

SOLITONIC BRANES AND WRAPPING RULES

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We show that the solitonic branes of ten-dimensional IIA/IIB string theory must satisfy, upon toroidal compactification, a specific wrapping rule in order to reproduce the number of half-supersymmetric solitonic branes that follows from a supergravity analysis. The realization of this wrapping rule suggests that IIA/IIB string theory contains a whole class of so-called «non-standard» Kaluza–Klein monopoles.

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INTRODUCTION

It is by now well-understood that branes form a crucial ingredient of string theory. For instance, they have been used to calculate the entropy of certain black holes [1] and they are at the heart of the AdS/CFT correspondence [2]. In general, branes are massive objects that divide space-time into a number of world volume and transverse directions. For instance, a ten-dimensional string corresponds to 2 worldvolume and 8 transverse directions. The question we would like to address in this talk is: *what can we learn about branes by using as input supergravity as a low-energy approximation to string theory?* Often, the presence of a p -brane in string theory can be deduced from the presence of a rank $(p + 1)$ -form potential in the corresponding supergravity theory. At first sight, the relation between the branes of string theory and the potentials of its supergravity approximation could have been investigated many years ago. The new twist we want to give to this old question is to make use of the relatively new insight that the potentials of a given supergravity theory are not only the ones that describe the physical degrees of freedom of the supermultiplet. It turns out that the supersymmetry algebra allows additional high-rank potentials that do not describe any degree of freedom but, nevertheless, play an important role in describing the coupling of branes to background fields.

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One can divide branes into *standard* branes, with $T \geq 3$ transverse directions, and *non-standard* branes, with $0 \leq T \leq 2$ transverse directions. The standard branes are asymptotically flat. The remaining set of non-standard branes are not asymptotically flat. The consistency of these non-standard branes requires to consider a given number of them, in combination with a so-called orientifold. In this talk we will not pursue this but, instead, consider single branes only and see whether they satisfy some *half-supersymmetric brane criterion*, to be defined later on in this talk*. It is easy to see that the standard branes always couple to potentials that describe physical degrees of freedom. For instance, in $D = 10$ dimensions, the standard p -branes, with $0 \leq p \leq 6$, couple to physical $(p + 1)$ -form potentials, which include the dual potentials. The highest-rank potential is a 7-form potential which is dual to a vector. The non-standard branes with $T = 2$ transverse directions are special in the sense that they couple to $(D - 2)$ -form potentials that are dual to the scalars of the supergravity non-linear sigma models. Due to the non-linearity of the scalars, this duality is non-trivial and unusual in the sense that the number of physical scalars and dual potentials are not the same. For the exact relation between the numbers, we refer to [3] where the branes with $T = 2$ have been denominated «defect branes» since they include objects such as four-dimensional cosmic strings and ten-dimensional Dirichlet 7-branes. The non-standard branes with $T = 1$ transverse directions are domain-walls and they couple to $(D - 1)$ -form potentials. One can view these potentials as being the duals of an integration constant such as the massive Romans parameter in IIA supergravity or any gauge coupling constant in gauged supergravity. Finally, the non-standard branes with zero transverse directions are called «space-filling» branes. They are special in the sense that they only allow a double dimensional reduction to a lower-dimensional space-filling brane. These space-filling branes play an important role in describing superstring theories with less than the maximum number of supercharges.

All potentials, whether describing physical degrees of freedom or not, can be classified according to the allowed U-duality representations. The U-duality representations of the physical potentials have been classified a long time ago, and they follow from the representation theory of the supersymmetry algebra. The physical potentials of the different maximal supergravity theories are related to each other via toroidal reduction. The lower-dimensional ones all follow from the reduction of the ten-dimensional IIA or IIB potentials. Remarkably, the U-duality representations of the remaining higher-rank potentials that do not describe physical degrees of freedom have also been classified recently [4–6]. In principle, these representations can be derived by the requirement that the supersymmetry

*By *half-supersymmetric* we mean invariance under 16 of the 32 supercharges. In this talk we do not consider branes with less or no supersymmetry.

algebra is realized on these fields. This has been explicitly verified in $D = 10$ dimensions in which case the physical potentials of IIA and IIB supergravity can be extended with the potentials given in Table 1 [7].

Table 1. This table lists the U-duality representations of all potentials, both physical and un-physical, that are consistent with the IIA or IIB supersymmetry algebra. The representations in the IIB case refer to the $SL(2, \mathbb{R})$ S-duality group

D	p									
	0	1	2	3	4	5	6	7	8	9
IIA	1	1	1		1	1	1	1	1	2×1
IIB		2		1		2		3		$4 \oplus 2$

A distinguishing feature of the un-physical potentials is that, when considered in different dimensions, they are not related to each other by toroidal compactification. This is unlike the «physical» potentials, including the dual potentials, whose numbers are fixed by the representation theory of the supersymmetry algebra. Supergravity is therefore not complete in the sense that the lower-dimensional supergravity theories, including the un-physical potentials, do not follow from the reduction of the ten-dimensional supergravity theory. It is this incomplete nature of supergravity that will lead us to suggest at the end of this talk a class of non-standard Kaluza–Klein (KK) monopoles in string theory.

In this talk we will consider the supersymmetric branes of the IIA/IIB string theory compactified on a torus, which couple to the fields of the corresponding maximal supergravities. As mentioned above, these fields do not only include the physical potentials, i.e., the p -forms with $0 \leq p \leq D - 2$, but also the un-physical potentials, i.e., $(D - 1)$ -forms (which are dual to constant parameters) and D -forms (that have no field strength). While standard branes are automatically classified because their number coincides with the dimension of the U-duality representation of the corresponding field, we find that this is in general not true for the non-standard branes. In fact we find two new features for the non-standard branes:

- 1) Not every U-duality representation corresponds to *half-supersymmetric* branes;
- 2) Not each component of a U-duality representation corresponds to a half-supersymmetric brane.

For instance, of all potentials corresponding to the non-standard branes in $D = 10$ dimensions, see Table 1 for $p = 7, 8$ and 9, only a subset, see Table 2, corresponds to a half-supersymmetric brane.

To determine whether a given potential couples to a half-supersymmetric brane or not, we first construct a gauge-invariant Wess–Zumino (WZ) term, which is always possible at the cost of having to introduce a number of world-volume potentials. Next, we impose the following *half-supersymmetric brane criterion* [8, 9]: a potential can be associated to a half-supersymmetric brane if

Table 2. This table shows that the only supersymmetric non-standard branes in $D = 10$ dimensions are the D7-brane and its S-dual (IIB), the D8-brane (IIA) and the D9-brane and its S-dual (IIB)

D	p		
	7	8	9
IIA	$0 \subset \mathbf{1}$	$\mathbf{1}$	$0 \subset 2 \times \mathbf{1}$
IIB	$2 \subset \mathbf{3}$		$(2 \subset \mathbf{4}) \oplus (0 \subset \mathbf{2})$

the corresponding gauge-invariant WZ term requires the introduction of world-volume fields that fit within the bosonic sector of a suitable supermultiplet with 16 supercharges.

Since many different branes will pass by in this talk, it is useful to classify them in different ways. We already discussed the distinction between standard branes, with $T \geq 3$ transverse directions, and the non-standard ones, with $0 \leq T \leq 2$ transverse directions. These are the defect branes ($T = 2$), the domain walls ($T = 1$) and the space-filling branes ($T = 0$). Another useful way to classify the branes of string theory is according to the way that the string tension T scales with the string coupling constant g_s . Introducing an integer number $\alpha \leq 0$, this scaling is given by

$$T \sim (g_s)^\alpha. \quad (1)$$

This leads us to fundamental branes ($\alpha = 0$), Dirichlet branes ($\alpha = -1$), solitonic branes ($\alpha = -2$), etc. To determine the value of α corresponding to a given potential, it is easiest to decompose in each dimension $D = 10 - d$ the U-duality representations in terms of T-duality representations as

$$\text{U-duality} \supset SO(d, d) \times \mathbb{R}^+. \quad (2)$$

The value of α then follows from the \mathbb{R}^+ -weight of the corresponding potential.

We now want to proceed with the analysis of the non-standard solitonic supersymmetric branes of maximal supergravity theories. Before discussing these branes, we will first discuss all the standard ones in the next section. These branes have $\alpha = 0, -1$ and -2 , and we will show how the counting of the half-supersymmetric branes leads to interesting so-called «wrapping rules» for each value of α . We will then discuss the non-standard solitonic, i.e. $\alpha = -2$, branes in Sec. 2, and see how the wrapping rules apply to these branes as well by means of generalized KK monopoles.

1. THE «STANDARD» BRANES

It is well-known that both the IIA and the IIB string theory have a single fundamental string that couples to the NS-NS 2-form potential. Since strings

can wrap we have in $D < 10$ dimensions both strings and wrapped strings, i.e., 0-branes, which couple to 2-forms and 1-forms, respectively. Naively, one would expect one wrapped string for each compactified direction. Instead, we end up with *two* 0-branes for each compactified direction. This is due to the fact that IIA/IIB string theory also contains a pp-wave which, upon reduction, gives rise to an additional 0-brane. Effectively, we therefore end up with *two* 0-branes for each compactified direction. This is precisely what we need in order that the corresponding 1-forms $B_{1,A}$ ($A = 1, \dots, 2d$) organize themselves as a vector of the T-duality group $SO(d, d)$.

It turns out that in each dimension $D < 10$ the T-duality singlet 2-form B_2 and the T-duality vector $B_{1,A}$ transform under each other's gauge transformation and together form a « p -form algebra». Therefore, both are needed to construct a gauge-invariant WZ term. To construct a gauge-invariant WZ term, we need to introduce a T-duality vector $b_{0,A}$ of additional world volume scalars:

$$\mathcal{L}_{WZ}(D < 10) = B_2 + \eta^{AB} \mathcal{F}_{1,A} B_{1,B}, \quad \mathcal{F}_{1,A} = db_{0,A} + B_{1,A}. \quad (3)$$

Together with the embedding scalars, these «extra» scalars will not fit into a world volume scalar multiplet. To get the correct counting, we need to impose a self-duality condition on the extra scalars like in doubled geometry [10].

The lower-dimensional fundamental branes (0-branes $F0_A$ and string $F1$) can be nicely understood as the result of the following simple «wrapping rule»:

$$\begin{aligned} \text{wrapped} &\rightarrow \text{doubled}, \\ \text{unwrapped} &\rightarrow \text{undoubled}, \end{aligned} \quad (4)$$

when applied to the single fundamental IIA/IIB string, see Table 3.

Table 3. Applying the fundamental wrapping rule (4) to the IIA/IIB fundamental string gives rise, in each dimension $3 \leq D \leq 9$, to a singlet fundamental string $F1$ and a T-duality vector of 0-branes $F0_A$

Fp -brane	IIA/IIB	9	8	7	6	5	4	3
0		2	4	6	8	10	12	14
1	1/1	1	1	1	1	1	1	1

Similarly, one can analyze the Dirichlet branes, i.e., the branes with $\alpha = -1$. One can show that they satisfy the wrapping rule

$$\begin{aligned} \text{wrapped} &\rightarrow \text{undoubled}, \\ \text{unwrapped} &\rightarrow \text{undoubled}. \end{aligned} \quad (5)$$

Unlike the fundamental branes, the D-branes are complete by themselves in the sense that the realization of the D-brane wrapping rule (5) does not require the input of any gravitational solutions.

The third set of standard branes we consider are the standard solitons, i.e., branes with $\alpha = -2$ and $T \geq 3$ transverse directions. The IIA/IIB string theory has a single solitonic NS–NS 5-brane. Upon wrapped reduction, it gives rise to a single $D = 9$ solitonic 4-brane and upon unwrapped reduction it leads to a single $D = 9$ solitonic 5-brane. It turns out that in this case the solitonic 5-brane is doubled due to the presence of a single Kaluza–Klein monopole in the IIA/IIB string theory. This leads to the following dual or solitonic wrapping rule:

$$\begin{aligned} \text{wrapped} &\rightarrow \text{undoubled}, \\ \text{unwrapped} &\rightarrow \text{doubled}. \end{aligned} \tag{6}$$

When applied to the solitonic NS–NS 5-brane of the IIA/IIB string theory, it gives rise to a singlet $S(D - 5)$ -brane soliton and a T-duality vector $S(D - 4)_A$ of brane-solitons, see Table 4.

Table 4. Applying the solitonic wrapping rule (6) to the NS–NS solitonic 5-brane of the IIA/IIB string theory leads to a lower-dimensional singlet $S(D - 5)$ -brane soliton and a vector $S(D - 4)_A$ of brane-solitons

Sp -brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	
1					1	10		
2				1	8			
3			1	6				
4		1	4					
5	1/1	2						

This finishes our discussion of the standard branes. The question is now what happens with the non-standard solitonic branes, i.e., the $\alpha = -2$ branes with $T \leq 2$ transverse directions. We will discuss this in the next section.

2. THE «NON-STANDARD» SOLITONIC BRANES

Our discussion on the solitonic branes, started in the previous section, has not finished yet. A supergravity analysis, making use of the decomposition (2), shows that there are more «non-standard» solitonic branes, i.e., branes with $0 \leq T \leq 2$ transverse directions. They occur as antisymmetric tensor representations of the T-duality group, see Table 5. This table should be read as follows. In each dimension D there are solitonic branes in antisymmetric representations of

increasing rank, starting with rank 0 at the top row up to a maximum rank r_{\max} given by $r_{\max} = d$ if $D \geq 6$, which always corresponds to a solitonic 5-brane, and $r_{\max} = 4$ if $D \leq 6$. The solitonic 5-brane with maximum rank representation decomposes into a self-dual and anti-self-dual representation of the $SO(d, d)$ T-duality group. One of these representations has a world volume vector multiplet, while the other has a world volume self-dual tensor multiplet.

Table 5. This table indicates all D -dimensional solitonic branes, standard as well as non-standard ones. For a given dimension D the maximum rank under T-duality is given by $r_{\max} = d$ if $D \geq 6$, corresponding to a solitonic 5-brane, and $r_{\max} = 4$ if $D \leq 6$

$S(D - 5)$ -brane
$[S(D - 4)$ -brane] $_A$
$[S(D - 3)$ -brane] $_{AB}$
$[S(D - 2)$ -brane] $_{ABC}$
$[S(D - 1)$ -brane] $_{ABCD}$

As we already anticipated in the introduction, the non-standard branes behave differently than the standard ones in the sense that not each component of the antisymmetric tensor representations occurring in Table 5 corresponds to a *half-supersymmetric* solitonic brane. Imposing our half-supersymmetric brane criterion discussed in the introduction leads to the correct number of supersymmetric branes. The result of this analysis can be found in Table 6, which contains the half-supersymmetric standard solitonic branes as well.

Table 6. This table indicates the number of half-supersymmetric solitonic branes, both the standard and the non-standard ones for dimensions $3 \leq D \leq 10$

Sp -brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	160	560
3			1	6	24	80	240	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

Surprisingly, we find that the numbers of half-supersymmetric solitons, given in Table 6, are *precisely* the same as the ones one obtains by extending the solitonic wrapping rule (6) from standard solitons only to standard as well as non-standard half-supersymmetric solitons! Strictly speaking, without saying explicitly we also extended the wrapping rule for the fundamental branes and D-branes from standard to non-standard ones. The difference is that in that case *all*

components of the scalar, vector and spinor T-duality representations involved correspond to supersymmetric branes. In the case of fundamental branes, non-standard strings only happen for $D \leq 4$ dimensions, while non-standard 0-branes only occur in $D = 3$ dimensions. Non-standard D-branes already occur in $D = 10$ dimensions.

We are now faced with the following question: where do the solitons that realize the solitonic wrapping rule (6) come from? In the case of the standard solitons, the answer to this question is that the solitonic wrapping rule can be realized due to the presence of the Kaluza–Klein (KK) monopole in $D = 10$ dimensions. The difference between the KK monopole and the other branes is that these monopoles divide space-time into *three* in-equivalent directions. Besides the world volume and transverse directions, which we already encountered with the branes, there is a third so-called «isometry» direction. We call the KK monopole a «standard» KK monopole because it has three transverse directions. It turns out that in the same way that the standard KK monopole is needed to realize the solitonic wrapping rule (6) and obtain the standard solitons, a new kind of so-called «non-standard» KK monopoles, with $T \leq 2$ transverse directions, are needed to realize the same wrapping rule and obtain the non-standard solitonic branes. Precisely this class of non-standard KK monopoles have been analyzed and classified some time ago in [11]. They are local solutions of the corresponding supergravity theory. It remains to be seen whether they can be realized as non-singular finite-energy solutions within string theory.

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