increase of the number of levels on the branches of the fermionic spectrum that correspond to the imaginary values of $k_\pm$ in (4.17), where $0 < \nu_\pm \le M$. Directly for Eq. (4.17) this effect shows up in an intricate enough way due to the presence of separate branches for imaginary k_+ and k_- and therefore can be analyzed in detail only numerically, but its essence could be understood quite simply, if we neglect for a while the difference between k_+ and k_-. Then we are left with only one branch with $0 < \nu_n \le M$ determined from Eq. (3.13). For $\alpha \to \infty$ the spectrum of energy levels belonging to this branch becomes quasicontinuous with the interval between the levels of order π/α, hence $\sum_n \varepsilon_n$ over this branch can be approximated by the integral

$$\sum_{0<\nu_n\le M} \varepsilon_n \to -\frac{1}{\pi} \int_0^1 dx \ \sqrt{1-x^2} \ \left[-\frac{\alpha x}{\sqrt{1-x^2}} + \right.$$
$$\left. + \frac{x\left(\beta + 1/(1-x^2)\right) - \sinh(\beta x+\gamma)/\sqrt{1-x^2}}{\cosh(\beta x + \gamma) - \sqrt{1-x^2}}\right] . \tag{4.22}$$

From (4.22) one can easily see that for $\alpha \to \infty$ the contribution of these levels to $\sum_n \varepsilon_n$ takes the form $\alpha/2\pi$ + finite terms depending on β only. Transforming further the relevant terms in the subtracted sum to the integral, one obtains

$$\int_{\pi/2}^{\alpha} dx \ \left(\frac{x}{\rho} + \frac{1+\beta/2}{x}\right) , \tag{4.23}$$

whence it follows that for $\alpha \to \infty$ the main terms in the subtracted sum should be

$$\frac{\alpha^2}{2\pi\rho} + \frac{1+\beta/2}{\pi} \ln \alpha . \tag{4.24}$$

The leading terms in Eqs. (4.22), (4.24) cancel each other, so in this limit after subtraction the contribution of the branch with $0 < \nu_n \le M$ to the renormalized $\tilde{\mathcal{E}}_\psi$ becomes $(1/\pi)(1 + \beta/2)\ln \alpha$ + finite terms. For the case of separate branches for k_+ and k_- the general features of their asymptotic behavior for $\alpha \to \infty$ remain the same. As a result, after combining

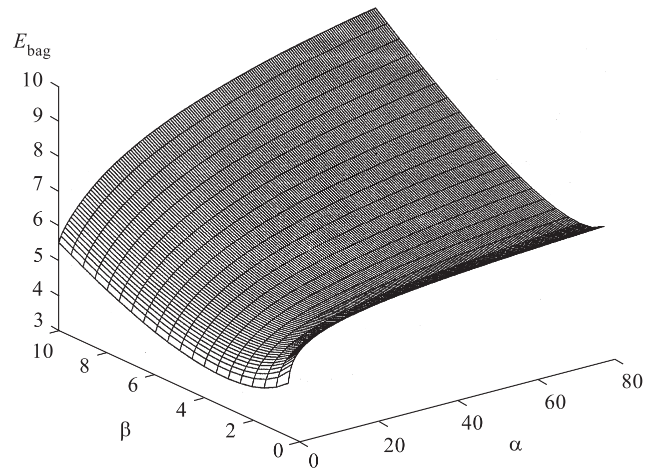

Fig. 4. The profile of the surface $\mathcal{E}_{\rm bag}(\alpha,\beta)$

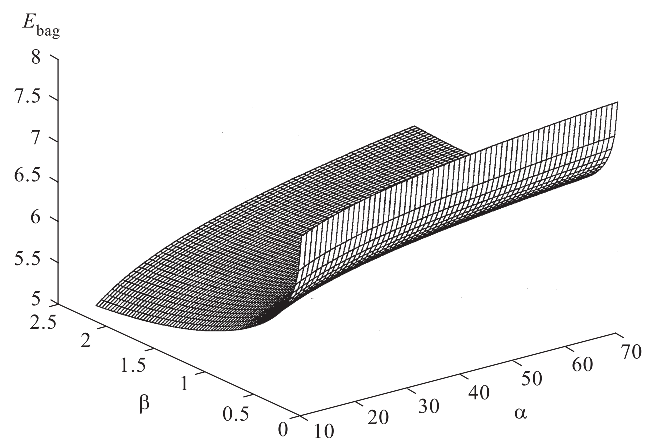

Fig. 5. The profile of the surface $\mathcal{E}_{\rm bag}(\alpha,\beta)$, rescaled to observe the behavior for small β

this asymptotics with the corresponding input of the logarithmic term in the UV asymptotics (4.21), for the leading term in $\mathcal{E}_\psi$ for $\alpha \to \infty$ one finds

$$\frac{1}{\pi}\left(1+\beta/2+\cos 2\lambda d\right)\ \ln\alpha, \quad \alpha\to\infty, \tag{4.25}$$

which is definitely positive for all β. So in this limit the bag's energy also grows, but now proportionally to $\ln\alpha$.

The numerical calculation, performed for the same values $\mu = 0.25$ and $g = 1$ as for the topological bags, confirms such behavior of $\tilde{\mathcal{E}}_\psi$ and $\mathcal{E}_{\rm bag}$. Moreover, the calculation shows that there is not any nontrivial minimum in the total energy for the nontopological case at all, while the minimal value of energy is achieved for the configuration with vanishing size of the phase of asymptotic freedom and for finite nonzero β, which is clearly seen from Figs. 4, 5, where the profiles of the 2D surfaces $\mathcal{E}_{\rm bag}(\alpha, \beta)$ are presented in different scales. So for the bags with zero topological charge the considered three-phase model predicts that the main role should be played by the intermediate phase of constituent quarks, which is quite consistent with semiphenomenological quark models of mesons [9, 33].

CONCLUSION

This work is aimed at the construction of a three-phase version of a hybrid chiral bag, wherein first the initially free, almost massless current quarks transmute into «dressed», due to interaction, massive constituent quarks with the same quantum numbers, and only afterwards there emerges a purely mesonic colorless phase. Our results show that such a model can be formulated in a quite consistent fashion and leads to reasonable behavior of the total bag's energy as a function of its size, which takes the form of an infinitely deep potential well with a distinct minimum in the topological case, whereas in the nontopological case the minimal energy of the bag corresponds to the configuration in which the phase of asymptotic freedom disappears.

The specific feature of this model is a substantially enhanced influence of the fermion vacuum polarization on the bag properties. In particular, in this case the Dirac sea polarization itself produces the infinite increase of energy at large distances. Another essential trait is the appearance of infinite interaction energy between fermions and bag boundaries (i. e., confining potential) for $d \neq 0$, which means that the size of the intermediate domain does not actually vanish, although on the level of the initial Lagrangian the formal limit $d \to 0$ exists and describes a two-phase HCBM. In other words, such a three-phase model cannot be continuously transformed into a two-phase one, which is the ultimate reason of its remarkably different features.

It is worthwhile to mention once more the question of the choice of method for calculation of the Dirac sea averages for fermion bags. The method we used is based on the discreteness of the fermionic energy spectrum, which by means of quite obvious considerations leads to very simple solution of coupled equations of the bag in the intermediate domain. Let us remark, however, that, despite arguments in favor of such a method of calculation of sea averages, we cannot completely reject alternative methods like the thermal regularization. The question of which one is more adequate to the physics of the problem should be answered only through detailed study of realistic models.

It should also be emphasized that by constructing such a three-phase model we have substantially leant on the requirement of Lorentz covariance. The initial formulation of the model, where θ fields are restored, is a local field theory [10], and, regardless of the diversity of classical solutions one needs to deal with, the covariance is broken only spontaneously and so can be freely restored by means of methods of Refs. [21] based on the covariant center-of-mass variables for a localized quantum-field system. However, such an explicitly covariant framework requires some essential changes in the calculation techniques, since the invariant dynamics of fields acquires a specific finite-difference form [21], and so will be considered separately.

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REFERENCES

1. *Bogoliubov P. N.* // Ann. Inst. H. Poincare. 1967. V. 8. P. 163;
   Bogoliubov P. N., Dorokhov A. E. // Sov. J. Part. Nucl. 1987. V. 18. P. 391.
2. *Chodos A. et al.* // Phys. Rev. D. 1974. V. 9. P. 3471;
   Chodos A. et al. // Ibid. V. 10. P. 2559;
   DeGrand T. et al. // Phys. Rev. D. 1975. V. 12. P. 2060.
3. *Chodos A., Thorn C. B.* // Phys. Rev. D. 1975. V. 11. P. 2733;
   Inoue T., Maskawa T. // Progr. Theor. Phys. 1975. V. 54. P. 1853.
4. *Bardeen W. A. et al.* // Phys. Rev. D. V. 11. 1975. P. 1094.
5. *Brown G. E., Rho M.* // Phys. Lett. B. 1979. V. 82. P. 177;
   Vento V. et al. // Nucl. Phys. A. 1980. V. 345. P. 413.
6. *Rho M.* // Phys. Rep. 1994. V. 240. P. 1.
7. *Hosaka H., Toki O.* // Phys. Rep. 1996. V. 277. P. 65.
8. *Zahed I., Brown G. E.* // Phys. Rep. 1986. V. 142. P. 1;
   Holzwarth G., Schwesinger B. // Rep. Progr. Phys. 1986. V. 49. P. 825.
9. *Gasiorowicz S., Rosner J. L.* // Am. J. Phys. 1981. V. 49. P. 1954;
   Montanet L. et al. // Phys. Rev. D. 1994. V. 50. P. 1173;
   Povh B. et al. Particles and Nuclei. Berlin: Springer, 1995.
10. *Sveshnikov K., Silaev P.* // Theor. Meth. Phys. 1998. V. 117. P. 263.
11. *Cherednikov I. et al.* // Nucl. Phys. A. 2000. V. 676. P. 339.
12. *Théberge S., Thomas A. W., Miller G. A.* // Phys. Rev. D. 1980. V. 22. P. 2838; 1981. V. 24. P. 216.
13. *Thomas A. W.* // Adv. Nucl. Phys. 1984. V. 13. P. 1.
14. *Miller G. A.* // Int. Rev. Nucl. Phys. 1984. V. 2. P. 190.
15. *Kahana S., Ripka G., Soni V.* // Nucl. Phys. A. 1984. V. 415. P. 351.
16. *Banerjee M. K., Broniowski W., Cohen T. D.* Chiral Solitons / Ed. by K. F. Liu. Singapore: World Scientific, 1987.
17. *Diakonov D. I., Petrov V. Yu., Povilitsa P.V.* // Nucl. Phys. B. 1988. V. 306. P. 809.
18. *Birse M. C.* // Progr. Part. Nucl. Phys. 1991. V. 25. P. 1.
19. *Alkofer R., Reinhardt H., Weigel H.* // Phys. Rep. 1996. V. 265. V. 139.
20. *Creutz M.* // Phys. Rev. D. 1974. V. 10. P. 1749;
    Creutz M., Soh K. // Phys. Rev. D. 1975. V. 12. P. 443.

21. *Sveshnikov K.* // Phys. Lett. A. 1989. V. 136. P. 1; Teor. Mat. Fiz. 1990. V. 82. P. 55.

22. *Perry R., Rho M.* // Phys. Rev. D. 1986. V. 34. P. 1169.

23. *Campbell D. K., Liao Y.-T.* // Phys. Rev. D. 1976. V. 14. P. 2093.

24. *Niemi A. J., Semenoff G. W.* // Phys. Rep. 1986. V. 135. P. 99.

25. *Malakhov I., Sveshnikov K.* To be published in «Theor. Math. Phys.».

26. *Daschen R., Hasslacher B., Neveu A.* // Phys. Rev. D. 1975. V. 12. P. 2443.

27. *Jackson A. D., Rho M.* // Phys. Lett. B. 1986. V. 173. P. 217; 220;
Jackson A. D., Vepstas L. // Phys. Rep. 1990. V. 187. P. 109.

28. *Mulders P. J.* // Phys. Rev. D. 1984. V. 30. P. 1073.

29. *Plumien G., Muller B., Greiner W.* // Phys. Rep. 1986. V. 134. P. 87.

30. *Elizalde E., Bordag M., Kirsten K.* // J. Math. Phys. 1996. V. 37. P. 895;
Elizalde E. et al. // J. Phys. A. 1998. V. 31. P. 1743.

31. *Falomir H., DeFrancia M., Santangelo E. M.* // Phys. Lett. B. 1996. V. 371. P. 285.

32. *Bordag M., Mohideen U., Mostepanenko V.* // Phys. Rep. 2001. V. 353. P. 1.

33. *Lavelle M., McMillan D.* // Phys. Rep. 1997. V. 279. P. 1.

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