ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2012. Т. 43. ВЫП. 5

# ON LAGRANGIAN FORMULATIONS FOR ARBITRARY BOSONIC HS FIELDS ON MINKOWSKI BACKGROUNDS

*A. A. Reshetnyak*[*]

Laboratory of Nonlinear Media Physics,
Institute of Strength Physics and Materials Science, Tomsk, Russia

We review the details of unconstrained Lagrangian formulations for Bose particles propagated on an arbitrary dimensional flat space-time and described by the unitary irreducible integer higher-spin representations of the Poincare group subject to Young tableaux $Y(s_1, \ldots, s_k)$ with $k$ rows. The procedure is based on the construction of scalar auxiliary oscillator realizations for the symplectic $sp(2k)$ algebra which encodes the second-class operator constraints subsystem in the HS symmetry algebra. Application of a universal BRST approach reproduces gauge-invariant Lagrangians with reducible gauge symmetries describing the free dynamics of both massless and massive bosonic fields of any spin with appropriate number of auxiliary fields.

PACS: 11.15.-q

## INTRODUCTION

Growth of the interest to higher-spin (HS) field theory is mainly stipulated by the hopes to reconsider the problems of a unique description of variety of elementary particles and all known interactions especially due to expected output of LHC on the planned capacity. Remind, that it suspects both the proof of supersymmetry display, the answer on the question on existence of Higgs boson, and probably a new insight on origin of Dark Matter ([1]). Because of HS field theory is closely related to superstring theory, which operates with an infinite tower of bosonic and fermionic HS fields, it can be treated as an approach to study the structure of superstring theory from field-theoretic viewpoint. Some of the aspects of current state of HS field theory are discussed in the reviews [2]. The paper considers the last results of constructing Lagrangian formulations (LFs) for free integer both massless and massive mixed-symmetry tensor HS fields on flat $\mathbb{R}^{1,d-1}$-space-time subject to arbitrary Young tableaux (YT) $Y(s_1, \ldots, s_k)$ in Fronsdal metric-like formalism within BFV–BRST approach [3], and based on the results presented in [4] (see [4], for detailed bibliography).

---

[*]E-mail: reshet@ispms.tsc.ru

It is known that, for $d>4$ space-time dimensions, there appear, besides totally symmetric irreducible representations of Poincare or (Anti)-de-Sitter ((A)dS) algebras, the mixed-symmetry representations determined by more than one spin-like parameters [5, 6]. Whereas for the former ones, the LFs both for massless and massive free higher-spin fields are well enough developed [7–11] as well as on the base of BFV–BRST approach, e.g., in [12–15], for the latter the problem of their field-theoretic description is not completely solved. So, the main result within the problem of LF for arbitrary massless mixed-symmetry HS fields on a Minkowski space-time was obtained in [16] with the use of unfolded form of equations of motion for the field in the «frame-like» formulation. In the «metric-like» formulation corresponding Lagrangians were derived in closed manner for only reducible Poincare group $ISO(1,d-1)$ representations in [17].

## 1. INTEGER HS SYMMETRY ALGEBRA FOR BOSONIC FIELDS

A massless integer spin Poincare group irrep in $\mathbb{R}^{1,d-1}$ is described by rank $\sum\limits_{i\geqslant 1}^{k} s_i$ tensor field $\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},\ldots,(\mu^k)_{s_k}} \equiv \Phi_{\mu^1_1\ldots\mu^1_{s_1},\mu^2_1\ldots\mu^2_{s_2},\ldots,\mu^k_1\ldots\mu^k_{s_k}}(x)$ with generalized spin $\mathbf{s}=(s_1,s_2,\ldots,s_k)$, $(s_1\geqslant s_2\geqslant\ldots\geqslant s_k>0, k\leqslant[d/2])$ subject to a YT with $k$ rows of lengths $s_1,s_2,\ldots,s_k$

$$\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},\ldots,(\mu^k)_{s_k}} \longleftrightarrow \begin{array}{|c|c|c|c|c|c|c|c|}\hline \mu^1_1 & \mu^1_2 & \cdot\ \ \cdot\ \ \cdot & \cdot & \cdot & \cdot & \cdot & \mu^1_{s_1} \\ \hline \mu^2_1 & \mu^2_2 & \cdot\ \ \cdot\ \ \cdot & \cdot & \cdot & \mu^2_{s_2} & \multicolumn{2}{c}{} \\ \cline{1-6} \cdots & \multicolumn{7}{c}{} \\ \cline{1-5} \mu^1_k & \mu^k_2 & \cdot\ \ \cdot\ \ \cdot & \cdot & \mu^k_{s_k} & \multicolumn{3}{c}{} \\ \cline{1-5} \end{array} \qquad (1)$$

The field is symmetric with respect to the permutations of each type of Lorentz indices $\mu^i$, (for $\eta_{\mu\nu}=\mathrm{diag}\,(+,-,\ldots,-)$, $\mu,\nu=0,1,\ldots,d-1$) and obeys to the Klein–Gordon, divergentless (2), traceless (3) and mixed-symmetry equations (4) (for $i,j=1,\ldots,k$; $l_i,m_i=1,\ldots,s_i$):

$$\partial^\mu\partial_\mu\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},\ldots,(\mu^k)_{s_k}}=0,\quad \partial^{\mu^i_{l_i}}\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},\ldots,(\mu^k)_{s_k}}=0, \qquad (2)$$

$$\eta^{\mu^i_{l_i}\mu^i_{m_i}}\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},\ldots,(\mu^k)_{s_k}}=\eta^{\mu^i_{l_i}\mu^j_{m_j}}\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},\ldots,(\mu^k)_{s_k}}=0,\quad l_i<m_i, \qquad (3)$$

$$\Phi_{(\mu^1)_{s_1},\ldots,\{(\mu^i)_{s_i}\underbrace{,\ldots,\mu^j_1\ldots}\mu^j_{l_j}\}\ldots\mu^j_{s_j},\ldots(\mu^k)_{s_k}}=0,\quad i<j,\quad 1\leqslant l_j\leqslant s_j, \qquad (4)$$

where the bracket below denotes that the indices in it are not included in symmetrization.

Simultaneous description of all $ISO(1,d-1)$ group irreps may be reformulated in a standard manner with an auxiliary Fock space $\mathcal{H}$, generated by $k$ pairs of bosonic creation $a^i_{\mu^i}(x)$ and annihilation $a^{j+}_{\nu^j}(x)$ operators, $i,j=1,\ldots,k$, $\mu^i,\nu^j=0,1\ldots,d-1$: $[a^i_{\mu^i},a^{j+}_{\nu^j}]=-\eta_{\mu^i\nu^j}\delta^{ij}$ and a set of constraints for an arbitrary string-like (the so-called *basic*) vector $|\Phi\rangle\in\mathcal{H}$,

$$|\Phi\rangle=\sum_{s_1=0}^{\infty}\sum_{s_2=0}^{s_1}\cdots\sum_{s_k=0}^{s_{k-1}}\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},\ldots,(\mu^k)_{s_k}}(x)\prod_{i=1}^{k}\prod_{l_i=1}^{s_i}a_i^{+\mu^i_{l_i}}|0\rangle, \tag{5}$$

$$\begin{gathered}(l_0,l^i,l^{ij},t^{i_1j_1})|\Phi\rangle=\left(\partial^\mu\partial_\mu,-ia^i_\mu\partial^\mu,\frac{1}{2}a^i_\mu a^{j\mu},a^{i_1+}_\mu a^{j_1\mu}\right)|\Phi\rangle=0,\\ i\leqslant j;\quad i_1<j_1.\end{gathered} \tag{6}$$

The set of $(k(k+1)+1)$ primary constraints (6), $\{o_\alpha\}=\left\{l_0,l^i,l^{ij},t^{i_1j_1}\right\}$, are equivalent to Eqs. (2)–(4) for all spins. In turn, additional condition, $g^i_0|\Phi\rangle=(s_i+d/2)|\Phi\rangle$ for number particles operators, $g^i_0=-a^{i+}_\mu a^{\mu i}+d/2$, makes (6) to be equivalent to (2)–(4) for a given spin $\mathbf{s}$.

The procedure of LF implies the Hermiticity of BFV–BRST operator $Q$, $Q=C^\alpha o_\alpha+\ldots$, that means the extension of the set $\{o_\alpha\}$ up to one of $\{o_I\}=\{o_\alpha,o^+_\alpha;g^i_0\}$, which is closed with respect to Hermitian conjugation related to standard scalar product on $\mathcal{H}$ and commutator multiplication $[\ ,\ ]$. Operators $o_I$ satisfy the Lie-algebra commutation relations, $[o_I,\ o_J]=f^K_{IJ}o_K$, for structure constants $f^K_{IJ}=-f^K_{JI}$, to be determined from the multiplication Table.

The products $B^{i_2j_2}_{i_1j_1},A^{i_2j_2,i_1j_1},F^{i_1j_1,i},L^{i_2j_2,i_1j_1}$ in the Table are given by the relations,

$$B^{i_2j_2}_{i_1j_1}=(g^{i_2}_0-g^{j_2}_0)\,\delta^{i_2}_{i_1}\,\delta^{j_2}_{j_1}+(t^{j_2}_{j_1}\,\theta^{j_2}_{j_1}+t^{j_2+}_{j_1}\,\theta^{j_2}_{j_1})\,\delta^{i_2}_{i_1}-(t^{+i_2}_{i_1}\,\theta^{i_2}_{i_1}+t^{i_2}_{i_1}\,\theta^{i_2}_{i_1})\,\delta^{j_2}_{j_1}, \tag{7}$$

$$A^{i_2j_2,i_1j_1}=t^{i_1j_2}\,\delta^{i_2j_1}-t^{i_2j_1}\,\delta^{i_1j_2},\quad F^{i_2j_2,i}=t^{i_2j_2}\,(\delta^{j_2i}-\delta^{i_2i}), \tag{8}$$

$$\begin{aligned}L^{i_2j_2,i_1j_1}=\frac{1}{4}\Big\{&\delta^{i_2i_1}\,\delta^{j_2j_1}\left[2g^{i_2}_0\delta^{i_2j_2}+g^{i_2}_0+g^{j_2}_0\right]-\\ &-\left(\delta^{j_2\{i_1}\left[t^{j_1\}i_2}\theta^{i_2j_1\}}+t^{i_2j_1\}+}\theta^{j_1\}i_2}\right]+(j_2\leftrightarrow i_2)\right)\Big\},\end{aligned} \tag{9}$$

with Heaviside $\theta$-symbol $\theta^{ij}$. From the Hamiltonian analysis of the dynamical systems the operators $\{o_I\}$ contain $2k^2$ second-class $\{o_a\}=\{l^{ij},t^{i_1j_1},l^+_{ij},t^+_{i_1j_1}\}$, $(2k+1)$ first-class $\{l_0,l^i,l^+_j\}$ constraints subsystems and $k$ elements $g^i_0$ forming matrix $\Delta_{ab}(g^i_0)$ in $[o_a,o_b]\sim\Delta_{ab}$.

We called in [4] the algebra of the operators $O_I$ as *integer higher-spin symmetry algebra in Minkowski space with a Young tableaux having $k$ rows* and denoted it as $\mathcal{A}(Y(k),\mathbb{R}^{1,d-1})$.

**HS symmetry algebra $\mathcal{A}(Y(k),\mathbb{R}^{1,d-1})$**

| $[\downarrow,\rightarrow]$ | $t^{i_1j_1}$ | $t^+_{i_1j_1}$ | $l_0$ | $l^i$ | $l^{i+}$ | $l^{i_1j_1}$ | $l^{i_1j_1+}$ | $g_0^i$ |
|---|---|---|---|---|---|---|---|---|
| $t^{i_2j_2}$ | $A^{i_2j_2,i_1j_1}$ | $B^{i_2j_2}{}_{i_1j_1}$ | $0$ | $l^{j_2}\delta^{i_2i}$ | $-l^{i_2+}\delta^{j_2i}$ | $l^{\{j_1j_2}\delta^{i_1\}i_2}$ | $-l^{i_2\{i_1+}\delta^{j_1\}j_2}$ | $F^{i_2j_2,i}$ |
| $t^+_{i_2j_2}$ | $-B^{i_1j_1}{}_{i_2j_2}$ | $A^+_{i_1j_1,i_2j_2}$ | $0$ | $l_{i_2}\delta^i_{j_2}$ | $-l^+_{j_2}\delta^i_{i_2}$ | $l_{i_2}{}^{\{j_1}\delta^{i_1\}}_{j_2}$ | $-l_{j_2}{}^{\{j_1+}\delta^{i_1\}}_{i_2}$ | $-F_{i_2j_2}{}^{i+}$ |
| $l_0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $l^j$ | $-l^{j_1}\delta^{i_1j}$ | $-l_{i_1}\delta^j_{j_1}$ | $0$ | $0$ | $l_0\delta^{ji}$ | $0$ | $-\frac{1}{2}l^{\{i_1+}\delta^{j_1\}j}$ | $l^j\delta^{ij}$ |
| $l^{j+}$ | $l^{i_1+}\delta^{j_1j}$ | $l^+_{j_1}\delta^j_{i_1}$ | $0$ | $-l_0\delta^{ji}$ | $0$ | $\frac{1}{2}l^{\{i_1}\delta^{j_1\}j}$ | $0$ | $-l^{j+}\delta^{ij}$ |
| $l^{i_2j_2}$ | $-l^{j_1\{j_2}\delta^{i_2\}i_1}$ | $-l_{i_1}{}^{\{i_2+}\delta^{j_2\}}_{j_1}$ | $0$ | $0$ | $-\frac{1}{2}l^{\{i_2}\delta^{j_2\}i}$ | $0$ | $L^{i_2j_2,i_1j_1}$ | $l^{i\{i_2}\delta^{j_2\}i}$ |
| $l^{i_2j_2+}$ | $l^{i_1\{i_2+}\delta^{j_2\}j_1}$ | $l_{j_1}{}^{\{j_2+}\delta^{i_2\}}_{i_1}$ | $0$ | $\frac{1}{2}l^{\{i_2+}\delta^{ij_2\}}$ | $0$ | $-L^{i_1j_1,i_2j_2}$ | $0$ | $-l^{i\{i_2+}\delta^{j_2\}i}$ |
| $g_0^j$ | $-F^{i_1j_1,j}$ | $F_{i_1j_1}{}^{j+}$ | $0$ | $-l^i\delta^{ij}$ | $l^{i+}\delta^{ij}$ | $-l^{j\{i_1}\delta^{j_1\}j}$ | $l^{j\{i_1+}\delta^{j_1\}j}$ | $0$ |

The subsystem of the second-class constraints $\{o_a\}$ together with $\{g_0^i\}$ forms the subalgebra in $\mathcal{A}(Y(k), \mathbb{R}^{1,d-1})$ to be isomorphic to symplectic $sp(2k)$ algebra (see details in [4]).

Having constructed the HS symmetry algebra, we cannot still construct BRST operator $Q$ with respect to the elements $o_I$ from $\mathcal{A}(Y(k), \mathbb{R}^{1,d-1})$ due to second-class constraints $\{o_a\}$ presence in it. One should convert symplectic algebra $sp(2k)$ of $\{o_a, g_0^i\}$ into enlarged set of operators $O_I$ with only first-class constraints.

## 2. NEW OSCILLATOR REALIZATION FOR $sp(2k)$

Consider an additive conversion procedure developed within BRST approach, (see, e.g., [13]), which implies the enlarging of $o_I$ to $O_I = o_I + o'_I$, with additional parts $o'_I$ to be given on a new Fock space $\mathcal{H}'$. Now, the elements $O_I$ are given on $\mathcal{H} \otimes \mathcal{H}'$ so that a requirement for $O_I$, $[O_I,\ O_J] \sim O_K$ leads to the same algebraic relations for $O_I$ and $o'_I$ as those for $o_I$.

Leaving aside the details of Verma module (special representation space [12]) construction for the symplectic algebra $sp(2k)$ of new operators $o'_I$ considered in [4], we present here their explicit oscillator form in terms of new $2k^2$ creation and annihilation operators $(B^c; B_d^+) = (b_{ij}^+, d_{rs}^+; b_{ij}, d_{rs}),\ i, j, r, s = 1, \ldots, k$; $i \leqslant j$; $r < s$ as follows (for $k_0 \equiv l$)

$$g_0^{\prime i} = \sum_{l \leqslant m} b_{lm}^+ \, b_{lm} \, (\delta^{il} + \delta^{im}) + \sum_{r<s} d_{rs}^+ \, d_{rs} \, (\delta^{is} - \delta^{ir}) + h^i, \tag{10}$$

$$l_{ij}^{\prime +} = b_{ij}^+, \quad t_{lm}^{\prime +} = d_{lm}^+ - \sum_{n=1}^{l-1} d_{nl} \, d_{nm}^+ - \sum_{n=1}^{k} (1 + \delta_{nl}) b_{nm}^+ \, b_{ln}, \tag{11}$$

$$t'_{lm} = -\sum_{n=1}^{l-1} d_{nl}^+ \, d_{nm} + \sum_{p=0}^{m-l-1} \sum_{k_1=l+1}^{m-1} \ldots \sum_{k_p=l+p}^{m-1} C^{k_p m} \, (d^+, d) \prod_{j=1}^{p} d_{k_{j-1} \, k_j} - \\ - \sum_{n=1}^{k} (1 + \delta_{nm}) b_{nl}^+ \, b_{nm}. \tag{12}$$

Note, first, that $B_c, B_d^+$ satisfy the standard commutation relations, $[B_c, B_d^+] = \delta_{cd}$; second, the arbitrary parameters $h^i$ in (10) serve to reproduce correct LF fo HS field with given spin $\mathbf{s}$, whereas the form of the rest elements $l'_{ij}$, for $i \leqslant j$, to be expressed by means of $C^{lm}(d^+, d)$ as well as the property of Hermiticity for them may be found in [4]*.

## 3. BRST–BFV OPERATOR AND LAGRANGIAN FORMULATIONS

Because the algebra of $O_I$ under consideration is a Lie algebra $\mathcal{A}(Y(k), \mathbb{R}^{1,d-1})$, the BFV–BRST operator $Q'$ can be constructed in the standard way as

$$Q' = O_I \mathcal{C}^I + \frac{1}{2} \mathcal{C}^I \mathcal{C}^J f^K_{JI} \mathcal{P}_K \tag{13}$$

with the constants $f^K_{JI}$ from the Table, constraints $O_I = (L_0, L^+_i, L_i, L_{ij}, L^+_{ij}, T_{ij}, T^+_{ij}, G^i_0)$, fermionic ghost fields and conjugated to them momenta $(C^I, \mathcal{P}_I) = ((\eta_0, \mathcal{P}_0);\ (\eta^i, \mathcal{P}^+_i);\ (\eta^+_i,\ \mathcal{P}_j);\ (\eta^{ij}, \mathcal{P}^+_{ij});\ (\eta^+_{ij}, \mathcal{P}_{ij});\ (\vartheta_{rs}, \lambda^+_{rs});\ (\vartheta^+_{rs}, \lambda_{rs});\ (\eta^i_G, \mathcal{P}_G))$ with the properties

$$\begin{gathered} \eta^{ij} = \eta^{ji}, \quad \vartheta_{rs} = \vartheta_{rs}\, \theta^{sr}, \\ \{\vartheta_{rs}, \lambda^+_{tu}\} = \delta_{rt}\, \delta_{su}, \quad \{\mathcal{P}_j, \eta^+_i\} = \delta_{ij}, \quad \{\eta_{lm},\, \mathcal{P}^+_{ij}\} = \delta_{li}\, \delta_{jm} \end{gathered} \tag{14}$$

and nonvanishing anticommutators $\{\eta_0, \mathcal{P}_0\} = \imath$, $\{\eta^i_{\mathcal{G}}, \mathcal{P}^j_{\mathcal{G}}\} = \imath\delta^{ij}$ for zero-mode ghosts**.

To construct LF for bosonic HS fields in a $\mathbb{R}^{1,d-1}$ Minkowski space, we partially follow the algorithm of [13, 14], which is a particular case of our construction, corresponding to $s_3 = 0$. First, we extract the dependence of $Q'$ (13) on the ghosts $\eta^i_G, \mathcal{P}^i_G$, to obtain the BRST operator $Q$ only for the system of converted first-class constraints $\{O_I\} \setminus \{G^i_0\}$ and generalized spin operator $\sigma^i$:

$$Q' = Q + \eta^i_G(\sigma^i + h^i) + \mathcal{A}^i \mathcal{P}^i_G, \quad \text{with some } \mathcal{A}^i, \text{ where} \tag{15}$$

$$Q = \left( \frac{1}{2}\eta_0\, L_0 + \eta^+_i L^i + \sum_{l \leqslant m} \eta^+_{lm} L^{lm} + \sum_{l<m} \vartheta^+_{lm}\, T^{lm} + \text{h.c.} \right) + \frac{1}{2}\, \widehat{\mathcal{C}}^I\, \widehat{\mathcal{C}}^J\, f^K_{JI}\, \widehat{\mathcal{P}}_K, \tag{16}$$

---

*The case of the massive bosonic HS fields, whose system of second-class constraints contains additionally to elements of $sp(2k)$ algebra the constraints of isometry subalgebra of Minkowski space $l^i, l^+_i, l_0$, may be treated via procedure of dimensional reduction of the algebra $\mathcal{A}(Y(k), \mathbb{R}^{1,d})$ for massless HS fields to one $\mathcal{A}(Y(k), \mathbb{R}^{1,d-1})$ for massive HS fields, (see [4]). Now, the wave equation in (2) is changed on Klein–Gordon equation corresponding to the constraint $l_0$ ($l_0 = \partial^\mu \partial_\mu + m^2$) acting on the same *basic* vector $|\Phi\rangle$ (5).

**The ghosts possess the standard ghost number distribution, $\mathrm{gh}\,(\mathcal{C}^I) = -\mathrm{gh}\,(\mathcal{P}_I) = 1 \Longrightarrow \mathrm{gh}\,(Q') = 1$.

$$\sigma^i = G_0^i - h^i - \eta_i \mathcal{P}_i^+ + \eta_i^+ \mathcal{P}_i + \sum_m (1+\delta_{im}) (\eta_{im}^+ \mathcal{P}^{im} - \eta_{im} \mathcal{P}_{im}^+) + \\ + \sum_{l<i} \left[\vartheta_{li}^+ \lambda^{li} - \vartheta^{li} \lambda_{li}^+\right] - \sum_{i<l} \left[\vartheta_{il}^+ \lambda^{il} - \vartheta^{il} \lambda_{il}^+\right], \quad (17)$$

where $\{\widehat{\mathcal{C}}^I\} \equiv \{\mathcal{C}^I\} \setminus \{\eta_G^i\}$, $\{\widehat{\mathcal{P}}^I\} \equiv \{\mathcal{P}^I\} \setminus \{\mathcal{P}_G^i\}$. Then, we choose a representation of $\mathcal{H}_{\rm tot}$: $(\eta_i, \eta_{ij}, \vartheta_{rs}, \mathcal{P}_0, \mathcal{P}_i, \mathcal{P}_{ij}, \lambda_{rs}, \mathcal{P}_G^i)|0\rangle = 0$ and suppose that the field vectors $|\chi\rangle$ as well as the gauge parameters $|\Lambda\rangle$ do not depend on ghosts $\eta_G^i$

$$|\chi\rangle = \sum_n \prod_{i\leqslant j, r<s}^k (b_{ij}^+)^{n_{ij}} (d_{rs}^+)^{p_{rs}} (\eta_0^+)^{n_{f0}} \times \\ \times \prod_{i,j,l\leqslant m, n\leqslant o} (\eta_i^+)^{n_{fi}} (\mathcal{P}_j^+)^{n_{pj}} (\eta_{lm}^+)^{n_{flm}} (\mathcal{P}_{no}^+)^{n_{pno}} \times \\ \times \prod_{r<s, t<u} (\vartheta_{rs}^+)^{n_{frs}} (\lambda_{tu}^+)^{n_{\lambda tu}} \left|\Phi(a_i^+)^{n_{f0}(n)_{fi}(n)_{pj}(n)_{flm}(n)_{pno},(n)_{frs}(n)_{\lambda tu}}_{(n)_l (n)_{ij} (p)_{rs}}\right\rangle {}^* \quad (18)$$

We denote by $|\chi^k\rangle$ the state (18) satisfying $\mathrm{gh}\,(|\chi^k\rangle) = -k$. Thus, the physical state having the ghost number zero is $|\chi^0\rangle$, the gauge parameters $|\Lambda\rangle$ having the ghost number $-1$ is $|\chi^1\rangle$ and so on. The vector $|\chi^0\rangle$ must contain physical string-like vector $\Big|\Phi\rangle = |\Phi(a_i^+)^{(0)_{fo}(0)_{fi}(0)_{pj}(0)_{flm}(0)_{pno}(0)_{frs}(0)_{\lambda tu}}_{(0)_{ij}(0)_{rs}}\Big\rangle$:

$$|\chi^0\rangle = |\Phi\rangle + |\Phi_A\rangle, \quad \text{where} \quad |\Phi_A\rangle\Big|_{[B_a^+ = C^I = \mathcal{P}_I = 0]} = 0. \quad (19)$$

Independence of the vectors (18) on $\eta_G^i$ transforms the equation for the physical state $Q'|\chi^0\rangle = 0$ and the BRST complex of the reducible gauge transformations, $\delta|\chi\rangle = Q'|\chi^1\rangle$, $\delta|\chi^1\rangle = Q'|\chi^2\rangle, \ldots, \delta|\chi^{(r-1)}\rangle = Q'|\chi^{(r)}\rangle$, to the relations:

$$1)\ Q|\chi^0\rangle = 0, \qquad 2)\ \delta|\chi^0\rangle = Q|\chi^1\rangle, \qquad \ldots \quad r)\ \delta|\chi^{r-1}\rangle = Q|\chi^{(r)}\rangle, \quad (20)$$

$$1)\ (\sigma^i + h^i)|\chi^0\rangle = 0, \quad 2)\ (\sigma^i + h^i)|\chi^1\rangle = 0, \quad \ldots \quad r)\ (\sigma^i + h^i)|\chi^r\rangle = 0, \quad (21)$$

where $r-1 = k(k+1)-1$ is the stage of reducibility both for massless and for the massive bosonic HS field. Resolving the spectral problem from Eqs. (21) we

---

*The brackets $(n)_{fi}, (n)_{pj}, (n)_{ij}$ in (18) mean, e.g., for $(n)_{ij}$ the set of indices $(n_{11}, \ldots, n_{1k}, \ldots, n_{k1}, \ldots, n_{kk})$. The sum above is taken over $n_l, n_{ij}, p_{rs}$ and running from 0 to infinity, and over the rest $n$'s from 0 to 1.

determine the eigenvectors of the operators $\sigma^i$: $|\chi^0\rangle_{(n)_k}$, $|\chi^1\rangle_{(n)_k}, \ldots, |\chi^s\rangle_{(n)_k}$, $n_1 \geqslant n_2 \geqslant \ldots n_k \geqslant 0$ and corresponding eigenvalues of the parameters $h^i$ (for massless HS fields),

$$-h^i = n_i + \frac{d-2-4i}{2}, \quad i = 1, \ldots, k, \quad n_1, \ldots, n_{k-1} \in \mathbb{Z}, \quad n_k \in \mathbb{N}_0. \quad (22)$$

Let us fix some values of $n_i = s_i$. Then one should substitute $h^i$ corresponding to the chosen $s_i$ (22) into $Q$ (16) and relations (20). Thus, e.g., the equation of motion (20) corresponding to the field with given spin $(s_1, \ldots, s_k)$ has the form, $Q_{(n)_k}|\chi^0\rangle_{(n)_k} = 0$, with nilpotent $Q_{(n)_k}$ and the same for the rest relations in (20).

Following to bosonic one- [12] and two-row cases, [13,14] one can show that last equation may be derived from the Lagrangian action for fixed spin $(n)_k = (s)_k$,

$$\mathcal{S}_{(s)_k} = \int d\eta_{0\,(s)_k} \langle\chi^0|K_{(s)_k} Q_{(s)_k}|\chi^0\rangle_{(s)_k} \left(\Longrightarrow \frac{\delta\mathcal{S}_{(s)_k}}{\delta_{(s)_k}\langle\chi^0|} = Q_{(s)_k}|\chi^0\rangle_{(s)_k} = 0\right), \quad (23)$$

where the standard scalar product for the creation and annihilation operators in $\mathcal{H}_{\rm tot} = \mathcal{H} \otimes \mathcal{H}' \otimes \mathcal{H}_{\rm gh}$ is assumed, and nondegenerate operator $K_{(s)_k}$ provides reality of the action.

Concluding, one can prove that the action (23) indeed reproduces the basic conditions (2)–(4) for massless (massive) HS fields. General action (23) gives, in principle, a straight recept to obtain the Lagrangian for any component field from general vector $|\chi^0\rangle_{(s)_k}$.

**Acknowledgements.** The author is thankful to the organizers of the International Workshop «SQS'11» for the hospitality. A. R. is grateful to K. Alkalaev, D. Francia, Yu. M. Zinoviev for valuable discussions and to I. L. Buchbinder for collaboration. The work was supported by the RFBR grant, project No. 12-02-00121 and by LRSS grant No. 224.2012.2.

REFERENCES

1. *Feldman D., Perez P. F., Nath P.* $R$-Parity Conservation via the Stueckelberg Mechanism: LHC and Dark Matter Signals // JHEP. 2012. V. 1201. P. 038.
2. *Vasiliev M.* Higher Spin Gauge Theories in Various Dimensions // Fortsch. Phys. 2004. V. 52. P. 702–717;
   *Sorokin D.* Introduction to the Classical Theory of Higher Spins // AIP Conf. Proc. 2005. V. 767. P. 172–202;
   *Bouatta N., Compère G., Sagnotti A.* An Introduction to Free Higher-Spin Fields. Preprint [arXiv:hep-th/0409068];
   *Fotopoulos A., Tsulaia M.* Gauge Invariant Lagrangians for Free and Interacting Higher Spin Fields. A Review of BRST Formulation // Intern. J. Mod. Phys. A. 2008. V. 24. P. 31–60.

3. *Fradkin E. S., Vilkovisky G. A.* Quantization of Relativistic Systems with Constraints // Phys. Lett. B. 1975. V. 55. P. 224–226;
*Batalin I. A., Fradkin E. S.* Operator Quantization of Dynamical Systems with First Class Constraints // Phys. Lett. B. 1983. V. 128. P. 303.
4. *Buchbinder I. L., Reshetnyak A. A.* General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry: I. Bosonic Fields. Preprint [arXiv:1110.5044[hep-th]].
5. *Labastida J. M. F.* Massless Particles in Arbitrary Representations of the Lorentz Group // Nucl. Phys. B. 1989. V. 322. P. 185–209.
6. *Metsaev R. R.* Massless Mixed Symmetry Bosonic Free Fields in $D$-Dimensional Anti-de Sitter Space-Time // Phys. Lett. B. 1995. V. 354. P. 78–84.
7. *Fronsdal C.* Massless Fields with Integer Spin // Phys. Rev. D. 1978. V. 18. P. 3624–3629.
8. *Fronsdal C.* Singletons and Massless, Integer-Spin Fileds on de Sitter Space // Phys. Rev. D. 1979. V. 20 P. 848–856;
*Vasiliev M. A.* «Gauge» Form of Description of Massless Fields with Arbitrary Spin (in Russian) // Yad. Fiz. 1980. V. 32. P. 855–861.
9. *Lopatin V. E., Vasiliev M. A.* Free Massless Bosonic Fields of Arbitrary Spin in $D$-Dimensional de Sitter Space // Mod. Phys. Lett. A. 1988. V. 3. P. 257–265.
10. *Singh L. P. S., Hagen C. R.* Lagrangian Formulation for Arbitrary Spin: 1 The Bosonic Case // Phys. Rev. D. 1974. V. 9. P. 898–909.
11. *Zinoviev Yu. M.* On Massive High-Spin Particles in AdS. Preprint [arXiv: hep-th/0108192];
*Metsaev R. R.* Massive Totally Symmetric Fields in AdS($d$) // Phys. Lett. B. 2004. V. 590. P. 95–104.
12. *Burdik C., Pashnev A., Tsulaia M.* Auxiliary Representations of Lie Algebras and the BRST Constructions // Mod. Phys. Lett. A. 2000. V. 15. P. 281–291.
13. *Burdik C., Pashnev A., Tsulaia M.* On the Mixed Symmetry Irreducible Representations of Poincare Group in the BRST Approach // Mod. Phys. Lett. A. 2001. V. 16. P. 731–746.
14. *Buchbinder I., Krykhtin V., Takata H.* Gauge Invariant Lagrangian Construction for Massive Bosonic Mixed-Symmetry Higher-Spin Fields // Phys. Lett. B. 2007. V. 656. P. 253.
15. *Buchbinder I. L., Krykhtin V. A., Reshetnyak A. A.* BRST Approach to Lagrangian Construction for Fermionic HS Fields in AdS Space // Nucl. Phys. B. 2007. V. 787. P. 211–240.
16. *Skvortsov E. D.* Frame-Like Actions for Massless Mixed-Symmetry Fields in Minkowski Space // Nucl. Phys. B. 2009. V. 808. P. 569.
17. *Campoleoni A. et al.* Unconstrained Higher Spins of Mixed Symmetry: I. Bose Fields // Nucl. Phys. B. 2009. V. 815. P. 289–357.