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ABELIAN 3-FORM GAUGE THEORY: SUPERFIELD APPROACH

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We discuss a D-dimensional Abelian 3-form gauge theory within the framework of Bonora–Tonin's superfield formalism and derive the off-shell nilpotent and absolutely anticommuting Becchi–Rouet–Stora–Tyutin (BRST) and anti-BRST symmetry transformations for this theory. To pay our homage to V. I. Ogievetsky (1928–1996), who was one of the inventors of Abelian 2-form (antisymmetric tensor) gauge field, we go a step further and discuss the above D-dimensional Abelian 3-form gauge theory within the framework of BRST formalism and establish that the existence of the (anti-)BRST invariant Curci–Ferrari (CF)-type of restrictions is the hallmark of any arbitrary p-form gauge theory (discussed within the framework of BRST formalism).

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INTRODUCTION

In recent years, the study of higher p-form ($p = 2, 3, 4, \ldots$) gauge theories has become quite fashionable because of its relevance in the context of (super)string theories and related extended objects (see, e.g., [1, 2]). It is worthwhile to mention, in this context, that Ogievetsky and Polubarinov [3] were the first to study the 2-form antisymmetric tensor gauge field (way back in 1966–1967). Our presentation is a tribute to V. I. Ogievetsky (1928–1996) because we go a step further in the direction of the study of higher p-form ($p \geqslant 2$) gauge theories and discuss the Abelian 3-form gauge theory in arbitrary D-dimensions of spacetime within the framework of superfield approach to Becchi–Rouet–Stora–Tyutin (BRST) formalism, proposed in [4, 5].

We derive the proper (i.e., off-shell nilpotent and absolutely anticommuting) (anti-)BRST symmetry transformations for the above 3-form ($A^{(3)} = (1/3!)(dx^\mu \wedge dx^\nu \wedge dx^\eta) A_{\mu\nu\eta}$) totally antisymmetric tensor gauge field $A_{\mu\nu\eta}$ which appears

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in the quantum excitations of the (super)strings. Furthermore, we also obtain the proper (anti-)BRST transformations associated with the (anti-)ghost fields of the theory. Our main goal is to establish that the existence of the Curci–Ferrari (CF)-type restrictions [6] is the hallmark of any arbitrary p-form ($p = 1, 2, 3, \ldots$) gauge theory when it is discussed within the framework of superfield approach to BRST formalism. In fact, we show that the derivation of the CF-type condition(s) is a very natural consequence of the application of the superfield approach [4, 5] to BRST formalism.

Our present write-up is organized as follows. In Sec. 1, we recapitulate the bare essentials of the horizontality condition and apply it to the Abelian 3-form gauge theory. In the next section, we derive the proper (anti-)BRST symmetry transformations and corresponding coupled (but equivalent) Lagrangian densities for the present theory. Section 3 deals with the critical and crucial comments on the CF-type restrictions. Finally, in Conclusion, we summarize our key results and make some concluding remarks.

1. HORIZONTALITY CONDITION

Let us begin with the starting Lagrangian density of the Abelian 3-form field

$$\mathcal{L}_0 = \frac{1}{24} H^{\mu\nu\eta\kappa} H_{\mu\nu\eta\kappa}, \quad H_{\mu\nu\eta\kappa} = \partial_\mu A_{\nu\eta\kappa} - \partial_\nu A_{\eta\kappa\mu} + \partial_\eta A_{\kappa\mu\nu} - \partial_\kappa A_{\mu\nu\eta}, \quad (1)$$

where the 4-form $H^{(4)} = dA^{(3)} \equiv (1/4!)(dx^\mu \wedge dx^\nu \wedge dx^\eta \wedge dx^\kappa) H_{\mu\nu\eta\kappa}$ defines the curvature tensor $H_{\mu\nu\eta\kappa}$ which is derived from the exterior derivative $d = dx^\mu \partial_\mu$ ($d^2 = 0, \mu, \nu, \eta, \ldots = 0, 1, 2, \ldots, D-1$) and the 3-form $A^{(3)}$ that encodes the totally antisymmetric tensor gauge connection $A_{\mu\nu\eta}$. It can be easily seen that, under the following local gauge symmetry transformations δ_g (with the local infinitesimal antisymmetric gauge parameter $\Lambda_{\mu\nu} = -\Lambda_{\nu\mu}$):

$$\delta_g A_{\mu\nu\eta} = \partial_\mu \Lambda_{\nu\eta} + \partial_\nu \Lambda_{\eta\mu} + \partial_\eta \Lambda_{\mu\nu}, \quad (2)$$

the Lagrangian density and the curvature tensor $H_{\mu\nu\eta\kappa}$ remain invariant. Thus, the gauge invariant curvature tensor $H_{\mu\nu\eta\kappa}$, derived from the 4-form $H^{(4)}$, is a physical quantity (in some sense). One of the most intuitive approaches to covariantly quantize any arbitrary p-form gauge theory is the BRST formalism where the local gauge symmetry transformations (e.g., (2)) are traded with the proper (i.e., off-shell nilpotent and absolutely anticommuting) (anti-)BRST symmetry transformations.

The above (anti-)BRST symmetries can be derived by exploiting the geometrical superfield formalism [4, 5] where any arbitrary D-dimensional gauge theory is generalized onto the $(D, 2)$-dimensional supermanifold as [7]:

$$\begin{aligned} d \to \tilde{d} \equiv dZ^M \partial_M = dx^\mu \, \partial_\mu + d\theta \, \partial_\theta + d\bar{\theta} \, \partial_{\bar{\theta}}, \quad x^\mu \to Z^M = (x^\mu, \theta, \bar{\theta}), \\ A^{(3)} \to \tilde{A}^{(3)} = \frac{(dZ^M \wedge dZ^N \wedge dZ^K)}{3!} \, \tilde{A}_{MNK}, \quad \partial_M = (\partial_\mu, \partial_\theta, \partial_{\bar{\theta}}). \end{aligned} \tag{3}$$

Here the superspace variables $Z^M = (x^\mu, \theta, \bar{\theta})$ are the generalization of the ordinary D-dimensional coordinates x^μ that incorporate a pair of Grassmannian variables θ and $\bar{\theta}$, too, which satisfy $\theta^2 = \bar{\theta}^2 = 0, \theta\bar{\theta} + \bar{\theta}\theta = 0$. In the horizontality condition (HC), we require the physical (geometrical) quantity $H^{(4)}$ to remain independent of the Grassmannian variables, namely:

$$\tilde{H}^{(4)} = H^{(4)} \Longrightarrow \tilde{d}\tilde{A}^{(3)} = dA^{(3)}. \tag{4}$$

This leads, automatically, to the derivation of the off-shell nilpotent $(s_{(a)b}^2 = 0)$ and absolutely anticommuting $(s_b s_{ab} + s_{ab} s_b = 0)$ (anti-)BRST symmetry transformations $(s_{(a)b})$ for the gauge field and corresponding (anti-)ghost fields of the theory which we discuss, in detail, in the following section.

2. PROPER (ANTI-)BRST SYMMETRIES

It is clear from equation (4) that the r.h.s. of it contains only the space-time differentials (i.e., $dA^{(3)} = (1/4!)(dx^\mu \wedge dx^\nu \wedge dx^\eta \wedge dx^\kappa) H_{\mu\nu\eta\kappa}$). However, the l.h.s. contains the space-time differentials together with the Grassmann differentials. Thus, it is evident that the HC amounts to setting equal to zero the coefficient of all the differentials that contain Grassmannian variables. To check the above statement, it is imperative to compute the l.h.s. explicitly. Towards this goal, it can be seen that equation (3) implies [7]

$$\begin{aligned} \tilde{A}^{(3)} = \frac{1}{3!}(dx^\mu \wedge dx^\nu \wedge dx^\eta)\tilde{A}_{\mu\nu\eta} + \frac{1}{2}(dx^\mu \wedge dx^\nu \wedge d\theta)\tilde{A}_{\mu\nu\theta} + \\ + \frac{1}{2}(dx^\mu \wedge dx^\nu \wedge d\bar{\theta})\tilde{A}_{\mu\nu\bar{\theta}} + \frac{1}{3!}(d\theta \wedge d\theta \wedge d\theta)\tilde{A}_{\theta\theta\theta} + \frac{1}{3!}(d\bar{\theta} \wedge d\bar{\theta} \wedge d\bar{\theta})\tilde{A}_{\bar{\theta}\bar{\theta}\bar{\theta}} + \\ + (dx^\mu \wedge d\theta \wedge d\bar{\theta})\tilde{A}_{\mu\theta\bar{\theta}} + \frac{1}{2}(dx^\mu \wedge d\theta \wedge d\theta)\tilde{A}_{\mu\theta\theta} + \frac{1}{2}(d\theta \wedge d\theta \wedge d\bar{\theta})\tilde{A}_{\theta\theta\bar{\theta}} + \\ + \frac{1}{2}(dx^\mu \wedge d\bar{\theta} \wedge d\bar{\theta})\tilde{A}_{\mu\bar{\theta}\bar{\theta}} + \frac{1}{2}(d\theta \wedge d\bar{\theta} \wedge d\bar{\theta})\tilde{A}_{\theta\bar{\theta}\bar{\theta}}. \end{aligned} \tag{5}$$

Keeping in mind the nature of the superfields, we make the suitable identifications: $\tilde{A}_{\mu\nu\eta} = \tilde{\mathcal{A}}_{\mu\nu\eta}(x, \theta, \bar{\theta})$, $\tilde{A}_{\mu\nu\theta} = \tilde{\bar{\mathcal{F}}}_{\mu\nu}(\S, \theta, \bar{\theta})$, $\tilde{A}_{\mu\nu\bar{\theta}} = \tilde{\mathcal{F}}_{\mu\nu}(x, \theta, \bar{\theta})$, $\tilde{A}_{\mu\theta\bar{\theta}} = \tilde{\Phi}_\mu(x, \theta, \bar{\theta})$, $(1/3!)\tilde{A}_{\theta\theta\theta} = \tilde{\bar{\mathcal{F}}}_2(x, \theta, \bar{\theta})$, $(1/3!)A_{\bar{\theta}\bar{\theta}\bar{\theta}} = \tilde{\mathcal{F}}_2(x, \theta, \bar{\theta})$, $(1/2)\tilde{A}_{\theta\bar{\theta}\bar{\theta}} = \tilde{\mathcal{F}}_1(x, \theta, \bar{\theta})$, $(1/2)\tilde{A}_{\theta\theta\bar{\theta}} = \tilde{\bar{\mathcal{F}}}_1(x, \theta, \bar{\theta})$, $(1/2)\tilde{A}_{\mu\bar{\theta}\bar{\theta}} = \tilde{\beta}_\mu(x, \theta, \bar{\theta})$ and $(1/2)\tilde{A}_{\mu\theta\theta} = \tilde{\bar{\beta}}_\mu(x, \theta, \bar{\theta})$ as the generalization of the D-dimensional local fields $A_{\mu\nu\eta}$, $\bar{C}_{\mu\nu}$,

$C_{\mu\nu}, \phi_\mu, \bar{C}_2, C_2, C_1, \bar{C}_1, \beta_\mu, \bar{\beta}_\mu$ of the (anti-)BRST invariant local D-dimensional ordinary theory onto the $(D, 2)$-dimensional supermanifold (within our superfield formalism).

The super-expansions of the above superfields, along the Grassmannian directions of the $(D, 2)$-dimensional supermanifold are as follows [7]:

$$
\begin{aligned}
\tilde{\mathcal{A}}_{\mu\nu\eta}(x,\theta,\bar{\theta}) &= A_{\mu\nu\eta}(x) + \theta\ \bar{R}_{\mu\nu\eta}(x) + \bar{\theta}\ R_{\mu\nu\eta}(x) + i\ \theta\ \bar{\theta}\ S_{\mu\nu\eta}(x),\\
\tilde{\beta}_\mu(x,\theta,\bar{\theta}) &= \beta_\mu(x) + \theta\ \bar{f}^{(1)}_\mu(x) + \bar{\theta} f^{(1)}_\mu(x) + i\theta\bar{\theta} b_\mu(x),\\
\tilde{\bar{\beta}}_\mu(x,\theta,\bar{\theta}) &= \bar{\beta}_\mu(x) + \theta \bar{f}^{(2)}_\mu(x) + \bar{\theta} f^{(2)}_\mu(x) + i\theta\bar{\theta}\bar{b}_\mu(x),\\
\tilde{\Phi}_\mu(x,\theta,\bar{\theta}) &= \phi_\mu(x) + \theta \bar{f}^{(3)}_\mu(x) + \bar{\theta} f^{(3)}_\mu(x) + i\theta\bar{\theta} b^{(3)}_\mu(x),\\
\tilde{\mathcal{F}}_{\mu\nu}(x,\theta,\bar{\theta}) &= C_{\mu\nu}(x) + \theta \bar{B}^{(1)}_{\mu\nu}(x) + \bar{\theta} B^{(1)}_{\mu\nu}(x) + i\theta\bar{\theta} s_{\mu\nu}(x),\\
\tilde{\bar{\mathcal{F}}}_{\mu\nu}(\S,\theta,\bar{\theta}) &= \bar{C}_{\mu\nu}(x) + \theta \bar{B}^{(2)}_{\mu\nu}(x) + \bar{\theta} B^{(2)}_{\mu\nu}(x) + i\theta\bar{\theta} \bar{s}_{\mu\nu}(x),\\
\tilde{\mathcal{F}}_1(x,\theta,\bar{\theta}) &= C_1(x) + \theta \bar{b}^{(1)}_1(x) + \bar{\theta} b^{(1)}_1(x) + i\theta\bar{\theta} s_1(x),\\
\tilde{\bar{\mathcal{F}}}_\infty(\S,\theta,\bar{\theta}) &= \bar{C}_1(x) + \theta \bar{b}^{(2)}_1(x) + \bar{\theta} b^{(2)}_1(x) + i\theta\bar{\theta} \bar{s}_1(x),\\
\tilde{\mathcal{F}}_2(x,\theta,\bar{\theta}) &= C_2(x) + \theta \bar{b}^{(1)}_2(x) + \bar{\theta} b^{(1)}_2(x) + i\theta\bar{\theta} s_2(x),\\
\tilde{\bar{\mathcal{F}}}_\in(\S,\theta,\bar{\theta}) &= \bar{C}_2(x) + \theta \bar{b}^{(2)}_2(x) + \bar{\theta} b^{(2)}_2(x) + i\theta\bar{\theta} \bar{s}_2(x),
\end{aligned}
\tag{6}
$$

where $A_{\mu\nu\eta}$ is the gauge field; ϕ_μ is the vector bosonic field; $(\bar{C}_{\mu\nu})C_{\mu\nu}$ are the pair of fermionic antisymmetric (anti-)ghost fields; $(\bar{\beta}_\mu)\beta_\mu$ are the bosonic ghost-for-ghost (anti-)ghost fields; $(\bar{C}_2)C_2$ and $(\bar{C}_1)C_1$ are the Lorentz scalar fermionic (anti-)ghost fields. The above fields are required for the proof of unitarity in the theory. The rest of the fields, on the r.h.s. of Eq. (6), are secondary fields that have to be determined in terms of the basic and auxiliary fields of the D-dimensional ordinary theory by exploiting the HC.

Explicit computation of (4) and setting equal to zero all the coefficients of the Grassmannian differentials of the super 4-form of the l.h.s., leads to

$$
\begin{gathered}
b^{(1)}_2 = 0, \quad s_2 = 0, \quad \bar{b}^{(2)}_2 = 0, \quad \bar{s}_2 = 0, \quad \bar{s}_1 = 0, \quad s_1 = 0,\\
\bar{b}^{(1)}_2 + b^{(1)}_1 = 0, \quad b^{(2)}_1 + \bar{b}^{(1)}_1 = 0, \quad \bar{f}^{(2)}_\mu = \partial_\mu \bar{C}_2, \quad f^{(1)}_\mu = \partial_\mu C_2,\\
\bar{b}_\mu = -i\partial_\mu b^{(2)}_2, \quad b^{(3)}_\mu = -i\partial_\mu b^{(2)}_1, \quad B^{(1)}_{\mu\nu} = \partial_\mu\beta_\nu - \partial_\nu\beta_\mu, \quad \bar{f}^{(2)}_\mu = \partial_\mu\bar{C}_2,\\
\bar{B}^{(2)}_{\mu\nu} = \partial_\mu\bar{\beta}_\nu - \partial_\nu\bar{\beta}_\mu, \quad s_{\mu\nu} = i(\partial_\mu \bar{f}^{(1)}_\nu - \partial_\nu \bar{f}^{(1)}_\mu) \equiv -i(\partial_\mu f^{(3)}_\nu - \partial_\nu f^{(3)}_\mu),\\
\bar{s}_{\mu\nu} = +i(\partial_\mu \bar{f}^{(3)}_\nu - \partial_\nu \bar{f}^{(3)}_\mu) \equiv -i(\partial_\mu f^{(2)}_\nu - \partial_\nu f^{(2)}_\mu), \quad b_\mu = i\partial_\mu \bar{b}^{(1)}_2, \qquad (7)\\
R_{\mu\nu\eta} = \partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}, \quad \bar{R}_{\mu\nu\eta} = \partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu},\\
S_{\mu\nu\eta} = -i(\partial_\mu B^{(2)}_{\nu\eta} + \partial_\nu B^{(2)}_{\eta\mu} + \partial_\eta B^{(2)}_{\mu\nu}) \equiv +i(\partial_\mu \bar{B}^{(1)}_{\nu\eta} + \partial_\nu \bar{B}^{(1)}_{\eta\mu} + \partial_\eta \bar{B}^{(1)}_{\mu\nu}),\\
b^{(2)}_2 + \bar{b}^{(2)}_1 = 0.
\end{gathered}
$$

In addition to the above results, we obtain the following Curci–Ferrari-type restrictions from the HC illustrated in (4), namely:

$$f_\mu^{(2)} + \bar{f}_\mu^{(3)} = \partial_\mu \bar{C}_1, \quad \bar{f}_\mu^{(1)} + f_\mu^{(3)} = \partial_\mu C_1, \quad \bar{B}_{\mu\nu}^{(1)} + B_{\mu\nu}^{(2)} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu, \quad (8)$$

which ensure the consistency of the *three* equivalences shown in (7). At this stage, a couple of remarks are in order. First, the above restrictions emerge from setting the specific coefficients of the 4-form differentials (e.g., $(dx^\mu \wedge d\theta \wedge d\theta \wedge d\bar{\theta})$, $(dx^\mu \wedge d\theta \wedge d\bar{\theta} \wedge d\bar{\theta})$, $(dx^\mu \wedge dx^\nu \wedge d\theta \wedge d\bar{\theta})$) of the l.h.s. of the HC. Second, it is worth pointing out that the coefficients of the differentials $(dx^\mu \wedge dx^\nu \wedge dx^\eta \wedge dx^\kappa)$ from the l.h.s. and r.h.s. of the condition $\tilde{d}\tilde{A}^{(3)} = dA^{(3)}$ match due to the precise form of $R_{\mu\nu\eta}, \bar{R}_{\mu\nu\eta}$, and $S_{\mu\nu\eta}$, quoted in (7).

To make the notations cute and a bit simpler, we identify: $b_1^{(2)} = B_1$, $b_2^{(2)} = B_2$, $\bar{b}_2^{(1)} = \bar{B}$, $\bar{f}_\mu^{(1)} = F_\mu$, $f_\mu^{(2)} = \bar{F}_\mu$, $f_\mu^{(3)} = f_\mu$, $\bar{f}_\mu^{(3)} = \bar{f}_\mu$, $B_{\mu\nu}^{(2)} = B_{\mu\nu}$, $\bar{B}_{\mu\nu}^{(1)} = \bar{B}_{\mu\nu}$. As a consequence, the celebrated CF-type restrictions become $B_{\mu\nu} + \bar{B}_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$, $f_\mu + F_\mu = \partial_\mu C_1$, $\bar{f}_\mu + \bar{F}_\mu = \partial_\mu \bar{C}_1$. Furhermore, the proper off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetry transformations that emerge from HC (cf. Eq. (4)) are [7, 8]

$$\begin{gathered}
s_{ab} A_{\mu\nu\eta} = \partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu}, \quad s_{ab} \bar{C}_{\mu\nu} = \partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu, \\
s_{ab} \bar{\beta}_\mu = \partial_\mu \bar{C}_2, \quad s_{ab} \bar{C}_2 = 0, \quad s_{ab} \bar{B}_{\mu\nu} = 0, \quad s_{ab} C_1 = -B_1, \\
s_{ab} \bar{C}_1 = -B_2, \quad s_{ab} \bar{B} = 0, \quad s_{ab} C_2 = \bar{B}, \quad s_{ab} \beta_\mu = \bar{F}_\mu, \quad s_{ab} \bar{F}_\mu = 0, \qquad (9) \\
s_{ab} \bar{f}_\mu = 0, \quad s_{ab} F_\mu = -\partial_\mu B_2, \quad s_{ab} f_\mu = -\partial_\mu B_1, \quad s_{ab} B_{\mu\nu} = \partial_\mu \bar{f}_\nu - \partial_\nu \bar{f}_\mu, \\
s_{ab} C_{\mu\nu} = \bar{B}_{\mu\nu}, \quad s_{ab} B_1 = 0, \qquad s_{ab} B_2 = 0, \quad s_{ab} \phi_\mu = \bar{f}_\mu,
\end{gathered}$$

$$\begin{gathered}
s_b A_{\mu\nu\eta} = \partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}, \quad s_b C_{\mu\nu} = \partial_\mu \beta_\nu - \partial_\nu \beta_\mu, \\
s_b \beta_\mu = \partial_\mu C_2, \quad s_b C_2 = 0, \quad s_b B_{\mu\nu} = 0, \quad s_b C_1 = -\bar{B}, \\
s_b \bar{C}_1 = B_1, \quad s_b B_1 = 0, \quad s_b \bar{C}_2 = B_2, \quad s_b \bar{\beta}_\mu = F_\mu, \quad s_b F_\mu = 0, \qquad (10) \\
s_b f_\mu = 0, \quad s_b \bar{F}_\mu = -\partial_\mu \bar{B}, \quad s_b \bar{f}_\mu = \partial_\mu B_1, \quad s_b \bar{B}_{\mu\nu} = \partial_\mu f_\nu - \partial_\nu f_\mu, \\
s_b \bar{C}_{\mu\nu} = B_{\mu\nu}, \quad s_b \bar{B} = 0, \qquad s_b B_2 = 0, \quad s_b \phi_\mu = f_\mu.
\end{gathered}$$

It is elementary to check that the above (anti-)BRST symmetry transformations are off-shell nilpotent of order two (i.e., $s_{(a)b}^2 = 0$).

The above transformations have been obtained from the superfield formalism without any knowledge of the (anti-)BRST invariant Lagrangian density. This is due to the fact that the substitution of the results of (7) into (6) leads to the following superexpansion of the superfields in the language of the nilpotent (anti-)BRST symmetry transformations [7]

$$
\begin{aligned}
\tilde{\mathcal{A}}^{(h)}_{\mu\nu\eta}(x,\theta,\bar\theta) &= A_{\mu\nu\eta}(x) + \theta(s_{ab}A_{\mu\nu\eta}(x)) + \bar\theta(s_b A_{\mu\nu\eta}(x)) + \theta\bar\theta(s_b s_{ab} A_{\mu\nu\eta}(x)),\\
\tilde{\beta}^{(h)}_{\mu}(x,\theta,\bar\theta) &= \beta_\mu(x) + \theta(s_{ab}\beta_\mu(x)) + \bar\theta(s_b\beta_\mu(x)) + \theta\bar\theta(s_b s_{ab}\beta_\mu(x)),\\
\tilde{\bar{\beta}}^{(h)}_{\mu}(x,\theta,\bar\theta) &= \bar\beta_\mu(x) + \theta(s_{ab}\bar\beta_\mu(x)) + \bar\theta(s_b\bar\beta_\mu(x)) + \theta\bar\theta(s_b s_{ab}\bar\beta_\mu(x)),\\
\tilde{\Phi}^{(h)}_{\mu}(x,\theta,\bar\theta) &= \phi_\mu(x) + \theta(s_{ab}\phi_\mu(x)) + \bar\theta(s_b\phi_\mu(x)) + \theta\bar\theta(s_b s_{ab}\phi_\mu(x)),\\
\tilde{\mathcal{F}}^{(h)}_{\mu\nu}(x,\theta,\bar\theta) &= C_{\mu\nu}(x) + \theta(s_{ab}C_{\mu\nu}(x)) + \bar\theta(s_b C_{\mu\nu}(x)) + \theta\bar\theta(s_b s_{ab} C_{\mu\nu}(x)),\\
\tilde{\bar{\mathcal{F}}}^{(h)}_{\mu\nu}(x,\theta,\bar\theta) &= \bar C_{\mu\nu}(x) + \theta(s_{ab}\bar C_{\mu\nu}(x)) + \bar\theta(s_b \bar C_{\mu\nu}(x)) + \theta\bar\theta(s_b s_{ab} \bar C_{\mu\nu}(x)),\\
\tilde{\mathcal{F}}^{(h)}_{1}(x,\theta,\bar\theta) &= C_1(x) + \theta(s_{ab}C_1(x)) + \bar\theta(s_b C_1(x)) + \theta\bar\theta(s_b s_{ab} C_1(x)),\\
\tilde{\bar{\mathcal{F}}}^{(h)}_{1}(x,\theta,\bar\theta) &= \bar C_1(x) + \theta(s_{ab}\bar C_1(x)) + \bar\theta(s_b \bar C_1(x)) + \theta\bar\theta(s_b s_{ab} \bar C_1(x)),\\
\tilde{\mathcal{F}}^{(h)}_{2}(x,\theta,\bar\theta) &= C_2(x) + \theta(s_{ab}C_2(x)) + \bar\theta(s_b C_2(x)) + \theta\bar\theta(s_b s_{ab} C_2(x)),\\
\tilde{\bar{\mathcal{F}}}^{(h)}_{2}(x,\theta,\bar\theta) &= \bar C_2(x) + \theta(s_{ab}\bar C_2(x)) + \bar\theta;(s_b \bar C_2(x)) + \theta\bar\theta(s_b s_{ab} \bar C_2(x)),
\end{aligned}
\tag{11}
$$

where the proper (anti-)BRST symmetry transformations are denoted by $s_{(a)b}$, and the superscript (h), on the superfields, stands for the superexpansions of these superfields obtained after the application of HC (cf. (4)).

Furthermore, it can be checked that the anticommutativity property (i.e., $s_b s_{ab} + s_{ab} s_b = 0$) of $s_{(a)b}$ on the following basic fields [7, 8]:

$$
\{s_b, s_{ab}\}\, A_{\mu\nu\eta} = 0, \quad \{s_b, s_{ab}\}\, C_{\mu\nu} = 0, \quad \{s_b, s_{ab}\}\, \bar C_{\mu\nu} = 0, \tag{12}
$$

is true only when the Curci–Ferrari-type restrictions (8) are satisfied. The property of the anticommutativity of the (anti-)BRST symmetry transformations is trivially obeyed in the case of the rest of the fields of our present D-dimensional Abelian 3-form gauge theory. Finally, one can write down the coupled (but equivalent) (anti-)BRST invariant Lagrangian densities for the above Abelian 3-form gauge theory as (see, e.g., [8] for details)

$$
\begin{aligned}
\mathcal{L}_B = \frac{1}{24} H^{\mu\nu\eta\kappa} H_{\mu\nu\eta\kappa} + s_b s_{ab}\bigg(\frac{1}{2}\bar C_2 C_2 - \frac{1}{2}\bar C_1 C_1 + \frac{1}{2}\bar C_{\mu\nu} C^{\mu\nu} -\\
- \bar\beta^\mu \beta_\mu - \frac{1}{2}\phi^\mu\phi_\mu - \frac{1}{6} B^{\mu\nu\eta} B_{\mu\nu\eta}\bigg),
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\mathcal{L}_{\bar B} = \frac{1}{24} H^{\mu\nu\eta\kappa} H_{\mu\nu\eta\kappa} - s_{ab} s_b\bigg(\frac{1}{2}\bar C_2 C_2 - \frac{1}{2}\bar C_1 C_1 + \frac{1}{2}\bar C_{\mu\nu} C^{\mu\nu} -\\
- \bar\beta^\mu \beta_\mu - \frac{1}{2}\phi^\mu\phi_\mu - \frac{1}{6} B^{\mu\nu\eta} B_{\mu\nu\eta}\bigg).
\end{aligned}
\tag{14}
$$

The first Lagrangian density $\mathcal{L}_B$ is trivially invariant under the BRST transformations s_b. On the other hand, the second Lagrangian density $\mathcal{L}_{\bar{B}}$ is trivially invariant under the anti-BRST symmetry transformations s_{ab}. One can check that, under s_{ab}, the first Lagrangian density $\mathcal{L}_B$ transforms to a total derivative plus terms that are zero on the constrained surface defined by the CF-type restrictions (8). Precisely, similar is the situation with the Lagrangian density $\mathcal{L}_{\bar{B}}$ under the nilpotent BRST transformations s_b.

3. COMMENTS ON CF-TYPE RESTRICTIONS

It is well known that a gauge theory is always endowed with a local gauge symmetry that is generated by the first-class constraints in the language of Dirac's prescription for the classification scheme. Thus, the decisive feature of a gauge theory is the existence of the first-class constraints on the theory. When any arbitrary p-form gauge theory is discussed, within the framework of the BRST formalism, the above local gauge symmetry is traded with the supersymmetric-type (anti-)BRST symmetries $s_{(a)b}$ which turn out to be nilpotent ($s_{(a)b}^2 = 0$) of order two. Furthermore, the other sacrosanct feature of the latter symmetries is the absolute anticommutativity (i.e., $s_b s_{ab} + s_{ab} s_b = 0$). The anticommutativity property is achieved only due to the presence of CF-type restrictions. Thus, the clinching feature of any arbitrary p-form gauge theory, within the framework of BRST formalism, is the existence of the (anti-)BRST invariant CF-type restrictions. For the Abelian 1-form gauge theory, the CF-type restriction is *trivial*. However, it is *nontrivial* for all the rest of the gauge theories. Finally, the first-class constraints of the original gauge theory are encoded in the physicality criteria $Q_b|\text{phys}\rangle = 0$, where Q_b is the conserved and nilpotent BRST charge. This condition, in BRST formalism, enforces all the physical quantum states to be annihilated by the operator form of the first-class constraints of the original theory.

CONCLUSIONS

In this presentation, it has been emphasized that the Bonora–Tonin's superfield approach [4, 5] to BRST formalism always leads to the derivation of the proper (i.e., off-shell nilpotent and absolutely anticommuting) (anti-)BRST symmetry transformations for a given p-form gauge theory in any arbitrary D-dimensions of spacetime. Furthermore, this geometrical superfield formalism [4, 5] *necessarily* entails upon any arbitrary D-dimensional p-form gauge theory to be endowed with the (anti-)BRST invariant CF-type restriction(s) which, ultimately, lead to the absolute anticommutativity of the (anti-)BRST symmetry transformations and the derivation of the coupled (but equivalent) Lagrangian densities. It turns out that the CF condition, for the simple case of Abelian $U(1)$ 1-form gauge theory, is trivial. As a consequence, there is a single Lagrangian density for this theory that respects the (anti-)BRST symmetries *together*. This is *not* the case, however, for even the non-Abelian $SU(N)$ 1-form gauge theory and all the rest of the (non-)Abelian higher p-form ($p \geqslant 2$) gauge theories in any arbitrary D-dimensions of space-time.

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