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UNPARTICLES IN GAUGE THEORY

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We develop the model where the correct description of particle physics in the TeV energy scale needs to account for degrees of freedom obeying the conformal invariance. In this respect, the existing of Georgi's unparticles is strongly argued. We present the gauge model of scalar unparticles. The ground state of the scalar unparticles staff — the scalar un-Higgs, which is a dipole field — in the continuum is restricted by the vacuum expectation value of the Standard Model (SM) Higgs boson and the noncanonical scaling dimension d of the unparticle staff.

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1. The unparticle phenomenon [1] and several points concerning unparticle phenomenology have been widely overlooked in the literature (see, e.g., the references in [2]). However, to the author's opinion, a set of matter things has not been settled in a way suitable to more better understanding of the subject. This letter is an attempt to clarify the unparticle nature in gauge theories, e.g., within the invariance under gauge transformation of the second kind. A gauge model of unparticles contains the criteria relevant to the SM pattern, however, to keeping in mind noncanonical scaling dimension of the unparticle staff.

In general, a conformal theory is the one where there is an exact scale invariance (apart from more technical aspects). We have to deal with the theory so that predictions are possible in case this theory is renormalizable. The Higgs field is necessary to guarantee the renormalizability. On the other hand, at high energies, the Higgs field can weakly interact with a hidden scale-invariant sector realized through the unparticles staff which would be relevant since we do hope to observe the Higgs particle in an experiment. The Higgs field can serve as a portal to a hidden sector of the SM matter.

We suppose the effective field theory (EFT) containing a hidden sector lying beyond the SM. The hidden sector is modeled using an arbitrary field operator $O(M)$ on the running mass scale M in interaction with a heavy state that may occur in the ultraviolet (UV) region at high scale $\Lambda \gg \Lambda_{\rm SM}$, where $\Lambda_{\rm SM} \sim O(v)$ with $v \simeq 246$ GeV being the vacuum expectation value (v.e.v.) of the Higgs field in the SM. The simplest form of the relevant Lagrangian density (LD) is

$$L_{\rm EFT}(M) = c(M,\Lambda)\, O(M) = \frac{c_0(M)}{\Lambda^{d-4}}\, O(M), \qquad (1)$$

where d is the scaling dimension of $O(M)$ and the effect of heavy state is encoded in $c(M, \Lambda)$. Physically interesting goal is to develop the theory near its infrared (IR) fixed point $M_{\rm IR} \simeq \Lambda_{\rm SM}$.

Somewhat surprisingly is the possibility of having a model that has basically the singlet-doublet mixing. One can start with an interaction of the form $\sim O_U HH^+ + O_U^2 HH^+$, where O_U is the new singlet-like unparticle field, or even the un-Higgs staff (a part of the approximate conformal field theory) with $1 < d < 2$, and H stands for the SM Higgs field with ${\rm Re}\, H = (h+v)/\sqrt{2}$.

We find that the behavior of the un-Higgs field is compatible with that of a dipole field. The model is looked as a first attempt to start with an effective Lagrangian at energies E covering the region between the IR scale $\tilde{\Lambda}_U$ and the UV scale, $\tilde{\Lambda}_U < E < \Lambda_U$. To our knowledge, irrespective of the validity of the conjecture of references (see, e.g., the references in [2]), a discussion concerning the unparticle states of dipole field models with the dimension d is lacking in current literature. In this letter we give a positive contribution to this direction by making use of the standard Lagrangian approach to quantum field theory.

2. We start with the effective LD $L = L_1 + L_2$, where

$$L_1 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha^2 \, D_\mu O_U \, \bar{D}^\mu O_U^* - \lambda^2 \left|O_U\right|^4 + \mu_0^2 \left|O_U\right|^2, \qquad (2)$$

$$L_2 = a \left|H\right|^2 O_U + b \left|H\right|^2 \left|O_U\right|^2. \qquad (3)$$

In formulas (2) and (3), $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with the gauge field A_μ; $D_\mu = \partial_\mu + ieA_\mu$, $\bar{D}_\mu = \partial_\mu - ieA_\mu$; α, λ, μ_0, a and b are d-dependent in mass units: $1-d$, $2(1-d)$, $2-d$, $2-d$ and $2(1-d)$, respectively.

There are three characteristic scales Λ at high energies in the theory:

a) M_U scale: hidden sector (unparticles) is coupled to SM through nonrenormalizable couplings at M_U. This leads to the interaction form $M_U^{-n} O_{\rm SM} O_{\rm BZ}$ with the positive numbers n, where $O_{\rm SM}$ is the SM operator, while $O_{\rm BZ}$ stands for Banks–Zaks theory [3] operator (massless fermions case). Hence, the UV sector is coupled to SM through the exchange of messenger fields with large mass scale M_U.

b) Λ_U scale: assume that unparticle sector becomes conformal at $\Lambda_U < M_U$, and couplings to SM preserve conformality in IR region that gives the operator form $M_U^{-n} O_{\rm SM} O_{\rm BZ} \to {\rm const}\, \Lambda^{-d-d_{\rm SM}+4} O_{\rm SM} O_U$.

c) Electroweak symmetry breaking leads to conformal symmetry breaking through the superrenormalizable operator $a\left|H\right|^2 O_U$ at the third scale $\tilde{\Lambda}_U = (av^2)^{1/(4-d)} < \Lambda_U$. Thus, when the Higgs field accepts its v.e.v., the operator $a|H|^2 O_U$ brings a scale (occurrence of a tadpole term in L_2 (3)). A sector of unparticle staff leaves the conformal-fixed point and a theory becomes nonconformal-invariant at the scale $\tilde{\Lambda}_U$ below which the unparticle staff transforms into a standard particle sector.

One can conclude that unparticle physics is only possible in the conformal window. However, the width of this window depends on d, $\tilde{\Lambda}_U$, Λ_U, M.

The nature of unparticle staff is unknown, and to find the nontrivial result, in particular, to get the v.e.v. of unparticle staff, we suggest to follow the following scheme:

For the structure of O_U operator we choose the approach with the infinite tower of N scalars ϕ_k $(k = 1, 2, \ldots, \infty)$ [4]

$$O_U \to O = \sum_{k=1}^{N=\infty} f_k\, \phi_k, \tag{4}$$

where ϕ_k are characterized by the mass squared $m_k^2 = k\,\Delta^2$ as $\Delta \to 0$. In the LD $L = -(1/4)F_{\mu\nu}F^{\mu\nu} + \alpha^2 D_\mu O \bar{D}^\mu O^* - V(O,H)$ the potential $V(O,H)$ is given in the deconstructed conventional form

$$V(O,H) = \frac{1}{4}\xi \left(\sum_{k=1}^{N} \phi_k^2\right)^2 - \frac{1}{2}\sum_{k=1}^{N} m_k^2 \phi_k^2 - a\,|H|^2 \sum_{k=1}^{N} f_k \phi_k - b'|H|^2 \sum_{k=1}^{N} \phi_k^2 \tag{5}$$

with new coefficients ξ and b' in zero mass units. The minimization equation for ϕ_k fields looks like at $\langle \phi_k \rangle = \sigma_k$ and $\langle H \rangle = v/\sqrt{2}$

$$\sigma_k = \frac{1}{2}\frac{-av^2 f_k}{m_k^2 - (\xi \sum_{l=1}^{N} \sigma_l^2 - b' v^2)}. \tag{6}$$

It is evident from (6) that the interaction term $a|H|^2 O_U$ in (3) with $a \neq 0$ ensures $\sigma_k \neq 0$.

The free propagator of the field O_U is

$$D(p^2,d) = \int_0^\infty \frac{dm^2}{2\pi}\, \frac{\rho_0(m^2,d)}{p^2 - m^2 + i\,\epsilon}, \tag{7}$$

where in accordance with the scale invariance, the spectral density $\rho_0(m^2,d)$ is

$$\rho_0(m^2,d) = A_d \left(m^2\right)^{d-2}, \quad A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d}} \frac{\Gamma(d+1/2)}{\Gamma(d-1)\,\Gamma(2d)}. \tag{8}$$

On the other hand, $\rho_0(m^2)$ is expressed in the form:

$$\rho_0(m^2) = 2\,\pi \sum_\omega \delta(m^2 - m_\omega^2)\, |\langle 0|O(0)|\omega\rangle|^2, \tag{9}$$

where the sum is over all relativistic states $|\omega\rangle$ normalized by relativistic manner. From (8) and (9) one finds ($|\langle 0|O(0)|\omega_k\rangle|^2 = f_k^2$): $f_k^2 = A_d \left(m_k^2\right)^{d-2} \Delta^2/(2\pi)$.In the continuum

$$\langle O_U\rangle = \int_0^\infty \bar{\sigma}(s)\bar{f}(s)\,ds = \frac{v^2}{2}\int_0^\infty \frac{\bar{f}^2(s)}{z-s}ds = \\ = \frac{A_d}{4\pi} a v^2 (-1)^{d-1} z^{d-2}\Gamma(d-1)\Gamma(2-d), \quad (10)$$

where $z = b'\,v^2 - \xi \sum_{l=1}^{N} \sigma_l^2$ is the infrared regularized (IRR) mass square (mass gap), induced by interaction $|H|^2 O_U$ in the unparticle continuum.

3. The gauge un-Higgs model is naturally obtained of itself by considering the LD (2) which is invariant under the transformations $A_\mu(x) \to A_\mu(x) + \partial_\mu \vartheta(x)$, $O_U(x) \to \exp\left[-ie\vartheta(x)\right] O_U(x)$, where $\vartheta(x)$ is a real function obeying the equation $\nabla^2\vartheta(x) = 0$ ($\nabla \equiv \partial/\partial x_\mu$).

The generating current $K_\mu(x;\Lambda) = \nabla^2 A_\mu(x) - \beta_U\,\partial^\nu\partial_\mu A_\nu(x)$ with $\Lambda > \tilde{\Lambda}_U$ and $\beta_U \neq 1$ is closely related to a model introduced in [5] in a study of the Higgs phenomenon, from which the present model is distinguished by coupling to un-Higgs staff. Obviously, the hidden parameter $\beta_U = 1$ if $\Lambda < \tilde{\Lambda}_U$.

To find a solution we introduce the real fields ψ and χ in $O_U = (1/\sqrt{2})(\sigma + \psi + i\,\chi)$, $O_U^* = (1/\sqrt{2})(\sigma + \psi - i\,\chi)$, where $(\Omega, \chi\Omega) = 0$, $(\Omega, (\sigma+\psi)\Omega) = \sigma \neq 0$, and Ω is the vacuum. In terms of new real fields ψ and χ, the LD (2) of the order $e\sigma$ and $\lambda\sigma$ becomes

$$\tilde{L}_1 = -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}\beta_U \partial_\mu A_\nu \partial^\nu A^\mu - \frac{1}{2}m^2 A_\mu^2 + \frac{1}{2}\alpha^2[(\partial_\mu\chi)^2 + (\partial_\mu\psi)^2] + \\ + m\alpha A^\mu \partial_\mu \chi - \frac{1}{2}(3\lambda^2\sigma^2 - \mu_0^2)\psi^2 - \frac{1}{2}(\lambda^2\,\sigma^2 - \mu_0^2)\chi^2, \quad m = e\alpha\sigma. \quad (11)$$

The equations of motion lead to the dipole-type equation

$$\lim_{\kappa^2 \to 0} (\nabla^2 + \kappa^2)^2\,\chi = 0, \quad (12)$$

which means that the scale-invariant particle staff is almost massless. The spectrum of the model consists of the unparticle stuff and massive bosons:

$$A_\mu(x) = B_\mu(x) - \frac{\alpha}{m}\,\partial_\mu \left[1 - \frac{1-\beta_U}{m^2}\,\nabla^2\right]\chi(x), \quad (13)$$

where the field B_μ obeys $(\nabla^2 + m^2)B_\mu = 0$, $\partial_\mu B^\mu = 0$, and $[B_\mu(x), \chi(y)] = 0$.

The generating current looks like:

$$K_\mu(x) = -m^2 \left[B_\mu(x) + \frac{1-\beta_U}{m^3}\, \alpha\, \partial_\mu \nabla^2 \chi(x) \right], \tag{14}$$

where σ is the solution of (10).

4. The propagator of the field $\chi(x)$ is defined as the time-ordered two-point Wightman function (see for details [6-8]):

$$W(x) = \langle 0|T\chi(x)\chi(0)|0\rangle = \frac{(e\sigma)^2}{(1-\beta_U)} \frac{1}{(4\pi)^2} \ln \frac{l^2}{-x^2+i\epsilon} + \text{const.} \tag{15}$$

Actually, at energy $E < \tilde{\Lambda}_U$ the parameter β_U is equal to one, while $\sigma = 0$ because the interaction between the Higgs boson and the unparticle staff disappears ($a = 0$) in the continuum limit (see formulas (3) and (10)).

The desired propagator $\tilde{W}(p)$ in the four-momentum p space is given in the form

$$\tilde{W}(p) = -\frac{1}{4}\, i\varepsilon \frac{\partial^2}{\partial p^2} \left\{ \frac{\ln\left[\mathrm{e}^{2\gamma}(-p^2 l^2 - i\epsilon)/4\right]}{-p^2 - i\epsilon} \right\}, \tag{16}$$

where ς is the regularized length. On the other hand, let us consider the equation

$$\lim_{\lambda^2\to 0} \int d_4 p\, \mathrm{e}^{-i\,p\,x} \frac{1}{(p^2-\lambda^2+i\epsilon)^2} = \frac{i}{8\pi^2} K_0\left(\lambda\sqrt{-x^2+i\epsilon}\right). \tag{17}$$

The naive use of (15) leads to appearance of $\ln\lambda$ divergence, because at small z the following behavior of the Bessel function $K_0(z)$, $\lim_{z\to 0} K_0(z) \simeq \ln(2/z) - \gamma + O(z^2, z^2 \ln z)$, is correct. Thus, the propagator for the un-Higgs field $\chi(x)$ in the sense of distributions becomes

$$\tilde{W}(p) = \frac{(e\sigma)^2}{(1-\beta_U)} \lim_{\lambda^2\to 0} \left[\frac{1}{i\,(p^2-\lambda^2+i\epsilon)^2} + \frac{1}{(4\pi)^2} \ln \frac{\lambda^2}{\kappa^2} \delta^4(p) \right]. \tag{18}$$

A consequence of $\tilde{W}(p)$ (16) is that the lowest order energy (potential) of a «charge » in the static limit has the following Fourier transform ($|\mathbf{x}| \equiv r$)

$$E(r;\sigma) = i \int d_3\mathbf{p}\, \mathrm{e}^{i\mathbf{p}\mathbf{x}} \tilde{W}(p^0=0,\mathbf{p};\sigma) =$$

$$= \frac{(e\sigma)^2\, r}{8\pi(1-\beta_U)} \left[\frac{9}{2}\ln 2 - 2 + 3\ln(\kappa r) \right]. \tag{19}$$

The energy (19) grows as r at large distances.

5. In summary, we have presented the gauge model of scalar unparticles. From the phenomenological point of view, the most physical motivated way to explore the unparticle sector is the interaction with the Higgs field operator $|H|^2$. We have clarified how the scale-invariant self-coupling ξ generates the mass gap squared $m^2_{\rm IRR} = b'\,v^2 - \xi\,\sigma^2$ for unparticles that acts as an infra-red cutoff to give a nonzero $\sigma = \langle O \rangle$.

The new dipole-type behavior of the propagator for the un-Higgs field has been carried out (a ghost-like field) in the energy region $\tilde{\Lambda}_U < E < \Lambda_U$. Moreover, the vector field becomes massive.

The behavior of (19) is the most general and compelling argument in favor of the (free) energy of un-Higgs field.

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