ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2004. Т. 35. ВЫП. 7

УДК 539.12: 539.171

INTERACTIONS AT LARGE DISTANCES AND SPIN EFFECTS IN NUCLEON–NUCLEON AND NUCLEON–NUCLEI SCATTERING

J.-R. Cudell

Institut de Physique, Bât. B5a, Université de Liège, Liège, Belgium

E. Predazzi

Dipartimento di Fisica Teorica — Unversità di Torino and Sezione INFN di Torino, Italy

*O. V. Selyugin**

Institut de Physique, Bât. B5a, Université de Liège, Liège, Belgium
Joint Institute for Nuclear Research, Dubna

The momentum-transfer dependence of the slopes of the spin-non-flip and spin-flip amplitudes is analyzed. It is shown that the long tail of the hadronic potential in impact parameter space leads for hadron–hadron interactions to a larger value of the slope for the reduced spin-flip amplitude than for the spin-non-flip amplitude. It is shown that the preliminary measurement of A_N obtained by the E950 Collaboration confirms such a behaviour of the hadron spin-flip amplitude.

Исследуется зависимость амплитуд с переворотом и без переворота спина от переданного импульса. Показано, что хвост потенциала в пространстве прицельных параметров ведет к бóльшему наклону амплитуды с переворотом спина и что это соответствует предварительным измерениям A_N, проведенным на E950.

The diffractive polarized experiments at HERA and RHIC allow one to study the spin properties of the quark–pomeron and proton–pomeron vertices, and to search for a possible odderon contribution. This provides an important test of the spin properties of QCD at large distances. In all of these cases, pomeron exchange is expected to contribute to the observed spin effects at some level [1].

In the general case, the form of the analyzing power A_N and the position of its maximum depend on the parameters of the elastic scattering amplitude $\sigma_{\rm tot}$, $\rho(s,t)$, on the Coulomb-nucleon interference phase $\varphi_{\rm cn}(s,t)$ and on the elastic slope $B(s,t)$. The Coulomb-hadron phase was calculated in the entire diffraction domain taking into account the form factors of the nucleons [2].

---

*E-mail: selugin@thsun1.jinr.ru

The dependence of the hadron spin-flip amplitude on the momentum transfer at small angles is tightly connected with the basic structure of the hadrons at large distances. We show that the slope of the «reduced» hadron spin-flip amplitude (the hadron spin-flip amplitude without the kinematic factor $\sqrt{|t|}$) can be larger than the slope of the hadron spin-non-flip amplitude as was observed long ago [3].

The first RHIC measurements at $p_L = 22$ GeV/c [4] in p^{12}C scattering indicated that A_N may change sign already at very small momentum transfer. Such a behaviour cannot be described by the Coulomb-Nuclear Interference effect alone, and requires some contribution of the hadron spin-flip amplitude.

The total helicity amplitudes can be written as

$$\Phi_i(s,t) = \phi_i^h(s,t) + \phi_i^{\rm em}(t)\exp\left[i\alpha_{\rm em}\varphi_{\rm cn}(s,t)\right],$$

where $\phi_i^h(s,t)$ comes from the pure strong interaction of hadrons; $\phi_i^{\rm em}(t)$ from the electromagnetic interaction of hadrons ($\alpha_{\rm em} = 1/137$ is the electromagnetic constant), and $\varphi_{\rm cn}(s,t)$ is the electromagnetic-hadron interference phase factor. So, to determine the hadron spin-flip amplitude at small angles, one should take into account all electromagnetic and interference effects.

As usual, the slope B of the scattering amplitudes is defined as the derivative of the logarithm of the amplitudes with respect to t. For an exponential dependence on t, this coincides with the standard slope of the differential cross sections divided by 2. If we define the forms of the separate hadron scattering amplitude as:

$$\begin{aligned} &\operatorname{Im}\mathcal{A}_{nf}(s,t) \sim \exp\left(B_1^+ t\right), \quad \operatorname{Re}\mathcal{A}_{nf}(s,t) \sim \exp\left(B_2^+ t\right), \\ &\operatorname{Im}\widetilde{\mathcal{A}_{sf}}(s,t) \sim \exp\left(B_1^- t\right), \quad \operatorname{Re}\widetilde{\mathcal{A}_{sf}}(s,t) \sim \exp\left(B_2^- t\right), \end{aligned} \tag{1}$$

($\mathcal{A}_{nf}(s,t)$ and $\widetilde{\mathcal{A}_{sf}}(s,t)$ are non-flip and «reduced» spin-flip amplitudes, respectively), then, at small t ($\sim 0{-}0.1$ GeV^2), most phenomenological analyses assume $B_1^+ \approx B_2^+ \approx B_1^- \approx B_2^-$. Actually, we can take the eikonal representation for the scattering amplitude

$$\begin{aligned} \phi_1^h(s,t) &= -ip\int_0^\infty \rho d\rho J_0(\rho q)[\mathrm{e}^{\chi_0(s,\rho)} - 1], \\ \phi_5^h(s,t) &= -ip\int_0^\infty \rho d\rho J_1(\rho q)\chi_1(s,\rho)\,\mathrm{e}^{\chi_0(s,\rho)}, \end{aligned} \tag{2}$$

where $q = \sqrt{-t}$ and $\chi_0(s,\rho)$ represents the corresponding interaction potential $V_i(\rho,z)$ in impact parameter space. If the potentials V_0 and V_1 are assumed to have a Gaussian form in the first Born approximation, ϕ_1^h and ϕ_5^h will have the same Gaussian form

$$\phi_1^h(s,t) \sim \int_0^\infty \rho d\rho J_0(\rho q)\, \mathrm{e}^{-\rho^2/2R^2} = R^2\, \mathrm{e}^{R^2 t/2},$$
$$\phi_5^h(s,t) \sim \int_0^\infty \rho^2 d\rho J_1(\rho q)\, \mathrm{e}^{-\rho^2/(2R^2)} = qR^4\, \mathrm{e}^{R^2 t/2}. \qquad (3)$$

In this special case, the slopes of the spin-flip and of the «residual» spin-non-flip amplitudes are indeed the same.

However, a Gaussian form for the potential is at best adequate to represent the central part of the hadronic interaction. This form cuts off the Bessel function and the contributions at large distances. If we keep only the first two terms in a small-x expansion of the J_i,

$$J_0(x) \simeq 1 - (x/2)^2; \quad \text{and} \quad 2J_1/x = (1{-}0.5(x/2)^2), \qquad (4)$$

the corresponding integrals have the same behaviour in q^2 [5]. So, the integral representation for spin-flip and spin-non-flip amplitudes will be the same as in (3). If, however, the potential (or the corresponding eikonal) has a long tail (exponential or power) in impact parameter, then the approximation (4) for the Bessel functions does not lead to a correct result and one has to perform the full integration.

Let us examine the contribution of the large distances. The Hankel asymptotics of the Bessel functions at large distances are

$$J_\nu(z) = \sqrt{2/\pi z}[P(\nu,z)\, \cos\, \chi(\nu,z) - Q(\nu,z)\, \sin\, \chi(\nu,z)],$$
$$P(\nu,z) \sim \sum_{k=0}^{\infty} (-1)^k \frac{(\nu,2k)}{(2z)^{2k}}, \quad Q(\nu,z) \sim \sum_{k=0}^{\infty} (-1)^k \frac{(\nu,2k+1)}{(2z)^{2k+1}} \qquad (5)$$

with $\chi(\nu,z) = z - (\nu/2 + 1/4)$ and $P_0(x)$ and $Q_i(x)$ some polynomials of x. The leading behaviour at large x will thus be proportional to $1/\sqrt{q\rho}$.

Let us calculate the corresponding integrals in the case of large distances

$$\phi_1^h(s,t) \sim \frac{1}{q^2}\int_0^\infty \sqrt{x}\left[\left(1 - \frac{0.125}{x}\right)\cos x + \left(1 + \frac{0.125}{x}\right)\sin x\right] \times$$
$$\times \exp\left(-\frac{x^2}{2R^2q^2}\right) dx \approx \frac{R}{q}\ {}_1F_1(3/4, 1/2, -q^2R^2/2), \qquad (6)$$

$$\frac{\phi_5^h(s,t)}{q} \sim \frac{1}{q^4}\int_0^\infty x^{3/2}\left[\left(\frac{0.375}{x} - 1\right)\cos x + \left(1 - \frac{0.375}{x}\right)\sin x\right] \times$$
$$\times \exp\left(-\frac{x^2}{2R^2q^2}\right) dx \approx \frac{R^{3/2}}{q^{5/2}}\ {}_1F_1(3/4, 1/2, -q^2R^2/2). \qquad (7)$$

The exponential asymptotics of both representations are the same, but the additional $q^{3/2}$ in the denominator of (7) leads to a larger slope for the residual spin-flip amplitude. So, although the integrals have the same exponential behaviour asymptotically, the additional inverse power of q leads to a larger effective slope for the residual spin-flip amplitude at small q although we take a Gaussian representation in impact parameter.

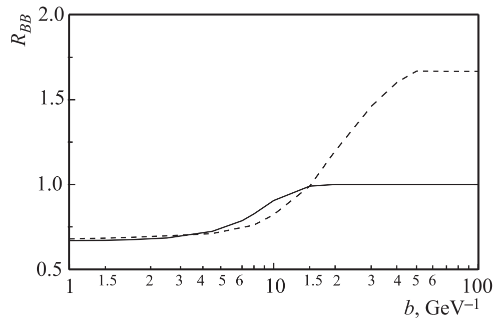

Fig. 1. The ratio of the effective slopes — R_{BB} for the case $n = 1$ (dashed line) and for $n = 2$ (solid line) as function of the upper bound of the integrals b

These investigations are confirmed numerically. We calculate the scattering amplitude in the Born approximation in the cases of exponential and Gaussian form factors in impact parameter space as a function of the upper limit b of the corresponding integral

$$\phi_1^h(t) \sim \int_0^b \rho d\rho J_0(\rho\Delta) f_n, \quad \phi_5^h(t)/q \sim \int_0^b \rho^2 d\rho J_1(\rho\Delta) f_n, \tag{8}$$

with $f_n = \exp\,[-(\rho/5)^n]$, and $n = 1, 2$. We then calculate the ratio of the slopes of these two amplitudes $R_{BB} = B^{sf}/B^{nf}$ as a function of b for these two values of n. The result is shown in Fig. 1. We see that at small impact parameter the value of R_{BB} is practically the same in both cases and depends weakly on the value of b. But at large distances, the behaviour of R_{BB} is different. In the case of the Gaussian form factor, the value of R_{BB} reaches its asymptotic value ($= 1$) quickly. But in the case of the exponential behaviour, the value R_{BB} reaches its limit $R_{BB} = 1.7$ only at large distances. These calculations confirm our analytical analysis of the asymptotic behaviour of these integrals at large distances.

In [5], it was shown that in the case of an exponential tail for the potentials, $\chi_i(b, s) \sim H\,\mathrm{e}^{-a\rho}$, one obtains

$$\mathcal{A}_{nf}(s,t) \sim \frac{1}{a\sqrt{a^2+q^2}}\,\mathrm{e}^{-Bq^2}, \quad \sqrt{|t|}\widetilde{\mathcal{A}_{sf}}(s,t) \sim \frac{3aqB^2}{\sqrt{a^2+q^2}}\,\mathrm{e}^{-2Bq^2}. \tag{9}$$

In this case, therefore, the slope of the «residual» spin-flip amplitude exceeds the slope of the spin-non-flip amplitude by a factor of two. Hence, a long-tail hadron potential implies a significant difference in the slopes of the «residual» spin-flip and of the spin-non-flip amplitudes.

Recently there has been very few experimental data for hadron–hadron scattering at large energy. Of course, it will be very interesting to obtain data from the $PP2PP$ experiment at RHIC. But now, we only have the preliminary data of A_N in proton–Carbon elastic scattering. Despite the fact that these data have bad normalization conditions, the slope of the analysing power is very interesting.

In our analysis, the scattering amplitude is $\mathcal{A}_i(s,t) = \mathcal{A}_i^h(s,t) + \mathcal{A}_i^{\rm em}(t)\,{\rm e}^{i\delta}$, $(i = nf, sf)$, where each term includes a hadronic and an electromagnetic contribution with the Coulomb-nuclear phase [2]. The electromagnetic form factor $F_{\rm em}^{^{12}\rm C}$ was obtained from the electromagnetic density of the nucleus. We parametrize the spin-non-flip and spin-flip parts of $p^{12}\rm C$ scattering as

$$\begin{aligned} \mathcal{A}_{nf}^{pA}(s,t) &= (1+\rho^{pA})\frac{\sigma_{\rm tot}^{pA}(s)}{4\pi}\,\exp\left(\frac{B^+}{2}t\right), \\ \mathcal{A}_{sf}^{h}(s,t) &= (k_2+ik_1)\frac{\sqrt{|t|}\sigma_{\rm tot}^{pA}(s)}{4\pi}\,\exp\left(\frac{B^-}{2}t\right). \end{aligned} \tag{10}$$

We take $\rho^{pA} = \rho^{pp}/2$, as a_2 and ρ contributions decrease in the nucleus. It is possible that in hadron scattering the ratio of the spin-non-flip to the asymptotic part of the spin-flip amplitude decreases very slowly with energy. In this case, if we take in our analysis only this part of the spin-flip amplitude, we cannot make its real part proportional to ρ_{pp} in this energy region.

For the determination of $\mathcal{A}_{nf}^{pA}(s,t)$, we rely on the data obtained by the SELEX Collaboration [7]. We also will consider the possibility of normalizing B^+ on the experimental data of [6]. We assume that the slope slowly rises with $\ln s$ in a way similar to the pp case.

According to the above analysis we investigate two variants for the slope of the spin-flip amplitude: case I — $B^- = B^+$; case II — $B^- = 2B^+$. The coefficients k_1 and k_2 are chosen to obtain the best description of A_N

$$A_N\frac{d\sigma}{dt} = -4\pi[{\rm Im}\,(\mathcal{A}_{nf})\,{\rm Re}\,(\mathcal{A}_{sf}) - {\rm Re}\,(\mathcal{A}_{nf})\,{\rm Im}\,(\mathcal{A}_{sf})] \tag{11}$$

at $p_L = 24,\ 100$ GeV/c. Of course, we only aim at a qualitative description as the data are only preliminary and as they are normalized to those at $p_L = 22$ GeV/c [4]. In Figs. 2 and 3, the calculations are made at $p_L =\ 100$ GeV/c for the different normalization of the slope B^+ (on data of [6] and [7]). At $p_L = 100$ GeV/c, they give $B^+ = 58.3$ GeV^{-2} and $B^+ = 72.1$ GeV^{-2}, respectively. It is clear that this difference changes the size of A_N at $|t| \geqslant 0.02$ GeV^2 only slightly. We can see that in both cases we obtain a small energy dependence. In case I, the t-dependence of A_N is weaker immediately after the maximum. But at large $|t| \geqslant 0.01$ GeV^2, the behaviour of A_N is very different: we obtain different signs for A_N at $|t| \approx 0.06$ GeV^2. In case I, when $B^- = B^+$, A_N changes its sign in the region $|t| \approx 0.02$ and then grows in magnitude.

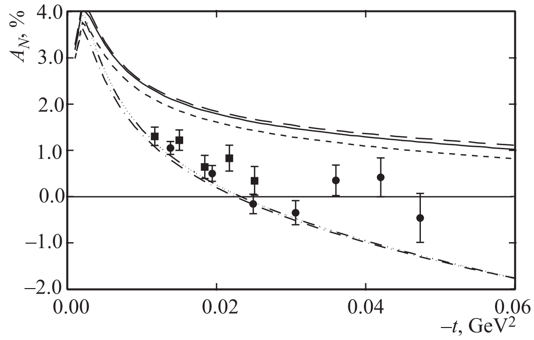

Fig. 2. The A_N without (upper 3 curves) and with hadron-spin-flip amplitude in case I ($B^- = B^+$) for $p_L = 24, 100, 250$ GeV/c (dash-dot, dashed-dots, and dots correspondingly)

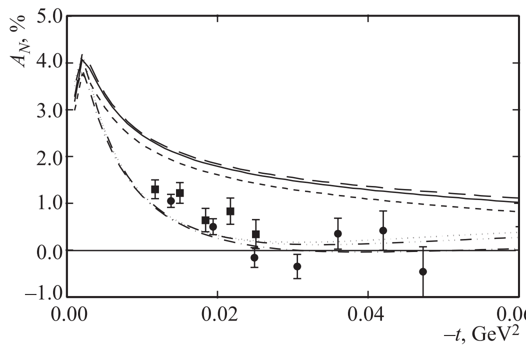

Fig. 3. The A_N with hadron-spin-flip amplitude in case II ($B^- = 2B^+$) for $p_L = 24, 100, 250$ GeV/c (dash-dot, dashed-dots, and dots correspondingly)

In case II, when $B^- = 2B^+$, A_N approaches zero and then grows positive again. It is interesting to note that in more complex cases [8], where one investigates the analyzing power for p^{12}C reaction in case I, but with a more complicated form factor, one again obtains the possibility that the slope of the hadron spin-flip exceeds the value 60 GeV^{-2}, and one can show that both slopes at very small momentum transfer are equal to about 90 GeV^{-2}. Of course, such a large slope for the spin-non-flip amplitude requires additional explanations and cannot be obtained in the standard Glauber approach.

We should note that all our considerations are based on the usual assumptions that the imaginary part of the high-energy scattering amplitude has an exponential behaviour. The other possibility, that the slope changes slightly when $t \to 0$, requires a more refined discussion that will be the subject of a subsequent paper.

REFERENCES

1. *Martini A. F., Predazzi E.* // Phys. Rev. D. 2002. V. 66. P. 034029.
2. *Selyugin O. V.* // Phys. Rev. D. 1999. V. 60. P. 074028.
3. *Predazzi E., Soliani G.* // Nuovo Cim. A. 1967. V. 2. P. 427;
Hinotani K. et al. // Nuovo Cim. A. 1979. V. 52. P. 363.

4. *Tojo J. et al. (BNL-AGS E250 Collab.)* // Phys. Rev. Lett. 2002. V. 89. P. 052302; hep-ex/0206057.
5. *Predazzi E., Selyugin O. V.* // Eur. Phys. J. A. 2002. V. 13. P. 471.
6. *Schiz A. et al.* // Phys. Rev. D. 1980. V. 21. P. 3010.
7. *Dersch U. et al. (SELEX Collab.)* // Nucl. Phys. B. 2000. V. 579. P. 277; hep-ex/9910052.
8. *Selyugin O. V., Cudell J.-R.* hep-ph/0301048; Proc. «Spin and Symmetry», Prague, July 12–19, 2003.