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TUNNELING IN SUPERCONDUCTING STRUCTURES

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Here we review our results on the breakpoint features in the coupled system of IJJ obtained in the framework of the capacitively coupled Josephson junction model with diffusion current. A correspondence between the features in the current voltage characteristics (CVC) and the character of the charge oscillations in superconducting layers is demonstrated. Investigation of the correlations of superconducting currents in neighboring Josephson junctions and the charge correlations in neighboring superconducting layers reproduces the features in the CVC and gives a powerful method for the analysis of the CVC of coupled Josephson junctions. A new method for determination of the dissipation parameter is suggested.

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INTRODUCTION

Physics of tunneling in superconducting structures is closely related to the name of N. N. Bogolyubov. It belongs to the intrinsic Josephson effect in high-T_c superconductors as well, which attracts a wide interest today due to the observed powerful coherent terahertz radiation from the stack of intrinsic Josephson junctions (IJJ) [1]. The radiation is related to the same region in CVC, where the breakpoint and the parametric resonance in the IJJ were predicted [2, 3]. The experimental manifestation [4] of the breakpoint and the breakpoint region in the CVC of IJJ in $Bi_2Sr_2CaCu_8O_y$ gives an impulse for further investigations in this field.

The resistively and capacitively shunted junction (RCSJ) model and its different modifications are well known to describe the properties of single Josephson junctions, giving a clear picture of the role of quasiparticle and superconducting currents in the formation of CVC. In the case of a stack of intrinsic Josephson junctions (IJJ) the situation is cardinally different. The system of the coupled Josephson junctions has a multiple branch structure and it has additional characteristics: the breakpoint current, the transition current to another branch and the breakpoint region (BPR) width [2, 3, 5]. It was demonstrated that the CVC of the stack exhibits a fine structure in the BPR [6].

The breakpoint current characterizes the resonance point, at which the longitudinal plasma wave (LPW) is created in stacks, with a given number and distribution of the rotating and oscillating IJJ. These notions should be taken into account to have a correct interpretation of the experimental results. The investigation of the coupled system of Josephson junctions with a small value of the coupling parameter (as in the case of capacitive coupling) allowed us to understand in a significant way the influence of the coupling between junctions on physical properties of the system. The capacitive coupling is realized in nanojunctions if the length of the junctions is comparable to, or smaller than the Josephson penetration depth. Coupling between intrinsic Josephson junctions leads to the interesting features which are absent in single Josephson junction. This problem is complex enough for the experimental study and many questions concerning the effect of interjunction coupling on CVC are open till now. Here we discuss the models of IJJ in HTSC, their CVC, the charge and current correlations and new methods for determination of the model parameters.

1. MODEL AND METHOD

In capacitively coupled Josephson junctions model (CCJJ model) [7–9] the IJJ in HTSC are considered as a coupled system. They are formed from the atomic scale superconducting layers (S-layers) whose thickness is comparable with the Debye screening length [7]. Because the layers are very thin, the electric charge does not screen and it leads to the capacitive coupling between junctions. For the small-size stacks the inductive coupling of IJJ in the absence of magnetic field can be neglected and the phase dynamics is determined by capacitive coupling only. The absence of the complete screening of the charge in the superconducting layer ρ_l leads to the generalized scalar potential Φ_l: $\rho_l = -(1/4\pi r_D^2)\Phi_l$, where r_D is the Debye screening length and Φ_l is expressed through a scalar potential ϕ_l and derivative of phase θ by $\Phi_l(t) = \phi_l - V_0\dot{\theta}_l$ [7]. The dot above θ means the time derivative. Using Maxwell equation $\mathrm{div}\,(\varepsilon\varepsilon_0 E) = \rho$, where ε and ε_0 are relative dielectric and electric constants, we express the charge density Q_l in the S-layer l by the voltages V_l and V_{l+1} in the neighbor insulating layers $Q_l = Q_0\alpha(V_{l+1}-V_l)$, where $Q_0 = \varepsilon\varepsilon_0 V_0/r_D^2$. The charge on S-layer is expressed by the voltage difference of adjacent junctions and the study of the charge dynamics allows us to predict new physical properties of IJJ. The Josephson equation is generalized in this case and phase dynamics in CCJJ model is described by the system of equations $\dot{\varphi}_l = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l), I = \dot{V}_l + \sin\varphi_l + \beta V_l$. In comparison with RCSJ model, the total current in CCJJ model

$$I = \ddot{\varphi}_l + \sin\varphi_l + \beta\dot{\varphi}_l + \alpha(\sin\varphi_{l+1}\sin\varphi_l) + \alpha(\sin\varphi_{l-1} - \sin\varphi_l) \quad (1)$$

has additional terms which depend on the coupling constant and are proportional to the difference of the superconducting currents in the considered and in the adjusted junctions. The CVC in CCJJ model are characterized by branching at $I = I_c$ and strong branching in the hysteresis region: the number of branches exceeds essentially the number of junctions.

The diffusion current $J_D^l = \Phi_{l+1} - \Phi_l$ is added in CCJJ+DC model [10, 11] and the phase dynamics in this case is described by the system of equations including the GJR. The expression for current through the stack of junctions is

$$I = \dot{V}_l + \sin\varphi_l + \beta\varphi_l. \tag{2}$$

In CCJJ model the third term in the right-hand side is equal to V_l/R. The total current in CCJJ+DC model

$$I = \ddot{\varphi}_l + \sin\varphi_l + \beta\dot{\varphi}_l + \alpha(\sin\varphi_{l+1} - \sin\varphi_l) + \alpha(\sin\varphi_{l-1} - \sin\varphi_l) + \\ + \alpha\beta(\dot{\varphi}_{l+1} - \dot{\varphi}_l) + \alpha\beta(\dot{\varphi}_{l-1} - \dot{\varphi}_l) \tag{3}$$

has additional terms in comparison with CCJJ model which are proportional to the product of coupling and dissipation parameters.

We solve a system of dynamical equations for the gauge-invariant phase differences $\varphi_l(\tau) = \theta_{l+1}(\tau) - \theta_l(\tau) - \frac{2e}{\hbar}\int\limits_l^{l+1} dz A_z(z,\tau)$ between superconducting layers (S-layers) for the stacks with a different number of IJJ in the framework of the capacitively coupled Josephson junctions model with diffusion current (CCJJ+DC model) [10, 11], where θ_l is the phase of the order parameter in the S-layer l, A_z is the vector potential in the barrier. The details of the method are described in [8, 9]. Let us now discuss shortly the main features of the phase dynamics in the coupled system of Josephson junctions.

2. CVC IN CCJJ + DC MODEL

In Fig. 1 we show the CVC for CCJJ+DC model. We stress here two main differences in comparison with CCJJ model. First, there is switching to the higher branches at $I \geqslant I_c$ in CCJJ model, but in CCJJ+DC model a transition to the

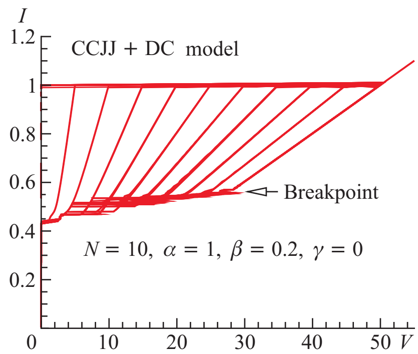

Fig. 1. The CVC of IJJ in CCJJ+DC model

outermost branch is observed. Second, number of branches in CCJJ+DC model in wide region of parameters is equal to the number of junctions, while in CCJJ model it much exceeds this number. The experimental CVC mostly corresponds to the CCJJ+DC model [4].

3. THE BREAKPOINT

As we have seen in Fig. 1, the CVC demonstrates a breakpoint in the outermost branch. In Fig. 2 the profile of time dependence of the charge in the S-layers in the stack with 9 IJJ is combined with the CVC of the outermost branch at $\alpha = 1$, $\beta = 0.2$ and periodic BC. We show the profile of the charge oscillations in the seventh layer. We see that the features of the CVC are in correlation with the features of time dependence of the charge on the S-layer. This fact is in agreement with the idea of the parametric resonance at the breakpoint. As was shown in [7], the system of equations for CCJJ has a solution corresponding to the LPW propagating along the c-axis. A frequency of the LPW at $I = 0$ and $\beta = 0$ is $\omega_{\rm LPW}(k) = \omega_p\sqrt{1 + 2\alpha(1 - \cos kd)}$, where k is wave vector of the LPW. We found that at the breakpoint the Josephson oscillations excite the LPW by their periodical actions. The frequency of Josephson oscillations is determined by the voltage value in the junction, so at $\omega_J = 2\omega_{\rm LPW}$ the parametric resonance is realized and the LPW is created.

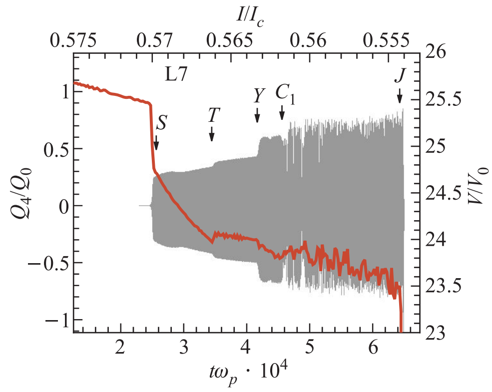

Fig. 2. Profile of time dependence of the charge in the S-layers in the stack with 9 IJJ in the seventh layer. The line shows the CVC of this stack in the BPR

4. CORRELATIONS IN THE COUPLED SYSTEM OF JJ

To understand the origin of the CVC features in the BPR, we study [12] the correlation functions $C^s_{j,j+1}$, describing the correlations of the superconducting currents in the neighboring junctions j and $j+1$:

$$C^s_{j,j+1} = \langle \sin\varphi_j(\tau)\sin\varphi_{j+1}(\tau)\rangle =$$
$$= \lim_{(T_m-T_i)\to\infty} \frac{1}{(T_m-T_i)} \int_{T_i}^{T_m} \sin\varphi_j(\tau)\sin\varphi_{j+1}(\tau)d\tau, \quad (4)$$

where the brackets $\langle\rangle$ mean the averaging over time, and T_i is the starting point in time domain from which we collect the data for the averaging. In our simulations we take $T_i = 50$. It was checked that any further increase in T_i keeps the CVC practically intact. The $C^s_{j,j+1}$ as functions of bias current I for the stack with 9 IJJ are presented in Fig. 3 for $j = 2$ and $j = 4$. The black curve shows the outermost branch of the CVC. We see that the features of the correlation functions coincide with the features of CVC, i.e., they manifest themselves in the CVC curve. The arrows show the characteristic points. Study of $C^s_{j,j+1}$ allowed us to find additional features in CVC which were not noticed in the previous studies [6].

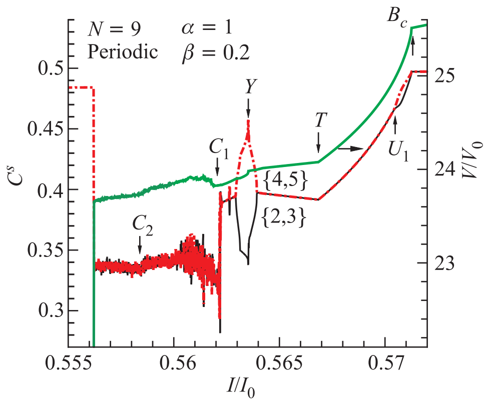

Fig. 3. The correlation of superconducting currents in the stack with 9 IJJ: the curves plot the correlation functions $C^s_{2,3}$ (solid) and $C^s_{4,5}$ (dash-dotted) as a function of bias current. The thick curve shows the corresponding CVC

5. DETERMINATION OF THE DISSIPATION PARAMETER β

The study of the breakpoint features in the CVC of IJJ in HTSC allows us to develop a new method for determination of the dissipation parameters of the system. In the case of coupled system of junctions, parameter β cannot be determined usual way (by the return current), because the return current depends now on two parameters, as β and α. The CVC of the stacks with odd number N of IJJ at periodic boundary condition demonstrates the same behavior for $I_{\rm bp}(N)$ and BPR width $w_{\rm bp}(N)$ as in the nonperiodic case, but for the stacks with even N the value of $I_{\rm bp}$ does not depend on N and the BPR for these stacks is absent [3]. We may estimate the value of β, using the results of these simulations of CVC for stacks with different number of junctions. At $\alpha = 1$, $\beta = 0.2$ and periodic boundary conditions the stacks with even number of junctions have the same value of $I_{\rm bp} = 0.576$, because the same π-mode is created at the breakpoint. In the Table we present the values of $V_{\rm bp}$, found from the simulation, and the values of β, calculated at $I_{\rm bp} = 0.576$ by the formula

$$\beta = N\frac{I_{\rm bp}}{V_{\rm bp}}, \tag{5}$$

which follows from the breakpoint position in the IVC.

The values of the breakpoint voltage $V_{\rm bp}$ from the numerical simulation and dissipation parameter β calculated by formula (5)

N	$V_{\rm bp}$	β
4	11.352	0.2029
6	17.158	0.2014
8	22.667	0.2033
10	28.595	0.2014
12	34.300	0.2015
14	39.870	0.2022
16	45.684	0.2017
18	51.474	0.2014

The absolute error (absolute accuracy) in this calculation consists of $1-2$ percents of the β value. The estimation gives the conservative value. The same order of the absolute error we have for the stacks with odd number of junctions. Particularly, for $N = 5$, $I_{\rm bp} = 0.5567$ and $V_{\rm bp} = 13.794$, we get $\beta = 0.2018$. For $N = 11$, $I_{\rm bp} = 0.5721$ and $V_{\rm bp} = 13.794$, and we have $\beta = 0.2017$.

CONCLUSIONS

The breakpoint phenomenon in the coupled system of Josephson junctions opens a wide field for new investigations. Some important unsolved problems are: (i) the influence of the inductive and other types of coupling between junctions on the breakpoint features; (ii) a question: Can we predict the features of the fine structure in the BPR for stacks with different number of IJJ; (iii) the influence of the parametric resonance in the coupled system of IJJ on their synchronization and coherent electromagnetic radiation in the terahertz region; (iiii) the experimental testing of the idea concerning the temperature variation of the LPW number.

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