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ON THE MASS SPECTRA OF THE PSEUDOSCALAR MESONS IN THE RELATIVISTIC INDEPENDENT QUARK MODEL

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In the framework of the relativistic independent quark model with the QCD-motivated static potential, the masses of the ground states of pseudoscalar mesons and their radial excitations are calculated for both observed mesons and unobserved ones. The strength of the spin–spin interaction and the magnitude of the mean field contribution are estimated for both the light and heavy 0^{-+} mesons. The calculated masses are in agreement with experimental values within an accuracy of $30 \div 40$ MeV, and the predictions are obtained for the mass values of a number of unobserved yet radial excitations of pseudoscalar mesons.

В рамках релятивистской модели независимых кварков с КХД-мотивированным статическим потенциалом вычислены массы основных состояний псевдоскалярных мезонов и их радиальных возбуждений как для известных, так и для неизвестных еще состояний. Оценены величина спин-спинового взаимодействия и значение вклада среднего поля для легких и для тяжелых 0^{-+}-мезонов. Вычисленные массы согласуются с экспериментальными значениями с погрешностью $30 \div 40$ МэВ. Получены также предсказания значений масс для ряда не открытых пока радиальных возбуждений псевдоскалярных мезонов.

INTRODUCTION

Among a variety of phenomenological hadron models which are used for the interpretation of the extensive amount of data [1], the relativistic model of quasi-independent quarks seems to be one of the most interesting for the description of the hadron spectroscopical properties [2]. In the framework of this model the spectroscopical characteristics can be calculated with sufficient precision by means of the consideration of a system of independent quarks (or quasi-independent ones with weak residual interaction), which move in a certain mean field. It is possible that the independent quark model is the relevant approximation to the QCD bound state problem as well as that the parton model is an adequate approximation for the perturbative QCD in hard processes.

In the papers [3–6] the translation invariant version of the relativistic independent quark model was developed for the description of spectroscopical properties of both light and heavy vector mesons. In order to determine the quark or antiquark motion in the mean field inside the

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meson, the Dirac equation with a static QCD-motivated potential was used. The model does not contradict the first principles of QCD and allows one in a simple fashion, to carry out the numerical calculations of meson characteristics, such as their masses, the average separations between quarks and antiquarks and the e^+e^--decay widths of vector mesons. The value of the confining potential coefficient σ evaluated on the basis of 1^{--} meson mass spectra was found to be (0.197 ± 0.005) GeV^2 for quark–antiquark interaction independently of their flavours. Besides that, the obtained value of the quasi-Coulombic potential strength α_s decreases with the diminution of the quark and antiquark mean distances [6], which corresponds to the QCD asymptotic freedom behaviour.

In the present paper we calculate the pseudoscalar mesons mass spectra using the potential parameters and the quark masses obtained in Ref. 6 on the basis of the spectroscopical data for 1^{--} mesons. In order to obtain the consistent treatment of 1^{--} and 0^{-+} mesons, it is nessasary to take into account the spin–spin interaction between quark and antiquark as well as the mean field energy for meson mass evaluations in the framework of the considered model. Note that the account of these terms is most important in the case of the pseudoscalar mesons. It is well known that the explicit inclusion of contact particle–particle interaction lies beyond the mean field approximation. But as has been found in the present paper, the magnitude of spin–spin interaction is small enough compared with the mean field contribution, so its account has been performed with the help of perturbation theory. Then the calculations produced for two families of 1^{--} and 0^{-+} mesons are on the same level of accuracy as for 1^{--} mesons [6].

When we apply the model for the description of pseudoscalar meson mass spectra, we separate the π-meson mass problem from the beginning; i. e., we perform the fitting of the model parameters including the spin–spin interaction terms without the π-meson mass. When the values of model parameters have been obtained, we find the value of the π-meson mass with the help of the determination of the appropriate value of mean field contribution. Evaluated magnitudes of spin–spin interaction turn out to be sufficiently small not only for the heavy mesons, but for the light ones, as well. It should be noted that, as usual in other variants of quark model (see, e. g., [7, 8]), the magnitude of spin–spin interaction is considerably large, especially for light mesons.

We exclude from our consideration η and η' mesons, too, because at present sufficiently large uncertainty takes place for their constituent content due both to the strong mixing between the $(\bar{u}u + \bar{d}d)/\sqrt{2}$ and $\bar{s}s$ states (and states with charm quark and antiquark), and to the possible contributions from gluonic degrees of freedom. Taking into account these observations, we perform the calculations of pseudoscalar meson masses both for the ground states and for their radial excitations.

In the framework of the model [3–6] the mass M_m of the $\bar{q}q$ meson can be evaluated as

$$M_m = E_{0m} + E_1(n_1^r, j_1) + E_2(n_2^r, j_2), \tag{1}$$

where $E_i(n_i^r, j_i)$, $i = 1, 2$, are the energy spectral functions or the mass terms for the i-th quark (antiquark) which represent the relativistic effective energy of the i-th quark/antiquark moving in the mean field inside the meson. Here, n_i^r and j_i are the radial quantum number and the quantum number of the angular moment, correspondingly, for the i-th constituent moreover, $n_1^r = n_2^r = n-1$ for $n^{2S+1}L_J$ state. The term E_{0m} contains the mean field energy and the possible nonpotential corrections, which cannot be taken into account in the frame of mean field approximation. This term has purely phenomenological origin (cf. [9]) and has nonzero value only for ground states of some mesons. The terms $E_i(n_i^r, j_i)$, which represent the energies of the constituents in the mean field, should be determined from the solution of the Dirac equation:

$$\sqrt{\lambda_i + m_i^2}\psi_i(\mathbf{r}_i) = \left[(\boldsymbol{\alpha}_i \mathbf{p}_i) + \beta_i(m_i + V_0) + V_1\right]\psi_i(\mathbf{r}_i), \tag{2}$$

with $E_i(n_i^r, j_i) = \sqrt{\lambda_i + m_i^2}$, $V_0(r) = \sigma r/2$ and $V_1(r) = -2\alpha_s/3r$, where the model parameters m_i, σ and α_s have meanings of the quark mass, string tension and the strong coupling constant at small distances, correspondingly, and λ_i are found from the solution of the radial equation of the model. Note that the addition of some constant to the scalar linear confining potential $V_0(r)$ is equivalent to the addition of the same constant to the quark mass and vice versa, while the addition of some constant to the vector quasi-Coulombic potential $V_1(r)$ is equivalent to the energy shift of the opposite sign.

It is well known that the solutions of Eq. (2) with the total angular momentum j and its projection m can be represented as

$$\psi(\mathbf{r}) \propto \begin{pmatrix} f(r)\Omega_{jl}^m(\mathbf{n}) \\ -ig(r)(\boldsymbol{\sigma}\mathbf{n})\Omega_{jl}^m(\mathbf{n}) \end{pmatrix},$$

where $\mathbf{n} = \mathbf{r}/r$, the subscript i here and below is omitted.

Further on we restrict ourselves only to the evaluation of the S-wave $\bar{q}q$-mesons characteristics (i. e., pseudoscalar and vector mesons, $j_i = j_2 = 1/2$), because they are described by the simplest version of the model and the results obtained can be compared with the extensive set of accurate data [1], especially for heavy vector mesons. Using Eq. (2), one can derive the second-order equation for the «large» component $f(r)$, and then, making a substitution

$$\varphi(r) = rf(r)\left[V_0(r) - V_1(r) + m + \sqrt{\lambda + m^2}\right]^{-1/2},$$

one comes on to the model S-wave radial equation for $\varphi(r)$ in the following form:

$$\varphi'' + \lambda\varphi = \left\{ -\frac{4\alpha_s\sqrt{\lambda + m^2}}{3r} - \left(\frac{2\alpha_s}{3r}\right)^2 + m\sigma r + \left(\frac{\sigma r}{2}\right)^2 - \right.$$
$$\left. - \frac{\left[\frac{\sigma r}{2}\left(m + \sqrt{\lambda + m^2}\right) + \left(\frac{\sigma r}{4}\right)^2 + \frac{5\alpha_s\sigma}{6} - \frac{\alpha_s^2}{3r^2}\right]}{\left[\frac{2\alpha_s}{3} + r\left(m + \sqrt{\lambda + m^2}\right) + \frac{\sigma r^2}{2}\right]^2} \right\}\varphi. \tag{3}$$

It should be noted that the terms $E_i(n_i^r, j_i)$, $i = 1, 2$, and E_{0m} in Eq. (1) represent the main contributions to the energy of the system due to the interaction of the constituents with the mean field and the mean field energy, respectively. However, it is obvious that the constituent energies $E_i(n_i^r, 1/2)$ are the same for the 1^{--} and 0^{-+} mesons with the identical radial quantum numbers, and the mass differences are the result of the spin–spin interaction and the different values of the E_{0m} terms for some mesons.

In order to estimate the spin–spin interaction between quark and antiquark, we take into account the following expression for the interaction of this type, which has been used in the relativized quark model [10]:

$$V_{S_1 S_2} = \frac{32\pi\alpha_s \mathbf{S}_1 \mathbf{S}_2}{9E_1 E_2}\delta^3(\mathbf{r}), \tag{4}$$

where $\mathbf{S}_1$ and $\mathbf{S}_2$ are the spin operators and E_1 and E_2 are the energies of quark and antiquark, respectively. Evaluation of the expectation value of this operator in the first order of perturbation theory for the wave functions of quark and antiquark shows that this expectation value is a slowly varying function versus E_1 and E_2 and depends mainly on the total spin of quark–antiquark system and the strong coupling constant α_s. So one may use the modified mass formula for S-wave 1^{--} and 0^{-+} mesons, which takes into account spin–spin interaction between quark and aniquark, in the following form:

$$M_m = E_{0m} + E_1(n_1^r, 1/2) + E_2(n_2^r, 1/2) + 4\langle \mathbf{S}_1 \mathbf{S}_2 \rangle_{q_1 \bar{q}_2} V_{SS}, \tag{5}$$

where V_{SS} is proportional to α_s with some constant v_{0S}. As our calculations show, the E_{0m} values are the same and equal zero for the majority of mesons from 0^{-+} and 1^{--} families, both for ground states and for their radial excitations. The exclusions take place only for some special cases, which will be discussed below.

During evaluation of spin–spin interaction contribution and determination of the model parameters, one should take into account systematic errors of the considered model in meson masses calculations. These errors are due to theoretical approximations (note that the isotopic mass splitting for u and d quarks is neglected) and experimental errors of meson masses values. These estimations give the systematic errors of this model at the level of $30 \div 40$ MeV. Although in some cases for vector heavy mesons the contributions of spin–spin interaction are less than the model precision, nevertheless for pseudoscalar mesons, especially for light ones, the spin–spin contributions should be taken into account. It is known that in the case of isoscalar mesons, which are neutral with respect to flavour quantum numbers, the additional terms due to the annihilation interaction for quark and antiquark appear [9–11]. But in the framework of the model under consideration there is a possibility of including these contributions into E_{0m} terms.

When evaluating the masses for the pseudoscalar mesons, we follow the procedure described above and the results obtained in Ref. 6. The string tension has been put to (0.20 ± 0.01) GeV^2 and the joint fitting of vector and pseudoscalar mesons mass spectra leads to the presented below values of model parameters:

Evaluated masses in MeV for the ground states and the radial excitations of the pseudoscalar $\bar{q}q'$ mesons in comparison with the data from Ref. 1

Meson	$M_n^{\rm exp}$	$M_n^{\rm th}$	Meson	$M_n^{\rm exp}$	$M_n^{\rm th}$	Meson	$M_n^{\rm exp}$	$M_n^{\rm th}$
π	138 ± 3.1	120	π'	1300 ± 100	1290	π''	1801 ± 13	1810
K	495 ± 3	500	K'	—	1400	K''	—	1850
D	1867.7 ± 0.5	1850	D'	—	2450	D''	—	2880
D_s	1968.6 ± 0.6	1990	D_s'	—	2560	D_s''	—	2950
η_c	2979.8 ± 2.1	2990	η_c'	3594 ± 5	3600	η_c''	—	3970
B	5279 ± 1.8	5250	B'	—	5650	B''	—	6070
B_s	5369.6 ± 2	5370	B_s'	—	5750	B_s''	—	6130
B_c	6400 ± 100	6450	B_c'	—	6800	B_c''	—	7150
η_b	9300 ± 40	9330	η_b'	—	9960	η_b''	—	10320

$$
\begin{aligned}
\bar{m}_{u,d} &= (0.01 \pm 0.008)\ \text{GeV}, \quad & \alpha_s^{u,d} &= 0.60 \pm 0.15, \\
m_s &= (0.17 \pm 0.05)\ \text{GeV}, \quad & \alpha_s^{s} &= 0.47 \pm 0.10, \\
m_c &= (1.40 \pm 0.05)\ \text{GeV}, \quad & \alpha_s^{c} &= 0.35 \pm 0.03, \\
m_b &= (4.75 \pm 0.10)\ \text{GeV}, \quad & \alpha_s^{b} &= 0.25 \pm 0.02.
\end{aligned}
$$

Moreover, we have found that a suitable fit within the model accuracy for masses of pseudoscalar mesons, which are composed of quark and antiquark with u, d, s, c or b flavours [1], is provided by the following values of E_{0m} parameter:

$$E_0(\eta_c) = E_0(J/\Psi) = -150 \text{ MeV}, \quad E_0(K) = -200 \text{ MeV},$$

$$E_0(\pi) = E_0(\eta_b) = E_0(\Upsilon) = -450 \text{ MeV}.$$

The values of E_{0m} for the ground states 0^{-+} and 1^{--} mesons, which were not pointed out above, as well as for the radial excitations of 0^{-+} and 1^{--} mesons within the model accuracy may be set to zero. Besides that, the magnitude of the spin–spin interaction constant v_{0S} is equal to 100 MeV.

The results obtained for meson masses $M_m^{\rm th}$ are presented in the table. Let us consider the η_b-meson mass value, which can be estimated in the framework of this model. In the cases of η_b and Υ mesons, in order to calculate their masses with the expression (5) one should take nonzero and equal value of E_{0m} term. The same situation occurs for η_c and J/Ψ mesons. So the η_b-meson mass must be less than the Υ-meson mass due to spin–spin interaction and its value must be of the order of 9.3 GeV. The same conclusion has been made in the framework of other models too; for instance, in Ref. 10 the value of η_b-meson mass is 9.4 GeV. The η_b meson has been observed for the first time recently [1] and the experimental value of its mass is equal to 9.3 GeV.

It is useful to compare the mass values of the radial excitations evaluated with the formula (5) and shown in the table with the values obtained in the framework of other theoretical approaches [10, 12–14]. In Ref. 12 on the basis of finite energy sum rules and the $1/N_c$ expansion, the mass formula for the radial excitations of π meson was found in the following form: $m^2_{\pi_n} = nm^2_{\pi'}$, $n = 1, 2, ...$, where $m_{\pi'}$ is the mass value of π', the first radial excitation of π meson. According to this formula, the mass values for the three first π-meson excitations are 1.3 [1], 1.84 and 2.25 GeV. In Refs. 13, 14 the mass values for the radial excitatious for light mesons were obtained with the help of the radial Regge trajectories. For instance, the masses for π-meson radial excitations are 1.37, 1.8, 2.07 GeV. In Ref. 10 the evaluation of the radial excitations masses was performed by means of the rest-frame Schrödinger-type equation with the one-gluon-exchange-plus-linear-confinement potential with relativistic corrections. It is worth listing the obtained in Ref. 10 values for some pseudoscalar mesons excitations: π-meson excitations are 1.3, 1.88 GeV, K-meson excitations are 1.45, 2.0 GeV, D-meson excitation is 2.58 GeV, D_s-meson excitation is 2.67 GeV, B-meson excitation is 5.9 GeV, B_s-meson excitation is 5.98 GeV and B_c-meson excitation is 6.85 GeV. Note that, if the second π-meson radial excitation has its mass value in the 1800 MeV region, than the second ρ-meson radial excitation must lie in the 1900 MeV region. The up-to-date situation with the spectroscopical identification of the second ρ-meson radial excitation is discussed in Ref. 15.

We come to the conclusion that the values of masses for the pseudoscalar mesons obtained with suitable accuracy in the framework of our model do not contradict similar values obtained with the help of other methods and, moreover, in some cases they are in better agreement with the existing data. The calculated mass values can be used for search of unobserved radial excitations of pseudoscalar mesons.

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REFERENCES

1. *Hagiwara K. et al.* (Particle Data Group) // Phys. Rev. D. 2002. V. 66. P. 010001.
2. *Bogolyubov P. N., Dorokhov A. E.* // Part. and Nucl. 1987. V. 8. P. 917.
3. *Khruschev V. V.* // Yad. Fiz. 1987. V. 46. P. 219; 1992. V. 55. P. 773.
4. *Semenov S. V., Khruschev V. V.* // Yad. Fiz. 1993. V. 56. P. 218.
5. *Khruschev V. V., Savrin V. I., Semenov S. V.* // Phys. Lett. B. 1996. V. 374. P. 159.
6. *Khruschev V. V., Savrin V. I., Semenov S. V.* // Phys. Lett. B. 2002. V. 525. P. 283.
7. *Ivanshin Yu. I. et al*. JINR Commun. P2-83-727. Dubna, 1983.
8. *Galkin V. O., Mishurov A. Yu., Faustov R. N.* // Yad. Fiz. 1992. V. 55. P. 2175.
9. *Filippov A. T.* // Yad. Fiz. 1979. V. 29. P. 1035.
10. *Godfrey S., Isgur N.* // Phys. Rev. D. 1985. V. 32. P. 189.

11. *Hirata K., Kobayashi T., Takaiwa Y.* // Phys. Rev. D. 1978. V. 18. P. 236.

12. *Kataev A. L., Krasnikov N. V., Pivovarov A. A.* // Phys. Lett. B. 1983. V. 123. P. 93.

13. *Anisovich A. V., Anisovich V. V., Sarantsev A. V.* // Phys. Rev. D. 2000. V. 62. P. 051502; *Anisovich V. V.* Systematics of $\bar{q}q$-States, Scalar Mesons and Glueballs. hep-ph/0110326. Oct. 2001.

14. *Badalian A. M., Bakker B. L. G., Simonov Yu. A.* Light Meson Radial Regge Trajectories. hep-ph/0204088. April 2002.

15. *Achasov N. N., Kozhevnikov A. A.* // Yad. Fiz. 2002. V. 65. P. 158.

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