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UNIVERSALITY OF T-ODD FRAGMENTATION FUNCTIONS

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Recently, a lot of attention has been devoted to the influence of collinear gluon exchange in deep inelastic scattering and related processes. The most striking observable effect of the gluon exchange are transverse single spin asymmetries. In factorization formulae this effect is encoded in the gauge link appearing in the operator definition of parton distributions and fragmentation functions. The presence of the gauge link, however, makes the universality of such correlation functions a nontrivial issue. The main steps of these recent developments are summarized with a particular emphasis on the question of universality of T-odd fragmentation. A one-loop calculation suggests that T-odd fragmentation functions, in contrast to the corresponding parton distributions, are universal.

Приведен краткий обзор влияния глюонной связи на универсальность T-нечетных функций фрагментации. Явные однопетлевые расчеты подтверждают, что T-нечетные функции фрагментации, в противоположность соответствующим функциям распределения, являются универсальными.

INTRODUCTION

For inclusive DIS the importance of gluon exchange between the struck quark and target spectators has been emphasized recently [1]. It has been demonstrated that this rescattering effect causes (additional) on-shell intermediate states in the Compton amplitude, resulting in a shadowing contribution to the DIS cross section. In Feynman gauge, this shadowing effect is described by the gauge link appearing in the definiton of parton distributions.

Subsequently, the effect of rescattering has also been investigated in the case of semi-inclusive DIS [2]. Using a simple model, it has been shown that a transverse single target-spin asymmetry arises from the interference between the tree-level amplitude of the fragmentation process and the imaginary part of the one-loop amplitude, where the latter describes the gluon exchange between the struck quark and the target system. This asymmetry has been interpreted as a model for the time-reversal odd (T-odd) and transverse momentum dependent ($k_\perp$-dependent) Sivers function [3] including its gauge link [4].

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Since the gauge link in DIS and in the Drell–Yan process runs along a different path, one deals a priori with different parton distributions in both cases. Nevertheless, one can relate both definitions by applying time-reversal. This leads to a very interesting consequence which has been discussed in Ref. 4: the Sivers asymmetry in semi-inlcusive DIS has the opposite sign compared to the one in Drell–Yan, i.e., the Sivers function is nonuniversal.

In the following these issues will be treated in more detail [5]. (We do not discuss here an interesting possible relation of the Sivers asymmetry to a spin-dependent generalized parton distribution, which has been suggested very recently [6].) The main part of this note concerns the study of universality for T-odd spin-dependent fragmentation functions, where we focus on the fragmentation of an unpolarized quark into a transversely polarized spin-$1/2$ hadron [7] — the fragmentation counterpart of the Sivers parton distribution. To this end we investigate the gluon exchange incorporated in the gauge link of the fragmentation functions for both e^+e^- annihilation and semi-inclusive DIS. In a one-loop model calculation we find universality of T-odd fragmentation [7]. We point out that a corresponding calculation leads also to universality of the T-odd Collins fragmentation function [8].

1. GAUGE-INVARIANT CORRELATION FUNCTIONS

In inclusive DIS the longitudinal momentum fraction of the struck parton is fixed by external kinematics, while its transverse momentum cannot be measured but is rather integrated over. This situation changes in semi-inclusive lepton nucleon scattering (like $e^- p \to e^- HX$), where in addition to the scattered electron a hadron H is detected in the final state. If the cross section is kept differential in the transverse momentum $\mathbf{P}_{H\perp}$ of the hadron, one is sensitive to both the transverse momentum on the parton distribution side (transverse momentum of the quark relative to the target) and on the fragmentation side. (We limit ourselves to the kinematics $|\mathbf{P}_{H\perp}| \ll Q$, with Q denoting the mass of the exchanged gauge boson.) At tree level, it is relatively easy to derive a factorization formula for this process [9].

There are two more reactions which can be described within the same formalism. One is the Drell–Yan process (like $pp \to \mu^+\mu^- X$), where the transverse momentum of the lepton-pair is small compared to the mass of the timelike photon. The other is $e^+e^- \to H_1 H_2 X$, where the two detected hadrons have a low transverse momentum relative to each other. The factorization for this class of processes becomes highly nontrivial once gluonic corrections are included. One source of the complications is the presence of soft gluons, which typically lead to an additional nonperturbative factor (Sudakov factor) in a factorization formula. (In the case of the one-loop calculations for T-odd correlation functions to be discussed below, these difficulties don't show up yet. Therefore, we refrain from elaborating more on this issue here.) A treatment to all orders in the strong coupling constant has been given at least for the e^+e^--annihilation process [10].

We now specify the gauge invariant correlation functions entering a factorized description of this type of processes. The correlator through which the $k_\perp$-dependent parton distributions are defined reads

$$\Phi_{ij}(x, \mathbf{k}_\perp, S) = \int \frac{d\xi^- \, d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} \, \mathrm{e}^{ik\cdot\xi} \, \langle P, S \,|\, \bar{\psi}_j(0) \, \mathcal{W}(0,\xi) \, \psi_i(\xi) \,|\, P, S\rangle \Big|_{\xi^+=0} . \quad (1)$$

The target state is characterized by its four-momentum P and the covariant spin vector S ($P^2 = M^2$, $S^2 = -1$, $P \cdot S = 0$). The quark momentum is k with $k^+ = xP^+$. The quark fields carry a Dirac index, while flavor and color indices are suppressed. The color gauge invariance of the correlator is ensured by the Wilson line $\mathcal{W}(0,\xi)$, where the suitable path of this line is discussed below. The gauge link encodes the reinteraction of the struck quark with the target system via the exchange of collinear gluons. The $k_\perp$-dependent parton distributions can be obtained from Eq. (1) using suitable projections. For instance, the unpolarized quark distribution is given by $f_1(x, \mathbf{k}_\perp^2) = \mathrm{Tr}\,(\Phi\gamma^+)/2$.

In contrast to the correlation functions which show up in inclusive processes, with two quark fields separated only along one light-cone direction (here the ξ^- direction), the quark fields in Eq. (1) have also a separation in transverse position space. Moreover, a proper definition of $k_\perp$-dependent parton distributions requires that the gauge link in (1) has the form [4, 11, 12]

$$\mathcal{W}(0,\xi)_{\mathrm{DIS}} = [0,0,\mathbf{0};0,\infty,\mathbf{0}] \times [0,\infty,\mathbf{0};0,\infty,\boldsymbol{\xi}_\perp] \times [0,\infty,\boldsymbol{\xi}_\perp;0,\xi^-,\boldsymbol{\xi}_\perp]. \quad (2)$$

In this equation, $[a^+, a^-, \mathbf{a}_\perp; b^+, b^-, \mathbf{b}_\perp]$ denotes the Wilson line connecting the points $a^\mu = (a^+, a^-, \mathbf{a}_\perp)$ and $b^\mu = (b^+, b^-, \mathbf{b}_\perp)$ along a straight line.

It is now important to note that the path of the gauge link depends on the process. For DIS the future-pointing Wilson line in (2) has to be taken, while for Drell–Yan one has a past-pointing line [4, 12, 13],

$$\mathcal{W}(0,\xi)_{\mathrm{DY}} = [0,0,\mathbf{0};0,-\infty,\mathbf{0}] \times \\ \times [0,-\infty,\mathbf{0};0,-\infty,\boldsymbol{\xi}_\perp] \times [0,-\infty,\boldsymbol{\xi}_\perp;0,\xi^-,\boldsymbol{\xi}_\perp]. \quad (3)$$

As a consequence, the definitions of $k_\perp$-dependent parton distributions are different for both processes. In other words, one is dealing with a potential nonuniversality for these objects. The same feature appears in the case of fragmentation functions, which a priori have a different gauge link in semi-inclusive DIS and in e^+e^- annihilation [13]. Whether and how one can still obtain universality for parton distributions and fragmentation functions will be discussed in the following two sections.

Another interesting discussion arises when considering the correlator in Eq. (1) for different gauges. For example in Feynman gauge the Wilson line at the light-cone infinity (second piece on the r.h.s. in Eqs. (2), (3)) can be neglected. In contrast, this line needs to be taken into account in light-cone gauge [12], in which the transverse gluon potential doesn't vanish at the light-cone infinity.

For the subsequent discussion, T-odd correlators will be of particular importance. It turns out that two (out of eight) $k_\perp$-dependent parton distributions are T-odd and can appear at leading order in a $1/Q$-expansion (twist-expansion) of observables. The same holds for T-odd fragmentation functions. The following list (in the notation of Ref. 14) summarizes these functions and their meaning:

- $f_{1T}^{\perp}$: distribution of an unpolarized quark in a transversely polarized target [3];
- $h_1^{\perp}$: distribution of a transversely polarized quark in an unpolarized target [15];
- $D_{1T}^{\perp}$: fragmentation of an unpolarized quark into a transversely polarized hadron [14];
- $H_1^{\perp}$: fragmentation of a transversely polarized quark into an unpolarized hadron [8].

Typically, T-odd correlation functions give rise to azimuthal/single spin asymmetries. In 1993, in connection with the Sivers function $f_{1T}^{\perp}$, it was proven that T-odd parton distributions should vanish because of time-reversal invariance [8]. (This claim later on needed to be revised as will be explained in Sec. 2.) In contrast, T-odd fragmentation functions may well exist because of final state interactions taking place in the fragmentation process, which give rise to the required nontrivial phase (imaginary part) in the scattering amplitude.

2. RESCATTERING AND T-ODD PARTON DISTRIBUTIONS

We now turn attention to possible observable effects of the Wilson line, i.e., the rescattering of the struck quark via the exchange of collinear gluons. It was demonstrated in 2001, that even for inclusive DIS this rescattering influences the cross section [1]. The gluon exchange leads to additional on-shell intermediate states in the forward Compton amplitude, yielding a reduction of the cross section which may be interpreted as shadowing effect [1].

One may wonder now whether an observable exists, which is *entirely* connected with the gauge link. Such a situation was discussed in the context of a simple spectator model calculation for a transverse single target spin asymmetry in DIS [2]. The relevant process is

$$\gamma^* + p \to q + s, \tag{4}$$

and is shown in Fig. 1. The quark q may fragment or form a jet. It is only important that the transverse momentum of the outgoing quark is detected. The spectator s is a scalar diquark. The proton is polarized transversely with respect to its momentum. A tree level calculation, i.e., taking only the diagram in Fig. 1, *a*, leads to a vanishing asymmetry because the scattering amplitude has no imaginary part. In order to obtain an imaginary part, loop corrections need to be included. (The gluon loops have been modelled by an Abelian gauge field in Ref. 2.) It is easy to see that at one-loop only the box graph in Fig. 1, *b*, which describes the reinteraction of the struck quark with the target system to lowest order, gives rise to an imaginary part. Now one indeed obtains a nonzero transverse single spin asymmetry which is given by the interference of the tree-level amplitude A with the imaginary part of the one-loop amplitude B,

$$\mathcal{A}_{\perp} = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}} = A \times \operatorname{Im} B \neq 0. \quad (5)$$

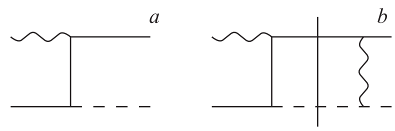

Fig. 1. Tree-level (*a*) and relevant one-loop (*b*) contribution for single spin asymmetry in DIS in spectator model (see also text). The spectator is indicated by a dashed line. The thin line characterizes the possible on-shell intermediate state

We re-emphasize that the spin asymmetry would be zero if there were no reinteraction of the struck quark. In other words, for this observable the rescattering enters in a maximal possible way.

In Ref. 2 it has been stated that the asymmetry in (5) is a new effect, and that this effect is not compatible with factorization. However, later on [4] it was found that the asymmetry obtained in [2] is nothing else but a model for the T-odd Sivers function including its gauge link. Therefore, the result of [2] is neither a new effect nor it is necessarily in contradiction with factorization. However, in Ref. 2 it has been demonstrated for the first time *explicitly*, that T-odd parton distributions can be nonzero. It turned out that the proof, according to which T-odd parton distributions should vanish [8], no longer holds once the path-ordered exponential is taken into account [4].

As mentioned in the previous section, the parton distributions in DIS and Drell–Yan have a different gauge link, which endangers the universality of these objects. Nevertheless, the two definitions can still be related using time-reversal. One finds that all six T-even $k_{\perp}$-dependent parton distributions are in fact universal, while a violation of universality appears only for the two T-odd functions in the sense that they have a reversed sign in both processes [4],

$$f_{1T}^{\perp}\Big|_{\text{DIS}} = -f_{1T}^{\perp}\Big|_{\text{DY}}, \quad (6)$$

$$h_{1}^{\perp}\Big|_{\text{DIS}} = -h_{1}^{\perp}\Big|_{\text{DY}}. \quad (7)$$

From a practical point of view this violation of universality has no direct consequence, since the possibility of relating cross sections of different processes is not spoiled. An experimental investigation of relations (6), (7) would serve as a very important check of our present-day understanding of the nature of T-odd parton distributions.

3. RESCATTERING AND T-ODD FRAGMENTATION FUNCTIONS

We now wish to investigate the corresponding influence of the Wilson line on the fragmentation process. Because of their operator structure, fragmentation functions in DIS and e^+e^- annihilation cannot be related using time reversal (see also [13]). Hence, one has to conclude that, unless another argument comes to our aid, fragmentation functions are nonuniversal.

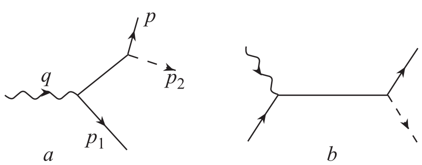

Fig. 2. Tree-level diagrams of fragmentation in e^+e^- annihilation (*a*) and semi-inclusive DIS (*b*). In both cases a quark fragments into a spin-$1/2$ hadron and a scalar remnant (dashed line)

To investigate this point we have performed a model calculation for T-odd fragmentation (unpolarized quark into transversely polarized hadron, i.e., model for $D_{1T}^{\perp}$) [7], which is quite similar to the calculation of Ref. 2 discussed in the previous section.

In Fig. 2, the tree-level diagrams of the fragmentation in e^+e^- annihilation and in DIS are displayed. For e^+e^- annihilation we consider the decay of a timelike virtual photon into a $q\bar{q}$ pair, where the quark fragments into a spin-$1/2$ hadron (e.g., a proton) and a scalar remnant, i.e.,

$$\gamma^*(q) \to \bar{q}(p_1, \lambda') + p(p, \lambda) + s(p_2). \quad (8)$$

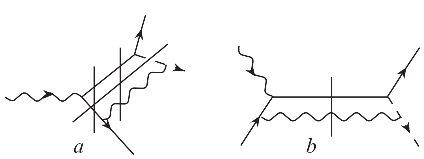

Fig. 3. One-loop diagrams of fragmentation in e^+e^- annihilation (*a*) and semi-inclusive DIS (*b*). The possible on-shell intermediate states are indicated by thin lines

The fragmentation of the quark is described in the model of Ref. 2.

The one-loop corrections are shown in Fig. 3. For e^+e^- annihilation (semi-inclusive DIS) a single photon is exchanged between the remnant and the antiquark (initial quark). These diagrams provide a simple Abelian model for the lowest order contribution of the gauge link of the fragmentation function. Two cuts (on-shell quark and antiquark, as well as on-shell antiquark and remnant) for e^+e^- annihilation have no counterpart in semi-inclusive DIS. The quark-photon cut in e^+e^- annihilation corresponds to the cut in DIS.

In contrast to T-odd parton distributions, there is another source — beyond the rescattering encoded in the gauge link — that leads to T-odd fragmentation. Because of final state interactions in the fragmentation process, imaginary parts can appear in the scattering amplitude for the fragmenting quark is necessarliy timelike (for those Feynman graphs in which the gauge link doesn't enter) [8,16]. Such effects are obviously universal, and therefore won't be discussed in the following.

To proceed with the calculation, we consider e^+e^- annihilation in the rest frame of the timelike photon. The proton in the final state has no transverse momentum, and its minus-momentum is given by zq^-, where q^- is the minus-momentum of the virtual photon. We fix the plus-momentum of the antiquark according to $p_1^+ \approx q^+$. The antiquark also carries a soft transverse momentum $-\mathbf{\Delta}_\perp$, implying that the fragmenting quark and the outgoing proton have a relative transverse momentum, which is necessary for the transverse spin asymmetry. These requirements specify the kinematics:

$$
\begin{aligned}
q &= \Big(Q, Q, \mathbf{0}_\perp\Big), & p_1 &= \Big(Q, \frac{\mathbf{\Delta}_\perp^2 + m_q^2}{Q}, -\mathbf{\Delta}_\perp\Big), \\
p &= \Big(\frac{M^2}{zQ}, zQ, \mathbf{0}_\perp\Big), & p_2 &= \Big(\frac{\mathbf{\Delta}_\perp^2 + m_s^2}{(1-z)Q}, (1-z)Q, \mathbf{\Delta}_\perp\Big).
\end{aligned} \tag{9}
$$

For simplicity, in (9) only the leading terms have been listed.

In the model of Ref. 2, the proton carries no electromagnetic charge. Therefore, the charge of the fragmenting quark (denoted by e_1) and the one of the remnant are equal. The interaction between the quark, the proton, and the remnant is described by a scalar vertex with the coupling constant g. The x component of the current for the diagram in Fig. 2, *a* reads

$$
\begin{aligned}
J^1_{(0)}(\lambda, \lambda') &= e_1\, g\, \frac{1}{s - m_q^2}\, \bar{u}(p,\lambda)\, (\not{q} - \not{p}_1 + m_q)\, \gamma^1\, v(p_1, \lambda') = \\
&= e_1\, g\, \frac{1-z}{\sqrt{z}}\, \frac{Q}{\mathbf{\Delta}_\perp^2 + \tilde{m}^2} \left[(\Delta^1 - i\lambda\Delta^2)\delta_{\lambda,-\lambda'} - \lambda \left(\frac{M}{z} + m_q \right) \delta_{\lambda,\lambda'} \right],
\end{aligned} \tag{10}
$$

$$
\text{with} \quad \tilde{m}^2 = \frac{1}{z} \Big(M^2\, \frac{1-z}{z} + m_s^2 - m_q^2\,(1-z) \Big),
$$

where use has been made of the relation

$$
s - m_q^2 = (q - p_1)^2 - m_q^2 = \frac{z}{1-z} \Big(\mathbf{\Delta}_\perp^2 + \tilde{m}^2 \Big), \tag{11}
$$

which connects the total energy $\sqrt{s}$ in the c.m. frame of the outgoing proton and scalar remnant with the variables z and $\boldsymbol{\Delta}_\perp^2$.

The transverse spin asymmetry $\mathcal{A}_x$ (polarization of the proton along x axes) is given by $\sigma_{\rm pol}/\sigma_{\rm unp}$, with the cross sections evaluated according to

$$\begin{aligned}
\sigma_{\rm unp} &\propto \frac{1}{2}\sum_{\lambda,\lambda'} J^1(\lambda,\lambda')\Big(J^1(\lambda,\lambda')\Big)^*, \\
\sigma_{\rm pol} &\propto \frac{1}{2}\sum_{\lambda'} \Big[J^1(s_x=\uparrow,\lambda')\Big(J^1(s_x=\uparrow,\lambda')\Big)^* - \\
&\qquad -J^1(s_x=\downarrow,\lambda')\Big(J^1(s_x=\downarrow,\lambda')\Big)^*\Big].
\end{aligned} \tag{12}$$

While $\sigma_{\rm unp}$ is computed using the tree-level current, in $\sigma_{\rm pol}$ the imaginary part of the one-loop amplitude is inlcuded to obtain a nonzero asymmetry.

It turns out that for e^+e^- annihilation the contributions to the spin asymmetry caused by two on-shell intermediate states (on-shell quark and antiquark, as well as on-shell antiquark and remnant) cancel against each other (to leading order in $1/Q$) [7]. Therefore, we are left only with the on-shell $q\gamma$ intermediate state, for which one obtains the following asymmetry [7]:

$$\begin{aligned}
\mathcal{A}_{x,q\gamma} = &-\frac{(e_1)^2}{8\pi}\,\frac{2(M/z)\Delta^2}{(M/z)^2+\boldsymbol{\Delta}_\perp^2}\,\frac{\boldsymbol{\Delta}_\perp^2+\tilde{m}^2}{\boldsymbol{\Delta}_\perp^2}\Bigg[\ln\frac{p_{20}-|\mathbf{p}_2|\cos\alpha}{m_s}+ \\
&+\cos\alpha\ln\frac{p_{20}+|\mathbf{p}_2|}{m_s}+\frac{1-\cos^2\alpha}{4(1-z)}\left(1-\frac{p_{20}}{|\mathbf{p}_2|}\ln\frac{p_{20}+|\mathbf{p}_2|}{m_s}\right)\Bigg],
\end{aligned} \tag{13}$$

where the result holds for $m_q = 0$. In Eq. (13) the energy and the three-momentum of the remnant, as well as the scattering angle (angle between the antiquark and the remnant) in the c.m. frame of the proton and the remnant appear. These variables can be expressed in terms of z and $\boldsymbol{\Delta}_\perp^2$ (see Ref. 7).

By explicit calculation we have shown that for semi-inclusive DIS the spin-asymmetry caused by the on-shell $q\gamma$ state coincides with the result in (13), i.e., the asymmetry in both processes has even the same sign. This means, we do not observe a sign-reversal for this contribution as was the case for T-odd parton distributions. The origin for the different behavior can be directly traced back to the different kinematics one has to deal with in fragmentation.

In summary, our calculation shows that the total transverse spin asymmetries in e^+e^- annihilation (summing the contributions of all three on-shell intermediate states) and in semi-inclusive DIS are equal, i.e., the T-odd fragmentation of an unpolarized quark into a transversely polarized spin-$1/2$ hadron is universal in a one-loop model. Therefore, we observe universality for the fragmentation function $D_{1T}^\perp$. The same conclusion holds for the Collins function as well, since the imaginary part due to the $q\gamma$ cut for the four independent helicity amplitudes is equal for both processes. Hence, we have the results

$$D_{1T}^{\perp}\Big|_{\mathrm{DIS}} = D_{1T}^{\perp}\Big|_{e^+e^-}, \tag{14}$$

$$H_{1}^{\perp}\Big|_{\mathrm{DIS}} = H_{1}^{\perp}\Big|_{e^+e^-}. \tag{15}$$

It is relatively easy to see that the universality we have obtained in this specific one-loop calculation is a general one-loop result, i.e., it doesn't depend on the model we are using for the fragmentation [7]. It remains to be seen whether universality of fragmentation functions holds to all orders. There is work in progress to clarify this important issue [17].

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