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ON BOUNCING SOLUTIONS IN NONLOCAL GRAVITY

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A nonlocal modified gravity model with an analytical function of the d'Alembert operator, is considered. This model has been recently proposed as a possible way of resolving the singularities problem in cosmology. We present exact bouncing solution, which is simpler compared to the already known one in this model, in the sense, it does not require an additional matter to satisfy all gravitational equations.

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INTRODUCTION

Modified gravity cosmological models have been proposed in the hope of finding solutions to the important open problems of the standard cosmological model. One possible modification, which allows one to improve ultraviolet behavior, and even to get renormalizable theory of quantum gravity is adding higher-derivative terms to the Einstein–Hilbert action (as one of the first papers we can mention [1]). Unfortunately, models with higher-derivative terms have ghosts. A way to overcome this problem is to consider nonlocal gravity.

The main theoretical motivation for studying cosmological models, with nonlocal corrections to the Einstein–Hilbert action, comes from the string field theory [2]. These corrections usually contain the exponential functions of the d'Alambertian operator and appear in such stringy models as tachyonic actions in string field theory framework. The majority of nonlocal cosmological models motivated by such structures explicitly include an analytic or meromorphic function of the d'Alembert operator [3–9].

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Usually both general relativity and modified gravity models are described by a nonintegrable system of equations and only particular exact solutions can be obtained. At the same time, exact solutions play an important role in the cosmological models since one must consider perturbations in order to claim the model is realistic. Needless to say exact solutions for nonlocal nonlinear equations is an extremely tough subject. Some studies for nonlocal gravitational models with exact solutions can be found in [5–8].

1. ACTION AND EQUATIONS OF MOTION

The nonlocal modification of the Einstein gravity, which has been proposed in [5, 6], is described by the following action:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{1}{2}R\mathcal{F}(\Box/M_*^2)R - \Lambda\right), \tag{1}$$

where M_P is the Planck mass. M_* is the mass scale at which the higher derivative terms in the action become important. An analytic function $\mathcal{F}(\Box/M_*^2) = \sum\limits_{n\geqslant 0} f_n\Box^n$ is an ingredient inspired by the SFT. The operator $\Box$ is the covariant d'Alembertian. In the case of an infinite series we have a nonlocal action.

Let us introduce dimensionless coordinates $\bar{x}_\mu = M_* x_\mu$ and $\bar{M}_P = M_P/M_*$. It is easy to see that $\mathcal{F}(\Box/M_*^2) = \mathcal{F}(\bar{\Box})$, where $\bar{\Box}$ is the d'Alembertian in terms of dimensionless coordinates. In the following formulae we omit bars, but use only dimensionless coordinates.

Straightforward variation of action (1) yields the following equations:

$$\begin{aligned}(M_P^2 + 2\mathcal{F}(\Box)R)\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = 2(D_\mu\partial_\nu - g_{\mu\nu}\Box)\mathcal{F}(\Box)R - \Lambda g_{\mu\nu} +{}\\ + \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1}\left[\partial_\mu\Box^l R\partial_\nu\Box^{n-l-1}R + \partial_\nu\Box^l R\partial_\mu\Box^{n-l-1}R -{}\right.\\ \left. - g_{\mu\nu}\left(g^{\rho\sigma}\partial_\rho\Box^l R\partial_\sigma\Box^{n-l-1}R + \Box^l R\Box^{n-l}R\right)\right] - \frac{1}{2}R\mathcal{F}(\Box)Rg_{\mu\nu},\end{aligned} \tag{2}$$

where D_μ is the covariant derivative. It is useful [6] to write down the trace equation:

$$M_P^2 R - \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1}\left(\partial_\mu\Box^l R\partial^\mu\Box^{n-l-1}R + 2\Box^l R\Box^{n-l}R\right) - 6\Box\mathcal{F}(\Box)R = 4\Lambda. \tag{3}$$

2. GENERAL ANSATZ FOR FINDING EXACT SOLUTIONS

It has been shown [5] that the following ansatz:

$$\Box R = r_1 R + r_2, \tag{4}$$

where $r_1 \neq 0$, is useful to find exact solutions. Using (4), the trace equation becomes

$$A_1 R + A_2 \left(2r_1 R^2 + \partial_\mu R \partial^\mu R\right) + A_3 = 0, \tag{5}$$

where

$$A_1 = -M_P^2 + 4\mathcal{F}'(r_1) r_2 - 2\frac{r_2}{r_1}(\mathcal{F}(r_1) - f_0) + 6\mathcal{F}(r_1) r_1,$$
$$A_2 = \mathcal{F}'(r_1),$$
$$A_3 = 4\Lambda + \frac{r_2}{r_1} M_P^2 + \frac{r_2}{r_1} A_1.$$

The simplest way to get a solution of equation (5) is to put all the above coefficients to zero. Relation $A_j = 0$, $j = 1, 2, 3$ determines values of r_1, r_2 and provides a constraint on the value of the cosmological constant:

$$\begin{gathered} \mathcal{F}'(r_1) = 0, \quad r_2 = -\frac{r_1[M_P^2 - 6\mathcal{F}(r_1) r_1]}{2[\mathcal{F}(r_1) - f_0]}, \\ \Lambda = -\frac{r_2 M_P^2}{4 r_1} = M_P^2 \frac{[M_P^2 - 6\mathcal{F}(r_1) r_1]}{8[\mathcal{F}(r_1) - f_0]}. \end{gathered} \tag{6}$$

3. EXACT SOLUTIONS AND THEIR APPLICATIONS

Let us consider solutions in the spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric with the interval $ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right)$.

The very important result for this kind of models was a construction of an analytic solution describing the nonsingular bounce

$$a(t) = a_0 \cosh(\lambda t), \tag{7}$$

where a_0 is an arbitrary constant and $\lambda = \sqrt{\Lambda}/3M_P$. To satisfy all equations (2) one should add a radiation to the model. This is the exact analytic result and we refer the reader to [5, 6] about all the details.

Let us consider another solution, which satisfies the ansatz (4). Namely,

$$a(t) = a_0 \, \exp\left(\frac{\lambda}{2} t^2\right), \tag{8}$$

where a_0 is an arbitrary constant. On this solution

$$\begin{gathered} H(t) = \lambda t, \quad R = 12\lambda^2 t^2 + 6\lambda, \quad \Box R = -72\lambda^3 t^2 - 24\lambda^2 \quad \Rightarrow \\ r_1 = -6\lambda, \quad r_2 = 12\lambda^2, \end{gathered} \tag{9}$$

where $H = \dot{a}/a$ is the Hubble parameter, differentiation with respect to time t is denoted by a dot. From relation $A_3 = 0$, we get $\Lambda = \lambda M_P^2/2$. From $A_1 = 0$ and $A_2 = 0$, we get the following conditions on the function $\mathcal{F}$ at the point r_1:

$$\mathcal{F}(r_1) = -\frac{M_P^2}{32\lambda} - \frac{f_0}{8}, \quad \mathcal{F}'(r_1) = 0. \tag{10}$$

There are two independent Einstein equations in the FLRW metric. Let us consider «00» component of system (2), which reads, after imposing the simplifying ansatz and using conditions $A_j = 0$, as the second-order differential equation for the Hubble parameter H:

$$\mathcal{F}(r_1)\left[H\ddot{H} + 3H^2\dot{H} - \frac{1}{2}\dot{H}^2 + \frac{r_1}{2}H^2 + \frac{r_2}{24}\right] = 0. \tag{11}$$

Substituting (9) we get that Eq. (11) is satisfied, so the function (8) is a solution of all Einstein equations. Note that we do not add any matter to get the exact solution.

We stress that a construction of exact solutions is obviously a nontrivial task and to the moment only one nontrivial exact analytic bouncing solution (7) in the class of models given by action (1) is analyzed [6]. We present here another bouncing solution (8) which is simpler compared to (7), in the sense, it does not require an additional matter to be present.

We leave open questions of perturbation spectrum for exact solutions in this nonlocal model and those applications to describing the bounce phase and the initial inflation stage. These questions will be addressed in the forthcoming publications.

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