ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2010. Т. 41. ВЫП. 6

CATALYSIS OF BLACK HOLES/WORMHOLES FORMATION IN HIGH-ENERGY COLLISIONS *

I. Ya. Aref'eva

Steklov Mathematical Institute, Russian Academy of Sciences, Moscow

The current paradigm suggests that BH/WH formation in particles collisions will happen when a center-of-mass energy of colliding particles is sufficiently above the Planck scale (the trans-Planckian region). We confirm the classical geometrical cross section of the BH production, reconsidering the process of two trans-Planckian particles collision in the rest frame of one of incident particles. This consideration permits us to use the standard Thorne's hoop conjecture for a *matter* compressed into a region to prove a variant of the conjecture dealing with a total amount of compressed *energy* in the case of colliding particles. We briefly mention that the process of BH formation is catalyzed by the negative cosmological constant and by a particular scalar matter, namely dilaton, while it is relaxed by the positive cosmological constant and at a critical value just turns off.

PACS: 97.60.Lf

1. INTRODUCTION

Gravity does not play a role in the usual high-energy terrestrial physics. However, in the TeV gravity scenario [1] the processes with energy about few TeV become trans-Planckian and the gravity is important.

Black hole formation in collisions of trans-Planckian particles is one of outstanding problems in theoretical physics. Our aim in this talk is to overview the current understanding of the problem.

Study of trans-Planckian collisions in gravity has a long history. In the 1980s–1990s the problem was discussed mainly in superstring theory frameworks [2–6] and was considered as an academical one, since the four-dimensional Planck scale $E_{\rm Pl}$ is $\approx 10^{19}$ GeV, and energies satisfying $\sqrt{s} > E_{\rm Pl}$ are wholly out of reach of terrestrial experiments.

The situation has been changed after the proposal of TeV gravity scenario [1]. The D-dimensional Planck energy $E_{{\rm Pl},D}$ plays the fundamental role in TeV gravity, it has the electroweak scale of $\sim$ TeV, as this would solve the hierarchy problem. The TeV gravity is strong enough to play a role in elementary particle collisions at accessible energies.

---

*Extended version of the talk is available at arXiv:0912.5481.

The TeV gravity assumes the brane world scenario [8] that means that all light particles (except gravity) are confined to a brane with the 4-dimensional world sheet embedded in the D-dimensional bulk. The collider signatures of such brane world scenarios would be energy nonconservation due to produced gravitons escaping into the bulk, signatures of new Kaluza–Klein particles as well as signatures of creating black holes (BH) [9–13] and more exotic objects, such as wormholes (WH) or time machines [14–16].

According to the common current opinion, the process of BH formation in trans-Planckian collisions of particles may be adequately described by the classical general relativity. We also believe that the same is true for the WH creation [14]. Calculations based on the classical general relativity support [17, 18] the simple geometrical cross section of black hole production in particles collisions, which is proportional to the area of the disk

$$\sigma = f\pi\, R_S^2(E), \tag{1}$$

where R_S is the Schwarzschild radius of the black hole formed in the particles scattering process and it is defined by the center-of-mass collision energy $E = \sqrt{s}$, and f is a formation factor of order unity. Colliding particles in hadron colliders are partons and the total cross section for black hole production is calculated using a factorization hypothesis in which the parton-level process is integrated over the parton density functions of the protons [19]. If the geometric cross section were true and colliding particles carried few TeV, the LHC would produce black holes at a rate of ~ 1 Hz for $M_{\mathrm{Pl},D} = 1$ TeV, becoming a black hole factory [11, 13].

However, BH formation in particle collisions is a threshold phenomena and the threshold is of order the Planck scale $M_{\mathrm{Pl},D}$ [20]. The exact value of the threshold is unknown since it depends on quantum gravity description of colliding particles. BH production rates depend on the value of $M_{\mathrm{Pl},D}$ [21]. Current bounds [22] are dimension-dependent but lie around $M_{\mathrm{Pl},D} \gtrsim 1$ TeV. Taking simple estimation for cross section (1) with $f \sim 1$ above the threshold, one can conclude that the cross section of semiclassical BH production above the threshold at the LHC varies between 15 and 1 nb for the Planck scale between 1 and 5 TeV. Note that this cross section is compatible, for example, with $t\bar{t}$ production [23]. Just after production BHs quickly ($\sim 10^{-26}$ s) evaporate via Hawking radiation [24] with a characteristic temperature of ~ 100 GeV [11, 13]. However, since produced BHs are light they decay into only a few high-energy particles and this would be difficult to disentangle from the background [25].

A natural question arises: can we catalyze the semiclassical process of the BH formation and increase the production factor f in (1)? This is the main question that we are going to discuss in this talk. Let us note that in this talk we are going to deal with semiclassical consideration and make a few notes of the region of its applicability. We will search for theoretical possibilities to increase the formation factor in the formula for the geometrical cross section. There are effects that work in the opposite direction and push the collision energy needed for BH formation considerably higher than $M_{\mathrm{Pl},D}$. These effects are related with the energy loss by colliding particles prior to the formation of the BH horizon [26] and the effects of the charge [27].

In fact there are few possibilities at our disposal to increase the formation factor. We are going to explore the following proposals:

• find effects related with nontrivial dynamics of 3-brane embedding in D-dimensional space-time;

• change the background (4-dimensional background or D-dimensional one), in particular, we can add the 4-dimensional cosmological constant (or cosmological constant in D-dimensional space-time);

• take into account that shock wave in D-dimensional space-time can be made of closed string excitations.

Some attempts toward these directions are presented in the extended version of this talk [28]. Here we show that the process of BH formation is catalyzed by the negative cosmological constant and by a special scalar matter. In contrast, it is relaxed by the positive cosmological constant and at a critical value just turns off [29,30]. Also, we note that the cross section is sensible to the compactification of extra dimensions and particular brane models, and this will be studied in a separate paper in detail.

In the last years numerous papers have been devoted to improvement calculations based on classical general relativity to get more precise estimates for the cross section (1) [27, 31]. In particular, numerical calculations have been performed and they confirmed (1) and gave the estimations for the production factor [32]. The effects of finite size [33,34], charge [27] and spin [35] have been considered. It has been found that the effects of mass, spin, charge and finite size of the incoming particles are rather small. The effects of the cosmological constant have been considered in particular in papers [29,30] (see references therein). In these papers, estimation of the cross section of the BH production for particles colliding in (A)dS backgrounds has been made. The AdS case has been studied mainly within the AdS/CFT context, and the dS case within possible cosmological applications [29, 30]. It has been found that the negative cosmological constant increases the cross section, meanwhile the positive cosmological constant works in the opposite direction, destroying the trapped surface at the critical value of the cosmological constant and for this reason presumably holding up the BH production.

Quantum field theory is a local theory in the Minkowski space [36, 37]. However, if we take into account effects of quantum gravity, then some form of nonlocality occurs. The problem of (non)locality in quantum gravity was addressed in [2, 4, 38], and more recently in [39].

2. BLACK HOLE PRODUCTION

2.1. D-dimensional Planckian Energy. In TeV gravity scenario we assume that all particles and fields (except gravity) are confined to a brane with 4-dimensional world sheet embedded in the D-dimensional bulk. Matter fields leave on the brane and do not feel extra dimensions, only gravity feels $n = D-4$ extra dimensions, see Fig. 1.

According to the common current opinion, the process of BH formation in trans-Planckian collision of particles, i.e., in regions where

$$\sqrt{s} \gg E_{\rm Pl}, \tag{2}$$

may be adequately described using classical general relativity. We also believe that the same concerns the WH production. This is because in the trans-Planckian region (2) the de Broglie wavelength of a particle

$$l_{\mathcal{B}} = \frac{\hbar c}{E} \tag{3}$$

is less than the Schwarzschild radius corresponding to this particle,

$$l_{\mathcal{B}} \ll R_{S,D}, \tag{4}$$

here $R_{S,D}$ is the D-dimensional Schwarzschild radius in TeV gravity. In phenomenologically reasonable models with $n \geqslant 2$ the Schwarzschild radius corresponding to colliding particles with energies ≈ 1 TeV is $R_{S,D} \gtrsim 10^{-16}$ cm. In the usual 4-dimensional gravity the Schwarzschild radius corresponding to the same particles is of order $R_{S,4} \sim 10^{-49}$ cm, that is, a negligible quantity compared with the de Broglie wavelength of particles with energy about few TeV.

Although these types of processes are classical, it is instructive to have a full picture starting from a general quantum field theory setup and pass explicitly to the classical description of processes in question. This point of view is useful to deal with effects on the boundary of the classical applicability. For this reason we start in the next subsection from this general setup [7]; for the brane extension of this approach, see [10, 28].

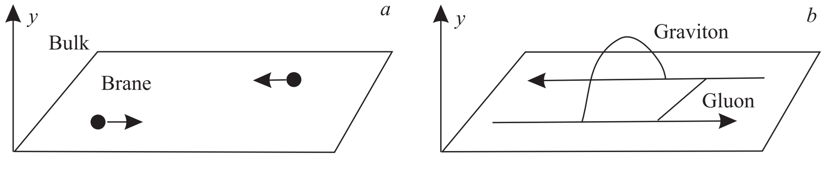

Fig. 1. *a*) Colliding particles on the brane. *b*) D-dimensional graviton and 4-dimensional gluon exchanges

2.2. Transition Amplitudes and Cross Section of the BH/WH Production. We start from quantum mechanical formula for the cross section σ_{AB} of a process $|A\rangle \Rightarrow |B\rangle$. To calculate this cross section, we calculate the transition amplitude between these states

$$\langle A|B\rangle = \int \Psi_A^*(X_A, t)\mathcal{K}(X_A, t; X_B, t')\Psi_B(X_B, t')\, dX_A\, dX_B, \tag{5}$$

where X are generalized coordinates, specifying the system, $\Psi_A(X, t)$ is a wave function of the state A including its asymptotical dynamics. The transition amplitude in the generalized coordinate representation is given by the Feynman integral. In our case we deal not only with particles but also with gravity. In particular, we discuss the process where the final state $|B\rangle$ is the state corresponding to the black hole. For this purpose we use a modification [7] of the standard formula [40]:

• For simplicity we work in $1+3$ formalism where space-time is presented as a set of slices (more general formulation is described in [7]). At the initial time t we deal with a slice Σ and at the final time t' with a slice Σ'.

• Generalized coordinates include a metric g and matter fields ϕ.

• The state at an initial time is specified by a three-metric h_{ij} and field ϕ and the final state by a three-metric h'_{ij} and ϕ'.

• The transition amplitude in this generalized coordinate representation is given by Feynman integral [7]

$$\mathcal{K}(h, \phi, t; h', \phi', t') = \int \mathrm{e}^{\frac{i}{\hbar}S[g,\phi]} \prod_{\substack{\phi|_{\tau=t} = \phi,\, g|_{\tau=t} = h \\ \phi|_{\tau=t'} = \phi',\, g|_{\tau=t'} = h'}} \mathcal{D}\phi(\tau)\ \mathcal{D}g(\tau), \tag{6}$$

where the integral is over all four-geometries and field configurations which match given values on two space-like surfaces, Σ and Σ' and matter on them, $S[g, \phi]$ is the action. The integral in (6) includes also summation over different topologies.

• The transition amplitude given by the functional integral includes gauge fixing and Faddeev–Popov ghosts (all these are omitted in (6) for simplicity).

• We are interested in the process of a black hole creation in particle collisions. Therefore,

— we specify the initial configuration h and ϕ on Σ without black holes; i.e., causal geodesics starting from Σ reach the future null infinity*;

*More precisely, this condition means that Σ is a partial Cauchy surface with asymptotically simple past in a strongly asymptotically predictable space-time.

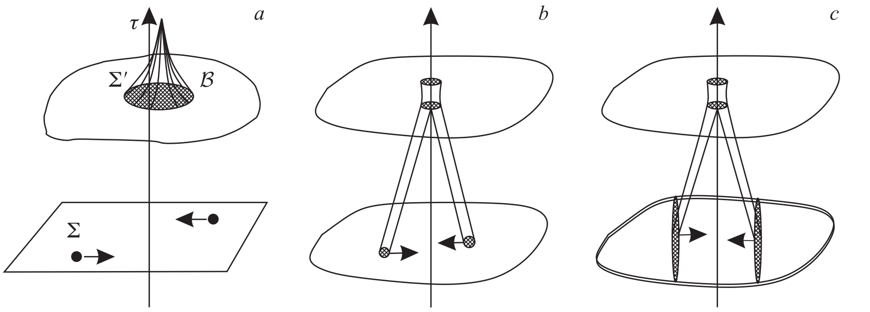

Fig. 2. *a*) A slice Σ at $\tau = t$ is an initial slice with particles and a slice Σ' at $\tau = t'$ is a slice with a black hole $\mathcal{B}$. Null geodesics started from the shaded region do not reach null infinity. *b*) Colliding two stars; the initial space-time is asymptotically flat. *c*) Colliding shock waves as models of ultrarelativistic particles; the initial space-time is not asymptotically flat

— we specify the final configuration h' and ϕ' on Σ' as describing black hole; i.e., Σ' contains a region from which the light does not rich the future null infinity*.

The explanation of notions used in the above footnotes is given in [7]; for more details, see [41, 42].

In Fig. 2, *a*, a slice with two colliding particles at $\tau = t$, and $\tau = t'$ with the BH area are presented. To describe such a process in the framework of a general approach (6), we have to find a classical solution of the Einstein equations with the matter, our moving particles, that corresponds to this picture, Fig. 2, and then study quantum fluctuations. We do not have analytical solutions describing this process.

Finding of such solutions is a very difficult problem. It is solved only in low-dimensional case, see [10, 43] and references therein. In 4-dimensional case this problem has been solved numerically only recently by Choptiuk and Pretorius [32]. The solution, as mentioned in Introduction, assumes a construction of a model for gravitational particles. We present this construction in the next subsection.

2.3. D-dimensional Gravitational Model of Relativistic Particles. To start a classical description of BH production in collision of elementary particles, we need a gravitational model of relativistic particles. At large distances the gravitational field of particle is the usual Newtonian field. The simplest way

*This means that Σ' is a partial Cauchy surface containing black hole(s); i.e., $\Sigma' - J^{-}(\mathcal{I}^{+})$ is non-empty.

to realize this is just to take the exterior of the Schwarzschild metric, i.e., in D-dimensional case away a particle we expect to have

$$ds^2 = \left(1 - \left(\frac{R_{S,D}}{R}\right)^{D-3}\right) dt^2 + \left(1 - \left(\frac{R_{S,D}}{R}\right)^{D-3}\right)^{-1} dR^2 + R^2 d\Omega^2_{D-2}, \tag{7}$$

where $R_{S,D}$ is the Schwarzschild radius

$$R^{D-3}_{S,D}(m) = \frac{16\pi G_D m}{(D-2)\Omega_{D-2}}, \quad \Omega_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}, \tag{8}$$

here G_D is D-dimensional Newton gravitational constant, here and in almost all formulas we take the speed of light $c = 1$. Γ is Euler's gamma function. We also use the expression of the Schwarzschild radius in terms of the Planck mass $M^{D-2}_{\mathrm{Pl},d} = 1/8\pi G_D$. For $D = 4$, $R_{S,4}(m) = 2G_4 m$, or $R_{S,4}(m) = m/4\pi M_4^2 = m/\bar{M}_4^2$.

The interior of the Schwarzschild metric is supposed to be filled with some matter. The simplest possibility is just to take a Tolman–Florides interior incompressible perfect fluid solution [44,45]. As another model of relativistic particles, one can consider a static spherical symmetric solitonic solution of gravity-matter equations of motion, the so-called boson stars (the authors of [46,47] deal with 4-dimensional space-time, but it is not a big deal to get D-dimensional extentions).

In the case of brane scenario a few comments are in order. In the simplest brane models we deal with matter only on the brane and we do not have matter out of the brane to fill the interior of the D-dimensional Schwarzschild solution. However, in the string scenario* there are closed string excitations which are supposed to be available in the bulk. One can assume that the matter in the bulk is a dilaton scalar field and deal with string inspired D-dimensinal generalization boson stars

$$ds^2 = \left(1 - \left(\frac{\mathcal{A}(R)}{R}\right)^{D-3}\right) dt^2 + \left(1 - \left(\frac{\mathcal{B}(R)}{R}\right)^{D-3}\right)^{-1} dR^2 + R^2 d\Omega^2, \tag{9}$$

where $\mathcal{A}(R) = A + A_1/R + \ldots$

2.4. Shock Wave as a Model of Ultrarelativistic Moving Particle. To consider ultrarelativistic moving particle, we have to make a boost of metric (9) with the large Lorentz boost factor $\gamma = 1/\sqrt{1 - v^2/c^2}$. The Schwarzschild sphere under this boost flattens up to an ellipsoid, see Fig. 3.

*Open string excitations are located on brane, closed string excitations propagate on the bulk.

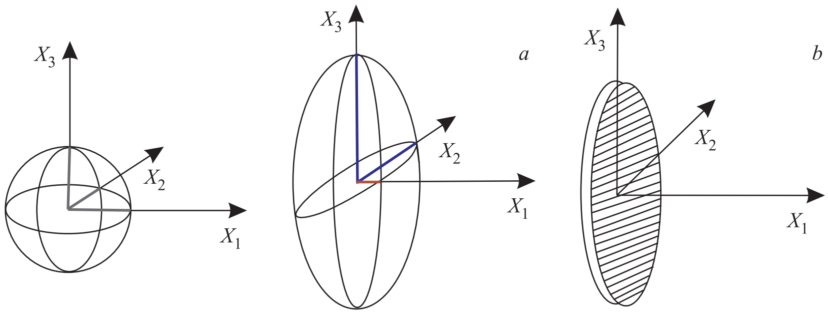

Fig. 3. *a*) Flattening of the Schwarzschild sphere in the boosted coordinates. *b*) Schematic picture for the shock wave as a flat disk

One can consider an approximation when γ is taken infinitely large and $E = \gamma A$ is fixed. The result metric is the Aichelburg–Sexl (AS) metric [48,49], a gravitational shock wave, where the nontrivial geometry is confined to a $(D-2)$-dimensional plane traveling at the speed of light, with Minkowski space-time on either side,

$$ds^2 = -2dU\,dV + dX_i^2 + F(X)\delta(U)\,dU^2, \quad i = 2, 3, \ldots, D-1, \tag{10}$$

where $V = (X^0 + X^1)/\sqrt{2}$, $U = (X^0 - X^1)/\sqrt{2}$. The form of the profile of the shock wave F depends on the behavior of $\mathcal{A}(r)$.

In particular, in the infinite boost limit where we also take $m \to 0$ and hold p fixed, the metric (7) reduces to an exact shock wave metric (10) with the shape function F being the Green function of the $(D-2)$-dimensional Laplace equation

$$\Delta_{R^{D-2}} F = -\frac{2p\sqrt{2}}{M_{\rm Pl}^{D-2}}\,\delta^{(D-2)}(X). \tag{11}$$

$F(X) = \dfrac{2\sqrt{2}p}{(D-4)\Omega M_{{\rm Pl},D}^{D-2}\rho^{D-4}}$, $\rho^2 = (X^2)^2 + \ldots (X^{D-1})^2$. For $D = 4$ the shape is

$$F(X) = -\frac{\sqrt{2}p}{\pi M_4^2}\ln\frac{\rho}{\varepsilon}. \tag{12}$$

Note that the metric (10) is obtained in the infinite boost limit when the source has zero rest mass. For fast particles of nonzero rest mass, the shock wave approximation breaks down far away from the moving particle, more precisely at transverse distances from the source which are of the order of

$$\ell \sim r_h(m)/\sqrt{1 - v^2}. \tag{13}$$

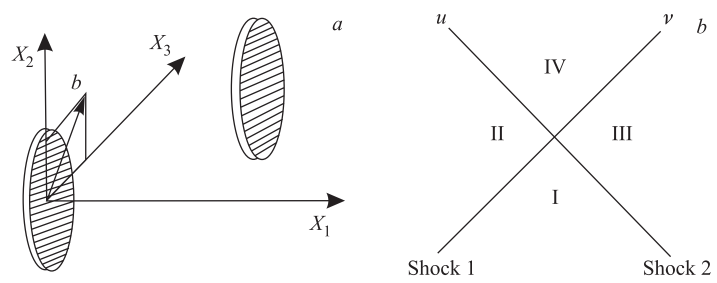

Fig. 4. *a*) Ultrarelativistic colliding particles in $(D-1)$-dimensional space; b is the $(D-2)$-dimensional impact vector. *b*) Ultrarelativistic colliding particles in U, V-plane

At these distances the field lines will spread out of the null transverse surface orthogonal to the direction of motion. But for $b \ll \ell$ one can use the shock wave field to extract the information about the black hole formation to the leading order in m/p. These shock waves are presented in Fig. 4 as sphere flattened up to the disk. Two such shock waves, moving in opposite directions (see Fig. 4, *b*), give the pre-collision geometry of the space-time. Though the geometry is not known to the future of the collision, since the shock wave solutions inevitably break down when the fields of different particles cross, at the moment of collision a trapped surface can be found [17, 18, 50].

According to [17, 18, 50], the trapped surfaces do form when $b \lesssim R_{S,D}$, and have the area of the order $\sim R^2_{S,D}$, where $R_{S,D}$ is the horizon radius given by (15) (see below).

Infinitely thin shock is an idealization. In reality the shocks will have a finite width w since γ is large but not infinite. The corresponding shocks have width $w_{\text{class}} \sim r/\gamma$, depending on the transverse distance r. Infinitely thin idealization leads to an appearance of a divergent curvature invariant in the intersection of the planes of the two shock waves [51]. In [52] it has been shown that this problem is an artifact of the unphysical classical point-particle limit and for a particle described by a quantum wavepacket, or for a continuous matter distribution, trapped surfaces indeed form in a controlled regime.

2.5. D-dimensional Thorne Hoop Conjecture and Geometrical Cross Section. We expect to get the BH formation due to nonlinear interaction of gravitational fields produced by particles. The BH formation in classical general relativity is controlled by the Thorne hoop conjecture [53]. According to the D-dimensional version of this conjecture, if a total amount of matter mass M is compressed into a spherical region of radius R, a black hole will form if R is less than the corresponding Schwarzschild radius

$$R < R_{S,D}(M), \tag{14}$$

here $R_{S,D}(M)$ is given by (8).

In the case of ultrarelativistic particle collisions, the main argument for black hole formation is based on a modification of Thorne's hoop conjecture. According to this modified conjecture, if a total amount of energy E is compressed into a spherical region of radius R, a black hole will form if R is less than the corresponding Schwarzschild radius

$$R < R_{S,D}(E) \equiv \left(\Omega_n \frac{G_D E}{c^4}\right)^{\frac{1}{n+1}}. \qquad (15)$$

Note that in this modified conjecture the horizon radius $R_{S,D}$ is set by the center-of-mass collision energy $E = \sqrt{s}$.

A few remarks are in order concerning this formulation. Literally speaking, as is formulated above, it is not applicable in all situations. But this conjecture is applicable for two colliding particles. There are several calculations and arguments supporting this conjecture:

• One set of arguments is related with examining trapped surfaces formation in collisions of ultrarelativistic particles [13, 17]. Note that commonly used evidence for black hole formation in collision of particles comes from the study of the collision of two Aichelburg–Sexl shock waves. This argument assumes that there is a solution interpolating between two shock waves and BH, Fig. 2, *c*. However, with this argument there is a problem that a space-time with a shock wave is not asymptotically flat, which is assumed in our scheme*.

• The same problem is also with colliding plane waves [7]. An advantage to deal with plane waves is that in this case one can construct explicitly the metric in the interacting region.

• There is a nontrivial possibility to reduce the proof Thorne's hoop conjecture for ultra colliding relativistic particles to Thorne's hoop conjecture for slow-moving relativistic particles (see Sec. 3 below).

• There are recent numerical calculations supporting (15) [32]. In [32] as a model of particles the boson star is taken [46]. Choptiuk and Pretorius have got a remarkable result that black holes do form at high velocities in boson star collisions and they found also that this happens already at a γ factor of roughly one-third predicted by the hoop conjecture.

On the modified Thorne's hoop conjecture for ultra colliding relativistic particles the so-called geometrical cross section of BH production is based. It estimates the black-hole-production cross section by the horizon area of a black hole whose horizon radius $R_{S,D}$ is set by the center-of-mass collision energy

*The AS metric also has a naked singularity at the origin. This is considered as an artifact of having used a black hole metric as the starting point, and assumed to be removed by taking a suitable mass distribution.

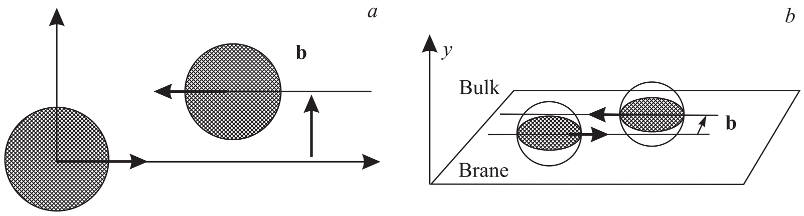

Fig. 5. *a*) Colliding particles in $(D-1)$-dimensional space: b is $(D-2)$-dimensional impact vector and $\sigma \approx \mathcal{D}R_{S,D}^{D-2}$. *b*) Colliding particles on the 3-brane: 2-dimensional impact vector b and $\sigma \approx \pi R_{S,D}^2$. A shaded region indicates the projection of $(D-1)$-dimensional Schwarzschild sphere onto the 3-brane

$E = \sqrt{s}$, Eq. (15). This estimation assumes that when the impact parameter b is smaller than $R_{S,D}$, then the probability of formation of a black hole is close to 1,

$$\sigma_{\text{BH},D} \approx \mathcal{D}_{D-2} R_{S,D}^{D-2}(E), \tag{16}$$

$\mathcal{D}_{D-2}$ is the volume of a plane cross section of the $(D-2)$-dimensional unit sphere, see Fig. 5, a where b is $(D-2)$-dimensional vector, i.e., the area of $(D-2)$-dimensional disk, $\mathcal{D}_D = \pi^{D/2}/\Gamma(1 + D/2)$.

In the 4-dimensional case this estimation gives

$$\sigma_{\text{BH},4} \approx \pi R_{S,4}^2(E). \tag{17}$$

For the 3-brane embedding in the D-dimensional space-time we have

$$\sigma_{\text{BH,3-brane}} \approx \pi R_{S,D}^2(E), \tag{18}$$

since our particles are restricted on the 3-dimensional brane and the impact vector b is a two-dimensional vector, see Fig. 5, *b*.

2.6. Looking from the Rest Frame of One of the Incident Particles. It is instructive to note that a similar analysis can be done in the rest frame of one of the incident particles [49]. This particle has a large de Broglie wavelength and has to be treated as a quantum particle. The gravitational field of the other, which is rapidly moving, looks like a gravitational shock wave, see Fig. 6.

Dynamics of the quantum particle can be described by a solution of the quantum Klein–Gordon equation in the shock wave background. This problem has been solved by 't Hooft [54]. Dynamics of the particle is given the eikonal approximation and is defined by the geodesics behavior near the shock wave. The approximation is valued for a large impact parameter. The shock wave focuses the geodesics down to a small impact parameter. Just in this region we expect the BH formation (see the next section) and in this region the eikonal approximation

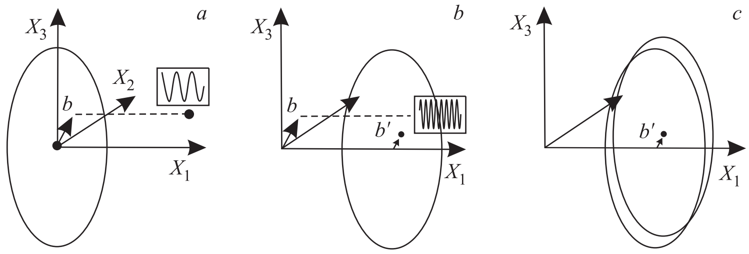

Fig. 6. *a*) Ultrarelativistic particle (shock wave) and a rest quantum particle, *b* is an impact vector. *b*) Quantum particle after a collision with the ultrarelativistic particle, its impact vector b' just after collision decreases, $|b'| \gg |b|$ and its frequency increases. *c*) After collision the particle which was in rest moves with an ultrarelativistic velocity and looks as a shock wave

is not nonapplicable. This gives an explanation why a straightforward eikonal approximation does not describe the BH production. But it is instructive to see what the eikonal approximation can give and this is a subject of the next subsection.

The picture presented in Fig. 6 is idealization. A more precise approach would be started from one moving particle with γ rather large, but $\gamma \neq \infty$ and the other particle in the rest. There should exist a classical solution that interpolates between this initial configuration and a configuration in the later time that represents two stars which are rather closed and move slowly with respect to each other. One can expect to estimate quantum fluctuations to such a classical configuration.

2.7. Black Hole Formation in Ultrarelativistic Particle Collision as a Classical Gravitational Collapse. We consider a collision of two massive particles with rest masses m and M, which move towards each other with relative velocity $\mathbf{v}$, and impact parameter b. Suppose that the particles in the rest frames are described by the Schwarzschild metric with the Schwarzschild radius, $R_{S,D}(m)$ and $R_{S,D}(M)$ given by (8).

For small relative velocity $v = |\mathbf{v}| \ll 1$, the cross section of the BH formation in the collision of these two BHs is of the order

$$\sigma \sim \mathcal{D}_{D-2}\, R_{S,D}^{D-2}(m), \tag{19}$$

where $\mathcal{D}_{D-2}$ is the area of $(D-2)$-dimensional disk. Here we assume $M \sim m$. The estimation (19) is based on the Thorne hoop conjecture. This conjecture says that an apparent horizon forms if and only if matter with a mass M gets compressed enough such that the circumference in all directions satisfies the condition of $\mathcal{C} \lesssim 4\pi M$.

At large relative velocities, $v \to 1$, the cross section is different and is expected to be defined not by the rest masses but by the energy in the c.m.f., Eq. (16). As has been mentioned in Sec. 2.5, the estimation (16) does not follow from the Thorne hoop conjecture.

Below we present arguments in favor of (16) based on study of the system of two colliding particles in the rest frame of one of them. Our consideration follows main steps of Kaloper and Terning [56]. In this paper the authors considered the 4-dimensional case and used the classical capture as a model of the black hole formation.

To show (16) following [49, 56], we go to the rest frame of one of two particles, say M. At large relative velocities, $v \to 1$, the gravitational field of the particle m is extremely strongly boosted in the rest frame of the particle M. In the infinite boost limit, where we also take $m \to 0$ and hold p fixed, the metric reduces to an exact shock wave metric [48,49], given by (10). The metric around the shock wave is just two pieces of the flat space separated by the shock wave, and test particles move freely except when they cross the shock wave front. This picture is similar to the picture of the electric field lines of a highly boosted charge where the lines are compressed into the directions transverse to its motion. Most of the scattering of a test particle takes place while it moves through this region with a more intense field and one can say that the shock wave behaves as a very thin gravitational lens.

Before the collision the particle M in its own rest frame stays at the point $X_0^1 = 0$, $X_0^2 = b$, $X_0^3 = 0$. We consider this particle as a test particle in the gravitation background (10) and, therefore, its movement after the collision is defined by the geodesics given by [49, 50, 57]

$$V = V_0 + V_1 U + V_f \theta(U) + V_d \theta(U) U, \tag{20}$$

$$X^i = X_0^i + X_1^i U + X_d^i \theta(U) U \tag{21}$$

with $V_f = (1/2)F$, $X_d^i = (1/2)F_{,i}$, $V_d = (1/2)F_{,i} \cdot X_1^i + (1/8)F_{,i}^2$ with the corresponding initial data. In X^0, X^1 coordinates this trajectory is $X^0_{(M)}(\tau) = \tau + \frac{F}{2\sqrt{2}}\theta(\tau) + \frac{F_{,i}^2}{16}\theta(\tau)\tau$, $X^1_{(M)}(\tau) = \frac{F}{2\sqrt{2}}\theta(\tau) + \frac{F_{,i}^2}{16}\theta(\tau)\tau$, $X^i_{(M)}(\tau) = b_i + \frac{1}{2\sqrt{2}}F_{,i}\theta(\tau)\tau$, here for simplicity we use $\tau = \sqrt{2}U$.

The m particle in the rest frame of the M particle moves along $U_{(m)}(\tau) = 0, X^i_{(m)}(\tau) = 0$, $i = 2, 3$. If the clocks for two particles are synchronized before the collision, i.e., $X^0_{(M)}(\tau) = X^0_{(m)}(\tau)$ for $\tau < 0$, we have

$$X^0_{(m)}(\tau) = X^1_{(m)}(\tau) = \tau + \frac{F}{2\sqrt{2}}\theta(\tau) + \frac{F_{,i}^2}{16}\theta(\tau)\tau, \tag{22}$$

$X_i = 0$, $i \geqslant 2$. The distance between the M and m particles after the collision is given by

$$\mathcal{R}(\tau)^2 = (X^2_{(M)}(\tau))^2 + (X^1_{(M)}(\tau) - X^1_{(m)}(\tau))^2 = b^2(1 - \tau\frac{v_f}{b})^2 + \tau^2; \quad (23)$$

here $v_f = -F_{,2}$. The minimal distance is achieved at $\tau_{\min} = bv_f/(1 + v_f^2)$ and is given by the formula $\mathcal{R}_{\min}(b) = b/\sqrt{1 + v_f^2}$. In a reasonable approximation $\mathcal{R}_{\min}(b) \approx \pi M^2_{\mathrm{Pl},4} b^2/p$.

The relative velocity of the m and M particles after the collision is

$$\mathbf{v}(\tau) = \left(\frac{-1}{1 + \frac{F^2_{,i}}{16}}, \frac{\frac{1}{2\sqrt{2}}F_{,2}}{1 + \frac{F^2_{,i}}{16}}, 0 \right). \quad (24)$$

Since $F \sim p$, for large values of p, the velocity $\mathbf{v}$ has small components. Therefore, after the collision, in the rest frame of the M particle we can use the nonrelativistic description and, in particular, apply the Thorne hoop conjecture. At this point our consideration is different from [56], where the capture process, related with the Laplace old idea [41], is interpreted as the BH production. In particular, we can say that if the minimal distance between particles after the collision is less than the Schwarzschild radius of the M particle (in the rest frame), then the m particle would be captured by the M particle and we interpret this as a BH formation. The requirement that the minimal distance between particles is smaller than the Schwarzschild radius of the M particle, $R_{S,D}(M) > \mathcal{R}_{\min}(b)$, gives a restriction on the impact parameter

$$b < b_*, \quad b_*^2 = \frac{2s}{M^4_{\mathrm{Pl},4}}; \quad (25)$$

here we use that $s = 2Mp$.

Hence, for all b satisfying (25) the M particle will capture the m particle and, interpreting this process as the black hole formation, we get a cross section

$$\sigma = \pi b_*^2 = \frac{s}{M^4_{\mathrm{Pl},4}}. \quad (26)$$

This answer is in agreement with estimations of the cross section based on the trapped surface area [18]. These estimations are based on the area theorem which states that the horizon area of the black hole must be greater than area of trapped surface, giving a lower bound on the mass of the black hole. Comparing (25) with the restriction of the validity of the classical description,

$$\frac{1}{M_{\mathrm{Pl},4}} < b \lesssim \frac{\sqrt{s}}{M^2_{\mathrm{Pl},4}}, \quad (27)$$

we see that the above considerations are valid only for the trans-Planckian energies $s > M^2_{\mathrm{Pl},4}$. The right estimation in (27) also means a validity of the shock wave approximation $b \ll l$ with l given by (13), since the RHS of (13) is nothing but $p/"8\pi" M_4^2$.

The above calculations are essentially more simple than the finding of the trapped surface in the case of non-head-on collision, and for this reason we call the above estimation the «express-check» of BH formation.

CONCLUSION

We rederive the classical geometrical cross section of the BH production, reconsidering the process of two trans-Planckian particle collision in the rest frame of one of incident particles. This consideration permits us to use the standard Thorne's hoop conjecture for a matter compressed into a region to prove a variant of Thorne's hoop conjecture dealing with a total amount of compressed energy in the case of colliding particles.

Acknowledgements. The author is grateful to I. V. Volovich for fruitful discussions. This work is supported in part by RFBR grants 08-01-00798 and 09-01-12179-ofi$_m$ and by the Russian Federal Agency for Science and Innovations state contract 02.740.11.5057.

REFERENCES

1. *Arkani-Hamed N., Dimopoulos S., Dvali G.* // Phys. Lett. B. 1998. V. 429. P. 263; *Antoniadis I. et al.* // Phys. Lett. B. 1998. V. 436. P. 257.
2. *'t Hooft G.* // Phys. Lett. B. 1987. V. 198. P. 61.
3. *Amati D., Ciafaloni M., Veneziano G.* // Phys. Lett. B. 1987. V. 197. P. 81; Intern. J. Mod. Phys. A. 1988. V. 3. P. 1615.
4. *Gross D. J., Mende P. F.* // Phys. Lett. B. 1987. V. 197. P. 129.
5. *Soldate M.* // Ibid. V. 186. P. 321; *Muzinich I. J., Soldate M.* // Phys. Rev. D. 1988. V. 37. P. 359.
6. *Amati D., Ciafaloni M., Veneziano G.* // Phys. Lett. B. 1989. V. 216. V. 41; Nucl. Phys. B. 1990. V. 347. P. 550; 1993. V. 403. P. 707.
7. *Aref'eva I. Y., Viswanathan K. S., Volovich I. V.* // Nucl. Phys. B. 1995. V. 452. P. 346; hep-th/9412157; *Arefeva I. Y., Viswanathan K. S., Volovich I. V.* // Intern. J. Mod. Phys. D. 1996. V. 5. P. 707.
8. *Rubakov V. A., Shaposhnikov M. E.* // Phys. Lett. B. 1983. V. 125. P. 136.
9. *Banks T., Fischler W.* hep-th/9906038.

10. *Aref'eva I. Ya.* // Part. Nucl. 2000. V. 31. P. 169.

11. *Dimopoulos S., Landsberg G.* // Phys. Rev. Lett. 2001. V. 87. P. 161602.

12. *Argyres P. C., Dimopoulos S., March-Russell J.* // Phys. Lett. B. 1998. V. 441. P. 96.

13. *Giddings S. B., Thomas S.* // Phys. Rev. D. 2002. V. 65. P. 056010.

14. *Aref'eva I. Y., Volovich I. V.* // Intern. J. Geom. Meth. Mod. Phys. 2008. V. 05. P. 641.

15. *Mironov A., Morozov A., Tomaras T. N.* arXiv:0710.3395.

16. *Nicolini P., Spallucci E.* arXiv:0902.4654.

17. *Penrose R.* Unpublished. 1974.

18. *Eardley D. M., Giddings S. B.* // Phys. Rev. D. 2002. V. 66. P. 044011.

19. *Pumplin J. et al.* // JHEP. 2002. V. 0207. P. 012.

20. *Gingrich D. M.* // Intern. J. Mod. Phys. A. 2008. V. 22. P. 5685; hep-ph/0706.0627.

21. *Amsler C. et al.* Particle Data Group // Phys. Lett. B. 2008. V. 667. P. 1.

22. *Abazov V. M. et al.* // Phys. Rev. Lett. 2009. V. 102. P. 051601;
Abazov V. M. et al. // Ibid. V. 103. P. 191803.

23. *Landsberg G.* hep-ex/0808.1867.

24. *Hawking S. W.* // Commun. Math. Phys. 1075. V. 43. P. 199.

25. *Dvali G., Sibiryakov S.* // JHEP. 2008. V. 0803. P. 007.

26. *Meade P., Randall L.* arXiv:0708.3017.

27. *Yoshino H., Mann R. B.* // Phys. Rev. D. 2006. V. 74. P. 044003.

28. *Aref'eva I. Ya.* arXiv:0912.5481.

29. *Aref'eva I. Y., Bagrov A. A., Guseva E. A.* // JEPH. 2009. P. 1130709;
Aref'eva I. Y., Bagrov A. A. // Theor. Math. Phys. 2009. V. 161.

30. *Aref'eva I. Y., Bagrov A. A., Joukovskaya L. V.* arXiv:0909.1294.

31. *Yoshino H., Rychkov V. S.* // Phys. Rev. D. 2005. V. 71. P. 104028.

32. *Choptuik M. W., Pretorius F.* arXiv:0908.1780.

33. *Kohlprath E., Veneziano G.* // JHEP. 2002. V. 0206. P. 057.

34. *Giddings S. B. Rychkov V. S.* // Phys. Rev. D. 2004. V. 70. P. 104026.

35. *Yoshino H., Zeinikov A., Frolov V. P.* gr-qc/0703127.

36. *Bogoliubov N. N., Shirkov D. V.* Introduction to the Theory of Quantized Fields. N. Y. Intersci. Publ. Inc., 1959.

37. *Bogolubov N. N. et al.* General Principles of Quantum Field Theory. Dordrecht: Kluwer Academic Publ., 1990.

38. *Arefeva I. Y. et al.* // Intern. J. Mod. Phys. A. 1991. V. 6. P. 4341.

39. *Giddings S. B.* arXiv:0911.3395.

40. *Arefeva I. Y., Faddeev L. D., Slavnov A. A.* // Theor. Math. Phys. 1975. V. 21. P. 1165.

41. *Hawking S. W., Ellis G. R. F.* The Large Scale Structure of Space-Time. Cambridge Univ. Press, 1973.

42. *Wald R. M.* General Relativity. Chicago: The Univ. of Chicago Press, 1984.

43. *'t Hooft G.* // Commun. Math. Phys. 1988. V. 117. P. 685.

44. *Tolman R. C.* // Phys. Rev. 1939. V. 55. P. 364.

45. *Florides P. S.* // Proc. Roy. Soc. (London). Ser. A. 1974. V. 337. P. 529.

46. *Colpi M., Shapiro S. L., Wasserman I.* // Phys. Rev. Lett. 1986. V. 57. P. 2485.

47. *Schunck F. E., Mielke E. W.* // Class. Quant. Grav. 2003. V. 20. P. R301.

48. *Aichelburg P. C., Sexl R. U.* // Gen. Rel. Grav. 1971. V. 2. P. 303.

49. *Dray T., 't Hooft G.* // Nucl. Phys. B. 1985. V. 253. P. 173;
Class. Quant. Grav. 1986. V. 3. P. 825.

50. *D'Eath P. D., Payne P. N.* // Phys. Rev. D. 1992. V. 46. P. 658; 675; 694.

51. *Rychkov V. S.* // Phys. Rev. D. 2004. V. 70. P. 044003.

52. *Giddings S. B., Rychkov V. S.* hep-th/0409131.

53. *Thorne K. S.* Nonspherical Gravitational Collapse — A Short Review // Magic without Magic / Ed. J. Klauder. San Francisco, 1972.

54. *'t Hooft G.* // Phys. Lett. B. 1987. V. 198. P. 61;
't Hooft G. // Nucl. Phys. B. 1988. V. 304. P. 867.

55. *Giddings S. B., Porto R. A.* The Gravitational S-matrix. 0908.0004.

56. *Kaloper N., Terning J.* // Gen. Rel. Grav. 2007. V. 39. P. 1525.

57. *Ferrari V., Pendenza P., Veneziano G.* // Gen. Rel. Grav. 1988. V. 20. P. 1185.