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ONE-LOOP EFFECTIVE ACTION IN THREE-DIMENSIONAL GENERAL CHIRAL SUPERFIELD MODEL

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Using the heat kernel approach we calculate one-loop Kähler effective potential for the general chiral superfield model in three dimensions.

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INTRODUCTION

The modern interest to three-dimensional supersymmetric models is caused by the achievements in the study of superconformal theories with Chern–Simons gauge fields interacting with matter [1, 2]. These models are devoted to developing a description of multiple M2 branes. It is known, that elimination of the Chern–Simons fields by fixing the gauge allows one to obtain three-dimensional supersymmetric nonlinear sigma-model for matter fields. And so, the quantization of three-dimensional nonlinear sigma-model perhaps helps to understand some of quantum aspects of M2 branes.

In the present work, we study the low-energy effective action in the three-dimensional general chiral superfield model for one chiral superfield φ with the classical action

$$S[\varphi,\bar{\varphi}] = -\int d^7 z K(\varphi,\bar{\varphi}) - \left(\int d^5 z W(\varphi) + \text{c.c.}\right), \quad (1)$$

where $K(\varphi,\bar{\varphi})$ is a supersmooth function called Kähler potential and $W(\varphi)$ is a chiral potential. We employ the superspace notations used in [3].

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We are interested in the effective potential. By the definition, this is a part of the effective action without derivatives of superfields. The effective potential of the model (1) consists of the effective Kähler potential and the effective chiral potential. The general structure of a supersymmetric effective potential in the four-dimensional case was discussed in detail in [4]. Our aim is to calculate a one-loop effective potential for the model under consideration. It is noticed that for a one-loop level it consists of the effective Kähler potential only, because there is no one-loop effective chiral potential. The computing of the effective chiral potential is a special problem demanding individual consideration and we don't discuss it here. It is obvious, that the Kähler potential contains derivatives of component fields. Therefore, the one-loop effective Kähler potential gives one-loop effective action of component fields.

The effective Kähler and chiral potentials for similar four-dimensional models were investigated in many works [4–11]. However, it should be noted that in three-dimensional case the one-loop effective Kähler potential does not contain ultraviolet divergences. We demonstrate this using the heat kernel approach in superspace (see, e.g., [12]).

1. ONE-LOOP EFFECTIVE ACTION IN GENERAL CHIRAL SUPERFIELD MODEL

The one-loop effective action of the model (1) depending on background superfield ϕ and $\bar{\phi}$ is defined by the standard expression

$$\Gamma^{(1)}[\phi,\bar{\phi}] = \frac{i}{2}\mathbf{s}\,\mathrm{Tr}\,\ln\,\mathbf{H}^{(\phi)} = -\frac{i}{2}\mathbf{s}\,\mathrm{Tr}\,\ln\,\mathbf{G}^{(\phi)}, \tag{2}$$

where the operator $\mathbf{H}^{(\phi)}$ is defined in the manner

$$\mathbf{H}^{(\phi)} = \begin{pmatrix} \dfrac{\delta^2 S}{\delta\phi(z')\delta\phi(z)} & \dfrac{\delta^2 S}{\delta\phi(z')\delta\bar{\phi}(z)} \\ \dfrac{\delta^2 S}{\delta\bar{\phi}(z')\delta\phi(z)} & \dfrac{\delta^2 S}{\delta\bar{\phi}(z')\delta\bar{\phi}(z)} \end{pmatrix} = $$

$$= \begin{pmatrix} -W''(\phi)\delta_+(z,z') & \dfrac{K_{\phi\bar{\phi}}}{4}\bar{D}^2\delta_-(z,z') \\ \dfrac{K_{\phi\bar{\phi}}}{4}D^2\delta_+(z,z') & \bar{W}''(\bar{\phi})\delta_-(z,z') \end{pmatrix}. \tag{3}$$

Here $\delta_+(z,z') = -(1/4)\bar{D}^2\delta^7(z-z')$ and $\delta_-(z,z') = -(1/4)D^2\delta^7(z-z')$ are (anti)chiral delta functions. The matrix superpropagator $\mathbf{G}^{(\phi)}$ meets the condition

$$\mathbf{H}^{(\phi)}\mathbf{G}^{(\phi)} = -\mathbb{I}, \quad \mathbb{I}(F,F') = \begin{pmatrix} \delta_+(z,z') & 0 \\ 0 & \delta_-(z,z') \end{pmatrix}.$$

Elements of matrix superpropagator $\mathbf{G}^{(\phi)}$ are denoted as follows:

$$\mathbf{G}^{(\phi)}(z,z') = \begin{pmatrix} G_{++}(z,z') & G_{+-}(z,z') \\ G_{-+}(z,z') & G_{--}(z,z') \end{pmatrix}. \quad (4)$$

Similarly as in the four-dimensional case [12], one can show that for the case of constant background superfields superpropagator $\mathbf{G}^{(\phi)}$ can be represented in the form

$$\mathbf{G}^{(\phi)}(z,z') = \frac{1}{16}\begin{pmatrix} \bar{D}_z^2 \bar{D}_{z'}^2 \mathbf{G}_V^{(\phi)}(z,z') & -\bar{D}_z^2 D_{z'}^2 \mathbf{G}_V^{(\phi)}(z,z') \\ -D_z^2 \bar{D}_{z'}^2 \mathbf{G}_V^{(\phi)}(z,z') & D_z^2 D_{z'}^2 \mathbf{G}_V^{(\phi)}(z,z') \end{pmatrix}, \quad (5)$$

where $\mathbf{G}_V^{(\phi)}(z,z')$ is a solution of the following equation:

$$\Delta \mathbf{G}_V^{(\phi)}(z,z') = -\delta^7(z,z'), \quad \Delta = K_{\phi\bar{\phi}}\Box - \frac{1}{4}W''(\phi)\bar{D}^2 + \frac{1}{4}\bar{W}''(\bar{\phi})D^2. \quad (6)$$

Here $\delta^7(z,z')$ is the delta function in the full $\mathcal{N}=2$, $d=3$ superspace. For such a representation (5), one can show that the effective action (2) can be written as (see [4] for the proof of this statement in the four-dimensional case)

$$\Gamma^{(1)} = -\frac{i}{2}\mathrm{Tr}\ln \mathbf{G}_V^{(\phi)}. \quad (7)$$

In contrast with (2), the functional trace in this representation is taken in the space of real superfields in the $\mathcal{N}=2$, $d=3$ superspace rather than in the (anti-)chiral ones.

To evaluate the one-loop affective action, we introduce the proper-time representation (see [12]) for the propagator (7)

$$\mathbf{G}_V^{(\phi)}(z,z') = i\int_0^\infty ds \;\; U_V^{(\phi)}(z,z'|s), \quad (8)$$

$U_V^{(\phi)}(z,z'|s)$ being unique solution of the equation

$$\left(i\frac{\partial}{\partial s} + \Delta\right) U_V^{(\phi)}(s) = 0, \quad U_V^{(\phi)}(z,z'|s\to +0) = \delta^7(z,z'). \quad (9)$$

Then the computation of the effective action (7) is reduced to finding the heat kernel at coincident superspace points,

$$\Gamma^{(1)}[\phi,\bar{\phi}] = -\frac{i}{2}\int_0^\infty \frac{d(is)}{is}\mathbf{s}\,\mathrm{Tr}\, U_V^{(\phi)}(z,z'|s). \quad (10)$$

The formal solution for (9) reads

$$U_V^{(\phi)}(z,z'|s) = \mathrm{e}^{is\Delta}\delta^7(z,z'). \tag{11}$$

We will assume that background superfields are constant ($\phi = \mathrm{const}$). The superkernel (11) can be represented in the form

$$\begin{aligned} U_V^{(\phi)}(z,z'|s) &= \exp\left[-\frac{is}{4}(W''\bar{D}^2 - \bar{W}''D^2)\right] U_V(z,z'|s), \\ U_V(z,z'|s) &= \mathrm{e}^{isK_{\phi\bar{\phi}}\Box}\delta^7(z-z'). \end{aligned} \tag{12}$$

Using useful identities (see [12])

$$\left(\frac{1}{16}\bar{D}^2D^2\right)^n = \Box^{n-1}\left(\frac{1}{16}\bar{D}^2D^2\right), \quad \left(\frac{1}{16}D^2\bar{D}^2\right)^n = \Box^{n-1}\left(\frac{1}{16}D^2\bar{D}^2\right),$$

we obtain that the heat kernel (11) can be written explicitly (at coincident points),

$$U_V^{(\phi)}(z,z|s) = \left[\frac{2}{\Box}(\cos(is\sqrt{W''\bar{W}''\Box}) - 1)\right] U_V(x,x'|s)\big|_{x=x'}, \tag{13}$$

$$U_V(x,x'|s) = \frac{1}{(4\pi K_{\phi\bar{\phi}} is)^{3/2}}\,\mathrm{e}^{-(1/4iK_{\phi\bar{\phi}}s)(x-x')^2}. \tag{14}$$

Substituting expressions (13) and (14) for the heat kernel into (10) and computing the integral over proper time s, we find the one-loop effective action,

$$\Gamma_K^{(1)} = \int d^3x d^4\theta\, K^{(1)}(\phi,\bar{\phi}), \quad K^{(1)}(\phi,\bar{\phi}) = \frac{1}{4\pi}\sqrt{\frac{W''\bar{W}''}{K^2_{\phi\bar{\phi}}}}. \tag{15}$$

The one-loop Kähler potential (15) is expressed in terms of classical potentials and represents the eventual form of the one-loop quantum contribution to the Kähler potential. Also we emphasize that the Kähler potential includes derivatives of component fields. Thus computation of the one-loop effective potential gives one-loop effective action of component fields.

We point out that the one-loop effective action (15) is UV finite. In three-dimensional field theories the divergences may appear only in the two-loop Feynman diagrams. Note also that there are no one-loop contributions to the effective chiral potential.

CONCLUSIONS

In the present work we computed one-loop quantum correction to the effective Kähler potential (15) using the heat kernel approach. In contrast to the four-dimensional case [6], the one-loop contribution to the effective potential (15) does not contain ultraviolet divergences.

As a natural continuation of the present work, it is interesting to compute the two-loop effective Kähler and chiral potentials in the general three-dimensional chiral superfield model and for the case when there are several interacting chiral superfields, Φ^i, $i = 1, \ldots, n$.

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