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FACTORIZATION AND UNIVERSALITY IN SPIN-DEPENDENT SIDIS

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The QCD factorization in SIDIS is considered in close analogy to the analysis of the Drell–Yan process. A special role of the (weighted) average over produced hadron transverse momentum is stressed. The case of Single Spin Asymmetry (SSA) due to the Collins-type fragmentation function is analyzed and its twist-3 nature is uncovered. The analysis of the sources of imaginary phases and respective cuts in hadronic kinematic variables leads to the effective character (nonuniversality) of T -odd distribution functions, contrary to universality of T -odd fragmentation functions.

КХД-факторизация в полуинклюзивном ГНР рассмотрена в тесной аналогии с анализом процесса Дрелла–Яна. Подчеркнута особая роль (взвешенных) средних по поперечному импульсу образованного адрона. Анализируется случай одиночной спиновой асимметрии, возникающей за счет функции фрагментации коллинзовского типа, показана ее природа твиста-3. Анализ источников мнимых фаз и соответствующих разрезов по адронным переменным ведет к эффективному характеру (неуниверсальности) T -нечетных функций распределения, в отличие от T -нечетных функций фрагментации.

INTRODUCTION

Semi-inclusive Deep Inelastic Scattering (SIDIS) is currently one of the main sources of the experimental information on spin asymmetries (in particular, Single Spin Asymmetries). The application of transverse momentum k_T -dependent Collins fragmentation function in combination with chiral-odd transversity distribution provides a reasonable description of experimental data [1].

At the same time, the status of QCD factorization is not very clear. Indeed, the rigorous results for semi-inclusive processes are based on the approach of Altarelli, Ellis and Martinelli, where the leading order radiative corrections to the semi-inclusive cross-sections integrated over hadron transverse momentum P_T are shown to be reduced to the anomalous dimensions of distribution and fragmentation function. This program is far from being accomplished in the case of Collins function, where integration should be of course the weighted one. As to the (unintegrated) k_T -dependent functions, some discussions of factorization by J. Collins also exist [2].

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The serious conceptual problem of factorization at low P_T , which will be the main object of our investigation, is the difficulty in identifying the short distance subprocess. This contrasts to the case of large P_T , where it is just the hard parton (gluon) balancing this large P_T which is providing the subprocess of interest.

This problem was solved in the case of Drell–Yan (DY) process by A. V. Efremov and A. V. Radyushkin more than 20 years ago [3]. It was shown that the integration over P_T provides an effective «propagator» of heavy photon constituting the hard subprocess.

Here I am applying the similar approach to the case of SIDIS. The resulting picture is even simpler: the effective propagator corresponds now to the quark so that the factorization of P_T -integrated SIDIS turns out to be an analog of the factorization in DIS.

By considering the weighted k_T averages, this approach can be easily generalized to the case of spin-dependent and T -odd fragmentation functions. For the later the definition in the coordinate space is suggested which does not require any specification of intrinsic k_T and is free from the ambiguities of the twist definition for that case. As a result, the analog of Collins function is of twist-3, although it reproduces some of the results with the standard k_T -dependent definition.

I would like to dedicate this paper to my teacher, Anatoli Vasil'evich Efremov, on the occasion of his 70th birthday. As it was already mentioned, and will be also clear of what follows, it is essentially based on his works and ideas from various years, including the factorization in DY process, twist-3 approach to single spin asymmetries, and his current work on the asymmetries in SIDIS.

1. FACTORIZATION IN p_T AVERAGED DY AND SIDIS

Let us recall the approach of Efremov and Radyushkin to the DY $N(p_1) + N(p_2) \rightarrow \gamma^*(q) + X$ process. It is based on the following representation of the transverse momentum averaged hadronic tensor (Fig. 1, *a*, *b*):

$$\begin{aligned} \overline{W}^{\mu\nu}(M^2, x_F) &= \int d^4q \delta(q^2 - M^2) \delta\left(\frac{2q(p_1 - p_2)}{s} - x_F\right) W^{\mu\nu}(p_1, p_2, q) = \\ &= \text{Disc}_s \int \frac{d^4q}{2\pi(q^2 - M^2)} W^{\mu\nu}(p_1, p_2, q) \delta\left(\frac{2q(p_2 - p_1)}{s} - x_F\right). \end{aligned} \quad (1)$$

The photon «propagator» in the r.h.s. marks the appearance of the hard subprocesses (Fig. 1, *b*), so that the dominant contribution to (1) is provided by the region $z^2 \sim 0$ in the coordinate representation.

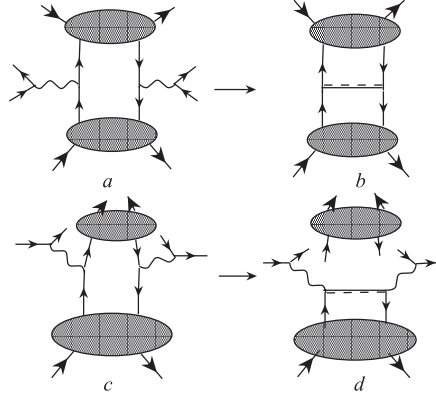


Fig. 1. Generation of hard subprocess by transverse momentum integration in DY process (a, b) and SIDIS (c, d)

As a result, at the leading twist level only the bilocal (anti)quark correlators $\hat{q}_i(z)$ contribute

$$W^{\mu\nu}(p_1, p_2, q) = \int \frac{d^4 z}{(2\pi)^4} e^{iqz} \text{Tr} [\hat{q}_1(z) \gamma^\mu \hat{q}_2(z) \gamma^\nu], \quad (2)$$

and one may assume their following standard parametrization:

$$\hat{q}_i(z) = \hat{p}_i \int_0^1 dy e^{ix_i p z} q_i(x_i), \quad i = 1, 2, \quad (3)$$

where $q_i(x_i)$ are the (anti)quark distributions in the colliding hadrons. Performing the integration over z one is recovering the DY formula

$$\begin{aligned} \overline{W}^{\mu\nu}(M^2, x_F) &= (g^{\mu\nu}(p_1 p_2) - p_1^\mu p_2^\nu - p_2^\mu p_1^\nu) \times \\ &\times \int dx_1 dx_2 \delta(s x_1 x_2 - M^2) \delta(x_1 - x_2 - x_F) q_1(x_1) q_2(x_2). \end{aligned} \quad (4)$$

The corresponding treatment of SIDIS $N(p_1) + \gamma^*(q) \rightarrow h(p_3) + X$, which is the main subject of this paper, is completely analogous (Fig. 1, c, d), except that integration over the produced hadron, rather than photon momentum should be performed. For spin-independent case

$$\begin{aligned} \overline{W}^{\mu\nu}(q^2, x_B, z) &= \int d^4 p_3 \delta(p_3^2) \delta\left(\frac{p_1 p_3}{p_1 q} - z\right) W^{\mu\nu}(p_1, p_3, q) \rightarrow \\ &\rightarrow \text{Disc}_s \int \frac{d^4 p_3}{2\pi p_3^2} W^{\mu\nu}(p_1, p_3, q) \delta\left(\frac{p_1 p_3}{p_1 q} - z\right). \end{aligned} \quad (5)$$

The «propagator» $1/p_3^2$ in this representation does not contain the large «mass», so it is not immediately clear what large parameter provides the light-cone dominance. In fact, it is also the photon «mass», and to see that one may assume this dominance and write the following expression:

$$W^{\mu\nu}(p_1, p_3, q) = \int \frac{d^4 t}{(2\pi)^4} e^{-iqt} \text{Tr} [\hat{q}_1(t) \gamma^\mu \hat{D}(t) \gamma^\nu]. \quad (6)$$

Here $\hat{D}(t)$ is the cutvertex describing the fragmentation of quark to hadron,

$$\hat{D}(t) = \hat{p}_3 \int_0^1 \frac{dz'}{z'^2} e^{ip_3 t/z'} D(z'), \quad (7)$$

where $D(z)$ is the spin-averaged fragmentation function. Let us now perform the integration over t and p_3 similar to the DY case. The resulting expression is

$$\begin{aligned} \overline{W}^{\mu\nu}(q^2, x_B, z) &= (g^{\mu\nu}(p_1 q) - 2xp_1^\mu p_1^\nu - q^\mu p_1^\nu - q^\nu p_1^\mu) \times \\ &\times \text{Disc}_s \int dx \frac{1}{2\pi(xp_1 + q)^2} q_1(x) z D(z). \end{aligned} \quad (8)$$

The effective propagator now assumed the form completely similar to standard DIS case leading to the very similar expression:

$$\overline{W}^{\mu\nu}(q^2, x_B, z) = (g^{\mu\nu}(p_1 q) - 2xp_1^\mu p_1^\nu - q^\mu p_1^\nu - q^\nu p_1^\mu) \frac{1}{2p_1 q} q_1(x_B) z D(z). \quad (9)$$

This proves *a posteriori* the self-consistency of the form (6). The possibility of providing the standard factorization only in such a way does not seem surprising. Indeed, there is another contribution with the same asymptotic behaviour, where the initial and final particles are described by the common nonperturbative object, fracture function [4]. Note, that due to the momentum sum rule for the fragmentation functions which has a partonic form due to the factor $1/z^2$ in (7)

$$\sum_i \int dz z D_i(z) = 1, \quad (10)$$

they provide the complete description of the SIDIS. The corresponding cross section, integrated over z and summed over all hadron species, is equal to the DIS one. As a result, fracture function contribution should be considered as a complementary, rather than the additive one, in order to avoid the double counting. In other words, one may speak about the fragmentation-fracture duality, when the factorization to the separate distribution and fragmentation functions is spoiled at low z , while in average, due to (10), these contributions are equal. This property may be used for the developing of the phenomenological models of the fracture functions.

2. WEIGHTED p_T AVERAGES AND SPIN-DEPENDENT SIDIS

Let us pass to the SSA in SIDIS involving the Collins fragmentation function. We use its analog in the coordinate space [5], so that the corresponding contribution to cutvertex takes the form:

$$\hat{H}(t) = iM\sigma_{\mu\nu}p_3^\mu z^\nu \int_0^1 \frac{dz}{z^2} e^{i(p_3 t)/z} I(z), \quad (11)$$

where M is the parameter of the order of jet mass, while incoming quark is described by transversity distribution

$$\hat{h}(z) = \sigma_{\mu\nu}\gamma_5 p_1^\mu S^\nu \int_0^1 dz e^{i(p_1 z)x} h(x), \quad (12)$$

where S is the target polarization.

The unintegrated hadronic tensor is now the following:

$$\Delta W^{\mu\nu}(p_1, p_3, q) = \int \frac{d^4 t}{(2\pi)^4} e^{-iqt} \text{Tr} [\hat{h}_1(t)\gamma^\mu \hat{H}(t)\gamma^\nu], \quad (13)$$

while the definition of the integrated tensor is ambiguous. The simplest possible way is to integrate over p_T like in the spin-independent case:

$$\Delta \bar{W}^{\mu\nu}(q^2, x_B, z) = \int d^4 p_3 \delta(p_3^2) \delta^4 \left(\frac{p_1 p_3}{p_1 q} - z \right) \Delta W^{\mu\nu}(p_1, p_3, q). \quad (14)$$

Substituting here (11)–(13) and performing integration over t one arrives at the expression

$$\begin{aligned} \Delta \bar{W}^{\mu\nu}(q^2, x_B, z) = M \int d^4 p_3 \delta(p_3^2) dx dz' \partial^\alpha \delta(xp_1 + q - p_3/z') h(x) z' I(z') \times \\ \times \text{Tr} [\gamma_5 \hat{p}_1 \hat{S} \gamma^\mu [\hat{p}_3 \gamma_\alpha] \gamma^\nu] \delta \left(\frac{p_1 p_3}{p_1 q} - z \right). \end{aligned} \quad (15)$$

The derivative acting to the δ function appeared due to the z factor in (11). It should be transferred to one of the p_3 -dependent factors. One can easily see that its action on the $\delta(p_3^2)$ and \hat{p}_3 in the trace does not contribute (in the latter case due to antisymmetrization denoted by square bracket). The only other possibility is its action on the other δ function fixing the produced particle momentum fraction z . The resulting form of hadronic tensor is not explicitly transverse, which should be imposed by requiring the validity of equations of motion for (11). However,

the relevant terms are proportional to q^μ, q^ν and do not contribute to the cross section after contraction with leptonic tensor*.

$$\Delta\sigma(q^2, x_B, z) = \frac{Mx_B^2 h(x_B)(zI(z))'}{(q^2)^2} (p_1 \bar{l}) \epsilon^{\bar{l} S p_1 q}, \quad (16)$$

where $\bar{l} = l_i + l_f$ is the average momentum of initial and final leptons. This expression in the target rest frame corresponds to the azimuthal asymmetry, proportional to $\sin \phi_S$, ϕ_S being the azimuthal angle between the direction of transverse polarization and lepton scattering plane.

Note also, that integration over z should lead to the zero result for each type of the produced hadron due to the sum rule

$$\int_0^1 dz (zI(z))' = 0. \quad (17)$$

The behaviour of I at large (small) z must guarantee the convergence of this sum rule, as in the z -integrated cross section the only δ function, providing the nonzero result is absent at all. Note that integration over z should lead, after summation over all the hadrons, to the inclusive DIS cross section (which for spin-independent case was guaranteed by (10)), where the T -odd asymmetry is absent. Here we see, that this actually happens for each hadron specie separately, which is easy to understand, as the z integration is sufficient in order to eliminate the kinematical variables producing the imaginary phase required for T -odd asymmetry, as will be discussed in some detail in the next section.

Let us now pass to another definition, corresponding to *weighted* average, which allows the consideration of other azimuthal angles:

$$\Delta_n \overline{W}^{\mu\nu}(q^2, x_B, z) = \int d^4 p_3 \delta(p_3^2) (p_3 n) \delta\left(\frac{p_1 p_3}{p_1 q} - z\right) \Delta W^{\mu\nu}(p_1, p_3, q), \quad (18)$$

where n is the unit transverse 4-vector ($np_1 = n_q = 0, n^2 = -1$). It is now obvious, that the derivative in

$$\begin{aligned} \Delta_n \overline{W}^{\mu\nu}(q^2, x_B, z) = & iM \int d^4 p_3 (p_3 n) \delta(p_3^2) dx dz' \partial^\alpha \delta(xp_1 + q - p_3/z') \times \\ & \times h(x) z' I(z') \text{Tr} [\gamma_5 \hat{p}_1 \hat{S} \gamma^\mu [\hat{p}_3 \gamma_\alpha] \gamma^\nu] \delta\left(\frac{p_1 p_3}{p_1 q} - z\right) \end{aligned} \quad (19)$$

*Neglecting the genuine twist-3 contributions will lead, after taking into account the equations of motion, to the zero result for the observable in question.

should be transferred only to the p_3 entering this weighting factor, so that

$$\Delta_n \overline{W}_n^{\mu\nu}(q^2, x_B, z) = \frac{M x_B h(x_B) z I(z)}{q^2} \times \\ \times (2x_B p_1^\mu \epsilon^{\nu]nSp_1} + p_1^\mu \epsilon^{\nu Sq_n} + q^\nu \epsilon^{\mu nSp_1} - S^\mu \epsilon^{\nu p_1 q_n} - n^\nu \epsilon^{\mu qSp_1}). \quad (20)$$

Let us note that this expression satisfies the electromagnetic gauge invariance. Moreover, it is actually equal to the standard expression for the contribution of Collins function, except that the role of intrinsic transverse momentum is played by the auxiliary transverse vector n so that the correspondence to standard expression, making also the mentioned gauge invariance obvious, is: $\text{Tr} [\hat{p}_1 \hat{S} \gamma_5 \gamma^\mu \hat{p}_3 \hat{n} \gamma^\nu] \rightarrow \text{Tr} [\hat{p}_1 \hat{S} \gamma_5 \gamma^\mu \hat{p}_3 \hat{k}_T \gamma^\nu]$. This does not change the azimuthal dependence, as the weighted integration corresponds to azimuthal average

$$\langle d\sigma(\phi_h) \cos(\phi_h - \phi_n) \rangle = \cos \phi_n \langle d\sigma(\phi_h) \cos(\phi_h) \rangle + \sin \phi_n \langle d\sigma(\phi_h) \sin(\phi_h) \rangle.$$

As a result the azimuthal dependence of cross section like $\sin \phi_h$, $\cos \phi_h$ is transferred to the same dependence on the angle ϕ_n , and $I(z)$ corresponds to the *moment* of the Collins function

$$I(z) \sim \int dk_T^2 \frac{k_T^2}{M^2} H_1(z, k_T^2). \quad (21)$$

The factor M^2 in the denominator of the r.h.s. is exactly the one resulting from the various appearance of M in the definitions of H and I [5].

At the same time, the weighting with the factor $|p_T|$ plays the crucial role, and the attempt to use other dependence would lead to the senseless singular expression. Postponing the further detailed studies of this expression, let us compare the two definitions of SSA from the point of view of their twist. While (16) contains extra factor q^2 in the denominator, coming from the differentiation of the relevant δ function, explicitly signalling on the twist-3 effect, it is absent in formula (19). However, its twist-3 character is expressed by the fact, that the dimension of the factor $|p_T|$ in the definition of the weighted integral is carried by M , rather than large scale Q . So one may consider that as a suppression with respect to the naive expectation only. This situation is quite general. We consider the higher twists for the spin independent Drell–Yan case, which was our starting point, and consider the kinematical higher twist corrections, manifested in the extra regular dependence of (anti)quark distributions on space-time interval z^2 :

$$\hat{q}_i(z) = \hat{p}_i \int_0^1 dy e^{ix_i py} q_i(x_i, M^2 z^2) = \sum_n a_n (M^2 z^2)^n \int_0^1 dy e^{ix_i py} q_i(x_i), \quad (22)$$

where, as before, the (logarithmic) dependence on the factorization scale μ , resulting in the extra argument $mu^2 z^2$ is not shown. To probe the higher twist contribution one may define the weighted average

$$\begin{aligned} \overline{W}^{\mu\nu m}(M^2, x_F) &= \\ &= \int d^4 q \delta(q^2 - M^2) W^{\mu\nu}(p_1, p_2, q) (qn)^{2m} \sim M^{2m} \sum_k a_1^k a_2^{m-k}. \end{aligned} \quad (23)$$

The most important property is the finiteness of all the $q_T = qn$ moments, implying that the cross section decreases faster than any power of q_T . So, the partial resummation of higher twists provides the natural explanation of the exponential fall-off of the cross sections.

3. T -ODD DISTRIBUTION FRAGMENTATION AND FRACTURE FUNCTIONS: EFFECTIVENESS AND UNIVERSALITY

Let us compare the above presentation with the complementary mechanism of generation of Single Spin Asymmetry in SIDIS, namely the Sivers function. As was already discussed earlier [6], the key role is played by the requirement for the existence of the imaginary phase to have the T -odd observables in T -invariant theories (while in the case of real T -violation their role is assumed by complex couplings). The Sivers function can be only *effective* (such a notion first suggested in [7]) or nonuniversal (like it is referred to now [2]), in the sense, that this imaginary phase emerges in the interaction, involving also the hard scattering and depending on its type. In other words, the respective cut, providing the imaginary phase, involves both hard and soft variables.

Let us compare [6] in more detail the possible nonperturbative inputs from the point of view of these imaginary cuts. The most widely known objects are parton distributions, describing the fragmentation of hadrons to partons and related to the forward matrix elements $\sum_X \langle P|A(0)|X\rangle \langle X|A(x)|P\rangle = \langle P|A(0)A(x)|P\rangle$ of renormalized nonlocal light-cone quark and gluon operators. As they do not contain any variable, providing the cut and corresponding imaginary phase (to put it in the dramatic manner, the proton is stable), the T -odd distribution functions cannot appear in the framework of the standard factorization scheme. At the same time, they may appear effective, when the imaginary phase is provided by the cut from the hard process, but may be formally attributed to the distribution [7]. Another way of treating the final state interaction, found in the explicit model calculations [8], as it was recently stressed by J. C. Collins [9], and elaborated by Belitsky, Ji and Yuan [10] is provided by the path-ordered gluonic exponential. However, in all cases, the T -odd distribution cannot be universal, as the imaginary phase appearance depends on the subprocess it is convoluted with. Practically,

the dependence on the subprocess enters through the specific choice of light cone vector playing the crucial role for the sign of the Siverson function contribution.

Let us also mention in this connection, that the similar to [8] calculation was performed earlier in twist-3 QCD [11] for the crossing related process of dilepton photoproduction. That result, when continued to the region $P_T \sim M$, will also not have any power suppression and looks formally as a twist-2 one.

As soon as the twist notion for Siverson function, like for any k_T -dependent function is ambiguous, we may write it also in the coordinate space as

$$\langle p, S | \psi(0) \gamma^\mu \psi(z) | p, S \rangle = M \epsilon^{\mu S p z} \int_0^1 dx e^{izpx} J(x). \quad (24)$$

Like the coordinate analog of Collins function, it is also of twist-3. To prove the nonuniversality of this T -odd distribution $J(x)$, it is sufficient to consider its contribution to the integrated asymmetry in semi-inclusive DIS (14), where the hadronic tensor is proportional to

$$\begin{aligned} \Delta \bar{W}^{\mu\nu}(q^2, x_B, z) &= iM \int d^4 p_3 \delta(p_3^2) dx dz' \partial^\alpha \delta(x p_1 + q - p_3/z') \times \\ &\times J(x) z' D(z') \epsilon^{\beta\alpha S p} \text{Tr} [\gamma_\beta \gamma^\mu \hat{p}_3 \gamma^\nu] \delta\left(\frac{p_1 p_3}{p_1 q} - z\right). \end{aligned} \quad (25)$$

One may see that the nonzero result is now only due to the action of the derivative to the $\delta(p_3^2)$ and the resulting expression is

$$\Delta \bar{W}^{\mu\nu}(q^2, x_B, z) = \frac{M x_B^3 J'(x_B) z D(z)}{2(q^2)^2} (p_1^\mu \epsilon^{\nu S p_1 q} + p_1^\nu \epsilon^{\mu S p_1 q}), \quad (26)$$

where we dropped the terms proportional to q^μ, q^ν , disappearing after contraction with the leptonic tensor. One may note the important difference with the similar expression (16), which is going to zero after the integration over z . Contrary to that, the current expression is nonzero after integration over z is performed. Performing in addition the summation over hadron species and taking into account the momentum sum rule (10), one is coming to the expression for *inclusive* DIS,

$$\Delta \bar{W}^{\mu\nu}(q^2, x_B) = \frac{M x_B^3 J'(x_B)}{2(q^2)^2} (p_1^\mu \epsilon^{\nu S p_1 q} + p_1^\nu \epsilon^{\mu S p_1 q}), \quad (27)$$

clearly requiring *real* T violation [12]. This proves the nonuniversality of Siverson function.

Let us note, for completeness, that the weighted average (18) receives the following contribution of Siverson function

$$\Delta_n \bar{W}_n^{\mu\nu}(q^2, x_B, z) = \frac{M x_B^2 J(x_B) z D(z)}{q^2} (p_1^{[\mu} \epsilon^{\nu] n S p_1} + g^{\mu\nu} \epsilon^{S q n p_1}). \quad (28)$$

At the same time, the electromagnetic gauge invariance requires to consider the contributions of quark-gluon correlators, leading to the appearance of any effective T -odd distribution, explicitly. Moreover, the fragmentation analog of Sivvers function was also shown [13] to be entirely related to quark gluon correlators.

Imaginary phase of the Collins function (or any other T -odd fragmentation function) constructed from the time-like cutvertices of the similar operators

$$\sum_X \langle 0|A(0)|P, X \rangle \langle P, X|A(x)|0 \rangle,$$

should come from the cut with respect to the jet mass, simulating the T violation. It is the same in the various hard processes, and T -odd fragmentation functions are therefore universal. The recent analysis [2] confirms this picture.

The FRACTURE Function (FF) [4], whose particular example is represented by the diffractive distribution (DD) [14], is related to the object

$$\sum_X \langle P_1|A(0)|P_2, X \rangle \langle P_2, X|A(x)|P_1 \rangle,$$

combining the properties of FRAGMENTATION and struCTURE functions. They describe the correlated fragmentation of hadrons to partons and vice versa. Originally this term was applied to describe the quantities integrated over the variable $t = (P_1 - P_2)^2$, while the fixed t case is described by the so-called extended fracture functions [15]. They may be also extended [16] to describe SSA in such processes. Namely, such functions can easily get the imaginary phase from the cut produced by the variable $(P_1 + k)^2$. Due to the extra momentum of produced hadron P_2 , the number of the possible T -odd combinations increases. Therefore, they may naturally allow for the T -odd counterparts. The T -odd fracture function may describe a number of SSA at HERMES and, especially, NOMAD [17]. The necessity of fracture functions, in particular of T -odd ones, is seen from the property of factorization in SIDIS mentioned above. Namely, that the appearance of separate distribution and fragmentation function cannot be proved in general, but rather assumed and justified a posteriori.

CONCLUSIONS

The factorization in P_T integrated SIDIS may be proved in a similar manner to the consideration of Drell–Yan process by A. V. Efremov and A. V. Radyushkin. The essential difference, however, is that factorization to separate distribution and fragmentation function cannot be proved in general, but rather justified a posteriori, leaving a room for the fracture function.

This proof may be easily generalized for the case of k_T -dependent fragmentation function, if the latter is defined in the co-ordinate space, when the explicit

definition of the transverse momentum notion occurs to be non-necessary. The respective analog of Collins function turns out to be of twist-3 order, which is expressed in the fact, that the dimension of $|P_T|$ in the weighted average is carried by the soft parameter M , rather than Q . This property may be generalized for arbitrary power of $|P_T|^{2n}$, so that the partial resummation of higher twist justifies the exponential decrease of cross sections with P_T .

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