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THE SINGLE STATE DOMINANCE IN $2\nu\beta\beta$-DECAY TRANSITIONS TO EXCITED 0^+ AND 2^+ FINAL STATES

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A single state dominance theoretical analysis of the two-neutrino double beta decay ($2\nu\beta\beta$ decay) transitions have been carried out for nuclear systems with $A = 110, 114, 116$ and 128. The energy denominators of the perturbation theory have been considered exactly. New results for the $2\nu\beta\beta$-decay transitions to the 0^+ and 2^+ excited states of the final nucleus are presented. A possibility of detecting these modes experimentally is addressed.

Проведен теоретический анализ двухнейтринного двойного бета-распада ($2\nu\beta\beta$-распад) в предположении доминантности основного состояния промежуточного ядра. Исследовались ядерные системы с $A = 110, 114, 116$ и 128. Энергетические знаменатели теории возмущений учитывались точно. Получены новые результаты для $2\nu\beta\beta$-переходов в возбужденные 0^+- и 2^+-состояния конечных ядер. Обсуждается возможность экспериментального детектирования этих процессов.

The two-neutrino double beta decay ($2\nu\beta\beta$ decay) remains at the forefront of nuclear physics [1–3]. The established $2\nu\beta\beta$-decay half-lifes for ground state to ground state transition for a couple of isotopes constrain nuclear theory and stimulate its further development.

Additional experimental information about the $2\nu\beta\beta$ decay and related processes is of great interest [3–6]. The attention of experimentalists is paid to the transitions to the 0^+ and 2^+ states of the final nucleus. The detection of these transitions has the advantage of additional experimental signature: By deexcitation of these nuclear states one or two gamma-quanta with strictly fixed energies are emitted. It is worth to notice that there is a first positive evidence for $2\nu\beta\beta$ decay of ^{100}Mo for transition to the 0^+_1 excited state of ^{100}Ru with $T^{2\nu}_{1/2} = 6.1^{+1.8}_{-1.2} \cdot 10^{20}$ y [4–7].

With present low-background detectors there is a chance to detect the $2\nu\beta\beta$ decay to the excited 0^+ and 2^+ states of the final nucleus at the level of 10^{21}–10^{22} years. In view of this fact the theoretical calculations of $2\nu\beta\beta$-decay half-lifes are highly required [8]. The aim of this contribution is to present theoretical predictions for some $2\nu\beta\beta$-decay transitions to excited 0^+ and 2^+ states.

The $2\nu\beta\beta$ decay process may occur in second order in standard theory of weak interaction. Thus the problem of the calculation of the $2\nu\beta\beta$-decay matrix elements consists in construction of the full set of virtual intermediate nuclear states of the double-odd nucleus. This is a complex task, which continues to be challenging for the specialists of different nuclear models.

There are few $2\nu\beta\beta$-decay nuclear systems where the spin-parity of the ground state of the intermediate nucleus is 1^+. Some times ago it was suggested that the $2\nu\beta\beta$-decay nuclear matrix elements governing these processes could be dominated by single transition through this $1^+_{\rm g.s.}$ intermediate state (the so-called Single State Dominance Hypothesis (SSD)) [9, 10].

This idea is supported by recent SSD calculations [10–12]. However, due to inaccurate experimental determination of both $2\nu\beta\beta$-decay half-lifes and $\log ft_{\rm EC}$ values for the electron capture it is not possible to decide whether SSD is realized or not. Recently, it was shown that by measuring differential decay rates the SSD hypothesis can be confirmed or ruled out [13] already by ongoing NEMO III experiment [14].

By assuming the SSD for the $2\nu\beta\beta$-decay half-life we can write

$$[T_{1/2}^{(2\nu-{\rm exc})}]^{-1}(0^+ \to J^\pi_f) = \frac{G_\beta^4 g_A^4 m_e^9}{96 \ln 2\, \pi^7} |M_1^i \; M_1^f|^2 H(T, J^\pi_f), \tag{1}$$

where the dimensionless integral over lepton phase space is

$$H^{\rm exc}(T, J^\pi_f) = \int_1^{T+1} d\varepsilon_1 \int_1^{T+2-\varepsilon_1} d\varepsilon_2 \int_0^{T+2-\varepsilon_1-\varepsilon_2} d\omega_1 \times \\ \times F(Z_f, \varepsilon_1) F(Z_f, \varepsilon_2) p_1 \varepsilon_1 p_2 \varepsilon_2 \omega_1^2 \omega_2^2 D(K, L). \tag{2}$$

Here, ω_i, p_i and ε_i $(i = 1, 2)$ are energies of antineutrinos, momenta and energies of electrons in units of the mass of electron m_e, respectively. $\omega_2 = T + 2 - \varepsilon_1 - \varepsilon_2 - \omega_1$ and the kinetic energy of leptons in final state is $T = (E_i - E_f - 2m_e)/m_e = Q_{\beta\beta}$. E_i (E_f) is the energy of the initial (final) nuclear state. $F(Z_f, \varepsilon)$ and g_A denote the relativistic Coulomb factor [1,2] and the vector axial coupling constant, respectively. The form of the factor $D(K, L)$ depends on the spin and parity of the final nuclear state. We have

$$\begin{aligned} D(K, L) &= K^2 + L^2 + KL \quad \text{if} \quad J^\pi_f = 0^+, \\ D(K, L) &= (K - L)^2 \qquad\quad\;\; \text{if} \quad J^\pi_f = 2^+. \end{aligned} \tag{3}$$

The K and L factors are built of the energy denominators of perturbation theory

$$\begin{aligned} K &= \frac{1}{\mu_1 + (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)/2} + \frac{1}{\mu_1 - (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)/2}, \\ L &= \frac{1}{\mu_1 + (\varepsilon_1 + \omega_2 - \varepsilon_2 - \omega_1)/2} + \frac{1}{\mu_1 - (\varepsilon_1 + \omega_2 - \varepsilon_2 - \omega_1)/2} \end{aligned} \tag{4}$$

with $\mu_1 = E_1 - (E_i + E_f)/2$. E_1 is the energy of the ground state of the intermediate nucleus with $J^\pi = 1^+$.

The beta transition amplitudes

$$M_1^i = \langle 1^+_{\rm g.s.} \parallel \hat{\beta}^- \parallel 0^+_i \rangle, \qquad M_1^i = \langle J^\pi_f \parallel \hat{\beta}^- \parallel 1^+_{\rm g.s.} \rangle \tag{5}$$

can be calculated in the framework of various nuclear models [1–3] or deduced from $\log ft$ values of electron capture and single beta decay processes as follows:

$$M_1^i = \frac{1}{g_A} \sqrt{\frac{3D}{ft_{\rm EC}}}, \quad M_1^f = \frac{1}{g_A} \sqrt{\frac{3D}{ft_{\beta^-}}}, \tag{6}$$

Table 1. **The calculated $2\nu\beta\beta$-decay half-lifes within SSD hypothesis with exact ($T_{1/2}^{(2\nu-\mathrm{exc})}$) and approximated ($T_{1/2}^{(2\nu-\mathrm{appr})}$) K and L factors**

Nucleus	Transition	$Q_{\beta\beta}$	μ_1	$\log ft_{\beta-}$	$T_{1/2}^{(2\nu-\mathrm{appr})}$	$T_{1/2}^{(2\nu-\mathrm{exc})}$
^{110}Pd, $\log ft_{\mathrm{EC}} = 4.08$	$0^+ \to 0^+_{\mathrm{g.s.}}$	3.913	3.703	4.66	$1.40 \cdot 10^{20}$	$1.31 \cdot 10^{20}$
	$0^+ \to 2^+_1$	2.627	3.060	5.528	$5.56 \cdot 10^{25}$	$4.64 \cdot 10^{25}$
	$0^+ \to 0^+_1$	1.031	2.262	6.8	$2.57 \cdot 10^{26}$	$2.54 \cdot 10^{26}$
	$0^+ \to 2^+_2$	1.025	2.259	7.39	$4.28 \cdot 10^{31}$	$4.08 \cdot 10^{31}$
^{114}Cd, $\log ft_{\mathrm{EC}} = 4.9$	$0^+ \to 0^+_{\mathrm{g.s.}}$	1.051	3.366	4.473	$1.26 \cdot 10^{25}$	$1.25 \cdot 10^{25}$
^{116}Cd, $\log ft_{\mathrm{EC}} = 4.39$	$0^+ \to 0^+_{\mathrm{g.s.}}$	5.489	3.664	4.662	$1.32 \cdot 10^{19}$	$1.14 \cdot 10^{19}$
	$0^+ \to 2^+_1$	2.958	2.399	5.85	$1.09 \cdot 10^{25}$	$7.30 \cdot 10^{24}$
	$0^+ \to 0^+_1$	2.051	1.945	5.88	$2.01 \cdot 10^{23}$	$1.88 \cdot 10^{23}$
	$0^+ \to 2^+_2$	1.356	1.598	6.31	$2.92 \cdot 10^{28}$	$2.45 \cdot 10^{28}$
	$0^+ \to 2^+_3$	1.134	1.487	6.4	$1.86 \cdot 10^{29}$	$1.62 \cdot 10^{29}$
^{128}Te, $\log ft_{\mathrm{EC}} = 5.05$	$0^+ \to 0^+_{\mathrm{g.s.}}$	1.697	3.298	6.08	$1.25 \cdot 10^{25}$	$1.23 \cdot 10^{25}$
	$0^+ \to 2^+_1$	0.830	2.865	6.38	$1.00 \cdot 10^{33}$	$9.84 \cdot 10^{32}$

where $D = (2\pi^3 \ln 2)/(G_\beta^2 m_e^5) = 6146.7$ s. In this case the exact form of the $2\nu\beta\beta$-decay half-life within the SSD is

$$T_{1/2}^{(2\nu-\mathrm{exc})}(0^+ \to J_f^\pi) = \frac{8\pi}{3\ln 2(\bar{\lambda}_C/c)} \frac{ft_{\mathrm{EC}} ft_{\beta-}}{H^{\mathrm{exc}}(T, J_f^\pi)} = \\ = 2.978 \cdot 10^{14} \frac{10^{\log ft_{\mathrm{EC}} + \log ft_{\beta-}}}{H^{\mathrm{exc}}(T, J_f^\pi)} \text{ y.} \quad (7)$$

Here, $\bar{\lambda}_C = \hbar/m_e c$ is the Compton wave length of the electron. We stress that the half-life $T_{1/2}^{(2\nu)}$ in Eq. (7) depends only on two unknown experimental quantities, which are $\log ft_{\mathrm{EC}}$ and $\log ft_{\beta-}$. We note that the half-life in Eq. (7) does not depend explicitly on G_β and g_A. There is a full cancellation of these factors appearing in Eq. (1) and in expressions for M_1^i, M_1^f in Eq. (6) through D-factor.

In the previous SSD calculations [10, 11] the K and L factors in Eq. (4) were replaced with their approximate values by assuming the energies of the outgoing leptons to be equal, i.e.,

$$D(K,L) = \frac{12}{\mu_1^2} \quad \text{if} \quad J_f^\pi = 0_f^+, \\ D(K,L) = \frac{4(\varepsilon_1 - \varepsilon_2)^2(\omega_1 - \omega_2)^2}{\mu_1^6} \quad \text{if} \quad J_f^\pi = 2_f^+. \quad (8)$$

In this way the dependence of $D(K,L)$ on lepton energies is ignored. This approximation is commonly used in the $2\nu\beta\beta$-decay calculations including contributions from a large number of intermediate nuclear states as it allows one to factorize the nuclear part and the integration over the phase space of outgoing leptons [1–3]. However, within the SSD there is no need for doing it. It was found that this approximation leads to a significant overestimation of $2\nu\beta\beta$-decay half-life in the case of ^{100}Mo [13]. There is an interest to examine this approximation also for other nuclear systems. By assuming Eq. (8) the integral over phase space in Eq. (2) can be written as

$$\begin{aligned} H^{\rm appr}(T,0^+_f) &= \frac{2}{5\mu_1^2}\int_1^{T+1} d\varepsilon_1 \int_1^{T+2-\varepsilon_1} d\varepsilon_2 F(Z_f,\varepsilon_1)\times \\ &\times \; F(Z_f,\varepsilon_2) p_1\varepsilon_1 p_2\varepsilon_2 (T+2-\varepsilon_1-\varepsilon_2)^5, \\ H^{\rm appr}(T,2^+_f) &= \frac{2}{105\mu_1^6}\int_1^{T+1} d\varepsilon_1 \int_1^{T+2-\varepsilon_1} d\varepsilon_2 F(Z_f,\varepsilon_1)\times \\ &\times \; F(Z_f,\varepsilon_2)(\varepsilon_1-\varepsilon_2)^2 p_1\varepsilon_1 p_2\varepsilon_2 (T+2-\varepsilon_1-\varepsilon_2)^7. \end{aligned} \tag{9}$$

We note that factors $H^{\rm appr}(T,0^+)$ and $H^{\rm appr}(T,2^+)$ are related with the common $G^{(2\nu)}(0^+)$ and $G^{(2\nu)}(2^+)$ factors [1,2] as follows:

$$\begin{aligned} G^{(2\nu)}(0^+) &= \frac{(G_\beta g_A)^4 m_e^9}{96\pi^7 \ln 2}\mu_1^2 H^{\rm appr}(T,0^+) = 2.414\cdot 10^{-23}\mu_1^2 H^{\rm appr}(T,0^+)\ {\rm y}^{-1}, \\ G^{(2\nu)}(2^+) &= \frac{(G_\beta g_A)^4 m_e^9}{32\pi^7 \ln 2}\mu_1^6 H^{\rm appr}(T,2^+) = 7.243\cdot 10^{-23}\mu_1^6 H^{\rm appr}(T,2^+)\ {\rm y}^{-1}. \end{aligned} \tag{10}$$

We have used $G_\beta = 1.149\cdot 10^{-5}$ GeV^{-2} and $g_A = 1.25$. Within the above approximation the $0\nu\beta\beta$-decay half-life takes the form

$$T^{(2\nu)}_{1/2}(0^+\to J^\pi_f) = \frac{8\pi^2}{3\ln 2(\bar\lambda_C/c)}\frac{ft_{\rm EC} ft_{\beta^-}}{H^{\rm appr}(T,J^\pi_f)} = 2.997\cdot 10^{14}\frac{10^{\log ft_{\rm EC}+\log ft_{\beta^-}}}{H^{\rm appr}(T,J^\pi_f)}\ {\rm y}. \tag{11}$$

The calculated $2\nu\beta\beta$-decay half-lifes of ^{110}Pd, ^{114}Cd, ^{116}Cd, and ^{128}Te for transitions to some of the lowest 0^+ and 2^+ states of the final nucleus are listed in the Table. By glancing the results in the Table we see that the $2\nu\beta\beta$-decay half-lifes for transitions to the excited states are significantly above the level of 10^{23} years. The only exception is the transition $0^+\to 0^+_1$ in the case of $2\nu\beta\beta$ decay of ^{116}Cd. The evaluated half-life $T^{2\nu}_{1/2}\approx 2.0\cdot 10^{23}$ favor this rare transition to be observed in the near future $2\nu\beta\beta$-decay experiments. We notice that for $A=116$ system the transition to 2^+_1 state is strongly suppressed in comparison with the transitions to 0^+_1 state even in spite of the fact it is favored by larger $Q_{\beta\beta}$ value. From the Table it follows that the ^{110}Pd, ^{114}Cd, ^{116}Cd, and ^{128}Te isotopes are not good candidates for near future experimental study of $2\nu\beta\beta$-decay transition to the 2^+ final state. We remark that the $0^+\to 2^+$ transition has been not observed yet.

We note that the SSD half-lifes in the Table for ground state to ground state transition differ slightly from those presented in Ref. 12. It is because in the present calculations we used $G_\beta = 1.149\cdot 10^{-5}$ GeV^{-2} [15] instead of $G_F\cos\theta_C$ in order to account for radiative corrections [16], which are included in expressions for $\log ft$ values. In addition, in Ref. 12 we employed $D(K,L) = 3(K+L)^2/4$ [1] instead of Eq. (8) what introduced inaccuracy of the order of a small factor $(K-L)^2$.

In our previous SSD calculation of $2\nu\beta\beta$ decay of ^{100}Mo we have found that the exact consideration of the energy denominators plays an important role [13]. The numerical study of this effect has shown that for the $0^+ \to 0_1^+$ transition the corresponding half-life time is corrected only by 20 percent. However, for $0^+ \to 2_1^+$ transition this effect has been found to be large (factor 2–3). For the nuclear systems discussed in the Table this effect is smaller. It is because the difference between the energies of 1^+ ground state of the intermediate nucleus (E_1) and the 0^+ ground state of the parent nucleus (E_i) is larger for these nuclear systems in comparison with $A = 100$ one. Nevertheless, one can see that the exact consideration of denominators entering the expressions for K and L in Eq. (4) leads always to a smaller value of $2\nu\beta\beta$-decay half-life. The relevant difference of ground state energies for studied nuclear systems are as follows:

$$\begin{aligned}
E_1 - E_i &= \quad 0.368\ \mathrm{MeV} \ \ (^{110}\mathrm{Pd}), \qquad 0.933\ \mathrm{MeV} \ \ (^{114}\mathrm{Cd}),\\
E_1 - E_i &= -0.042\ \mathrm{MeV} \ \ (^{116}\mathrm{Cd}), \qquad 0.748\ \mathrm{MeV} \ \ (^{128}\mathrm{Te}),\\
E_1 - E_i &= -0.342\ \mathrm{MeV} \ \ (^{100}\mathrm{Mo}).
\end{aligned} \tag{12}$$

It is obvious that more experimental information about $2\nu\beta\beta$-decay transitions to excited states is needed. It is especially important in connection with preparation of neutrinoless double beta ($0\nu\beta\beta$ decay) experiments measuring the transition to excited states, which perhaps will be able to improve the existing limits on different lepton number violating effective parameters (effective Majorana neutrino mass, parameters of right-handed currents, R-parity violating supersymmetry, etc.). In preparation of both $2\nu\beta\beta$- and $0\nu\beta\beta$-decay experiments the theoretical predictions are very useful. Till now there were missing information especially about $2\nu\beta\beta$-decay transitions to 2^+ excited states. In our contribution we presented exact SSD $2\nu\beta\beta$-decay half-lifes for transitions to both ground and excited final states for $A = 110, 114, 116$ and 128 nuclear systems. At present, it is not clear whether the SSD is realized or not. The chance to shed more light on this problem has the ongoing NEMO III experiment [13]. But even, if the SSD being only very approximate, it is expecting to give at least the correct order of the magnitude of $2\nu\beta\beta$-decay half-lifes of interest.

In summary, the $2\nu\beta\beta$-decay transitions to excited 0^+ and 2^+ final states have been studied in the framework of the Single State Dominance Hypothesis without any approximation. The calculated half-lifes for $2\nu\beta\beta$-decay of ^{110}Pd, ^{114}Cd, ^{116}Cd, and ^{128}Te are presented. We have found that perhaps the $2\nu\beta\beta$-decay $0^+ \to 0_1^+$ transition of ^{116}Cd is a good candidate for future experimental study.

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