ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2013. Т. 44. ВЫП. 1

NEUTRINO*

S. M. Bilenky

Joint Institute for Nuclear Research, Dubna

*Dedicated to the 100th anniversary of the birthday of Bruno Pontecorvo (1913–1993).

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Neutrinos are the only fundamental fermions which have no electric charges. Because of that neutrinos have no direct electromagnetic interaction and at relatively small energies they can take part only in weak processes with virtual $W^{\pm}$ and Z^0 bosons. Neutrino masses are many orders of magnitude smaller than masses of charged leptons and quarks. These two circumstances make neutrinos unique, special particles. The history of the neutrino is very interesting, exciting and instructive. We try here to follow the main stages of the neutrino history starting from the famous Pauli letter and finishing with the discovery and study of neutrino oscillations. Outstanding contribution to the neutrino physics of Bruno Pontecorvo is discussed in some detail.

Нейтрино являются единственными фундаментальными фермионами, не имеющими электрических зарядов. Вследствие этого нейтрино не имеют прямого электромагнитного взаимодействия и при относительно малых энергиях могут принимать участие только в слабых процессах с виртуальными $W^{\pm}$- и Z^0-бозонами. Массы нейтрино на много порядков меньше масс заряженных лептонов и кварков. Эти два обстоятельства делают нейтрино уникальными, специальными частицами. История нейтрино очень интересна и поучительна. Мы прослеживаем здесь основные ее этапы, начиная со знаменитого письма Паули и заканчивая открытием и исследованием осцилляций нейтрино. Детально обсуждается выдающийся вклад Бруно Понтекорво в физику нейтрино и слабого взаимодействия.

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INTRODUCTION

Neutrinos are unique particles. They played an extremely important role in the establishment of V–A current $\times$ current theory of the weak interaction and of the Standard Model (SM). Small neutrino masses and neutrino mixing discovered via observation of neutrino oscillations is the first particle-physics signature of the beyond the SM physics.

The first period of the history of neutrino was, in essence, the history of the weak interaction. This period started in 1930 with the famous Pauli letter in which idea of neutrino was proposed and finished with the creation of the SM.

The second period of neutrino history is the history of the development of ideas of neutrino masses and mixing and history of challenging solar, atmospheric,

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reactor and accelerator neutrino experiments in which neutrino oscillations driven by small neutrino masses were discovered and studied. This period started with ideas of neutrino oscillations put forward by Bruno Pontecorvo in 1957–1958 [1, 2], soon after the two-component neutrino theory was proposed.

Importance of neutrino for physics and astrophysics is emphasized by the fact that three Nobel Prizes were given for discoveries in neutrino physics.

In 1988, L. Lederman, M. Schwartz, and J. Steinberger were awarded by the Nobel Prize «for the neutrino beam method and the demonstration of the doublet structure of lepton through the discovery of the muon neutrino». The experiment with accelerator neutrinos which allowed to prove that ν_μ and ν_e are different particles was proposed by Bruno Pontecorvo in 1959 [3].

In 1995, F. Reines was awarded the Nobel Prize «for the detection of the neutrino». For the first time neutrinos were detected in the Reines and Cowan reactor neutrino experiments in the fifties. In 1946, Bruno Pontecorvo was the first who paid attention that reactors are very intensive sources of (anti)neutrinos, the most appropriate (at that time) for reactor neutrino experiments [4].

In 2002, the Nobel Prize was awarded to R. Davis and M. Koshiba «for pioneering contribution to astrophysics, in particular, for detection of cosmic neutrinos». In the Davis experiment the radiochemical method of the neutrino detection proposed by Bruno Pontecorvo in 1946 [4] was used.

1. NEUTRINO AND THE FIRST THEORY OF THE β DECAY

Development of the theory of weak interaction and neutrino started with famous Fermi paper [5] «Theory of the β rays» (1934). The theory was based on the Pauli assumption that in the β decay together with an electron a neutral, spin $1/2$, light particle (which after Fermi was called the neutrino) was emitted.

Fermi built the theory of the β decay assuming that nuclei are bound states of protons and neutrons. E. Amaldi and B. Pontecorvo remembered that there was a problem for Fermi to understand how an electron–neutrino pair was emitted by a nucleus which is a bound state of protons and neutrons. Fermi solved this problem on the basis of analogy with the emission of a photon by an electron in atom. He assumed that *the electron–neutrino pair was produced in the quantum transition of a neutron into a proton**

$$n \to p + e^- + \bar{\nu}. \quad (1)$$

*We know today that in the β decay together with the electron an antineutrino $\bar{\nu}$ is produced. Later we will explain the difference between neutrino and antineutrino.

The simplest electromagnetic Hamiltonian which provides the quantum transition

$$p \to p + \gamma \tag{2}$$

has the form of the scalar product of the electromagnetic (vector) current $\bar{p}(x)\gamma_\alpha p(x)$ and vector electromagnetic field $A^\alpha(x)$

$$\mathcal{H}^{\rm EM}(x) = e\,\bar{p}(x)\,\gamma_\alpha p(x)\,A^\alpha(x). \tag{3}$$

By analogy, Fermi assumed that the Hamiltonian of the decay (1) was the scalar product of the vectors $\bar{p}(x)\gamma_\alpha n(x)$ and $\bar{e}(x)\gamma_\alpha \nu(x)$ which could be built from electron and neutrino fields*

$$\mathcal{H}^{\beta}(x) = G_F\,\bar{p}(x)\,\gamma_\alpha n(x)\,\bar{e}(x)\,\gamma_\alpha \nu(x) + \text{h.c.}, \tag{4}$$

where G_F is a constant (which is called the Fermi constant).

Let us notice the important difference between the Hamiltonian of the electromagnetic interaction (3) and the Hamiltonian of the β decay (4). The electromagnetic Hamiltonian $\mathcal{H}^{\rm EM}$ is the Hamiltonian of the interaction of two fermion fields and a boson field, and $\mathcal{H}^{\beta}$ is the Hamiltonian of the interaction of four fermion fields. As a result of that, *the constants e and G_F have different dimensions.* In the system of units $\hbar = c = 1$, we use, the charge e is a dimensionless quantity and the Fermi constant G_F has a dimension M^{-2} (M is a mass). Later we will discuss the origin of the dimension of the constant G_F. We will see that the dimension of the constant G_F is connected with the fact that the Hamiltonian (4) is not a fundamental Hamiltonian of interaction but is an *effective Hamiltonian*.

Let us also stress that Fermi came to the unique expression for the Hamiltonian of the β decay assuming that:

1. The Hamiltonian of the β decay is the product of two vectors.
2. There are no derivatives of fields in the Hamiltonian.

Applying the methods of the Quantum Field Theory and using the Hamiltonian (4), Fermi calculated the energy spectrum of electrons emitted in the β decay and suggested a method of the measurement of the neutrino mass. He proposed to investigate the shape of the electron spectrum in the region near the maximal electron energy (which corresponds to the emission of nonrelativistic neutrinos). The same method of the measurement of the neutrino mass was proposed by Perrin [6].

It occurred that the investigation of the β decay of tritium

$$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu} \tag{5}$$

*The current $\bar{p}\gamma_\alpha n$ induces the transition $n \to p$. It changes the electric charge by one ($\Delta Q = 1$) and is called the hadronic charge current (CC). The current $\bar{e}\gamma_\alpha \nu$ provides the emission of the pair $(e^- - \bar{\nu})$. It is called the leptonic charge current.

is one the most sensitive ways of the measurement of the neutrino mass by the Fermi–Perrin method. This is connected with the fact that tritium has a convenient half-life $T_{1/2} = 12.3$ y, the energy release Q in the process (5) is small ($Q = 18.57$ keV), the nuclear matrix element of the process is a constant (^{3}H $\to$ ^{3}He is an allowed transition), etc.

The electron spectrum for the allowed transitions is determined by the phase-space factor

$$p_e E_e \, pE, \tag{6}$$

where E_e and E (p_e and p) are the energies (momenta) of the electron and the neutrino.

If we neglect the recoil of the final nucleus from the conservation of the energy, we have

$$Q = T_e + E, \tag{7}$$

where $T_e = E_e - m_e$ is the kinetic energy of the electron.

From (6), we obtain the following expression for the energy spectrum of the electrons in the decay (5)

$$\frac{dN}{dE} = Cp_e(T_e + m_e)(Q - T_e)\sqrt{(Q - T_e)^2 - m_\nu^2}\, F(T_e, Z), \tag{8}$$

where m_ν is the neutrino mass; $F(T_e, Z)$ is the Fermi function, which describes the Coulomb interaction of the final electron and nucleus; and C is a constant (which includes the modulus-squared of the nuclear matrix element).

The neutrino mass enters into the expression for the β spectrum through the neutrino momentum $p = \sqrt{(Q - T_e)^2 - m_\nu^2}$. From this expression it is obvious that the part of the spectrum in which $Q - T_e \simeq m_\nu$ is sensitive to the neutrino mass*.

The largest contributions to the β decay come from transitions in which electron and (anti)neutrino are produced in states with orbital momenta equal to zero (S states). Such transitions are called allowed. For allowed transitions it follows from the Fermi Hamiltonian (4) that spins and parities of the initial and final nuclei must be equal (Fermi selection rules):

$$\Delta J = 0, \quad \pi_i = \pi_f. \tag{9}$$

Here $\Delta J = J_f - J_i$, where J_i (J_f) is the spin of the initial (final) nucleus, and π_i (π_f) is the parity of the initial (final) nucleus.

From the conservation of the total momentum it follows that in the case of an allowed transition, which satisfies the Fermi selection rule, electron and

*In practice, for a neutrino mass $m_\nu \lesssim 1$ eV a much larger part of the spectrum is used for the analysis of experimental data (in order to increase the number of the events used in the analysis).

(anti)neutrino are produced in a state with the total spin S equal to zero (singlet state). If electron and (anti)neutrino are produced in the triplet spin state ($S = 1$), in this case for the allowed transition the total angular momentum of the final state is equal to $J_f = J_i \pm 1$ or $J_f = J_i$ (for $J_i = 0$ the total final angular momentum is equal to 1). We have in this case

$$\Delta J = \pm 1, 0 \quad \pi_i = \pi_f \quad (0 \to 0 \text{ is forbidden}). \tag{10}$$

The selection rules (10) are called the Gamov–Teller selection rules. They were introduced by Gamov and Teller in 1936 [7].

In the β-decay experiments, decays of nuclei which satisfy the Fermi and Gamov–Teller selection rules were observed. Thus, the total Hamiltonian of the β decay must include not only the Fermi Hamiltonian (4) but also an additional term (or terms).

The Fermi Hamiltonian is the product of two vectors. The most general Hamiltonian of the Fermi type, in which only fields but not their derivatives enter, has the form of the sum of the products of scalar $\times$ scalar, vector $\times$ vector, tensor $\times$ tensor, axial $\times$ axial, and pseudoscalar $\times$ pseudoscalar:

$$\mathcal{H}_I^{\beta}(x) = \sum_{i=S,V,T,A,P} G_i \bar{p}(x)\, O^i n(x)\, \bar{e}(x)\, O_i \nu(x) + \text{h.c.} \tag{11}$$

Here*

$$O^i \to 1(S), \gamma^{\alpha}(V), \sigma^{\alpha\beta}\ (T), \gamma^{\alpha}\gamma_5\ (A), \gamma_5\ (P) \tag{12}$$

and G_i are coupling constants which have dimensions $[M]^{-2}$. Let us notice that transitions, which satisfy the Fermi selection rules, are due to V and S terms and transitions, which satisfy the Gamov–Teller selection rules, are due to A and T terms.

In the Fermi Hamiltonian (4) only one fundamental constant G_F enters. The Hamiltonian (11) is characterized by five interaction constants. Analogy and economy which were the basis of the Fermi theory were lost.

There was a general belief that there are «dominant» terms in the interaction (11). Such terms were searched for during many years via analysis of the data of the β-decay experiments. This search did not lead, however, to a definite result: some experiments were in favor of V and A terms, other were in favor of S and T terms. Up to 1957, when violation of parity in the β decay and other weak processes was discovered, the situation with the Hamiltonian of the β decay remained uncertain.

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*Dirac matrices γ^{α} ($\alpha = 0, 1, 2, 3$) satisfy the relations $\gamma^{\alpha}\gamma^{\beta} + \gamma^{\beta}\gamma^{\alpha} = 2g^{\alpha\beta}$, where $g^{00} = 1, g^{ii} = -1$, and nondiagonal elements of $g^{\alpha\beta}$ are equal to zero. The matrix γ_5 is determined as follows: $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$. It satisfies the relations $\gamma^{\alpha}\gamma_5 + \gamma_5\gamma^{\alpha} = 0$, $\gamma_5\gamma_5 = 1$. Sixteen matrices $1, \gamma^{\alpha}, \sigma_{\alpha\beta} = (1/2)(\gamma^{\alpha}\gamma^{\beta} - \gamma^{\beta}\gamma^{\alpha}), \gamma^{\alpha}\gamma_5, \gamma_5$ form a complete system of 4×4 matrices.

2. THE FIRST ESTIMATE OF THE NEUTRINO–NUCLEUS CROSS SECTION

In the Fermi Hamiltonian (4) $e(x)$, $\nu(x)$, $n(x)$, and $p(x)$ are *quantum fields*. This means that the Hamiltonian (4) allows one to calculate not only the probability of the β^- decay

$$(A, Z) \to (A, Z+1) + e^- + \bar{\nu} \tag{13}$$

but also the probabilities of the β^+ decay and electron capture

$$(A, Z) \to (A, Z-1) + e^+ + \nu, \quad e^- + (A, Z) \to (A, Z-1) + \nu, \tag{14}$$

the cross sections of neutrino reactions

$$\bar{\nu} + (A, Z) \to e^+ + (A, Z-1), \tag{15}$$

$$\nu + (A, Z) \to e^- + (A, Z+1), \tag{16}$$

and other processes.

The first estimation of the cross section of the process (15) was done by Bethe and Peierls [8] soon after Fermi's paper appeared.

We will present here Bethe's and Peierls' arguments. At relatively small MeV energies, the nuclear matrix elements of the processes (13) and (15) are practically the same. Since the β-decay width $\Gamma = 1/T_{1/2}$ ($T_{1/2}$ is the half-life of the decay) and the cross section σ of the process (14) are proportional to the modulus-squared of the same nuclear matrix elements, we have

$$\sigma = \frac{A}{T_{1/2}}, \tag{17}$$

where A has a dimension (length)$^2 \times$time. The authors suggested that «the longest length and time which can possibly be involved are $\hbar/m_e c$ and $\hbar/m_e c^2$» and found the following upper bound:

$$\sigma < \frac{\hbar^3}{m_e^3 c^4 T_{1/2}}. \tag{18}$$

From this inequality for $T_{1/2} \simeq 3$ min, Bethe and Peierls found

$$\sigma < 10^{-44}\ \mathrm{cm}^2. \tag{19}$$

This bound corresponds to a neutrino absorption length in solid matter larger than 10^{14} km. On the basis of this estimate, Bethe and Peierls in their paper (with the title «The Neutrino») concluded «*. . . there is no practically possible way of observing the neutrino*».

For comparison, we will present the current calculations of the neutrino cross section. Let us consider the «elementary» process

$$\bar{\nu} + p \to e^+ + n. \tag{20}$$

Using the modern Hamiltonian of the weak interaction for the cross section of the process (20) we have

$$\sigma = 4\frac{G_F^2}{\pi} p_e E_e \simeq 9.5 \cdot 10^{-44} p_e E_e \ \mathrm{MeV}^{-2} \cdot \mathrm{cm}^2, \tag{21}$$

where E_e and p_e are the positron energy and the momentum. Neglecting the recoil of the final neutron, we have for the neutrino energy E the following expression:

$$E = E_e + \Delta, \tag{22}$$

where $\Delta = m_n - m_p \simeq 1.3$ MeV is the neutron–proton mass difference. For (anti)neutrinos with the energy $E \simeq 3$ MeV we find the value $\sigma \simeq 2.6 \cdot 10^{-43}$ cm^2. Correspondingly, the absorption length of (anti)neutrinos in water is given by

$$L_a = \frac{1}{n\sigma} \simeq 6 \cdot 10^{14} \ \mathrm{km}, \tag{23}$$

where n is the number density of protons (in the case of water $n \simeq 6.7 \times 10^{22}$ cm^{-3}). Thus, the present-day calculations confirm the Bethe and Peierls estimate.

After the Bethe and Peierls paper, there was a general opinion that the neutrino is an undetectable particle. The first physicist who challenged this general opinion was Bruno Pontecorvo [4].

In [4], Bruno Pontecorvo proposed radiochemical method of neutrino detection based on the observation of the decay of the daughter nucleus produced in the reaction

$$\nu + (A, Z) \to e^- + (A, Z+1).$$

An experiment based on the observation of ^{37}Ar atoms produced in the reaction

$$\nu + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar} \tag{24}$$

he considered as the most promising. Many years later, the Cl–Ar method of neutrino detection allowed R. Davis to observe solar neutrinos in the first solar neutrino experiment [9]. The radiochemical Ga–Ge method of neutrino detection based on the observation of ^{71}Ge produced in the process*

$$\nu + {}^{71}\mathrm{Ga} \to e^- + {}^{71}\mathrm{Ge} \tag{25}$$

was used in the GALLEX–GNO [11] and SAGE [12] solar neutrino experiments. We will discuss solar neutrinos and the Pontecorvo radiochemical method later.

*The reaction (25) was proposed in [10].

3. FIRST IDEAS OF $\mu{-}e$ UNIVERSAL WEAK INTERACTION

In 1947, Bruno Pontecorvo [13] came to an idea of existence of a universal weak interaction which governs not only the processes in which the electron–neutrino pair takes part (like the nuclear β decay) but also the processes in which the muon–neutrino pair participates. The process of such a type is μ capture

$$\mu^- + (A, Z) \rightarrow \nu + (A, Z-1). \tag{26}$$

B. Pontecorvo compared the probability of this process and the probability of the K capture

$$e^- + (A, Z) \rightarrow \nu + (A, Z-1) \tag{27}$$

and came to the qualitative conclusion that the constant of the interaction of the muon–neutrino pair with nucleons is of the same order as the Fermi constant.

The idea of $\mu{-}e$ universality of the weak interaction was also proposed by G. Puppi [14]. Puppi presented it in the form of a triangle («Puppi triangle»). He assumed that a universal weak interaction includes not only Hamiltonians of the β decay and μ capture but also the Hamiltonian of the μ decay

$$\mu^+ \rightarrow e^+ + \nu + \bar{\nu}. \tag{28}$$

Puppi suggested that different parts of the weak interaction form a triangle with vertices

$$(\bar{p}n) - (\bar{\nu}e) - (\bar{\nu}\mu), \tag{29}$$

and the Hamiltonian of the weak interaction is given by a sum of products of different vertices ($(\bar{p}n)\,(\bar{\nu}e)^\dagger$, etc.). The idea of a universal weak interaction was proposed also by O. Klein [15], and Yang and Tiomno [16].

Summarizing, let us stress that with the idea of universality there appeared a notion of *universal weak interaction.* The idea of universality was proposed, however, at the time where the form of the weak interaction was not known. It was, nevertheless, an extremely important general idea. We will see later, how it was implemented in the modern theory of the weak interaction.

4. VIOLATION OF PARITY IN THE β DECAY AND OTHER WEAK PROCESSES

Our understanding of the neutrino and the weak interaction has drastically changed after it was discovered in 1957–1958 that *in the β decay, the decay $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$ and in other weak processes the parity is not conserved.*

Strange particles were discovered in the fifties. The investigation of their decays created the so-called $\theta{-}\tau$ puzzle.

A strange particle which decays into π^+ and π^0 was called θ^+ ($\theta^+ \to \pi^+ + \pi^0$), and a strange particle which decays into π^+, π^-, and π^+ was called τ^+ ($\tau^+ \to \pi^+ + \pi^- + \pi^+$). From experimental data it followed that the *masses and lifetimes of θ^+ and τ^+ are the same*. Detailed investigations of the Dalitz-plot for the three-pions decay of τ^+ showed that the total momentum of the state of (π^+, π^-, π^+) was equal to zero and the parity (eigenvalue of the operator of the parity) was equal to -1. If τ^+ and θ^+ is the same particle, in this case its spin must be equal to zero. However, the parity of the two pions produced in the θ^+ decay is equal to $+1$ (the parity of two pions is equal to $I_\pi^2(-1)^l = (-1)^2(-1)^0 = 1$, where $I_\pi = -1$ is the internal parity of the pion and l is the orbital momentum of two pions). So if τ^+ and θ^+ is the same particle, we are confronted with the following problem: the same particle decays into states with different parities.

As one of the possible solutions of the $\theta - \tau$ problem, Lee and Yang [17] put forward the hypothesis of the nonconservation of parity (1956). They analyzed all existing experimental data and came to the conclusion that there was an evidence that parity was conserved in the strong and electromagnetic interactions, but there were no data that proved that parity was conserved in the β decay and other weak decays. («...as for weak interactions, parity conservation is so far the only extrapolated hypothesis unsupported by experimental evidence» [17].) Lee and Yang proposed different experiments which would allow one to test the hypothesis of the parity conservation in weak decays. The results of the first experiments in which large violation of parity in weak processes was observed were published by Wu et al. [18] and Lederman et al. [19] at the beginning of 1957.

We will discuss the experiment by Wu et al. [18] in which the β decay of polarized ^{60}Co was investigated (polarization of a nucleus is the average value of its spin). Let us consider the emission of the electron with momentum $\mathbf{p}$ in the β decay of a nucleus with polarization $\mathbf{P}$. We assume that only electron produced in the decay is observed. In this case, from the invariance under rotations (conservation of the total momentum), it follows that the decay probability can depend only on the scalar products $\mathbf{p} \cdot \mathbf{p}$ and $\mathbf{P} \cdot \mathbf{p}$. Taking into account that the decay probability depends linearly on the polarization of a nucleus, we obtain the following general expression for the probability of the emission of the electron with momentum $\mathbf{p}$ by a nucleus with polarization $\mathbf{P}$:

$$w_{\mathbf{P}}(\mathbf{p}) = w_0(1 + \alpha \mathbf{P} \cdot \mathbf{k}) = w_0(1 + \alpha P \cos\theta). \tag{30}$$

Here $\mathbf{k} = \mathbf{p}/p$ is a unit vector in the direction of the electron momentum; θ is the angle between the vectors $\mathbf{P}$ and $\mathbf{p}$; and w_0 and α are functions of p^2.

Under the inversion of a coordinate system (change of directions of all axes of the coordinate system), momentum $\mathbf{p}$ and polarization $\mathbf{P}$ are transformed *differently*. Namely, momentum is transformed as a vector

$$p_i' = -p_i, \tag{31}$$

while polarization is transformed as a pseudovector*

$$P_i' = +P_i. \tag{32}$$

Here p_i (P_i) are components of a vector of momentum (pseudovector of polarization) in some right-handed system and p_i' (P_i') are components of the same momentum (polarization) in the inverted (left-handed) system.

Relations (31) and (32) mean that under the inversion the vector of momentum does not change its position in space while polarization changes its direction to the opposite one.

From (31) and (32) it follows that under the inversion the scalar product $\mathbf{P}\cdot\mathbf{p}$ is transformed as a pseudoscalar (change sign)

$$\mathbf{P}'\cdot\mathbf{p}' = -\mathbf{P}\cdot\mathbf{p}, \tag{33}$$

while $\mathbf{p}\cdot\mathbf{p}$ is transformed as a scalar

$$\mathbf{p}'\cdot\mathbf{p}' = +\mathbf{p}\cdot\mathbf{p}. \tag{34}$$

If the invariance under the inversion holds (parity is conserved), in this case the decay probability in a right-handed system and in an inverted left-handed system is the same

$$w_{\mathbf{P}'}(\mathbf{p}') = w_{\mathbf{P}}(\mathbf{p}). \tag{35}$$

From (30), (31), (32), and (35), we conclude that in the case of conservation of parity $\alpha = 0$, and the probability of the emission of the electron by the polarized nucleus does not depend on the angle θ.

In the Wu et al. experiment [18] it was found that the parameter α was negative and $|\alpha| \geqslant 0.7$ (i.e., electrons are emitted (in the right-handed system) mainly in the direction opposite to the polarization of the nucleus)**. Thus, it was discovered that there was no invariance of the β-decay interaction under inversion (parity in the β decay is not conserved).

The paper of Wu et al. [18] was submitted to «Physical Review» on January 15, 1957. At the same day, another experimental paper [19] on the observation

*Notice that momentum, coordinates, electric field, etc., are vectors while angular momentum, polarization, magnetic field, etc., are pseudovectors.

**The sign of the parameter α depends on the handedness of the system. Conservation of parity means that such parameters cannot enter into measurable quantities. After Wu et al. and other experiments we know that this is not the case.

of the violation of parity in weak decays was submitted to the same journal. In the Lederman et al. experiment [19] strong violation of parity in the chain of the decays

$$\pi^+ \to \mu^+ + \nu \tag{36}$$

and

$$\mu^+ \to e^+ + \nu + \bar{\nu} \tag{37}$$

was observed.

If parity is violated, a muon produced in the decay (36) will be polarized in the direction opposite to the muon momentum*. Like in the case of the β decay, the dependence of the probability of the decay of polarized muons on the angle θ between muon polarization and electron momentum has the general form $(1 + a\cos\theta)$, where the second pseudoscalar term $(a\mathbf{P} \cdot k = a\cos\theta)$ is due to nonconservation of parity. In the Lederman et al. experiment [19] large asymmetry of e^+ was found $(|a| \simeq 1/2)$.

Let us discuss the Hamiltonian of the β decay. The Hamiltonian (11) is a scalar. It is invariant under the inversion. In order to take into account the results of the Wu et al. and other experiments, we must assume that the *Hamiltonian of the β decay is the sum of a scalar and a pseudoscalar*. In order to build such a Hamiltonian, we have to add to five scalars which enter into the Hamiltonian (11) additional five pseudoscalars which are formed from products of the scalar $\bar{p}(x)\,n(x)$ and pseudoscalar $\bar{e}(x)\gamma_5\nu(x)$, vector $\bar{p}(x)\gamma^\alpha n(x)$ and pseudovector $\bar{e}(x)\,\gamma^\alpha\gamma_5\nu(x)$, etc. The most general Hamiltonian of the β decay takes the form

$$\mathcal{H}_I^\beta(x) = \sum_{i=S,V,T,A,P} \bar{p}(x)\,O_i n(x)\,\bar{e}(x)\,O^i(G_i + G_i'\gamma_5)\,\nu(x) + \text{h.c.}, \tag{38}$$

where the constants G_i characterize the scalar part of the Hamiltonian; the constants G_i' characterize the pseudoscalar part, and the matrices O^i are given by (12).

The Hamiltonian (38) is characterized by 10 fundamental interaction constants. From the Wu et al. experiment it followed that scalar and pseudoscalar parts of the Hamiltonian must be of the same order. This means that the constants $|G_i|$ and $|G_i'|$ (at least for some i) must be of the same order.

In 1957–1958, enormous progress in the development of the theory of the β decay and other weak processes was reached. Soon after the discovery of the violation of parity the Hamiltonian of the weak interaction took a simple form

*Muon possesses longitudinal polarization if the probabilities of the emission of a muon with positive and negative helicities are different. This could happen only in the case if parity in the decay (36) is violated.

compatible with all experimental data. The new development of the theory of the weak interaction started with *the two-component theory of the neutrino.*

In conclusion, the conservation of parity (invariance under space inversion) was established for strong and electromagnetic processes. For many years physicists thought that the invariance under space inversion is a general law of nature valid for all interactions. The discovery of violation of parity in the β decay and other weak processes was a great surprise*. In the beginning it looked that this discovery made the theory of the β decay and other weak processes more complicated. In reality, as we will see later, this discovery allowed to build a simple, correct theory of the neutrino and weak interaction. The new development of the theory of the weak interaction started with *the two-component theory of the neutrino.*

5. MASSLESS TWO-COMPONENT NEUTRINO

Soon after the discovery of the parity violation, Landau [20], Lee and Yang [21], and Salam [22] came to an idea of *a possible connection of the violation of parity observed in the β decay and other weak processes with neutrinos.*

The neutrino field $\nu(x)$ satisfies the Dirac equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m_{\nu})\,\nu(x) = 0, \tag{39}$$

where m_{ν} is the neutrino mass.

Any fermion field $\psi(x)$ can be presented in the form

$$\psi(x) = \psi_L(x) + \psi_R(x), \tag{40}$$

where $\psi_L(x) = \left(\frac{1-\gamma_5}{2}\right)\psi(x)$ and $\psi_R(x) = \left(\frac{1+\gamma_5}{2}\right)\psi(x)$ are left-handed and right-handed components, respectively. For left-handed and right-handed components of the neutrino field, we obtain from (39) two equations:

$$i\gamma^{\alpha}\partial_{\alpha}\nu_L(x) - m_{\nu}\nu_R(x) = 0, \quad i\gamma^{\alpha}\partial_{\alpha}\nu_R(x) - m_{\nu}\nu_L(x) = 0. \tag{41}$$

The equations for $\nu_{L,R}(x)$ are coupled because of the mass term of the Dirac equation. Let us assume that $m_{\nu} = 0$. In this case for $\nu_L(x)$ and $\nu_R(x)$ we obtained two decoupled equations:

$$i\gamma^{\alpha}\partial_{\alpha}\nu_{L,R}(x) = 0. \tag{42}$$

Thus, in the case of $m_{\nu} = 0$ the neutrino field can be $\nu_L(x)$ (or $\nu_R(x)$).

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*The violation of parity in the weak interaction was one of the most important discoveries in the physics of the XX century. In 1957, Lee and Yang were awarded the Nobel Prize «for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles».

It is obvious that such a theory can be valid only if parity is violated. In fact, under the inversion of coordinates the field $\nu(x)$ is transformed as follows:

$$\nu'(x') = \eta\gamma^0\nu(x). \tag{43}$$

Here $x' = (x^0 - \mathbf{x})$ and η is a phase factor. Taking into account that $\gamma_5\gamma^0 = -\gamma^0\,\gamma_5$, from (43) we find

$$\nu'_{L(R)}(x') = \eta\gamma^0\nu_{R(L)}(x). \tag{44}$$

Hence, under the inversion a left-handed (right-handed) component is transformed into a right-handed (left-handed) component. This means that equations (42) are not invariant under the inversion*.

Landau [20], Lee and Yang [21], and Salam [22] assumed that the neutrino mass was equal to zero** and that the neutrino field is $\nu_L(x)$ or $\nu_R(x)$. For the reasons, which will be clear later, this theory is called *the two-component neutrino theory*.

There were two major consequences of the two-component neutrino theory.

1. Parity is strongly violated in the β decay and in other processes in which neutrino(s) participate.

The most general Hamiltonian of the β decay in the case of parity violation is given by expression (38). In this Hamiltonian enter five interaction constants G_i which characterize the scalar part of the Hamiltonian and five interaction constants G'_i which characterize the pseudoscalar part ($i = S, V, T, A, P$).

In the case of the two-component theory these constants are connected by the relations

$$G'_i = -G_i \quad \text{(if neutrino field is } \nu_L(x)\text{)} \tag{46}$$

and

$$G'_i = G_i \quad \text{(if neutrino field is } \nu_R(x)\text{)}. \tag{47}$$

The most general Hamiltonian of the β decay takes the form

$$\mathcal{H}_I^{\beta}(x) = \sum_{i=S,V,T,A,P} G_i\bar{p}(x)\,O_i n(x)\,\bar{e}(x)\,O^i(1 \mp \gamma_5)\,\nu(x) + \text{h.c.} \tag{48}$$

---

*Equations (42) for massless spin-$1/2$ particle were considered by H. Weil in 1929 [23]. However, as these equations did not conserve parity they were rejected. In [24], Pauli wrote «. . . because such equations are not invariant under space reflection they are not applicable to the physical reality».

**In 1957, from the investigation of the high-energy part of the tritium β spectrum, the following upper bound for the neutrino mass was obtained:

$$m_\nu \lesssim 200\ \text{eV} \simeq 4 \cdot 10^{-4} m_e. \tag{45}$$

Thus, it was found that the mass of the neutrino is much smaller than the mass of the electron, a particle which is emitted in the β decay together with the neutrino.

From this expression it follows that effects of violation of parity in the β decay will be large (maximal).

2. The neutrino helicity (projection of the spin onto the direction of momentum) is equal to -1 $(+1)$ in the case if the neutrino field is $\nu_L(x)$ $(\nu_R(x))$.

The spinor $u^r(p)$ which describes a massless neutrino with momentum p and helicity r satisfies the equations $\gamma \cdot p u^r(p) = (\gamma^0 p^0 - \boldsymbol{\gamma}\mathbf{p}) u^r(p) = 0$, $\boldsymbol{\Sigma} \cdot \mathbf{k} u^r(p) = r u^r(p)$. Here $\boldsymbol{\Sigma} = \gamma_5 \gamma^0 \boldsymbol{\gamma}$ is the spin operator and $\mathbf{k}$ is the unit vector in the direction of the momentum $\mathbf{p}$. From these equations we find $\gamma_5 u^r(p) = r u^r(p)$. In the expansion of the field $\nu_L(x)$ $(\nu_R(x))$ the spinor $u^r(p)$ is multiplied by the projection operator $\dfrac{1-\gamma_5}{2}$ $\left(\dfrac{1-\gamma_5}{2}\right)$. We have $\dfrac{1-\gamma_5}{2} u^r(p) = \dfrac{1-r}{2} u^r(p)$ $\left(\dfrac{1+\gamma_5}{2} u^r(p) = \dfrac{1+r}{2} u^r(p)\right)$. Thus, $r = -1$ $(r = 1)$ in the case if the neutrino field is $\nu_L(x)$ $(\nu_R(x))$.

In the general case of a Dirac particle with spin $1/2$, there are four states with momentum $\mathbf{p}$ and energy $E_p = \sqrt{p^2 + m^2}$: two particle states with helicities ± 1 and two antiparticle states with helicities ± 1. In the two-component theory with the neutrino field $\nu_L(x)$ $(\nu_R(x))$, only the state of the neutrino with helicity -1 $(+1)$ and the state of the antineutrino with helicity $+1$ (-1) are allowed.

It is easy to see that in the processes in which a two-component neutrino is emitted, large (maximal) violation of parity will be observed. In fact, let w_r^R be the probability to emit a neutrino with helicity r in a right-handed system. This probability is equal to the probability of the emission of a neutrino with helicity $-r$ in a left-handed system

$$w_r^R = w_{-r}^L. \tag{49}$$

If the parity is conserved,

$$w_r^R = w_r^L. \tag{50}$$

From (49) and (50), it follows that in the case of the conservation of parity the probabilities of the emission of neutrinos with helicities r and $-r$ must be equal

$$w_r^{L,R} = w_{-r}^{L,R}, \quad \text{i.e.,} \quad w_1^{L,R} = w_{-1}^{L,R}. \tag{51}$$

In the case of the two-component neutrino theory $w_1 = 0$ (or $w_{-1} = 0$). Thus, in the two-component theory relation (51) is maximally violated.

Let us notice that Landau [20], Lee and Yang [21], and Salam [22] had different arguments in favor of the two-component neutrino theory.

Landau assumed that the neutrino mass was equal to zero and for the neutrino field he chose $\nu_L(x)$ (or $\nu_R(x)$) assuming CP invariance of the weak interaction (C is charge conjugation, i.e., the operation of transition from particles to antiparticles). Lee and Yang assumed that the neutrino is a particle with helicity equal to -1 (or $+1$). This is possible only if the neutrino mass is equal to zero, parity is violated, and the neutrino field is $\nu_L(x)$ (or $\nu_R(x)$). Salam assumed invariance of the equation for the neutrino field under γ_5-transformation ($\nu \to \gamma_5 \nu$). From this requirement it follows that the neutrino mass is equal to zero and the neutrino field is $\nu_L(x)$ (or $\nu_R(x)$).

Summarizing, the discovery of the violation of the parity in the β decay and other weak processes triggered enormous progress in the understanding of the weak interaction. *This progress started with the theory of the two-component neutrino.* This theory of the neutrino became part of the universal $V-A$ theory of the weak interaction and the unified theory of the electromagnetic and weak interaction (Standard Model). The main idea of the two-component theory (left-handed component of the neutrino field in the interaction Hamiltonian) was generalized in the subsequent development of the theory of the weak interaction.

Let us stress that the two-component theory was based on the assumption that the neutrino is a massless particle. We know today that neutrinos have small but different from zero masses and that the two-component theory must be generalized.

6. MEASUREMENT OF NEUTRINO HELICITY. GOLDHABER ET AL. EXPERIMENT

Soon after the two-component neutrino theory had been proposed, the neutrino helicity was determined from the results of the spectacular Goldhaber, Grodzins, and Sunyar experiment [25].

In this experiment, the neutrino helicity was inferred from the measurement of the circular polarization of γ's produced in the chain of reactions

$$\begin{array}{l} e^- + {}^{152}\mathrm{Eu} \to \nu + {}^{152}\mathrm{Sm}^* \\ \qquad\qquad\qquad\qquad\quad \downarrow \\ \qquad\qquad\qquad\qquad {}^{152}\mathrm{Sm} + \gamma. \end{array} \tag{52}$$

The spins of ^{152}Eu and ^{152}Sm are equal to zero and the spin of $^{152}\mathrm{Sm}^*$ is equal to one. Since the orbital momentum of the initial electron is equal to zero (K capture), from the conservation of the projection of the total angular momentum on the neutrino momentum we have

$$\frac{1}{2}h + m = \pm\frac{1}{2},$$

where h is the neutrino helicity, and m is the projection of the spin of $^{152}\mathrm{Sm}^*$. From this relation we have

$$m = 0, -1 \quad \text{for} \quad h = 1, \quad m = 0, +1 \quad \text{for} \quad h = -1. \tag{53}$$

Thus, the circular polarization of γ's emitted in the direction of the $^{152}\mathrm{Sm}^*$ momentum is equal to the helicity of the neutrino. In the Goldhaber et al. experiment, the circular polarization of resonantly scattered γ's was measured (only γ's emitted in the direction of motion of $^{152}\mathrm{Sm}^*$ satisfy the resonance condition). The authors concluded: «... our result is compatible with 100% negative helicity of neutrino emitted in orbital electron capture».

Thus, the Goldhaber et al. experiment confirmed the two-component neutrino theory. It was established that from the two possibilities for the neutrino field ($\nu_L(x)$ or $\nu_R(x)$) the first possibility was realized.

7. UNIVERSAL CURRENT × CURRENT V–A THEORY

The most general Hamiltonian of the β decay in the case of the two-component neutrino is given by expression (48). It includes five terms (S, V, T, A, P). There were many attempts to determine the dominant terms of the Hamiltonian from the data of different β-decay experiments. However, the situation was contradictory. From the measurement of the angular electron–neutrino correlation in the decay $^6\mathrm{He} \to {}^6\mathrm{Li} + e^- + \bar{\nu}$ and from other data, it followed that S, T terms are the dominant ones. On the other side, the data on the measurement of electron–neutrino correlation in the decay $^{35}\mathrm{Ar} \to {}^{35}\mathrm{Cl} + e^+ + \nu$ and other data were in favor of V, A terms.

In this uncertain experimental situation in 1958 two fundamental theoretical papers by Feynman and Gell-Mann [26], and Marshak and Sudarshan [27] appeared. These authors postulated a principle which allowed them to build the simplest possible universal theory of the β decay and other weak processes.

Feynman and Gell-Mann, Marshak and Sudarshan assumed that *in the Hamiltonian of the weak interaction there enter only left-handed components of all fields**.

The Hamiltonian of the β decay has in this case the form

$$\mathcal{H}_I^\beta(x) = \sum_{i=S,V,T,A,P} G_i \bar{p}_L(x)\, O_i n_L(x)\; \bar{e}_L(x)\, O^i \nu_L(x) + \mathrm{h.c.} \tag{54}$$

We have

$$\bar{p}_L(x)\, O_i n_L(x) = \bar{p}(x) \frac{1+\gamma_5}{2} O_i \frac{1-\gamma_5}{2} n(x). \tag{55}$$

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*Feynman and Gell-Mann assumed that $\left(\frac{1-\gamma_5}{2}\right)\psi_a(x)$ enters into the Hamiltonian of the weak interaction because this field satisfies second-order equation and could be considered as a fundamental field. Marshak and Sudarshan came to left-handed components from the requirement of γ_5 invariance of the interaction (invariance under the change $\psi_a(x) \to -\gamma_5\psi_a(x)$).

Using the algebra of the Dirac matrices γ's, it is easy to show that

$$\frac{1+\gamma_5}{2}(1;\sigma_{\alpha\beta};\gamma_5)\frac{1-\gamma_5}{2}=0. \tag{56}$$

Hence, S, T, and P terms do not enter into the Hamiltonian (55). Moreover, A and V terms are connected by the relation:

$$\frac{1+\gamma_5}{2}\gamma_\alpha\gamma_5\frac{1-\gamma_5}{2}=-\frac{1+\gamma_5}{2}\gamma_\alpha\frac{1-\gamma_5}{2}. \tag{57}$$

The Hamiltonian of the β decay takes the simplest possible form*

$$\begin{aligned}\mathcal{H}_I^\beta(x)&=\frac{G_F}{\sqrt{2}}4\bar{p}_L(x)\,\gamma_\alpha n_L(x)\,\bar{e}_L(x)\,\gamma^\alpha\nu_L(x)+\text{h.c.},\\&=\frac{G_F}{\sqrt{2}}\bar{p}(x)\,\gamma_\alpha(1-\gamma_5)\,n(x)\,\bar{e}(x)\,\gamma^\alpha(1-\gamma_5)\,\nu(x)+\text{h.c.}\end{aligned} \tag{58}$$

The Hamiltonian (58), like the Fermi Hamiltonian (4), is characterized by only one interaction constant G_F**. There is, however, a crucial difference between the Hamiltonian (58) and the Fermi Hamiltonian. In the Hamiltonian (58) left-handed components of all fields enter. This means that the Hamiltonian (58) unlike the Fermi Hamiltonian does not conserve parity.

What about numerous experiments from which it followed that S and T terms are the dominant terms of the Hamiltonian of the β decay? In the Feynman and Gell-Mann paper it is written: «These theoretical arguments seem to the authors to be strong enough to suggest that the disagreement with ^{6}He recoil experiment and with some other less accurate experiments indicates that these experiments are wrong». This was a correct expectation: subsequent experiments did not confirm the results of the experiments which indicated in favor of the dominance of S and T terms.

Let us notice that the Hamiltonian (58) is a correct effective Hamiltonian of the β decay and other connected processes. It describes all existing β decay and other data.

With the Feynman–Gell-Mann, Marshak–Sudarshan prescription (left-handed components of all fields enter into the Hamiltonian of the weak interaction) it was easy to implement the Pontecorvo and others idea of the universal weak interaction which we discussed before.

*If we assume that left-handed fields enter into the Hamiltonian of the weak interaction, in this case only weak currents $\bar{p}(x)\gamma_\alpha(1-\gamma_5)n(x)$ and $\bar{e}(x)\gamma^\alpha(1-\gamma_5)\nu(x)$ survive. Because the interaction Hamiltonian is a scalar, only one possibility for the Hamiltonian (current $\times$ current) exists.

**Interesting that the title of the Feynman and Gell-Mann paper is «Theory of the Fermi Interaction».

For the Hamiltonian of the decay

$$\mu^+ \to e^+ + \nu + \bar{\nu}, \tag{59}$$

we have in this case

$$\mathcal{H}_I^{\mu\to e\nu\bar{\nu}}(x) = \frac{G_F}{\sqrt{2}} 4\bar{e}_L(x)\,\gamma_\alpha \nu_L(x)\,\bar{\nu}_L(x)\,\gamma^\alpha \mu_L(x) + \text{h.c.} \tag{60}$$

From (60), it follows that the lifetime of the muon is given by the expression

$$\tau_\mu = \frac{192\pi^3}{G_F^2 m_\mu^5}, \tag{61}$$

where m_μ is the mass of the muon.

Feynman and Gell-Mann demonstrated that if we take for G_F the value obtained from the superallowed $0^+ \to 0^+$ β decay of ^{14}O, we will find perfect agreement with the experimental lifetime of muon. This was an important confirmation of the hypothesis of the universality of the weak interaction*.

From the $\mu{-}e$ universality it followed that the Hamiltonian of the μ capture and other connected processes can be obtained from (58) by the change $e(x) \to \mu(x)$. We have

$$\mathcal{H}_I^{\mu}(x) = \frac{G_F}{\sqrt{2}} 4\bar{p}_L(x)\,\gamma_\alpha n_L(x)\,\bar{\mu}_L(x)\,\gamma^\alpha \nu_L(x) + \text{h.c.}. \tag{62}$$

At the time when Feynman and Gell-Mann, Marshak and Sudarshan wrote their papers there was a contradiction of the idea of $\mu{-}e$ universality of the weak interaction with the data on the measurement of the width of the decay $\pi^+ \to e^+ + \nu$. From (58) and (62), it follows that the ratio of the decay widths $R = \dfrac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)}$ is given by the expression

$$R = \frac{m_e^2}{m_\mu^2} \frac{(1 - m_e^2/m_\pi^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} \simeq 1.2 \cdot 10^{-4}. \tag{63}$$

On the other hand, in the experiment [29] the decay $\pi^+ \to e^+ + \nu$ was not observed and it was found that $R < 10^{-5}$. Feynman and Gell-Mann wrote «This is a very serious discrepancy. The authors have no idea of how it can be resolved».

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*This agreement was also an evidence in favor of the conserved vector current (CVC) hypothesis [28]. According to this hypothesis the weak vector current is the «charged» component of the isovector current which is conserved due to isotopic invariance. The conservation of the vector current ensures the fact that the Fermi constant is not renormalized by the strong interaction.

In 1958, a new experiment on the measurement of the $\pi^+ \to e^+ + \nu$ decay was performed at CERN [30]. In this experiment, perfect agreement with prediction (63) of the universal Feynman and Gell-Mann, Marshak and Sudarshan theory was obtained*.

In order to unify the interactions (58), (60), and (62), Feynman and Gell-Mann introduced the $\mu{-}e$ symmetric weak current

$$j^\alpha = 2(\bar{p}_L \gamma^\alpha n_L + \bar{\nu}_L \gamma^\alpha e_L + \bar{\nu}_L \gamma^\alpha \mu_L) \tag{64}$$

and assumed that the total Hamiltonian of the weak interaction had the simplest current $\times$ current form

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} j^\alpha j^+_\alpha, \tag{65}$$

where G_F was the Fermi constant.

Two remarks are in order.

1. The hadron part of the current has the form

$$j^\alpha = v^\alpha - a^\alpha,$$

where $v^\alpha = \bar{p}\gamma^\alpha n$ and $a^\alpha = \bar{p}\gamma^\alpha \gamma_5 n$ are the vector and axial currents**. Notice that Fermi β transitions of nuclei are due to the vector current, and Gamov–Teller transitions are due to the axial current.

2. The current j^α provides transitions $n \to p, e^- \to \nu$, etc., in which $\Delta Q = Q_f - Q_i = 1$ ($Q_i(Q_f)$ is the initial (final) charge). By this reason the current j^α is called the charged current (CC).

There are two types of terms in the Hamiltonian (65): nondiagonal and diagonal. The nondiagonal terms have the form

$$\mathcal{H}_I^{\rm nd} = \frac{G_F}{\sqrt{2}} 4 \Big\{ [(\bar{p}_L \gamma^\alpha n_L)(\bar{e}_L \gamma_\alpha \nu_L) + \text{h.c.}] + \\ + [(\bar{p}_L \gamma^\alpha n_L)(\bar{\mu}_L \gamma_\alpha \nu_L) + \text{h.c.}] + [(\bar{e}_L \gamma^\alpha \nu_L)(\bar{\nu}_L \gamma_\alpha \mu_L) + \text{h.c.}] \Big\}. \tag{66}$$

The first term of this expression is the Hamiltonian of β decay of the neutron $n \to p + e^- + \bar{\nu}$, of the process $\bar{\nu} + p \to e^+ + n$, and other processes. The second term of (66) is the Hamiltonian of the process $\mu^- + p \to \nu + n$, of the neutrino process $\nu + n \to \mu^- + p$, and other processes. Finally, the third term of (66) is

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*When this result was obtained, Feynman was visiting CERN. The news reached him when he was queuing in the CERN cafeteria. It is said that when Feynman learnt about the $\pi \to e\nu$ news he started to dance.

**This is the reason why the Feynman and Gell-Mann, Marshak and Sudarshan theory is called the $V{-}A$ theory.

the Hamiltonian of the μ decay (59), of the process $\nu + e^- \to \mu^- + \nu$, and other processes.

The diagonal terms of the Hamiltonian (65) are given by

$$\mathcal{H}^d = \frac{G_F}{\sqrt{2}}\, 4\Big[(\bar{\nu}_L\gamma^\alpha e_L)(\bar{e}_L\gamma_\alpha\nu_L) + (\bar{\nu}_L\gamma^\alpha\mu_L)(\bar{\mu}_L\gamma_\alpha\nu_L) + (\bar{p}_L\gamma^\alpha n_L)(\bar{n}_L\gamma_\alpha p_L)\Big]. \tag{67}$$

The first term of (67) is the Hamiltonian of the processes of elastic scattering of a neutrino and an antineutrino on an electron

$$\nu + e \to \nu + e \tag{68}$$

and

$$\bar{\nu} + e \to \bar{\nu} + e, \tag{69}$$

of the process $e^+ + e^- \to \bar{\nu} + \nu$, and other processes. Such processes were not known in the fifties. Their existence and the cross sections of these processes were *predicted by the current × current theory*.

The cross sections of the processes (68) and (69) are very small (at MeV's energies of the order of 10^{-45} cm^2). The observation of such processes was a challenge. After many years of efforts, the cross section of the process (69) was measured by F. Reines et al. [31] in an experiment with antineutrinos from a reactor. At that time the Standard Model already existed. According to the Standard Model, to the matrix elements of the processes (68) and (69) contribute the Hamiltonian (67) and an additional (the so-called neutral current) Hamiltonian. The result of the experiment by F. Reines et al. was in agreement with the Standard Model.

In the Feynman and Gell-Mann, and Marshak and Sudarshan papers decays of Λ-hyperon and other strange particles were also briefly discussed. However, weak interaction of the strange particles was included into the current × current Hamiltonian in 1963 by N. Cabibbo [32]. We will discuss Cabibbo's contribution to the theory of weak interaction later.

Summarizing, the $V-A$ current × current theory of the weak interaction signified a great progress in the understanding of the weak interaction and neutrino. The Feynman and Gell-Mann, Marshak and Sudarshan idea of the left-handed components of all fields in the CC Hamiltonian was triggered mainly by some experimental data, success of the two-component neutrino theory, and great intuition. The idea of the left-handed components complemented with the idea of the universality of the weak interaction allowed one to build the simplest possible CC Hamiltonian of the weak interaction which is characterized by only one (Fermi) constant. Authors of this theory were courageous enough to state that some experimental data which existed at that time but contradicted this theory were wrong. Further experiments showed that the authors were correct: current × current V–A theory is in perfect agreement with all existing CC data.

8. INTERMEDIATE VECTOR W BOSON

In the Feynman and Gell-Mann paper, which we discussed in the previous section, it was mentioned that the current×current Hamiltonian of the weak interaction (65) could originate from the exchange of a heavy intermediate charged vector boson*. We will discuss now the hypothesis of a charged intermediate vector boson. Let us assume that there exists a charged vector $W^{\pm}$ boson and that the fundamental Lagrangian of the weak interaction $\mathcal{L}_{\mathcal{I}}$, which is equal to $-\mathcal{H}_{\mathcal{I}}$, has the form of a scalar product of the current j^{α} given by Eq. (64) and the vector field W_{α}

$$\mathcal{L}_{\mathcal{I}} = -\frac{g}{2\sqrt{2}} j_{\alpha} W^{\alpha} + \text{h.c.}, \tag{70}$$

where g is a dimensionless interaction constant**.

If the Lagrangian of the weak interaction has the form (70), in this case the β decay of the neutron proceeds in the following three steps (Fig. 1): 1) neutron produces the virtual W^{-} boson and is transferred into a proton; 2) the virtual W^{-} boson propagates; 3) the virtual W^{-} boson decays into an electron and an antineutrino.

In the Feynman diagram, the propagator of the W boson contains a factor $\dfrac{-1}{Q^2 - m_W^2}$, where $Q = p_n - p_p$ is the momentum transfer and m_W is the mass of the W boson. If the W boson is a heavy particle (say, with a mass which is much larger than the mass of the proton), in this case Q^2 in the W propagator can be safely neglected and the matrix element of the β decay of the neutron can be obtained from the Hamiltonian (65) in which the Fermi constant is given by the relation

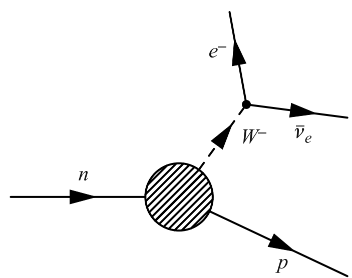

Fig. 1. Feynman diagram of the process $n \to p + e^{-} + \bar{\nu}$ in the theory with $W^{\pm}$ boson

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}. \tag{71}$$

In a similar way it can be shown that in the region of relatively small energies, the matrix elements of all weak processes with virtual (intermediate) charged W boson can be obtained from the current × current Hamiltonian (65) in which the Fermi constant is given by relation (71).

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*«We have adopted the point of view that the weak interactions all arise from the interaction of a current J_{α} with itself, possibly via an intermediate charged vector meson of high mass» [26].

**The Lagrangian (70) has the form analogous to the Lagrangian of the electromagnetic interaction $\mathcal{L}_I^{\text{EM}} = -e j_{\alpha}^{\text{EM}} A^{\alpha}$, where j_{α}^{EM} is the electromagnetic current, A^{α} is the electromagnetic field and e is the dimensionless electric charge.

From the point of view of the theory with the W boson, the current $\times$ current Hamiltonian with the Fermi constant (71) is the effective Hamiltonian of the weak interaction.

Thus, the theory with a vector $W^{\pm}$ boson could explain the current $\times$ current structure of the weak interaction Hamiltonian and the fact that the Fermi constant has the dimension $[M]^{-2}$.

We know now that the intermediate charged $W^{\pm}$ boson exists. The $W^{\pm}$ boson is one of the heaviest particles: its mass is equal to $m_W \simeq 80.4$ GeV. For the discovery of the $W^{\pm}$ boson and the Z^0 boson (see later) in 1984, C. Rubbia and S. van der Meer were awarded the Nobel Prize. As we will see later, the Lagrangian (70) is a part of the total Lagrangian of the Standard Model.

The first idea of the charged vector boson, mediator of the weak interaction, was discussed by O. Klein [33] in 1938, soon after the Fermi β-decay paper had appeared. Fermi built the first Hamiltonian of the β decay by analogy with electrodynamics. O. Klein noticed that the analogy would be more complete if a charged vector boson (analog of the γ quantum) exists and the weak interaction originated from an interaction which (like the electromagnetic interaction) had the form of a product of a current and a vector field. In order to build such a theory O.Klein assumed gauge invariance*.

9. THE PONTECORVO RADIOCHEMICAL METHOD OF NEUTRINO DETECTION

As we discussed before, because of the extreme smallness of the cross section for the absorption of neutrinos by nuclei for many years most physicists considered the neutrino as an undetectable particle.

The first method of neutrino detection was proposed by Bruno Pontecorvo in 1946 [4]. He wrote: «The object of this note is to show that the experimental observation of an inverse β process produced by neutrino is not out of the question with the modern experimental facilities, and to suggest a method which might make an experimental observation feasible».

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*The great Yukawa idea that the interaction between nucleons is due to the exchange of a meson (which allowed him to predict the π meson from the range of nuclear forces) was applied by Klein to the short range weak interaction. Klein assumed that the weak decay of the neutron was due to the exchange of a heavy charged vector boson between (np) and $(e\nu)$ pairs. It is impressive that this general quantum idea very early in the thirties allowed one to anticipate the existence of a very heavy particle which could be observed only many years later after modern high-energy accelerators were built.

Pontecorvo proposed radiochemical methods of neutrino detection. As an example, let us consider the reaction

$$\nu + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}. \tag{72}$$

The ^{37}Ar atoms decay (via K capture) with a lifetime of about 34 days.

After irradiation of a target (containing ^{37}Cl) by neutrinos for a relatively long time (say, one month), a few radioactive atoms of ^{37}Ar could be produced. As argon is a nobel gas, atoms of ^{37}Ar can be extracted from the target and can be placed into a proportional counter in which their decay will be detected. This is the main idea of Pontecorvo's radiochemical method. He discussed different reactions. Pontecorvo considered the Cl–Ar reaction (72) as very appropriate for the neutrino detection (a large volume of liquid Carbon Tetrachloride can be used as a target, ^{37}Ar atoms have a convenient lifetime, etc.).

In the report [4], B. Pontecorvo also pointed out the following intensive sources of neutrinos which existed at that time:

1. The Sun. The flux of the solar neutrinos is approximately equal to $6 \cdot 10^{10}$ cm$^{-2} \cdot$ s^{-1}.

2. Nuclear reactors*. The total flux of (anti)neutrinos from a reactor is equal to $2 \cdot 10^{20}$ s$^{-1} \cdot \mathrm{GW_{th}}$.

3. Radioactive sources which can be prepared in reactors.

Pontecorvo's radiochemical method of neutrino detection was used in solar neutrino experiments. The first experiment in which solar neutrinos were detected was performed by R. Davis and collaborators [9]. In this experiment solar neutrinos were detected via the observation of the Cl–Ar reaction (72). In 2002, R. Davis was awarded the Nobel Prize for this experiment.

10. DETECTION OF NEUTRINO. REINES AND COWAN EXPERIMENT

The first proof of the existence of neutrinos was obtained in 1953–1959 in the F. Reines and C. L. Cowan experiments [34]. In these experiments (anti)neutrinos from the Savannah River reactor** were detected through the observation of the process

$$\bar{\nu} + p \rightarrow e^+ + n. \tag{73}$$

*Pontecorvo wrote «Probably this is the most convenient neutrino source».

**In the beginning, Reines and Cowan planned to do an experiment with neutrinos from an atomic bomb explosion. Later they understood that an experiment with reactor antineutrinos was much more simpler and feasible. Reines remembered in his Nobel lecture «I have wandered since why it took so long for us to come to this now obvious conclusion and how it escaped others around us with whom we talked...».

Antineutrinos are produced in a reactor in a chain of β decays of neutron-rich nuclei, products of the fission of uranium and plutonium. The energies of antineutrinos from a reactor are less than 10 MeV. About $2.3 \cdot 10^{20}$ antineutrinos per second were emitted by the Savannah River reactor. The flux of $\bar{\nu}_e$'s in the Reines and Cowan experiment was about 10^{13} cm$^{-2} \cdot$ s^{-1}.

A liquid scintillator* $(1.4 \cdot 10^3$ l) loaded with $CdCl_2$ was used as a target in the experiment. A positron, produced in the process (73), slowed down in the scintillator and annihilated with an electron, producing two γ quanta with opposite momenta and each with energy $\simeq 0.51$ MeV.

A neutron, produced in the process (73), slowed down in the target and was captured by Cd within about 5 μs, producing a γ quantum in the capture $n + {}^{108}\mathrm{Cd} \to {}^{109}\mathrm{Cd} + \gamma$ (at small energies the cross section of this process is very large). The γ quanta were detected by 110 photomultipliers. Thus, the signature of the $\bar{\nu}$-event in the Reines and Cowan experiment was two γ quanta from the e^+–e^- annihilation in coincidence with a delayed γ quantum from the neutron capture by cadmium. For the cross section of the process (73), the value

$$\sigma_\nu = (11 \pm 2.6) \cdot 10^{-44} \text{ cm}^2 \tag{74}$$

was obtained in the latest measurements. This value was in agreement with the predicted value.

In the $V-A$ current $\times$ current theory the cross section of the process (73) is connected with the lifetime τ_n of the neutron by the relation

$$\sigma(\bar{\nu}_e p \to e^+ n) = \frac{2\pi^2}{m_e^5 f \tau_n} p_e E_e, \tag{75}$$

where $E_e \simeq E_{\bar{\nu}} - (m_n - m_p)$ is the energy of the positron; p_e is the positron momentum; $f = 1.686$ is the phase-space factor; m_n, m_p, m_e are the masses of the neutron, proton and electron, respectively. From (75) for the cross section of the process (73), averaged over the antineutrino spectrum, the value

$$\bar{\sigma}(\bar{\nu}_e p \to e^+ n) \simeq 9.5 \cdot 10^{-44} \text{ cm}^2 \tag{76}$$

was found. In 1995, the Nobel Prize in Physics was awarded to F. Reines «for the detection of the neutrino».

*Reines and Cowan were the first who understood that the phenomenon of scintillation of organic liquids, discovered at that time, could be employed in order to build a big (1 m^3) detector which was necessary to detect neutrinos.

11. LEPTON NUMBER CONSERVATION. DAVIS EXPERIMENT

The particle which is produced in the β decay together with the electron is called the antineutrino. It is a direct consequence of the quantum field theory that an antineutrino can produce a positron in the process (73) and other similar processes. Can antineutrinos also produce electrons in weak processes? This problem was investigated in an experiment which was performed in 1956 by Davis [35] at the Savannah River reactor. This was the first application of Pontecorvo's radiochemical method. Production of radioactive ^{37}Ar atoms which could be produced in the process

$$\bar{\nu} + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar} \tag{77}$$

were searched for in this experiment. No ^{37}Ar atoms were found. For the cross section of the process (77), the following upper bound was obtained:

$$\sigma(\bar{\nu} + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}) < 0.9 \cdot 10^{-45}\ \mathrm{cm}^2.$$

This bound is about five times smaller than the cross section of the corresponding reaction with the neutrino.

Thus, it was established that antineutrinos from a reactor can produce positrons (the Reines–Cowan experiment) but cannot produce electrons (the Davis experiment).

This result can be explained if we assume that there exists *conserving lepton number* L and ν and e^- have the same values of L (say, $L(\nu) = L(e^-) = 1$). The lepton numbers of antiparticles are opposite to the lepton numbers of particles. We have $L(\bar{\nu}) = L(e^+) = -1$. We also assume that the lepton numbers of proton, neutron and other hadrons are equal to zero. The conservation of the lepton number explains the negative result of the Davis reactor experiment.

12. DISCOVERY OF MUON NEUTRINO. BROOKHAVEN NEUTRINO EXPERIMENT

The authors of the universal V–A theory of the weak interaction considered only one type of neutrinos. There existed, however, an idea that neutrinos which take part in the weak interaction together with an electron and neutrino which take part in the weak interaction together with a muon could be different particles*.

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*Pontecorvo [36] remembered «...for people working with muons in the old times, the question about different types of neutrinos has always been present. True, later on many theoreticians forgot all about it and some of them «invented» again the two neutrinos».

Let us call neutrinos, which participate in weak processes together with electrons (muon), the electron (muon) neutrino (ν_e (ν_μ)). The charged current of the current $\times$ current theory takes in this case the form

$$j^\alpha = 2(\bar{p}_L\gamma^\alpha n_L + \bar{\nu}_{eL}\gamma^\alpha e_L + \bar{\nu}_{\mu L}\gamma^\alpha \mu_L). \tag{78}$$

Are ν_e and ν_μ the same or different particles? The answer to this fundamental question was obtained in the experiment which was proposed by B. Pontecorvo [3] in 1959 and was performed in Brookhaven in 1962 [37].

The first indication that ν_e and ν_μ are different particles was obtained from the data on the search for the decay $\mu \to e\gamma$*. If ν_e and ν_μ are identical particles, the $\mu \to e\gamma$ decay is allowed. The probability of the decay $\mu \to e\gamma$ in the theory with the W boson was calculated in [39]** soon after the $V-A$ theory has been proposed. It was found that the ratio R of the probability of the decay $\mu^+ \to e^+\gamma$ to the probability of the decay $\mu^+ \to e^+ + \nu + \bar{\nu}$ was given by the expression

$$R \simeq \frac{\alpha}{24\pi} \simeq 10^{-4}. \tag{79}$$

The decay $\mu \to e\gamma$ was not observed in experiment. At the time of the Brookhaven experiment, for the upper bound of the ratio R much smaller than (79), the value

$$R < 10^{-8} \tag{80}$$

had been found.

A direct proof of the existence of the second (muon) type of the neutrino was obtained by L. M. Lederman, M. Schwartz, J. Steinberger et al. in the first experiment with accelerator neutrinos in 1962. The idea of the experiment was proposed by B. Pontecorvo in 1959 [3]***.

A beam of π^+'s in the Brookhaven experiment was obtained by the bombardment of a Be target by protons with an energy of 15 GeV. In the decay channel (about 21 m long) practically all π^+'s decay. After the channel there was a shielding (13.5 m of iron), in which charged particles were absorbed. After the shielding there was neutrino detector (aluminium spark chamber, 10 t) in which the production of charged leptons was observed.

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*Let us notice that the first experiment on the search for the $\mu \to e\gamma$ decay was performed by Pontecorvo and Hincks in 1948 [38].

**Such a theory is a nonrenormalizable one. In [39], the cut-off $\Lambda \simeq m_W$ was applied.

***B. Pontecorvo came to the idea of such an experiment when thinking about a possible neutrino program at Meson Factories which were under construction at different places. «At the Laboratory of Nuclear Problems of JINR (Dubna) in 1958 a proton relativistic cyclotron was being designed with a beam energy 800 MeV and beam current 500 A. I started to think about experimental research program for such an accelerator» [36]. The Dubna Meson Factory eventually was not built.

The dominant decay channel of the π^+ meson is

$$\pi^+ \to \mu^+ + \nu_\mu. \tag{81}$$

According to the universal $V-A$ theory, the ratio R of the width of the decay

$$\pi^+ \to e^+ + \nu_e \tag{82}$$

to the width of the decay (81) is given by the relation (63) and is about $1.2 \cdot 10^{-4}$. Thus, the neutrino beam in the Brookhaven experiment was practically a pure ν_μ beam (with a small admixture of ν_e from decays of muons and kaons).

Neutrinos, emitted in the decay (81), produce μ^- in the process

$$\nu_\mu + N \to \mu^- + X. \tag{83}$$

If ν_μ and ν_e were the same particles, neutrinos from the decay (81) would produce also e^- in the reaction

$$\nu_\mu + N \to e^- + X. \tag{84}$$

Due to the $\mu - e$ universality of the weak interaction one could expect in this case to observe in the detector practically equal numbers of muons and electrons.

In the Brookhaven experiment, 29 muon events were detected. The observed six electron candidates could be explained by the background. The measured cross section was in agreement with the $V-A$ theory. Thus, it was proved that *ν_μ and ν_e were different particles.*

In 1963, with the invention of the magnetic horn at the CERN the intensity and purity of neutrino beams were greatly improved. In the 45 ton spark-chamber experiment and in the large bubble chamber experiment at CERN, the Brookhaven result was fully confirmed.

The results of the Brookhaven and other experiments can be explained if we introduce electron and muon lepton numbers L_e and L_μ and assume that the total electron and muon lepton numbers were conserved:

$$\sum_i L_e^{(i)} = \text{const}; \quad \sum_i L_\mu^{(i)} = \text{const}. \tag{85}$$

The flavor lepton numbers of particles are given in Table 1. The lepton numbers of antiparticles are opposite to the lepton numbers of the corresponding particles.

We know now that the notion of the flavor lepton number is an approximate one. It is valid only if we neglect small neutrino masses. The conservation laws (85) are violated in neutrino oscillations which are due to small neutrino masses and neutrino mixing. Later we will discuss neutrino masses, mixing and oscillations in detail.

Summarizing, the discovery of the second (muon) neutrino was a great event in physics. It was proved that two different (in mass) leptons e and μ corresponded

Table 1. **Flavor lepton numbers of particles**

Lepton number	$\nu_e e^-$	$\nu_\mu \mu^-$	Hadrons, γ
L_e	1	0	0
L_μ	0	1	0

to two different neutrinos ν_e and ν_μ. Now we know that with the discovery of ν_μ it was established that in addition to the first family of leptons (ν_e, e) there existed the second family (ν_μ, μ).

The Brookhaven neutrino experiment was the first experiment with high energy neutrinos originating from decays of pions, kaons, and muons produced at accelerators. As we will see later, important discoveries were made in such experiments.

12.1. Strange Particles. Quarks. Cabibbo Current. The current $\times$ current Hamiltonian (65) with CC current (78) is the Hamiltonian of such processes in which $p, n, \pi^\pm$ and other nonstrange particles take part. The strange particles were discovered in cosmic rays in the fifties. Their decays were studied in detail in accelerator experiments. From the investigation of the semileptonic decays

$$K^+ \to \mu^+ + \nu_\mu, \quad \Lambda \to n + e^- + \bar{\nu}_e,$$
$$\Sigma^- \to n + e^- + \bar{\nu}_e, \quad \Xi^- \to \Lambda + \mu^- + \bar{\nu}_\mu$$

and others, the following *phenomenological rules* were formulated.

1. The strangeness S in the decays of strange particles is changed by one

$$|\Delta S| = 1.$$

Here $\Delta S = S_f - S_i$, where S_i (S_f) is the initial (final) total strangeness of the hadrons. As an example, according to this rule, the decay $\Xi^- \to p + \pi^- + e^- + \bar{\nu}_e$, in which $\Delta S = 2$, is forbidden. From the data of experiments for the ratio R of the width of the decay $\Xi^- \to p + \pi^- + e^- + \bar{\nu}_e$ to the total decay width of Ξ^- the following upper bound was obtained: $R < 4 \cdot 10^{-4}$*.

2. The semileptonic decays of strange particles obey the rule

$$\Delta Q = \Delta S.$$

Here $\Delta Q = Q_f - Q_i$, where Q_i (Q_f) is the initial (final) total electric charge of hadrons (in the unit of the proton charge). According to this rule, the decay

*Here and below we present data given in «The Review of Particle Physics» [40].

Table 2. **Quantum numbers of quarks (Q is the charge, S is the strangeness, B is the baryon number)**

Quark	Q	S	B
u	$2/3$	0	$1/3$
d	$-1/3$	0	$1/3$
s	$-1/3$	-1	$1/3$

$\Sigma^+ \to n + e^+ + \nu_e$ is forbidden. From experimental data it follows that the ratio R of the width of this decay to the total width of Σ^+ is less than $5 \cdot 10^{-6}$.

3. The decays of strange particles are suppressed with respect to the decays of nonstrange particles.

In 1964, Gell-Mann and Zweig made the assumption that strange and nonstrange hadrons are bound states of u, d, and s quarks. The quantum numbers of the quarks are presented in Table 2.

From the point of view of the theory of quarks, $p, n, \Lambda, \Sigma^+, \Xi^-$ and other baryons are bound states of three quarks ($p = (uud)$, $n = (udd)$, $\Lambda = (uds)$, $\Sigma^+ = (uus)$, $\Xi^- = (dss)$, etc.), and π^+, K^-, $\bar{K}^0$ and other mesons are bound states of a quark and an antiquark ($\pi^+ = (u\bar{d})$, $K^- = (s\bar{u})$, $\bar{K}^0 = (d\bar{s})$, etc.).

One of the first arguments in favor of the quark structure of the hadrons was obtained from the study of the weak decays of strange particles. In expression (78) for the charged current enter the fields of protons and neutrons. If a proton and a neutron are bound states of quarks it is natural to assume that the *fundamental weak interaction is the interaction of charged leptons, neutrinos, and quarks*.

Let us build hadronic charged currents from the quark fields. The current (78) changes the electric charge by one. If we accept the Feynman–Gell-Mann, Marshak–Sudarshan prescription (the left-handed components of the fermion fields enter into the weak current), there are only two possibilities to build such currents from the fields of u, d, and s quarks:

$$j_\alpha^{\Delta S=0}(x) = 2\bar{u}_L(x)\gamma_\alpha d_L(x) \quad \text{and} \quad j_\alpha^{\Delta S=1}(x) = 2\bar{u}_L(x)\gamma_\alpha s_L(x). \tag{86}$$

The first current changes the charge by one and does not change the strangeness ($\Delta Q = 1$, $\Delta S = 0$). The second current changes the charge by one and the strangeness by one ($\Delta Q = 1$, $\Delta S = 1$). Thus, the matrix elements of these currents satisfy $|\Delta S| = 1$ and $\Delta Q = \Delta S$ rules.

In order to take into account the rule 3 (suppression of the decays with the change of the strangeness with respect to the decays in which the strangeness is not changed), N. Cabibbo [32] introduced a parameter (which is called the Cabibbo angle θ_C) and assumed that the hadronic charged current was the following combination of currents $j_\alpha^{\Delta S=0}$ and $j_\alpha^{\Delta S=1}$ *:

$$j_\alpha^{\text{Cabbibo}}(x) = 2(\cos\theta_C \bar{u}_L(x)\,\gamma_\alpha d_L(x) + \sin\theta_C \bar{u}_L(x)\,\gamma_\alpha s_L(x)). \quad (88)$$

It is obvious that the Cabibbo current can be written in the form

$$j_\alpha^{\text{Cabbibo}}(x) = \bar{u}_L(x)\gamma_\alpha d_L^{\text{mix}}(x), \quad (89)$$

where

$$d_L^{\text{mix}}(x) = \cos\theta_C d_L(x) + \sin\theta_C s_L(x) \quad (90)$$

is the «mixed» combination of $d_L(x)$ and $s_L(x)$ fields.

The total charged current took the form

$$j_\alpha(x) = 2(\bar{\nu}_{eL}(x)\,\gamma_\alpha e_L(x) + \bar{\nu}_{\mu L}(x)\,\gamma_\alpha \mu_L(x) + \bar{u}_L(x)\,\gamma_\alpha d_L^{\text{mix}}(x)). \quad (91)$$

As is seen from this expression, the lepton and quark terms have the same form and enter into the current with the same coefficients. However, there was asymmetry in the current (91): there are two lepton terms and one quark term. This asymmetry was connected with the fact that four leptons (e, ν_e, μ, ν_μ) and only three quarks (u, d, s) were known at that time.

12.2. Charmed Quark. Quark Mixing. Some years later it was found that the charged current (91) creates some problems. Namely, in the framework of gauge theories the current (91) generates neutral currents with $\Delta Q = 0$ and $|\Delta S| = 1$. Such a neutral current induces the decay

$$K^+ \to \pi^+ + \nu + \bar{\nu}, \quad (92)$$

with a decay rate which is many orders of magnitude larger than the upper bound obtained in experiments.

The solution of the problem was proposed in 1970 by Glashow, Illiopulos, and Maiani (GIM) [41]. They assumed that there existed a fourth «charmed» quark c with the charge $2/3$ and there was an additional term in the weak charged

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*Assuming a «weak universality», Cabibbo suggested that in the total hadronic current

$$j_\alpha^h(x) = a j_\alpha^{\Delta S=0}(x) + b j_\alpha^{\Delta S=1}(x) \quad (87)$$

the real coefficients a and b would satisfy the condition $a^2 + b^2 = 1$. From this condition it follows that $a = \cos\theta_C$ and $b = \sin\theta_C$. The Cabibbo paper was written before the quark hypothesis appeared. He assumed that the current which did not change strangeness and the current which changes the strangeness by one are the $1 + i2$ and $4 + i5$ components of the $SU(3)$ octet current. Cabibbo found that with the parameter he introduced it was possible to describe all data on the semileptonic decays of mesons and baryons. From analysis of the data he found that $\sin\theta_C \simeq 0.2$.

current in which enter the field of charmed quark $c_L(x)$ and the combination of $d_L(x)$ and $s_L(x)$ fields

$$s_L^{\mathrm{mix}}(x) = -\sin\theta_C d_L(x) + \cos\theta_C s_L(x),$$

which is orthogonal to the Cabibbo combination (90).

The total weak charged currents took the form

$$j_\alpha(x) = 2\Big(\bar{\nu}_{eL}(x)\,\gamma_\alpha e_L(x) + \bar{\nu}_{\mu L}(x)\,\gamma_\alpha \mu_L(x) + \\ + \bar{u}_L(x)\,\gamma_\alpha d_L^{\mathrm{mix}}(x) + \bar{c}_L(x)\,\gamma_\alpha s_L^{\mathrm{mix}}(x)\Big), \quad (93)$$

where

$$d_L^{\mathrm{mix}}(x) = \cos\theta_C d_L(x) + \sin\theta_C s_L(x), \\ s_L^{\mathrm{mix}}(x) = -\sin\theta_C d_L(x) + \cos\theta_C s_L(x). \quad (94)$$

Thus, the field of d and s quarks, which have the same charge $(-1/3)$ and differ in their masses, enter into the charged current (93) in the form of the orthogonal combinations $d_L^{\mathrm{mix}}(x)$ and $s_L^{\mathrm{mix}}(x)$ («*mixed form*»). The Cabibbo angle θ_C is the mixing angle.

With the additional c quark, the numbers of leptons and quarks are equal and there is a symmetry between lepton and quark terms in the current (93). It would be, however, a full lepton–quark symmetry of the charged current if the neutrino masses were different from zero and the fields of neutrinos with definite masses, like the fields of quarks, enter into the CC in a mixed form

$$\nu_{\mu L}(x) = \cos\theta\nu_{1L}(x) + \sin\theta\nu_{2L}(x), \\ \nu_{eL}(x) = -\sin\theta\nu_{1L}(x) + \cos\theta\nu_{2L}(x), \quad (95)$$

where $\nu_1(x)$ and $\nu_2(x)$ are the fields of neutrinos with masses m_1 and m_2, and θ is the neutrino mixing angle (generally different from θ_C).

We know now that mixing of quarks exists, neutrino masses are different from zero and neutrino mixing (in a more general form; see later) is confirmed by experiment. The lepton–quark symmetry arguments we presented above were early arguments in favor of the neutrino masses and mixing put forward in the seventies (see [42]).

If the c quark, a constituent of hadrons, exists, in this case must exist a new family of «charmed» particles. This prediction was perfectly confirmed by experiment. In 1974, the J/Ψ particles ($m_{J/\Psi} \simeq$ 3096.9 MeV), bound states of $(c\bar{c})$, were discovered. In 1976, $D^+ = (c\bar{d})$, $D^- = (\bar{c}d)$ ($m_{D^\pm} \simeq$ 1868.6 MeV), $D^0 = (c\bar{u})$, $\bar{D}^0 = (\bar{c}u)$ ($m_{D^0} \simeq$ 1864.8 MeV) were discovered.

Later many charmed bosons and baryons were found in experiment. All data obtained from the investigation of weak decays and neutrino reactions were in agreement with the current $\times$ current theory with the current given by (93).

Summarizing, with the idea of quarks, physics of elementary particles and, in particular, physics of the weak interaction and of the neutrino was changed. If the fundamental weak interaction is *the interaction of quarks and leptons*, the phenomenological rules $|\Delta S| = 1$ and $\Delta Q = \Delta S$, which were established for semileptonic decays of strange particles, have a natural explanation. The prediction of the charmed quark was motivated by the Cabibbo mixture of quarks, and the Cabibbo–GIM mixture of quarks implied a symmetry between the lepton and quark terms in the charged weak current. This symmetry was based on the fact that the number of lepton pairs $((\nu_e, e^-)$ and $(\nu_\mu, \mu^-))$ was equal to the number of quark pairs $((u, d)$ and $(c, s))$. Taking into account that fields of d_L and s_L quarks are mixed, it was natural to extend the lepton–quark symmetry of the charged current and to assume that neutrinos are also mixed. This implies the assumption that neutrinos have small, nonzero masses.

13. DISCOVERY OF THE THIRD CHARGED LEPTON τ. THE THIRD FAMILY OF LEPTONS AND QUARKS

We do not know why the muon, the particle which has the same interaction as the electron but with a mass 206.8 times larger than the electron mass, exists*. In such a situation it was natural to ask whether more heavier than μ (sequential) lepton(s) exist.

The answer to this question was obtained in experiments which were performed in 1975–1977 by M. Perl et al. at the e^+e^- collider in Stanford [43]. In these experiments, the third lepton $\tau^\pm$ was discovered**. The τ lepton decays into an electron (muon) and two neutrinos, pion(s) and neutrino, etc. Its mass $m_\tau = 1776.8$ MeV.

Let us combine a charged lepton, neutrino and quark fields in the following way:

1. $(\nu_e, e^-) \quad (u, d)$.
2. $(\nu_\mu, \mu^-) \quad (c, s)$.

In the first group (family, generation) enter the fields of the lightest leptons and quarks, and in the second family enter fields of heavier leptons and quarks***.

*The question which was put many years ago by Nobel Prize winner I. Rabi «Who ordered the μ-meson?» still has no answer. Now we can also ask, who ordered ν_μ, s, and c quarks...

**In 1995, M. Perl was awarded the Nobel Prize «for the discovery of the tau lepton».

***For quark masses we have: $m_u = 1.5$–3.3 MeV, $m_d = 3.5$–6.0 MeV, $m_s = 104^{+26}_{-34}$ MeV, $m_c = 1.27^{+0.07}_{-0.11}$ GeV.

The discovery of the τ could mean that there exists a third family of leptons and quarks. In this case, a third type of the neutrino ν_τ, which takes part in weak processes together with τ, and an additional pair of quarks (the top quark t with electric charge $2/3$ and bottom quark b with electric charge $-1/3$) must exist*.

All these expectations were perfectly confirmed by experiment. In 1977, Υ particles, a bound state of $(b-\bar{b})$, were discovered at the Fermilab ($m_\Upsilon \simeq$ 9460.3 MeV). Later $B^+ = (b\bar{u})$ ($m_{B^+} \simeq 5279.2$ MeV), $B^0 = (d\bar{b})$ ($m_{B^0} \simeq$ 5279.5 MeV) and other bottom bosons, $\Lambda_b^0 = (ubd)$ ($m_{\Lambda_b} \simeq 5629.2$ MeV) and other bottom baryons were detected and studied in many experiments. The mass of the b quark is equal to $m_b = 4.20^{+0.17}_{-0.07}$ GeV. In 1995, at the Fermilab the t quark was discovered. The t quark is the heaviest known elementary particle ($m_t = (171.2 \pm 2.1)$ GeV). The third type of neutrino ν_τ, the partner of the τ lepton, was observed in 2000 in an experiment performed by the DONUT Collaboration at Fermilab [44].

In this experiment, the production of τ in the process $\nu_\tau + (A,Z) \to \tau + \ldots$ was observed. At the energy of the experiment, the τ lepton decays, producing predominantly a single charged particle at an average distance of 2 mm from the production point. Nuclear emulsion was used to detect the τ production. A signature of the event in the emulsion was a track with a kink.

In the case of three generations the charged current takes the form

$$\begin{aligned} j_\alpha^{\mathrm{CC}}(x) = 2(&\bar{\nu}_{eL}(x)\,\gamma_\alpha e_L(x) + \bar{\nu}_{\mu L}(x)\,\gamma_\alpha \mu_L(x) + \bar{\nu}_{\tau L}(x)\,\gamma_\alpha \tau_L(x) + \\ &+ \bar{u}_L(x)\,\gamma_\alpha d_L^{\mathrm{mix}}(x) + \bar{c}_L(x)\,\gamma_\alpha s_L^{\mathrm{mix}}(x) + \bar{t}_L(x)\,\gamma_\alpha b_L^{\mathrm{mix}}(x)). \end{aligned} \quad (96)$$

The Cabibbo–GIM mixing of quarks (94) was generalized for the case of three families of quarks by Kobayashi and Maskawa in 1973 [45]. They assumed that «mixed» fields $d_L^{\mathrm{mix}}(x), s_L^{\mathrm{mix}}(x), b_L^{\mathrm{mix}}(x)$ were connected with the left-handed components of the fields of d, s, and b quarks by the unitary transformation:

$$\begin{gathered} d_L^{\mathrm{mix}}(x) = \sum_{q=u,s,b} V_{uq} q_L(x), \quad s_L^{\mathrm{mix}}(x) = \sum_{q=u,s,b} V_{cq} q_L(x), \\ b_L^{\mathrm{mix}}(x) = \sum_{q=u,s,b} V_{tq}\; q_L(x). \end{gathered} \quad (97)$$

The unitary 3×3 matrix V is called the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix. The matrix V is characterized by three mixing angles and one

*At the time when the τ lepton was discovered, the Standard Model of the electroweak interaction existed which we will discuss later. According to this theory the existence of the τ requires the existence of ν_τ, t, b.

phase which is responsible for CP violation*. On the basis of the lepton–quark symmetry, it was natural to assume that the neutrino fields $\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$ were also mixed (see [42]):

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\ \nu_{iL}(x), \quad l = e, \mu, \tau. \tag{98}$$

Here U is the unitary 3×3 neutrino mixing matrix.

In the theory with the intermediate vector boson $W^{\pm}$, the Lagrangian of the CC weak interaction has the form

$$\mathcal{L}_I^{\mathrm{CC}}(x) = -\frac{g}{2\sqrt{2}}\ j_\alpha^{\mathrm{CC}}(x) W^\alpha(x) + \mathrm{h.c.}, \tag{99}$$

where the charged current $j_\alpha^{\mathrm{CC}}(x)$ is given by expression (96).

14. NUMBER OF FAMILIES OF QUARKS AND LEPTONS

How many families of quarks and leptons exist in nature? The answer to this fundamental question was obtained in experiments made at SLC (Stanford) and LEP (CERN). In these experiments the width of the decay

$$Z^0 \to \nu_l + \bar{\nu}_l, \quad l = e, \mu, \tau, \ldots \tag{100}$$

was measured. The Z^0 boson has a mass $m_Z = (91.1876 \pm 0.0021)$ GeV. Different decay modes of the Z^0 boson ($Z^0 \to l^+ + l^-$ $(l = e, \mu, \tau)$, $Z^0 \to$ hadrons) were investigated in detail at (e^+e^-) colliders.

Neglecting small neutrino masses we have

$$\sum_l \Gamma(Z^0 \to \nu_l \bar{\nu}_l) = n_{\nu_f} \Gamma(Z^0 \to \nu\bar{\nu}), \tag{101}$$

where n_{ν_f} is the number of neutrino–antineutrino pairs, and $\Gamma(Z^0 \to \nu\bar{\nu})$ is the width of the decay of the Z^0 into a neutrino–antineutrino pair (this width is known from the Standard Model calculations).

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*Kobayashi and Maskawa showed that in the case of two generations of quarks it is impossible to explain CP violation which was observed in decays of neutral K mesons. This was a main motivation for the assumption of the existence of the third generation of quarks (before the τ lepton was discovered). In 2008, Kobayashi and Maskawa were awarded the Nobel Prize «for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature».

From (101), we find the following relation:

$$n_{\nu_f} = \frac{\sum\limits_l \Gamma(Z^0 \to \nu_l \bar{\nu}_l)}{\Gamma(Z^0 \to l\bar{l})} \left(\frac{\Gamma(Z^0 \to l\bar{l})}{\Gamma(Z^0 \to \nu\bar{\nu})} \right)_{\text{SM}}. \tag{102}$$

The first ratio is measured in experiments. The second ratio is known from the SM calculations $\left(\left(\frac{\Gamma(Z^0 \to \nu\bar{\nu})}{\Gamma(Z^0 \to l\bar{l})} \right)_{\text{SM}} = 1.991 \pm 0.001 \right)$.

From the data of four LEP experiments it was found [40]

$$n_{\nu_f} = 2.984 \pm 0.008. \tag{103}$$

Thus, it was established that the number of different types of neutrinos was equal to three (only ν_e, ν_μ, ν_τ exist in nature). Each family of leptons and quarks has its own neutrino. We conclude that *only three families of leptons and quarks exist in nature**.

15. UNIFIED THEORY OF WEAK AND ELECTROMAGNETIC INTERACTIONS. THE STANDARD MODEL

In 1967–1968, S. Weinberg [46] and A. Salam [47] proposed a new theory which unified the weak and electromagnetic interactions into one electroweak interaction. They built such a theory for the electron neutrino and the electron. Later, all three families of leptons and quarks were included in the theory. It is called the Standard Model (SM).

The Standard Model is one of the greatest achievements of particle physics of the XX century. It predicted a new class of the weak interaction (Neutral currents), the $W^\pm$ and Z^0 vector bosons and the masses of these particles, the existence of the third type of the neutrino ν_τ, the existence of the scalar Higgs boson, etc. All predictions of the Standard Model are in perfect agreement with existing experimental data. The search for the Higgs boson is one of the major aims of experiments at the LHC accelerator at CERN.

Neutrinos played an extremely important role in the establishment of the SM. In neutrino experiments, fundamental parameters of the theory were determined. Neutrinos played also an important role in the establishment of the quark structure of nucleons and its investigation.

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*From these data we cannot exclude, however, that there exist neutral leptons with masses larger than $m_Z/2$ which cannot be produced in decays of the Z^0 bosons. Thus, we cannot exclude from these experiments the existence of new families in which instead of neutrinos such heavy neutral leptons are present.

The V–A current $\times$ current theory of the weak interaction, which we discussed in the previous sections, has been a very successful theory. It allowed one to describe all experimental data, which existed in the sixties. However, the current $\times$ current theory and also the theory with the intermediate $W^{\pm}$ vector boson were unrenormalizable theories. The infinities at the higher orders of the perturbation theory could not be excluded in these theories by the renormalization of the masses and other physical parameters.

This was the main reason why, in spite of phenomenological success, the current $\times$ current theory of the weak interaction and the theory with the intermediate vector boson were not considered as satisfactory ones.

The Standard Model was born in the sixties in an attempt to build a renormalizable theory of the weak interaction. The only renormalizable physical theory, that was known at that time, was quantum electrodynamics. The renormalizable theory of the weak interaction was built in the framework of *the unification of the weak and electromagnetic interactions*. This theory was proposed by Weinberg [46] and Salam [47]. The same theory with the unification of the weak and electromagnetic interactions but without the mechanism of the spontaneous symmetry breaking (see later) was proposed by Glashow in 1961 [48]. Weinberg and Salam suggested that the SM would be a renormalizable theory but they did not prove that. The renormalizability of the SM was proved in 1971 by 't Hooft [49].

We will briefly discuss now the Standard Model of the electroweak interactions. The Standard Model is based on

1) phenomenological $V-A$ theory of the weak interaction;

2) local gauge $SU(2)\times U(1)$ invariance of the Lagrangian of fields of massless quarks, leptons, and vector bosons;

3) minimal interaction of fermions and vector bosons;

4) spontaneous breaking of symmetry and the Higgs mechanism of the generation of masses of quarks and leptons;

5) unification of the weak and electromagnetic interactions into one electroweak interaction.

The minimal group which ensures the CC interaction (99) of leptons and quarks with $W^{\pm}$ bosons is the local $SU(2)$ group. We assume that the left-handed components of the fields of quarks and leptons form doublets*

$$\psi_{1L} = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix}, \quad \psi_{2L} = \begin{pmatrix} c'_L \\ s'_L \end{pmatrix}, \quad \psi_{3L}(x) = \begin{pmatrix} t'_L \\ b'_L \end{pmatrix} \tag{104}$$

and

$$\psi_{eL} = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix}, \quad \psi_{\mu L} = \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix}, \quad \psi_{\tau L} = \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix}. \tag{105}$$

*The meaning of primes will be clear later.

We assume also that the the right-handed components of the fields of quark and leptons are singlets.

From the local $SU(2)$ invariance it follows that the minimal interaction includes only the left-handed components of quark and lepton fields and has the form

$$\mathcal{L}_I(x) = \left(-\frac{g}{2\sqrt{2}}\, j^{\rm CC}_\alpha(x) W^\alpha(x) + \text{h.c.}\right) - g j^3_\alpha(x) A^{\alpha 3}(x). \qquad (106)$$

Here

$$j^{\rm CC}_\alpha = 2(\bar{u}'_L \gamma_\alpha d'_L + \bar{c}'_L \gamma_\alpha s'_L + \bar{t}'_L \gamma_\alpha b'_L) + 2 \sum_{l=e,\mu,\tau} \bar{\nu}'_{lL} \gamma_\alpha l'_L \qquad (107)$$

is the charged current of the quarks and leptons,

$$j^3_\alpha = \sum_{a=1,2,3} \bar{\psi}_{aL} \frac{1}{3} \tau_3 \gamma_\alpha \psi_{aL} + \sum_{l=e,\mu,\tau} \bar{\psi}_{lL} \frac{1}{3} \tau_3 \gamma_\alpha \psi_{lL} \qquad (108)$$

(τ_3 is the third Pauli matrix) and g is a constant. The field $A^{\alpha 3}(x)$ is the field of neutral vector particles.

We would like to unify the weak and electromagnetic interactions. The first term of (106) is the Lagrangian of the CC weak interaction. However, the second term violates parity and cannot be identified with the Lagrangian of the electromagnetic interaction.

In order to unify the weak interaction (which maximally violates parity) and the electromagnetic interactions (which conserve parity) in one electroweak interaction, we must enlarge the symmetry group. The Standard Model is based on the local gauge $SU(2) \times U(1)$ invariance. This is a minimal enlargement of the $SU(2)$ group which generates the charge current weak interaction.

The $U(1)$ group is the group of the hypercharge Y which is determined by the Gell-Mann–Nishigima relation

$$Q = I_3 + \frac{1}{2} Y, \qquad (109)$$

where Q is the electric charge (in the unit of the proton charge) and I_3 is the third component of the isotopic spin of the $SU(2)$ group.

The invariance under the additional $U(1)$ group can be realized if, in addition to the vector W^α field (field of vector $W^\pm$ bosons) and the field of neutral vector particles $A^{\alpha 3}$, the *field of neutral vector particles* B^α *exists*.

The Lagrangian of the minimal interaction takes the form

$$\mathcal{L}_I(x) = \left(-\frac{g}{2\sqrt{2}}\, j^{\rm CC}_\alpha(x) W^\alpha(x) + \text{h.c.}\right) + \mathcal{L}^0_I(x). \qquad (110)$$

Here

$$\mathcal{L}_I^0(x) = -g j_\alpha^3(x) A^{\alpha 3}(x) - g' (j_\alpha^{\rm EM}(x) - j_\alpha^3(x))\, B^\alpha(x) \qquad (111)$$

is the Lagrangian of interaction of quarks and neutral vector particles, and

$$j_\alpha^{\rm EM} = \left(\frac{2}{3}\right) \sum_{q=u,c,t} \bar{q}' \gamma_\alpha q' + \left(-\frac{1}{3}\right) \sum_{q=d,s,b} \bar{q}' \gamma_\alpha q' + (-1) \sum_{l=e,\mu,\tau} \bar{l}' \gamma_\alpha l \qquad (112)$$

is the electromagnetic current of the quarks and leptons, and g' is a constant connected with the $U(1)$ group.

Up to now we considered fields of massless particles. The Standard Model is based on *the Higgs mechanism of the generation of masses* of $W^\pm$ bosons, quarks, and leptons.

We will assume that in our system of fields there are scalar complex charged and neutral Higgs fields (ϕ_+ and ϕ_0) and that these fields form the $SU(2)$ doublet

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}. \qquad (113)$$

The Lagrangian of the Higgs field is chosen in such a way that the energy of the field reaches a minimum when the value of the field is different from zero. This means that the Higgs vacuum is not an empty state. Moreover, due to the symmetry there are many (infinite) degenerate vacuum states. If we choose a definite vacuum field, say,

$$\phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \qquad (114)$$

we will violate the symmetry (v is a constant). Such a violation is called spontaneous.

Before spontaneous violation of the symmetry we had a massless complex (charged) W_α vector field and two massless real (neutral) vector fields A_α^3 and B_α. After spontaneous violation of the symmetry, the masses of the $W^\pm$ and Z^0 bosons are generated. The field of Z^0 bosons is the following combination of A_α^3 and B_α fields:

$$Z_\alpha = \frac{g}{\sqrt{g^2 + g'^2}} A_\alpha^3 - \frac{g'}{\sqrt{g^2 + g'^2}} B_\alpha. \qquad (115)$$

For the masses of the $W^\pm$ and Z^0 bosons we have the following relations:

$$m_W^2 = \frac{1}{4} g^2 v^2, \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2. \qquad (116)$$

After spontaneous violation of the symmetry, the mass of particles, quanta of the field

$$A_\alpha = \frac{g'}{\sqrt{g^2 + g'^2}} A^3_\alpha + \frac{g}{\sqrt{g^2 + g'^2}} B_\alpha, \tag{117}$$

which is an orthogonal to Z_α, remain equal to zero.

Let us introduce a weak (Weinberg) angle θ_W by the relation

$$\frac{g'}{g} = \tan\theta_W. \tag{118}$$

We have

$$A_\alpha = \cos\theta_W B_\alpha + \sin\theta_W A^3_\alpha, \quad Z_\alpha = -\sin\theta_W B_\alpha + \cos\theta_W A^3_\alpha. \tag{119}$$

From (111) and (119) we find the following expression for the Lagrangian of interaction of quarks and leptons with neutral vector particles:

$$\mathcal{L}^0_I = -\frac{g}{2\cos\theta_W} j^{\rm NC}_\alpha Z^\alpha - g\sin\theta_W j^{\rm EM}_\alpha, A^\alpha, \tag{120}$$

where

$$j^{\rm NC}_\alpha = 2j^3_\alpha - 2\sin^2\theta_W j^{\rm EM}_\alpha. \tag{121}$$

From (120) we can draw the following important conclusions:

1. The second term of (120) is the Lagrangian of the electromagnetic interaction of quarks and charged leptons if *the constants* g *and* $\sin\theta_W$ *satisfy the following (unification) condition*:

$$g\sin\theta_W = e, \tag{122}$$

where e is the proton charge.

2. The unification of the weak and electromagnetic interaction is possible if in addition to the charged vector $W^\pm$ boson, there exists a neutral vector Z^0 boson with a mass larger than the mass of the $W^\pm$ boson (see relation (116)). As a consequence of the unification *a new (neutral current) interaction of quarks, charged leptons, and neutrinos with the* Z^0 *boson appears*.

The Fermi constant is given by the relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m^2_W}. \tag{123}$$

From this relation and (116) it follows that the parameter v (vacuum expectation value), which characterizes the scale of the $SU(2)\times U(1)$ symmetry breaking, is given by

$$v = (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV}. \tag{124}$$

From the unification condition (122) and relations (116), it follows that the masses of the W and Z bosons are given by the following relations:

$$m_W = \left(\frac{\pi\alpha}{\sqrt{2}, G_F}\right)^{1/2} \frac{1}{\sin\theta_W}, \quad m_Z = \left(\frac{\pi\alpha}{\sqrt{2}G_F}\right)^{1/2} \frac{1}{\sin\theta_W \cos\theta_W}, \quad (125)$$

where $\alpha = e^2/4\pi$ is the fine structure constant.

The value of the parameter $\sin\theta_W$ can be determined from the study of neutral current (NC) processes. Thus, *the Standard Model predicts the masses of the $W^{\pm}$ and Z^0 bosons*. This prediction is in perfect agreement with experiment (see later).

We will now briefly discuss a much less predictive part of the SM, the Higgs mechanism of the generation of masses of quarks and leptons. In order to generate the masses of fermions, we need to assume that the total Lagrangian of the system contains an $SU(2) \times U(1)$ invariant Lagrangian of a Yukawa interaction of fermions and Higgs boson. For example, the Lagrangian

$$\mathcal{L}_Y^{\rm down}(x) = -\frac{\sqrt{2}}{v} \sum_{a=1,\ldots q'_R=d'_R,\ldots} \bar{\psi}_{aL}(x)\, M_{a;q}^{\rm down} q'_R(x)\, \phi(x) + \text{h.c.}, \quad (126)$$

after spontaneous violation of the symmetry, generates masses of the d, s, and b quarks. The matrix $M^{\rm down}$ in (126) is a complex 3×3 matrix. The Standard Model does not put any constraints on this matrix. After the diagonalization of the matrix $M^{\rm down}$ and another similar matrix $M^{\rm up}$ we find

$$q'_L = \sum_{q=d,\ldots} V_{q'_L q_L}^{\rm down} q_L (q'_L = d'_L, \ldots), \quad q'_L = \sum_{q=u,\ldots} V_{q'_L q_L}^{\rm up} q_L (q'_L = u'_L, \ldots). \quad (127)$$

Here $V^{\rm down}$ and $V^{\rm up}$ are the unitary 3×3 matrices, and q_L is the left-handed component of the field of q quark with mass m_q ($q = u, c, t, d, s, b$).

For the lepton fields we have

$$l'_L = \sum_{l=e,\mu,\tau} V_{l'_L l_L}^{\rm lep} l_L (l'_L = e'_L, \mu'_L, \tau'_L), \quad (128)$$

where l_L is the left-handed component of the field of the lepton l with mass m_l ($l = e, \mu, \tau$).

Similar relations connect the primed right-handed components of the fields of quarks and leptons and the right-handed components of fields of quarks and leptons with definite masses.

The matrices $V^{\rm down}$ and $V^{\rm up}$ are unitary and in general *different*. From (107), (127), and (128) we obtain the following expression for the charged current:

$$j_\alpha^{\rm CC} = 2(\bar{u}_L \gamma_\alpha d_L^{\rm mix} + \bar{c}_L \gamma_\alpha s_L^{\rm mix} + \bar{t}_L \gamma_\alpha b_L^{\rm mix}) + 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_\alpha l_L. \quad (129)$$

Here

$$q_L^{\text{mix}} = \sum_{q=d,s,b} V_{q_L^{\text{mix}} q_L} q_L, \quad q_L^{\text{mix}} = d_L^{\text{mix}}, s_L^{\text{mix}}, b_L^{\text{mix}}, \tag{130}$$

where

$$V = V^{L,\text{up}\dagger} V^{L,\text{down}} \tag{131}$$

is the 3×3 *unitary mixing Cabibbo–Kobayashi–Maskawa mixing matrix* and

$$\nu_{lL} = \sum_{l_1=e,\mu,\tau} (V^{\text{lep}})^{\dagger}_{l_L l_{1L}} \nu'_{l_1 L}. \tag{132}$$

Taking into account the unitarity of the matrices which connect $L(R)$-components of primed fields with the $L(R)$-components of the fields of particles with definite masses, it is easy to show that in the neutral current and in the electromagnetic current we must change primed fields of quarks, leptons, and neutrinos by the corresponding physical nonprimed fields. This means that the NC of the SM does not change strangeness, charm, etc.

From the consideration of the Higgs mechanism for quarks and charged leptons we could make the following conclusions:

1. The Higgs mechanism provides a *natural framework for the unitary CKM mixing of quarks* in the charged current. It leaves electromagnetic and neutral currents diagonal over fields.

2. However, the Standard Model cannot predict masses of quarks and charged leptons and CKM mixing angles. In the SM these quantities are parameters which have to be determined from experimental data.

What about neutrino masses and mixing in the Standard Model? Many people claim that in the Standard Model neutrinos are massless two-component particles. *If we assume that there are no right-handed fields* ν'_{lR}, in this case the corresponding Yukawa interaction cannot be built and flavor neutrinos ν_{lL} will be massless two-component particles. But this is equivalent to assume from the very beginning that neutrinos are the Landau, Lee, and Yang and Salam two-component massless particles*.

We can, however, generate neutrino masses by the standard Higgs mechanism in the same way as masses of quarks and charged leptons were generated. In this case neutrino masses would be proportional to the parameter v, and we could expect that they are of the same order of magnitude as the masses of other fermions, partners of neutrinos.

*Originally the Standard Model was built with massless two-component neutrinos. It was natural in 1967 for the authors of the Standard Model to make this simplest assumption.

Let us consider for illustration the masses of the quarks and leptons of the third family. We have

$$\begin{aligned} &m_t \simeq 1.7 \cdot 10^2 \text{ GeV}, \quad && m_b \simeq 4.7 \text{ GeV}, \\ &m_3 \leqslant 2.3 \cdot 10^{-9} \text{ GeV}, \quad && m_\tau \simeq 1.8 \text{ GeV}. \end{aligned} \tag{133}$$

The masses of t, b, and τ differ by not more than two orders of magnitude. The neutrino masses differ from the masses of quarks and charged leptons by (at least) nine–eleven orders of magnitude. *It is very unlikely that the masses of quarks, leptons, and neutrinos are of the same Higgs origin. For neutrino masses a new (or additional) mechanism is needed.* A possible mechanism of the generation of small neutrino masses will be discussed briefly later.

Summarizing, the unified theory of weak and electromagnetic interactions (Standard Model) is a theory of interaction of neutrinos, charged leptons, and quarks with the $W^{\pm}, Z^0$ bosons and γ quanta in a wide range of energies. This theory was perfectly confirmed by numerous experiments including very precise LEP (CERN) experiments.

The SM is based on the spontaneously broken local gauge $SU(2)\times U(1)$ symmetry and it is built in such a way to include the charged current of the phenomenological $V-A$ theory (assuming the existence of quarks) and the electromagnetic interaction of charged leptons and quarks.

The SM predicts the existence of the $W^{\pm}$ and Z^0 bosons and their masses. This prediction was perfectly confirmed by experiment.

Taking into account radiative corrections, for masses and decay widths of the $W^{\pm}$ and Z^0 bosons from the Standard Model it was obtained [40]

$$(m_W)_{\rm SM} = (80.420 \pm 0.031) \text{ GeV}, \qquad (\Gamma_W)_{\rm SM} = (2.0910 \pm 0.0007) \text{ GeV}, \tag{134}$$

$$(m_Z)_{\rm SM} = (91.1874 \pm 0.0021) \text{ GeV}, \quad (\Gamma_Z)_{\rm SM} = (2.4954 \pm 0.0009) \text{ GeV}. \tag{135}$$

These values are in agrement with the measured masses and decay widths:

$$m_W = (80.384 \pm 0.014) \text{ GeV}, \qquad \Gamma_W = (2.085 \pm 0.042) \text{ GeV}, \tag{136}$$

$$m_Z = (91.1876 \pm 0.0021) \text{ GeV}, \quad \Gamma_W = (2.4952 \pm 0.0023) \text{ GeV}. \tag{137}$$

The Standard Model predicts a new class of weak interactions: neutral currents. Numerous experimental data perfectly confirm this prediction. The standard neutral current is diagonal in quark, charged lepton, and neutrino fields and is characterized by $\sin^2\theta_W$. The values of this parameter determined from different data (e^+–e^-, deep-inelastic neutrino–nucleon scattering, P-odd asymmetry in deep-inelastic electron–nucleon scattering, etc.) are compatible with each other.

From the fit of all data it was found

$$\sin^2\theta_W = 0.23108 \pm 0.00005. \tag{138}$$

The Standard Model provides a natural framework for quark mixing. However, the SM cannot predict the masses of quarks and charged leptons and the CKM mixing angles.

Neutrino masses are not of the Standard Model Higgs origin. For the generation of small neutrino masses and neutrino mixing a new (or additional) mechanism is needed.

16. NEUTRINO AND DISCOVERY OF NEUTRAL CURRENTS

Neutral currents were discovered in 1973 at CERN. This was the first confirmation of the unified theory of weak and electromagnetic interactions.

Due to the exchange of the W boson between lepton and quark vertices muon neutrinos (antineutrino) produce μ^- (μ^+) in the inclusive processes

$$\nu_\mu + N \to \mu^- + X, \quad \bar{\nu}_\mu + N \to \mu^+ + X. \tag{139}$$

Here X means any possible final state of hadrons. If $Q^2 \ll m_W^2$ (Q^2 is the square of the momentum transfer), the effective Hamiltonian of the processes (139) has the form

$$\mathcal{H}^{\rm CC} = \frac{G_F}{\sqrt{2}} 2\bar{\mu}_L\gamma^\alpha\nu_{\mu L} j_\alpha^{\rm CC} + \text{h.c.}, \tag{140}$$

where $j_\alpha^{\rm CC}$ is the quark charged current, and G_F is the Fermi constant*. In the seventies the CC processes (139) were intensively studied in neutrino experiments at the Fermilab and CERN. These experiments were very important for the establishment of the quark structure of the nucleon.

If in addition to the CC interaction there exists also the NC interaction, in this case the processes

$$\nu_\mu + N \to \nu_\mu + X, \quad \bar{\nu}_\mu + N \to \bar{\nu}_\mu + X \tag{141}$$

induced by the exchange of the Z boson between neutrino and quark vertices become possible. The signature of such processes is hadrons in the final state (no muons). The effective SM Hamiltonian of the processes (141) has the form

$$\mathcal{H}^{\rm NC} = \frac{G_F}{\sqrt{2}} 2\bar{\nu}_{\mu L}\gamma^\alpha\nu_{\mu L} j_\alpha^{\rm NC} + \text{h.c.}, \tag{142}$$

*The Fermi constant has the following numerical value: $G_F = 1.166364(5) \cdot 10^{-5}$ GeV^{-2}.

where $j_\alpha^{\rm NC}$ is the neutral current of quarks. Thus, in the framework of the Standard Model, CC and NC interactions are characterized by the same Fermi constant. We can expect that the cross sections of the processes (139) and (141) are comparable.

The processes (141) were observed in the large bubble chamber Gargamelle at CERN in 1973. The bubble chamber Gargamelle (4.8 m long, 2 m in diameter, filled with 18 t of liquid freon) was built specially for the study of neutrino processes. At the first meeting of the collaboration in Milan (1968), where the neutrino program was discussed, the search for NC induced processes had the eighth priority. The main aim of the experiment was investigation of the structure of a nucleon through the observation of CC processes (139). The Glashow–Weinberg–Salam model was considered at that time as only one of the possibilities.

In the beginning of 1973, one event of the NC process

$$\nu_\mu + e \to \nu_\mu + e \tag{143}$$

was found in the Gargamelle chamber. Taking into account that the background for (143) is very small (less than 1%), this one event triggered the intensive search for hadronic NC-induced processes (139) which have cross sections about two orders of magnitude larger than the cross section of the NC leptonic process (143).

The main problem in the search for hadronic NC processes was a background from neutrons produced in CC neutrino interactions in the surrounding materials. The proof of the neutrino origin of NC hadronic events followed from the fact that the ratio of selected NC events and CC events did not depend on the longitudinal and radial distances, whereas hadronic events of neutron origin would have shown strong dependence on the distance. Obviously, the large size of the bubble chamber was very important for the detection of NC events. In the first Gargamelle publication [51] for the ratio R of the number of NC and CC events, the following values were given:

$$R_\nu = 0.21 \pm 0.03, \quad R_{\bar\nu} = 0.45 \pm 0.09. \tag{144}$$

In the beginning, these data were confirmed by the HPWF collaboration working at the Fermilab. However, later the HPWF collaboration modified their detector and for the ratio R_ν they announced a result compatible with zero ($R_\nu = 0.05 \pm 0.05$). For about one year, many people at CERN and other places did not believe in the correctness of the Gargamelle result.

By the middle of 1974, the Gargamelle collaboration doubled their statistics and confirmed their original result. The HPWF collaboration made a new measurement and also confirmed the Gargamelle finding. This result was confirmed by other Fermilab neutrino experiments. *The discovery of the neutral currents was firmly established.*

The eighties and nineties were years of intensive study of different NC-induced processes. The effects of neutral currents were observed in the experiments on the measurement of the asymmetry in the deep inelastic scattering of polarized electrons (and muons) on an unpolarized nucleon target and on the study of atomic processes*, in experiments on the study of $\nu_\mu(\bar\nu_\mu) + e \to \nu_\mu(\bar\nu_\mu) + e$ processes, etc. All these data were in perfect agreement with the SM. The values of the parameter $\sin^2\theta_W$ obtained from the data of different experiments are in good agreement with each other. From the measurement of the cross sections of NC neutrino reactions (141) and CC neutrino reactions (139), it was obtained

$$\sin^2\theta_W = 0.2335 \pm 0.0018. \quad (145)$$

Summarizing, the discovery of the neutral currents in the Gargamelle neutrino experiment at CERN in 1973 opened a new era in the physics of the weak and electromagnetic interactions. The Gargamelle result was the first confirmation of the approach based on the idea of the unification of these interactions.

At the beginning of the seventies, the Glashow–Weinberg–Salam model was considered as a correct strategy and one of the possible models. However, after the Gargamelle discovery of NC neutrino processes, detailed investigations of effects of NC in deep inelastic electron(muon)–nucleon scattering and in atomic transitions, the discovery of $W^\pm$ and Z^0 bosons and precise measurement of their masses, high precision studies of different electroweak processes at the e^+e^- colliders SLC (Stanford) and LEP (CERN) fully confirmed the minimal Glashow–Weinberg–Salam model. This model became the Standard Model of the weak and electromagnetic interactions. It perfectly describes the existing electroweak data.

Up to now, however, there is no proof of the correctness of the standard Higgs mechanism. The search for the scalar Higgs boson and the investigation of the mechanism of the symmetry breaking are first priority problems for experiments at the LHC collider at CERN.

17. NEUTRINO MASSES, MIXING AND OSCILLATIONS

17.1. Pontecorvo's Ideas of Neutrino Masses and Oscillations. The first idea of neutrino masses and oscillations was suggested in 1957–1958 by B. Pontecorvo [1, 2]. At that time, the Gell-Mann and Pais [52] theory of $K^0 \rightleftarrows \bar K^0$ mixing and oscillations was confirmed by experiment. Pontecorvo was fascinated by the idea of particle mixing and oscillations and thought about a possibility of

*In such experiments, the effect of interference of diagrams with the exchange of γ and Z was revealed.

oscillations in the lepton world. In such a way, he came to the idea of neutrino oscillations. This was a very courageous idea at the time when there was a common opinion that the neutrino is a two-component massless particle.

Before discussing neutrino oscillations, let us briefly consider ($K^0-\bar{K}^0$) mixing and oscillations which were studied in detail in many experiments. K^0 and $\bar{K}^0$ are particles with strangeness equal to $+1$ and -1, respectively. They are produced in hadronic processes ($\pi^- + p \to K^0 + \Lambda$, etc.) in which the strangeness is conserved. Assuming CPT invariance for the states of K^0 and $\bar{K}^0$ we have

$$H_0|K^0\rangle = m|K^0\rangle, \quad H_0|\bar{K}^0\rangle = m|\bar{K}^0\rangle. \tag{146}$$

Here H_0 is the sum of the free Hamiltonian and Hamiltonians of the strong and electromagnetic interactions, $|K^0\rangle$ and $|\bar{K}^0\rangle$ are states of K^0 and $\bar{K}^0$ (in the rest frame) and m is their mass. The arbitrary phases of $|K^0\rangle$ and $|\bar{K}^0\rangle$ states can be chosen in such a way that

$$|\bar{K}^0\rangle = CP|K^0\rangle. \tag{147}$$

The weak interaction does not conserve strangeness. Eigenstates of the total Hamiltonian, which includes the Hamiltonian of the weak interaction, are superpositions

$$|K^0_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle, \quad |K^0_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle. \tag{148}$$

From the normalization condition of the states $|K^0_{S,L}\rangle$, it follows that the coefficients p and q satisfy the condition $|p|^2 + |q|^2 = 1$. From (148), we find the following relations:

$$|K^0\rangle = \frac{1}{2p}(|K^0_S\rangle + |\bar{K}^0_L\rangle), \quad |\bar{K}^0\rangle = \frac{1}{2q}(|K^0_S\rangle - |\bar{K}^0_L\rangle). \tag{149}$$

Thus, the states of particles with definite strangeness K^0 and $\bar{K}^0$ are superpositions («mixtures») of the states of particles with definite masses and widths $K^0_{S,L}$, eigenstates of the total effective non-Hermitian Hamiltonian H:

$$H|K^0_{S,L}\rangle = \lambda_{S,L}|K^0_{S,L}\rangle. \tag{150}$$

Here

$$\lambda_{S,L} = m_{S,L} - \frac{i}{2}\Gamma_{S,L}, \tag{151}$$

where $m_{S,L}$ and $\Gamma_{S,L}$ are the mass and the total width of K^0_S (K^0_L). From experimental data it follows that the lifetimes of short-lived kaon K^0_S and long-lived kaon K^0_L are given by

$$\tau_S = \frac{1}{\Gamma_S} = (0.8953 \pm 0.0005) \cdot 10^{-10}\ \text{s}, \quad \tau_L = \frac{1}{\Gamma_L} = (5.116 \pm 0.021) \cdot 10^{-8}\ \text{s}. \tag{152}$$

States with definite masses and widths are evolved (in proper time t) as follows:

$$|K_S^0\rangle_t = \mathrm{e}^{-i\lambda_S t}\,|K_S^0\rangle, \quad |K_L^0\rangle_t = \mathrm{e}^{-i\lambda_L t}\,|K_L^0\rangle. \tag{153}$$

We will neglect small effects of CP violation. In this case, we have $p = q = 1/\sqrt{2}$ and

$$|K_S^0\rangle \simeq |K_1^0\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle), \quad |K_L^0\rangle \simeq |K_2^0\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle). \tag{154}$$

Thus, in the case of CP-conservation, we have the following mixing relations:

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1^0\rangle + |K_2^0\rangle), \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}(|K_1^0\rangle - |K_2^0\rangle). \tag{155}$$

Let us consider the evolution in time of a state $|K^0\rangle$. From (153) and (155) we find

$$|K^0\rangle_t = \frac{1}{\sqrt{2}}(\mathrm{e}^{-i\lambda_S t}\,|K_1^0\rangle + \mathrm{e}^{-i\lambda_L t}\,|K_2^0\rangle) = g_+(t)|K^0\rangle + g_-(t)|\bar{K}^0\rangle, \tag{156}$$

where

$$g_+(t) = \frac{1}{2}(\mathrm{e}^{-i\lambda_S t} + \mathrm{e}^{-i\lambda_L t}), \quad g_-(t) = \frac{1}{2}(\mathrm{e}^{-i\lambda_S t} - \mathrm{e}^{-i\lambda_L t}). \tag{157}$$

The state $|\bar{K}^0\rangle_t$ depends on time t in a similar way

$$|\bar{K}^0\rangle_t = \frac{1}{\sqrt{2}}(\mathrm{e}^{-i\lambda_S t}\,|K_1^0\rangle - \mathrm{e}^{-i\lambda_L t}\,|K_2^0\rangle) = g_+(t)|\bar{K}^0\rangle + g_-(t)|K^0\rangle. \tag{158}$$

Thus, because of the mixing (149) at $t > 0$ *the states* $|K^0\rangle_t$ *and* $|\bar{K}^0\rangle_t$ *are superpositions of* $|K^0\rangle$ *and* $|\bar{K}^0\rangle$. The probability of the transition $K^0 \to \bar{K}^0$ ($\bar{K}^0 \to K^0$) during the time t is given by the expression

$$P(K^0 \to \bar{K}^0; t) = |g_-(t)|^2 = \frac{1}{4}(\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - 2\mathrm{e}^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t}\cos\Delta m t), \tag{159}$$

where $\Delta m = m_L - m_S$. From (159) it follows that the oscillating term of the probability is determined by the mass difference of the K_L^0 and K_S^0 mesons. Let us stress that this term originates from the interference of the exponents in (158).

The study of the t-dependence of the probability $P(K^0 \to \bar{K}^0; t)$ in the region $\Delta m t \geqslant 1$ allows one to determine the mass difference Δm. From the analysis of the experimental data, it was found*

$$\Delta m = (3.483 \pm 0.006) \cdot 10^{-6}\ \text{eV}. \tag{160}$$

*This value is many orders of magnitude smaller than the masses of the neutral kaons $m_{K^0} = (497.614 \pm 0.022)$ MeV.

The measurement of such a small quantity became possible because of the interference nature of the $K^0 \to \bar{K}^0$ oscillations.

Let us now discuss Pontecorvo's idea of neutrino oscillations. Pontecorvo believed in the existence of symmetry between weak interaction of leptons and hadrons and he came first to the idea of muonium–antimuonum oscillations [1] which in the framework of the lepton–hadron symmetry are analogous to $K^0 \rightleftarrows \bar{K}^0$ oscillations (muonium is the bound state $(\mu^+ e^-)$ and antimuonum is the bound state of $(\mu^- e^+)$). In the paper [1], Pontecorvo also mentioned neutrino oscillations. This paper was written soon after the two-component theory of a massless neutrino was proposed and the neutrino helicity was measured in the Goldhaber et al. experiment. Only one type of neutrino was known at that time. According to the two-component neutrino theory, there were only two neutrino (antineutrino) states: ν_L and $\bar{\nu}_R$. Pontecorvo assumed that:

1. Neutrinos had small masses.

2. Lepton number was not conserved.

3. Additional neutrino states $\bar{\nu}_L$ and ν_R existed so that ν_L could be transferred into $\bar{\nu}_L$ and $\bar{\nu}_R$ could be transferred into ν_R.

Pontecorvo wrote in [1]: «If the two-component neutrino theory was not valid (which is hardly probable at present) and if the conservation law for neutrino charge did not take place, neutrino $\to$ antineutrino transitions in vacuum would be in principle possible».

A paper dedicated to the neutrino oscillations was published by B. Pontecorvo in 1958 [2]. At that time R. Davis was doing an experiment with reactor antineutrinos [35] with the aim to test the conservation of the lepton number L. Davis searched for the production of ^{37}Ar in the process

$$\bar{\nu} + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}, \qquad (161)$$

which is evidently forbidden if L is conserved. A rumor reached B. Pontecorvo that Davis had seen some events (161). B. Pontecorvo who had earlier been thinking about neutrino oscillations was very excited with a possibility of explaining Davis «events» by $\bar{\nu}_R \to \nu_R$ oscillations.

He wrote: «Recently, the question was discussed [1] whether there exist other *mixed* neutral particles beside the K^0 mesons, i.e., particles that differ from the corresponding antiparticles, with the transitions between particle and antiparticle states not being strictly forbidden. It was noted that the neutrino might be such a mixed particle, and consequently there existed a possibility of real neutrino $\rightleftarrows$ antineutrino transitions in vacuum, provided that lepton (neutrino) charge was not conserved. This means that the neutrino and antineutrino are *mixed* particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity».

So basically by analogy with the $K^0 - \bar{K}^0$ mixing (155), Pontecorvo assumed that

$$|\bar{\nu}_R\rangle = \frac{1}{\sqrt{2}}(|\nu_1\rangle + |\nu_2\rangle), \quad |\nu_R\rangle = \frac{1}{\sqrt{2}}(|\nu_1\rangle - |\nu_2\rangle), \tag{162}$$

where $|\nu_{1,2}\rangle$ are states of Majorana neutrinos $\nu_{1,2}$ with masses $m_{1,2}$.

Let us notice that if the lepton number L is violated, there is no way to distinguish a neutrino and an antineutrino: they are the same particles. A theory of such particles was proposed by E. Majorana [53].

In contrast to $K^0_{S,L}$ neutrinos, $\nu_{1,2}$ are stable particles*. From (162), we find (in the lab. system)

$$|\bar{\nu}_R\rangle_t = \frac{1}{\sqrt{2}}(\mathrm{e}^{-iE_1 t}\,|\nu_1\rangle + \mathrm{e}^{-iE_2 t}\,|\nu_2\rangle) = \frac{1}{2}(g_+(t)|\bar{\nu}_R\rangle + g_-(t)|\nu_R\rangle). \tag{163}$$

Here

$$g_\pm(t) = (\mathrm{e}^{-iE_1 t} \pm \mathrm{e}^{-iE_2 t}), \quad E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2E}, \tag{164}$$

where p is the neutrino momentum. In neutrino experiments we have $p \gg m_i$ and $p \simeq E$ (E is the neutrino energy).

From (163) and (164) for the transition probabilities, we obtain the following expressions:

$$P(\bar{\nu}_R \to \nu_R) = \frac{1}{2}\left(1 - \cos\frac{\Delta m^2 L}{2E}\right), \quad P(\bar{\nu}_R \to \bar{\nu}_R) = 1 - \frac{1}{2}\left(1 - \cos\frac{\Delta m^2 L}{2E}\right), \tag{165}$$

where $\Delta m^2 = m_2^2 - m_1^2$ and $L \simeq t$ is the distance between neutrino source and neutrino detector.

Thus, in the case of the neutrino oscillations, the probability for a reactor antineutrino to survive $P(\bar{\nu}_R \to \bar{\nu}_R)$ depends on the distance L. B. Pontecorvo wrote in [2]: «...the cross section of the production of neutrons and positrons in the process of the absorption of antineutrinos from a reactor by protons would be smaller than the expected cross section. It would be extremely interesting to perform the Reines–Cowan experiment at different distances from reactor».

If the value of the neutrino mass-squared difference Δm^2 is relatively large, the cosine terms in (165) disappear due to averaging over neutrino energies and distance. In this case, $P(\bar{\nu}_R \to \bar{\nu}_R) = P(\bar{\nu}_R \to \nu_R) = 1/2$. Discussing this case, Pontecorvo wrote: «...a beam of neutral leptons consisting mainly of antineutrinos when emitted from a nuclear reactor, will consist at some distance L from the reactor of half neutrinos and half antineutrinos».

*No indications in favor of neutrino decays were found.

If Δm^2 is very small, in this case the cosine terms are practically equal to one, and the effect of oscillations of reactor antineutrinos (with limited values of L) could not be observed. Pontecorvo noticed in [2]: «...effect of transformation of neutrino into antineutrino and vice versa may be unobservable in laboratory, but will certainly occur, at least, on an astronomic scale».

Let us stress again that the proposal of neutrino oscillations immediately after the great success of the two-component neutrino theory and in the situation when only one type of neutrino was known was a very nontrivial one. The Pontecorvo paper was written at the time when the Davis reactor experiment was not yet finished and candidate-events (161) existed. In order to explain them he had to assume that ν_R interacts with matter. He wrote: «...it is impossible to conclude a priori that the antineutrino beam which at first is essentially incapable of inducing the reaction in question transforms itself into a beam in which a definite fraction of particles can induce such reaction».

In spite of the fact that the candidate-events (161) disappeared and only an upper bound for the cross section of the process (161) was found in the Davis experiment, Pontecorvo continued to believe in neutrino oscillations. He liked the idea that neutrinos (antineutrinos) produced in weak processes can oscillate into antineutrinos (neutrinos) which have no (standard) weak interaction. He proposed to name such noninteracting neutrinos *sterile*. The idea of sterile neutrinos is very popular nowadays.

The program of the study of oscillations of reactor antineutrinos, which was outlined by B. Pontecorvo in the very first paper on neutrino oscillations, was realized in the KamLAND experiment about 40 years later. We will discuss this experiment in the next subsection.

After the first paper on the neutrino oscillations, Pontecorvo continued to think about this fascinating phenomenon. His belief in neutrino masses was based on the fact that there was no principle (like gauge invariance for photon) which requires the neutrino to be a massless particle*.

After the discovery of ν_μ in the Brookhaven experiment, Pontecorvo applied his idea of neutrino oscillations to the case of two types of neutrinos ν_e and ν_μ **. In the second paper on neutrino oscillations published in 1967 [54], Pontecorvo considered $\nu_e \rightleftarrows \nu_\mu$, $\nu_e \rightleftarrows \bar{\nu}_{eL}$ (sterile), $\nu_e \rightleftarrows \bar{\nu}_{\mu L}$ (sterile), etc., oscillations and applied the idea of neutrino oscillations to solar neutrinos.

At that time R. Davis started his famous experiment on the detection of the solar neutrinos in which the radiochemical method of neutrino detection,

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*In the sixties, B. Pontecorvo discussed the problem of neutrino masses with L. Landau, one of the authors of the two-component neutrino theory. Landau agreed that after the $V-A$ theory, it would be natural to assume that neutrino has a small mass.

**In reality, it was more natural and easier to introduce neutrino oscillations in this case: there was no necessity to assume the existence of sterile neutrinos.

proposed by B.Pontecorvo in 1946, was used. Solar neutrinos were detected in this experiment via the observation of the Pontecorvo–Davis reaction

$$\nu_e + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}. \tag{166}$$

In the paper [54], B. Pontecorvo wrote: «From an observational point of view, the ideal object is the sun. If the oscillation length is smaller than the radius of the sun region effectively producing neutrinos (let us say one tenth of the sun radius $R_\odot$ or 0.1 million km for ^{8}B neutrinos, which will give the main contribution in the experiments being planned now), direct oscillations will be smeared out and unobservable. The only effect on the earth's surface would be that the flux of observable sun neutrinos must be two times smaller than the total (active and sterile) neutrino flux».

The first Davis result was obtained in 1970. It was found that the upper bound of the observed flux of the solar ν_e's was 2–3 times smaller than the predicted flux. This result created «the solar neutrino problem». In the paper [54], *Pontecorvo envisaged the solar neutrino problem.* He understood, however, that the prediction of the flux of high-energy ^{8}B neutrinos, which gave the major contribution to the event rate in the Davis experiment, was an extremely difficult problem: «Unfortunately, the relative weight of different thermonuclear reactions in the sun and its central temperature are not known well enough to permit a comparison of the expected and observed solar neutrino intensities». It took many years of research to prove that the observed depletion of fluxes of solar neutrinos is an effect of neutrino transitions due to neutrino masses, mixing and interaction of neutrinos with matter which we will discuss briefly later.

The first phenomenological scheme of neutrino mixing was proposed by V. Gribov and B. Pontecorvo in 1969 [55]. They assumed that only the left-handed flavor fields $\nu_{eL}(x)$ and $\nu_{\mu L}(x)$ entered into the total Lagrangian. There was a widespread opinion at that time that in this case neutrino masses must be equal to zero. V. Gribov and B. Pontecorvo showed that this is not the case *if the total lepton number L is violated.* In this case

$$\begin{aligned}\nu_{eL}(x) &= \cos\theta\nu_{1L}(x) + \sin\theta\nu_{2L}(x),\\ \nu_{\mu L}(x) &= -\sin\theta\nu_{1L}(x) + \cos\theta\nu_{2L}(x),\end{aligned} \tag{167}$$

where $\nu_1(x)$ and $\nu_2(x)$ are the fields of *Majorana neutrinos with masses m_1 and m_2*; and θ is the mixing angle.

The scheme of two-neutrino mixing, proposed by V. Gribov and B. Pontecorvo, was the minimal one. In this scheme:

- The only possible oscillations are $\nu_e \rightleftarrows \nu_\mu$.
- There are no sterile neutrinos.

• To four states of flavor neutrinos and antineutrinos (left-handed ν_e, ν_μ and right-handed $\bar{\nu}_e$, $\bar{\nu}_\mu$) there correspond four states of two massive Majorana neutrinos with helicities ± 1.

In [55], the following general expression for the two-neutrino survival probability in vacuum was obtained*

$$P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta \left(1 - \cos\frac{\Delta m^2 L}{2E}\right). \tag{168}$$

In [55] and later in [56], the effect of vacuum $\nu_e \rightleftarrows \nu_\mu$ oscillations on the flux of solar ν_e's on the earth was discussed.

In the eighties, the Cabibbo–GIM mixing (94) of d and s quarks was fully established. In [57–59] the neutrino mixing

$$\begin{aligned} \nu_{eL}(x) &= \cos\theta\nu_{1L}(x) + \sin\theta\nu_{2L}(x), \\ \nu_{\mu L}(x) &= -\sin\theta\nu_{1L}(x) + \cos\theta\nu_{2L}(x) \end{aligned} \tag{169}$$

was introduced on the basis of the lepton–quark analogy. The main ideas were the following:

1. Neutrinos like all other fundamental fermions (charged leptons and quarks) are massive particles.

2. The mixing is a general feature of gauge theories with a mass generation mechanism based on the spontaneous violation of symmetry. Thus, *fields of neutrinos like fields of quarks enter into the charged current in a mixed form*.

In (169), $\nu_1(x)$ and $\nu_2(x)$ are the fields of neutrinos with masses m_1 and m_2. However, in contrast to the Gribov–Pontecorvo scheme, in this scheme the total lepton number is conserved and $\nu_{1,2}$ *are the Dirac particles* (like quarks). In [57–59], possible neutrino oscillations in reactor and accelerator neutrino experiments were discussed.

As we have seen earlier, the initial ideas of neutrino masses, mixing, and oscillations were based on symmetry (analogy) of weak interactions of leptons and hadrons (and later leptons and quarks). In 1962, Maki, Nakagawa, and Sakata [60] introduced the neutrino mixing in the framework of the Nagoya model in which the proton and other baryons were considered as bound states of neutrinos and a vector boson B^+, «a new sort of matter». At that time the Brookhaven experiment, in which it was proved that ν_e and ν_μ were different particles, was not yet finished. However, there was an indication, based on the fact that the decay $\mu^+ \to e^+ + \gamma$ was not observed, that ν_e and ν_μ are different types of neutrinos determined by the weak charged current

$$j_\alpha^{\mathrm{CC}} = 2(\bar{\nu}_{eL}\gamma_\alpha e_L + \bar{\nu}_{\mu L}\gamma_\alpha \mu_L). \tag{170}$$

*The expression (165) corresponds to the case of maximal mixing $\theta = \pi/4$.

The authors wrote: «We assume that there exists a representation which defines the true neutrinos ν_1 and ν_2 through orthogonal transformation»

$$\begin{aligned} \nu_1 &= \cos\delta\nu_e - \sin\delta\nu_\mu, \\ \nu_2 &= \sin\delta\nu_e + \cos\delta\nu_\mu, \end{aligned} \tag{171}$$

where δ is the Cabibbo angle. The authors of the paper [60] assumed that the «true neutrino» ν_1 was a constituent of baryons and possessed some mass m_1 and there existed an additional interaction of ν_2 with a field of heavy particles X which ensures the difference of masses of ν_2 and ν_1.

In contrast to [2, 54, 55], in [60] the quantum phenomenon of neutrino oscillations, based on the difference of phases which were gained in propagation of neutrinos with definite masses, was not considered. Nevertheless, $\nu_e \to \nu_\mu$ transitions were discussed in [60]. The authors wrote: «Weak neutrinos

$$\begin{aligned} \nu_e &= \cos\delta\nu_1 + \sin\delta\nu_2, \\ \nu_\mu &= -\sin\delta\nu_1 + \cos\delta\nu_2 \end{aligned} \tag{172}$$

are not stable due to the occurrence of virtual transition $\nu_e \rightleftarrows \nu_\mu$ caused by this additional interaction with ν_2». In connection with the Brookhaven neutrino experiment they noticed: «... a chain of reactions

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu + Z \to (\mu^- \text{ and/or}) \; e^- \tag{173}$$

is useful to check the two-neutrino hypothesis only when

$$|m_{\nu_2} - m_{\nu_1}| \leqslant \text{ eV} \tag{174}$$

under the conventional geometry of the experiments. Conversely, the absence of e^- will be able not only to verify the two-neutrino hypothesis but also to provide an upper limit of the mass of the second neutrino ν_2 if the present scheme should be accepted».

The papers [55, 57–60] were written at the time when only two types of flavor neutrinos ν_e and ν_μ were known. In [55], it was assumed that there was no conserved lepton number, and neutrinos with definite masses ν_1 and ν_2 were truly neutral Majorana particles. In [57–60], it was assumed that the total lepton number L was conserved, and ν_1 and ν_2 are Dirac particles ($L(\nu_i) = 1$, $L(\bar{\nu}_i) = -1$). After the discovery of the τ lepton, it was natural to assume that there existed (at least) three different types of neutrinos. The mixing relations (167) and (169) were generalized for an arbitrary number n of flavor neutrinos in the following way (see [61]):

$$\nu_{lL} = \sum_{i=1}^{n} U_{li}\nu_{iL}, \quad l = e, \mu, \ldots \tag{175}$$

Here U is a unitary $n \times n$ matrix ($U^{\dagger}U = 1$). The matrix U is called the mixing matrix. As we will see later, the existing neutrino oscillation data can be described if we assume that U is the 3×3 matrix. This matrix is usually called the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix in order to pay tribute to the pioneering contribution of these authors to the neutrino mixing and oscillations.

The mixing (175) is not the most general one. In the most general case we have (see [62])

$$\nu_{lL} = \sum_{i=1}^{n+n_{\rm st}} U_{li}\nu_{iL}, \quad \nu_{sL} = \sum_{i=1}^{n+n_{\rm st}} U_{si}\nu_{iL}, \tag{176}$$

where the index s takes $n_{\rm st}$ values and U is a unitary $(n+n_{\rm st}) \times (n+n_{\rm st})$ mixing matrix. The fields ν_{sL} are fields of the sterile neutrinos which have no standard weak interaction. Due to the mixing (176), transitions between flavor neutrinos $\nu_l \rightleftarrows \nu_{l'}$ as well as transitions between flavor and sterile neutrinos $\nu_l \rightleftarrows \nu_{sL}$ are possible.

In spite of the fact that in the seventies some plausible arguments for small nonzero masses were given and a general phenomenological theory of neutrino mixing and oscillations was developed, there was no so much interest in neutrino masses and oscillations at that time: the idea of massless two-component neutrinos was still the dominant one. In the first review on neutrino oscillations [61], only about ten neutrino oscillation papers existing at that time were referred to.

Summarizing, the earliest ideas of neutrino masses, mixing, and oscillations were based on arguments like an analogy between weak interactions of leptons and hadrons (quarks), the Nagoya model with the neutrino as a constituent of the proton and other baryons, etc.

After the great success of the theory of the two-component massless neutrinos, for many years these ideas were not shared by the majority of physicists.

Of course, it was absolutely unknown in the seventies whether neutrinos had small masses and, if they had masses, whether they were mixed. However, understanding of neutrino oscillations as an interference phenomenon made it clear (S. Bilenky and B. Pontecorvo [42]) that:

1. Experiments on the search for neutrino oscillations constitute the most sensitive way to look for small neutrino mass-squared differences.

2. Experiments with neutrinos from different facilities are sensitive to different values of neutrino mass-squared differences. Neutrino oscillations must be searched for in all possible neutrino experiments (solar, atmospheric, reactor, accelerator).

This strategy was summarized in [61]. After many years of efforts it brought success.

17.2. Neutrino Oscillations at the Time when Neutrino Masses Started to Be Considered as a Signature of Physics Beyond the SM. The situation with neutrino masses and the mixing drastically changed at the end of the seventies with the appearance of the models of grand unification (GUT). In these models leptons and quarks enter into the same multiplets, and the generation of masses of quarks and charged leptons in some models naturally leads to nonzero neutrino masses. At that time the famous seesaw mechanism of the neutrino mass generation [66], which could explain the smallness of the neutrino masses with respect to the masses of quarks and charged leptons, was proposed.

After the appearance of the GUT models and the seesaw mechanism of neutrino mass generation, *masses and mixing of neutrinos started to be considered as a signature of the physics beyond the Standard Model.* The problem of neutrino masses and oscillations attracted more and more the attention of theoreticians and experimentalists. Several short-baseline* experiments on the search for neutrino oscillations with reactor and accelerator neutrinos were performed in the eighties. No positive indications in favor of oscillations in these experiments with artificially produced neutrinos were found at that time**.

On the other hand, indications in favor of oscillations of solar neutrinos were strengthened in the eighties. The second solar neutrino experiment Kamiokande was performed [64]. In this experiment high-energy solar neutrinos from the decay $^8\text{B} \to {}^8\text{Be} + e^+ + \nu_e$ were detected via the observation of the recoil electrons from the elastic $\nu + e \to \nu + e$ scattering. The ratio of the observed flux of the solar neutrinos to the predicted flux obtained in the Kamiokande experiment was about $1/2$.

In the Kamiokande and IMB water Cherenkov detectors, high-energy muons and electrons produced by atmospheric muon and electron neutrinos were detected***. It was found in these experiments that the ratio of the numbers of the ν_μ and ν_e events was significantly smaller than the (practically model-independent) predicted ratio [65]. This effect was called the atmospheric neutrino anomaly.

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*Distances between sources and detectors in these experiments were a few hundred meters or less.

**Recently fluxes of $\bar{\nu}_e$'s from reactors were recalculated. It occurred that the fluxes are about 3% higher than the fluxes used in the analysis of old reactor neutrino oscillation data [67]. Thus, these data nowadays are interpreted as an indication in favor of short-baseline neutrino oscillations. New reactor and accelerator neutrino experiments are under preparation in order to check the hypothesis of short-baseline oscillations.

***Atmospheric neutrinos are produced mainly in decays of pions, produced in the processes of interactions of cosmic rays in the atmosphere, and muons which are produced in decays of pions ($\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$, $\mu^\pm \to e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$).

The anomaly could be explained by the disappearance of ν_μ due to transitions of ν_μ into other neutrino states.

At the beginning of the nineties, two new solar neutrino experiments GALLEX [11] and SAGE [12] were performed. In these experiments, like in the first Davis experiment, Pontecorvo's radiochemical method of neutrino detection was utilized. Solar ν_e's were detected via the observation of radioactive ^{71}Ge atoms produced in the process

$$\nu_e + {}^{71}\text{Ga} \to e^- + {}^{71}\text{Ge}. \tag{177}$$

There are three main sources of ν_e's in the sun:

1. the pp reaction $p + p \to d + e^+ + \nu_e$ ($E \leqslant 0.42$ MeV),
2. the ^{7}Be capture $e^- + {}^7\text{Be} \to {}^7\text{Li} + \nu_e$ ($E = 0.86$ MeV),
3. the ^{8}B decay $^8\text{B} \to {}^8\text{Be} + e^+ + \nu_e$ ($E \leqslant 15$ MeV).

The threshold of the Cl–Ar reaction (72) is equal to 0.81 MeV. Thus, in the Davis experiment mainly ^{8}B neutrinos can be detected. The threshold of the reaction (177) is equal to 0.23 MeV. This means that in the GALLEX and SAGE experiments neutrinos from all reactions of thermonuclear cycles in the sun including low-energy neutrinos from the $pp \to de^+\nu_e$ reaction, were detected. This reaction gives the largest contribution to the flux of the solar neutrinos. The flux of the pp neutrinos can be connected with the luminosity of the sun and can be predicted in a model-independent way.

The event rates measured in the GALLEX and SAGE experiments were approximately two times smaller than the predicted rates. Thus, in these experiments additional important evidence was obtained in favor of the disappearance of solar ν_e on the way from the central region of the sun, where solar neutrinos are produced, to the earth.

Solar ν_e's are produced in the central region of the sun and on the way to the earth pass about $7 \cdot 10^5$ km of the solar matter. It was discovered in the nineties by Wolfenstein, Mikheev, and Smirnov [68] that for neutrino propagation in matter not only masses and mixing but also coherent interaction are important. This interaction gives an additional contribution to the Hamiltonian of neutrino in matter which is determined by the electron number-density. If the electron density depends on the distance (as in the case of the sun), the transition probabilities between different flavor neutrinos in matter can have a resonance character (MSW effect).

Summarizing, in the eighties and nineties with new solar neutrino experiments and the increase in the number of detected atmospheric neutrino events, the evidence in favor of neutrino masses and oscillations, coming from these experiments, became stronger. However, the interpretation of the data of solar neutrino experiments depended on the Standard Solar Model. In experiments with neutrinos of terrestrial origin (reactor and accelerator neutrinos) no positive indications in favor of neutrino oscillations were found. In 1998, the situation with neutrino oscillations drastically changed.

17.3. Golden Years of Neutrino Oscillations (1998–2004). In 1998, in the Super-Kamiokande atmospheric neutrino experiment [69] (Japan) significant up-down asymmetry of the high-energy muon events was observed. Neutrinos produced in the earth atmosphere and coming from above pass distances from about 20 to 500 km. Neutrinos coming to the detector from below pass the earth and travel distances from 500 to about 12 000 km. It was discovered in the Super-Kamiokande experiment that the number of up-going high-energy muon neutrinos was about two times smaller than the number of the down-going high-energy muon neutrinos. Thus, it was proved that the number of observed muon neutrinos depended on the distance which neutrinos passed from a production point in the atmosphere to the detector.

The Super-Kamiokande atmospheric neutrino result was the first model-independent evidence of neutrino oscillations. This result marked a new era in the investigation of neutrino oscillations — an era of experiments with neutrinos from different sources which provide model-independent evidence of neutrino oscillations.

In 2002, in the SNO solar neutrino experiment [70] (Canada) a model-independent evidence of the disappearance of solar ν_e was obtained. In this experiment high-energy solar neutrinos from ^{8}B decay were detected through the observation of CC and NC reactions. The detection of solar neutrinos through the observation of the CC reaction allows one to determine the flux of solar ν_e on the earth, while the detection of solar neutrinos through the observation of the NC reaction allows one to determine the *flux of all flavor neutrinos (ν_e, ν_μ, and ν_τ)*. It was shown in the SNO experiment that the flux of the solar ν_e is approximately three times smaller than the flux of ν_e, ν_μ, and ν_τ. Thus, it was proved that solar ν_e's on the way from the sun to the earth are transferred to ν_μ and ν_τ.

In 2002–2004, the model-independent evidence of oscillations of reactor $\bar{\nu}_e$ was obtained in the KamLAND reactor experiment [71]. In this experiment $\bar{\nu}_e$'s from 55 reactors at an average distance of about 170 km from the large KamLAND detector were recorded. It was found that the total number of $\bar{\nu}_e$ events is about 0.6 of the number of the expected events. A significant distortion of the $\bar{\nu}_e$ spectrum with respect to the expected spectrum was observed in the experiment.

Neutrino oscillations were observed also in the long-baseline accelerator K2K experiment [72] (the distance L between source and detector is about 250 km) and in the MINOS accelerator neutrino experiment [73] (with a distance L of about 730 km). These experiments fully confirmed the results obtained in the atmospheric Super-Kamiokande experiment.

Thus, neutrino oscillations were discovered. It was proven that neutrinos have small masses and that the flavor neutrinos ν_e, ν_μ, ν_τ are «mixed particles». The analysis of existing data, which we will discuss in the next subsection, shows that the existing neutrino oscillation data are well described if we assume three-neutrino mixing.

Summarizing, it took about thirty years of heroic efforts of many physicists to discover neutrino oscillations envisaged in 1958 by B. Pontecorvo. The observation of neutrino oscillations is the most important recent discovery in high-energy physics.

The Standard Model is a beautiful theory which predicted families of new particles (charmed, bottom, top), new weak interaction (Neutral Current), new type of neutrino (ν_τ), etc. However, there are more than twenty parameters in this theory (masses of quarks and leptons, mixing angles, etc.) and there is the so-called hierarchy problem which is connected with the Higgs mass. For many years the main aim of high-energy physics was to find effects which cannot be explained by the SM and require a new theory. *The small neutrino masses and neutrino mixing is the first signature of the physics beyond the SM.*

17.4. Present Status of Neutrino Oscillations. In this subsection, we will briefly discuss the present status of neutrino mixing and oscillations. We will consider the case of three-neutrino mixing. «Mixed» flavor fields $\nu_{lL}(x)$ which enter into charged and neutral currents are given by the relations

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x), \quad l = e, \mu, \tau. \tag{178}$$

Here U is the 3×3 unitary PMNS mixing matrix and $\nu_i(x)$ is the field of neutrinos (Dirac or Majorana) with mass m_i.

There is a lot of discussions in the literature of the methods of the derivation of the expression for probability of the transition between different types of neutrinos. Different methods give the same expression for the transition probability.

Major assumption is that production (and detection) of neutrinos with different masses cannot be resolved and in CC weak processes together with a lepton l^+ the *flavor neutrino ν_l, which is described by the coherent superposition*

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^*|\nu_i\rangle, \tag{179}$$

is produced (and detected).

We are interested in neutrino beams. Thus, the states $|\nu_i\rangle$ in (179) are states of neutrinos ν_i with mass m_i, helicity -1, momentum $\mathbf{p}$, and energy $E_i = \sqrt{p^2 + m_i^2} \simeq p + \dfrac{m_i^2}{2p}$.

If at $t = 0$ a flavor neutrino ν_l was produced, for the neutrino state in vacuum at $t > 0$ we have

$$|\nu_l\rangle_t = \mathrm{e}^{-iH_0t} \sum_i |\nu_i\rangle, \quad U_{li}^* = \sum_i |\nu_i\rangle\, \mathrm{e}^{-iE_it} U_{li}^*, \tag{180}$$

where H_0 is the free Hamiltonian. Neutrinos are detected via the observation of weak processes in which flavor neutrinos take part ($\nu_{l'} + N \to l' + X$, etc.). Developing (180) over states $|\nu_{l'}\rangle$ we find

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle \sum_i U_{l'i}\, \mathrm{e}^{-iE_i t}\, U^*_{li}. \tag{181}$$

If $m_i = m$, in this case $E_i = E$, $\sum_i U_{l'i}\, U^*_{li} = \delta_{l'l}$ and $|\nu_l\rangle_t = \mathrm{e}^{-iEt}|\nu_l\rangle$. Thus, if all neutrino masses are equal, the produced ν_l will always remain ν_l. If neutrino masses are different, in this case the initial ν_l can be transferred into another flavor neutrino $\nu_{l'}$. The probability of the transition $\nu_l \to \nu_{l'}$ is given by the expression

$$P(\nu_l \to \nu_{l'}) = \left| \sum_i U_{l'i}\, \mathrm{e}^{-iE_i t} U^*_{li} \right|^2. \tag{182}$$

Taking into account that for ultrarelativistic neutrinos $t \simeq L$, where L is the distance between the neutrino source and the neutrino detector, and $E_i - E_2 = \dfrac{\Delta m^2_{2i} L}{2E}$, we can rewrite expression (182) in the form (see Appendix C)

$$P(\nu_l \to \nu_{l'}) = \left| \delta_{l'l} + \sum_{i \neq 2} U_{l'i} \left[\exp\left(-i \frac{\Delta m^2_{2i} L}{2E} - 1 \right) \right] U^*_{li} \right|^2. \tag{183}$$

It follows from this expression that the probability of the transition depends periodically on the parameter L/E. Expression (182) describes neutrino oscillations in vacuum. It is clear from (180) and (181) that neutrino oscillations happen if the states of neutrinos with different masses *gain different phases after the evolution of the neutrino beam during the time* t *(at the distance* L*)*.

The unitary 3×3 matrix U is characterized by four parameters: three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one phase δ. In the case of three neutrino masses there are two independent mass-squared differences Δm_{23} and Δm_{12}. Thus, in the general case the transition probability $P(\nu_l \to \nu_{l'})$ depends on six parameters.

It follows from the analysis of experimental data that two parameters are small:

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \simeq \frac{1}{30}, \quad \sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005. \tag{184}$$

In the first (leading) approximation we can neglect contributions of these parameters to neutrino transition probabilities. In this approximation a rather simple picture of neutrino oscillations emerges.

In the leading approximation in the atmospheric region of L/E $\left(\dfrac{\Delta m^2_{23} L}{2E} \gtrsim 1\right)$ $\nu_\mu \rightleftarrows \nu_\tau$ oscillations take place. In this case, the $\nu_\mu \to \nu_\mu$

survival probability has the simple two-neutrino form

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - P(\nu_\mu \to \nu_\tau) \simeq 1 - \frac{1}{2}\sin^2 2\theta_{23}\left(1 - \cos\Delta m^2_{23}\frac{L}{2E}\right). \quad (185)$$

Thus, in the leading approximation neutrino oscillations in the atmospheric region are characterized by the parameters Δm^2_{23} and $\sin^2 2\theta_{23}$.

In the KamLAND reactor region $\left(\frac{\Delta m^2_{12} L}{2E} \gtrsim 1\right)$ $\bar{\nu}_e \rightleftarrows \bar{\nu}_{\mu,\tau}$ take place. For the $\bar{\nu}_e \to \bar{\nu}_e$ survival probability, we have

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{12}\left(1 - \cos\Delta m^2_{12}\frac{L}{2E}\right). \quad (186)$$

Thus, in the leading approximation, neutrino oscillations in the KamLAND reactor region are characterized by the parameters Δm^2_{12} and $\sin^2 2\theta_{12}$.

Let us also notice that in the leading approximation the probability of the solar neutrinos to survive is given by the two-neutrino $\nu_e \to \nu_e$ survival probability in matter which depends on the parameters Δm^2_{12} and $\sin^2\theta_{12}$ and the electron number density.

The leading approximation gives the dominant contribution to the expressions for the neutrino transition probabilities. Until recently, in the analysis of neutrino oscillation data two-neutrino expressions (185) and (186) were used. Now with the improvement of the accuracy of the experiments three-neutrino transition probabilities are started to be used in the analysis of the data.

We will briefly discuss the results that were obtained in some neutrino oscillation experiments.

The SNO Solar Neutrino Experiment [70]. The SNO experiment was performed in the Creighton mine (Sudbury, Canada, depth 2092 m). Solar neutrinos were detected by a large heavy-water detector (1000 t of D_2O contained in an acrylic vessel 12 m in diameter). The detector was equipped with 9456 photomultipliers to detect light created by particles which are produced in neutrino interaction.

The high-energy ^{8}B neutrinos were detected in the SNO experiment. An important feature of the SNO experiment was the observation of solar neutrinos *via three different processes*.

1. The CC process

$$\nu_e + d \to e^- + p + p. \quad (187)$$

2. The NC process

$$\nu_x + d \to \nu_x + p + n \quad (x = e, \mu, \tau). \quad (188)$$

3. Elastic neutrino–electron scattering (ES)

$$\nu_x + e \to \nu_x + e. \tag{189}$$

The detection of solar neutrinos through the observation of the NC reaction (188) allows one to determine the total flux of ν_e, ν_μ, and ν_τ on the earth. In the SNO experiment it was found

$$\Phi^{\mathrm{NC}}_{\nu_{e,\mu,\tau}} = (5.25 \pm 1.6^{+0.11}_{-0.13}) \cdot 10^6 \text{ cm}^{-2} \cdot \text{s}^{-1}. \tag{190}$$

The total flux of all active neutrinos on the earth must be equal to the total flux of ν_e emitted by the sun (if there are no transitions of ν_e into sterile neutrinos). The flux measured by SNO is in agreement with the total flux of ν_e predicted by the Standard Solar Model:

$$\Phi^{\mathrm{SSM}}_{\nu_e} = (4.85 \pm 0.58) \cdot 10^6 \text{ cm}^{-2} \cdot \text{s}^{-1}. \tag{191}$$

The detection of the solar neutrinos via the reaction (187) allows one to determine the total flux of ν_e on the earth. It was found in the SNO experiments that the total flux of ν_e was about three times smaller than the total flux of all active neutrinos.

From the ratio of the fluxes of ν_e and ν_e, ν_μ, and ν_τ, the ν_e survival probability can be determined. It was shown in the SNO experiment that in the high-energy ^{8}B region the ν_e survival probability did not depend on the neutrino energy and was equal to

$$\frac{\Phi^{\mathrm{CC}}_{\nu_e}}{\Phi^{\mathrm{NC}}_{\nu_{e,\mu,\tau}}} = P(\nu_e \to \nu_e) = 0.317 \pm 0.016 \pm 0.009. \tag{192}$$

Thus, it was proved in a direct, model-independent way that solar ν_e on the way to the earth are transferred into ν_μ and ν_τ.

The KamLAND Reactor Neutrino Experiment [71]. The KamLAND detector is situated in the Kamioka mine (Japan) at a depth of about 1 km. The neutrino target is a 1 kiloton liquid scintillator which is contained in a 13 m-diameter transparent nylon balloon suspended in 1800 m^3 nonscintillating buffer oil. The balloon and buffer oil are contained in an 18 m-diameter stainless-steel vessel. On the inner surface of the vessel, 1879 photomultipliers are mounted.

In the KamLAND experiment, $\bar{\nu}_e$ from 55 reactors situated at distances of (175 ± 35) km from the Kamioka mine are detected.

Reactor $\bar{\nu}_e$'s are detected in the KamLAND experiment through the observation of the process

$$\bar{\nu}_e + p \to e^+ + n. \tag{193}$$

The signature of the neutrino event is a coincidence between two γ quanta produced in the annihilation of a positron (prompt signal) and a $\simeq 2.2$ MeV γ quantum produced by a neutron capture in the process $n + p \to d + \gamma$ (delayed signal).

The average energy of the reactor antineutrinos is 3.6 MeV. For such energies, distances of about 100 km are appropriate to study neutrino oscillations driven by the solar neutrino mass-squared difference Δm_{12}^2.

From March 2002 to May 2007 in the KamLAND experiment 1609 neutrino events were observed. The expected number of neutrino events (if there are no neutrino oscillations) is 2179 ± 89. Thus, it was proved that $\bar{\nu}_e$'s disappeared on the way from the reactors to the detector.

As the $\bar{\nu}_e$ survival probability depends on the neutrino energy, we must expect that the detected spectrum of $\bar{\nu}_e$ is different from the reactor antineutrino spectrum. In fact, in the KamLAND experiment a significant distortion of the initial antineutrino spectrum is observed (Fig. 2).

The data of the experiment are well described if we assume that two-neutrino oscillations take place. For the neutrino oscillation parameters, it was found

$$\Delta m_{12}^2 = (7.66^{+0.20}_{-0.22}) \cdot 10^{-5}\ \text{eV}^2, \quad \tan^2 2\theta_{12} = 0.52^{+0.16}_{-0.10}. \tag{194}$$

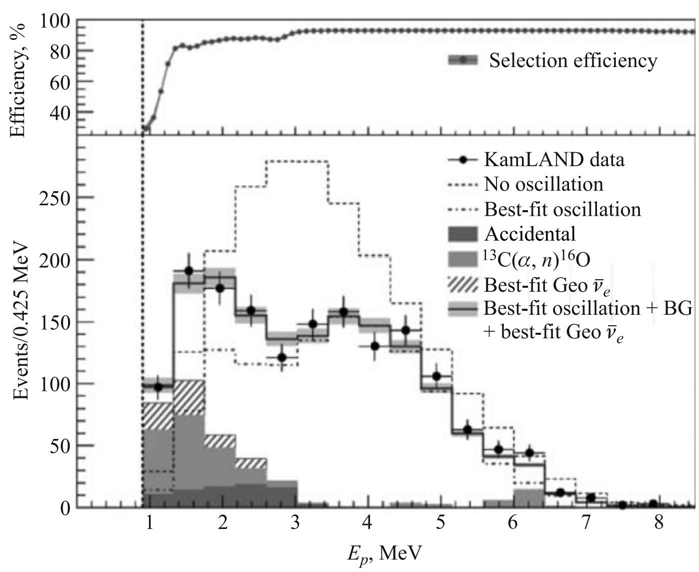

Fig. 2. Prompt event energy spectrum measured in the KamLAND experiment. The dashed line shows the predicted spectrum in the case of no oscillations. Best-fit oscillation curve is presented. In the shaded areas, different backgrounds are shown (arXiv:0801.4589)

From the three-neutrino analysis of all solar neutrino data and the data of the KamLAND reactor experiment for the neutrino oscillation parameters the following values were obtained:

$$\Delta m^2_{12} = (7.41^{+0.21}_{-0.19}) \cdot 10^{-5}\ \mathrm{eV}^2, \quad \tan^2\theta_{12} = 0.446^{+0.030}_{-0.029}, \quad \sin^2\theta_{13} < 0.053. \tag{195}$$

Super-Kamiokande Atmospheric Neutrino Experiment [69]. In the Super-Kamiokande atmospheric neutrino experiment the first model-independent evidence in favor of neutrino oscillations was obtained (1998). The Super-Kamiokande detector is situated in the same Kamioka mine as the KamLAND detector. It consists of two optically separated water-Cherenkov cylindrical detectors with a total mass of 50 kilotons of water. The inner detector with 11 146 photomultipliers has a radius of 16.9 m and a height of 36.2 m. The outer detector is a veto detector. It allows one to reject cosmic-ray muons. The fiducial mass of the detector is 22.5 kilotons.

In the Super-Kamiokande experiment atmospheric neutrinos in a wide range of energies from about 100 MeV to about 10 TeV are detected. Atmospheric ν_μ $(\bar\nu_\mu)$ and ν_e $(\bar\nu_e)$ are detected through the observation of μ^- (μ^+) and e^- (e^+) produced in the processes

$$\nu_\mu(\bar\nu_\mu) + N \to \mu^-(\mu^+) + X, \quad \nu_e(\bar\nu_e) + N \to e^-(e^+) + X. \tag{196}$$

For the study of neutrino oscillations it is important to distinguish electrons and muons produced in the processes (196). In the Super-Kamiokande experiment leptons are observed through the detection of the Cherenkov radiation. The shapes of the Cherenkov rings of electrons, and muons are completely different (in the case of electrons, the Cherenkov rings exhibit a more diffuse light than in the muon case). The probability of a misidentification of electrons and muons is below 2%.

A model-independent evidence of neutrino oscillations was obtained by the Super-Kamiokande Collaboration through the investigation of the zenith-angle dependence of the electron and muon events (Fig. 3). The zenith angle θ is determined in such a way that neutrinos going vertically downward have $\theta = 0$ and neutrinos coming vertically upward through the earth have $\theta = \pi$. At neutrino energies $E \gtrsim 1$ GeV the fluxes of muon and electron neutrinos are symmetric under the change $\theta \to \pi - \theta$. Thus, if there are no neutrino oscillations in this energy region, the numbers of electron and muon events must satisfy the relation

$$N_l(\cos\theta) = N_l(-\cos\theta), \quad l = e, \mu. \tag{197}$$

In the Super-Kamiokande experiment a large distortion of this relation for high-energy muon events was established (a significant deficit of upward-going muons was observed). The number of electron events satisfies the relation (197).

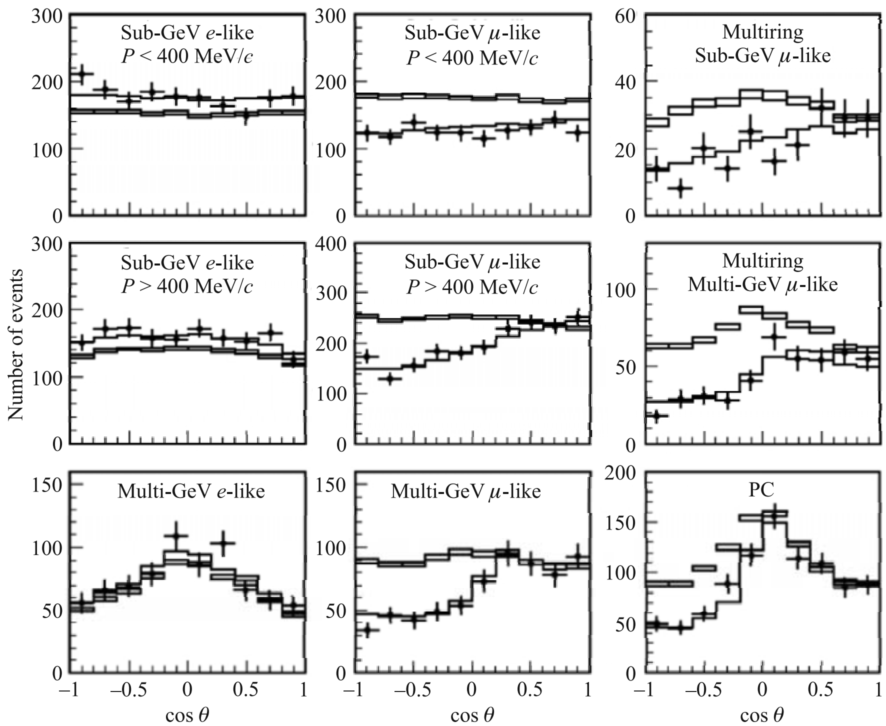

Fig. 3. Zenith-angle dependence of the numbers of electron and muon events measured in the Super-Kamiokande atmospheric neutrino experiment. Box histograms show expected numbers of events in the case of no oscillations. The best-fit two-neutrino oscillation curve is also plotted (arXiv:hep-ex/0501064)

This result can naturally be explained by the disappearance of muon neutrinos due to neutrino oscillations. The probability for ν_μ to survive depends on the distance between the neutrino source and the neutrino detector. Downward going neutrinos ($\theta \simeq 0$) pass a distance of about 20 km. On the other hand, upward going neutrinos ($\theta \simeq \pi$) pass a distance of about 13 000 km (earth diameter). The measurement of the dependence of the numbers of the electron and muon events on the zenith angle θ allows one to span the whole region of distances from about 20 km to about 13 000 km.

From the data of the Super-Kamiokande experiment for high-energy electron events was found

$$\left(\frac{U}{D}\right)_e = 0.961^{+0.086}_{-0.079} \pm 0.016. \qquad (198)$$

For high-energy muon events the value

$$\left(\frac{U}{D}\right)_\mu = 0.551^{+0.035}_{-0.033} \pm 0.004 \tag{199}$$

was obtained. Here U is the total number of upward going leptons ($-1 < \cos\theta < -0.2$) and D is the total number of downward going leptons ($0.2 < \cos\theta < 1$).

The data of the Super-Kamiokande atmospheric neutrino experiment are well described if we assume that ν_μ's disappear mainly due to $\nu_\mu \rightleftarrows \nu_\tau$ oscillations. From the three-neutrino analysis of the data for neutrino oscillation parameters in the case of normal (inverted) neutrino mass spectrum it was found

$$\begin{gathered} 1.9(1.7)\cdot 10^{-3} \leqslant \Delta m^2_{23} \leqslant 2.6(2.7)\cdot 10^{-3}\ \text{eV}^2, \\ 0.407 \leqslant \sin^2\theta_{23} \leqslant 0.583, \quad \sin^2\theta_{13} < 0.04(0.09). \end{gathered} \tag{200}$$

The result of the Super-Kamiokande atmospheric neutrino experiment was fully confirmed by the K2K and MINOS accelerator neutrino experiments [72, 73].

The Long-Baseline Accelerator Neutrino Experiments K2K and MINOS. In the MINOS experiment, muon neutrinos produced at the Fermilab Main Injector facility are detected. The MINOS data were obtained with neutrinos mostly with energies in the range $1 \leqslant E \leqslant 5$ GeV.

There are two identical neutrino detectors in the experiment. The near detector with a mass of 1 kt is at a distance of about 1 km from the target and about 100 m underground. The far detector with a mass of 5.4 kt is in the Sudan mine at a distance of 735 km from the target (about 700 m underground).

Muon neutrinos (antineutrinos) are detected in the experiment via the observation of the process

$$\nu_\mu(\bar{\nu}_\mu) + \text{Fe} \to \mu^-(\mu^+) + X. \tag{201}$$

The neutrino energy is given by the sum of the muon energy and the energy of the hadronic shower.

In the near detector the initial neutrino spectrum is measured. This measurement allows one to predict the expected spectrum of the muon neutrinos in the far detector in the case if there were no neutrino oscillations. A strong distortion of the spectrum of $\nu_\mu(\bar{\nu}_\mu)$ in the far detector was observed in MINOS experiment (Fig. 4).

From the two-neutrino analysis of the ν_μ data, for the neutrino-oscillation parameters the following values were obtained:

$$\Delta m^2_{23} = (2.32^{+0.12}_{-0.08})\cdot 10^{-3}\ \text{eV}^2, \quad \sin^2 2\theta_{23} > 0.90. \tag{202}$$

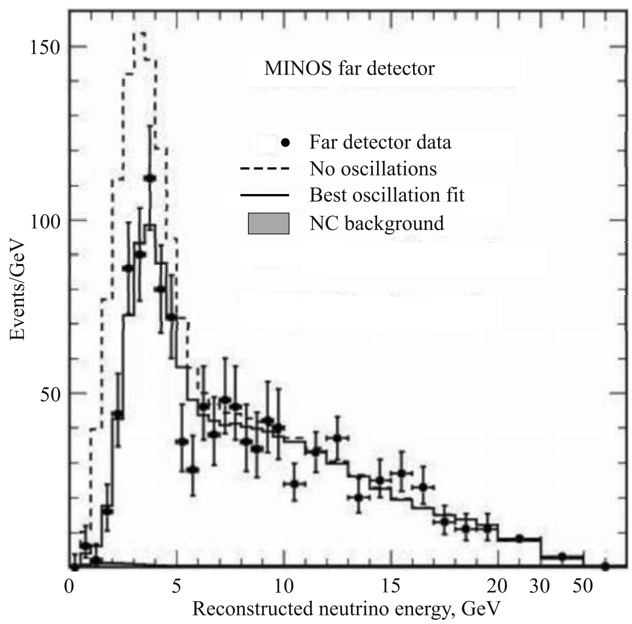

Fig. 4. Muon–neutrino energy spectrum measured in the MINOS experiment. The dashed curve shows the expected spectrum in the case of no oscillations. The neutrino oscillation best-fit spectrum is also shown (arXiv:0806.2237)

Measurements of the Angle θ_{13}. The value of the mixing angle θ_{13} is extremely important for the future of neutrino physics. If this angle is not equal to zero (and relatively large), in this case it will be possible to observe such a fundamental effect of the three-neutrino mixing as CP violation in the lepton sector. Another problem, the solution of which requires nonzero θ_{13}, is the problem of the neutrino mass spectrum. In the case of the three massive neutrinos, two neutrino mass spectra are possible:

1. Normal spectrum

$$m_1 < m_2 < m_3, \quad \Delta m_{12}^2 \ll \Delta m_{23}^2. \tag{203}$$

2. Inverted spectrum

$$m_3 < m_1 < m_2, \quad \Delta m_{12}^2 \ll |\Delta m_{13}^2|. \tag{204}$$

Let us notice that in order to have the same notation Δm_{12}^2 for the solar mass-squared difference for both spectra, the neutrino masses are usually labeled differently in the cases of the normal and inverted neutrino mass spectra; in the case of the normal spectrum $\Delta m^2_{23} > 0$ and in the case of the inverted spectrum $\Delta m^2_{13} < 0$.

For many years only an upper bound on the parameter $\sin^2\theta_{13}$ existed. This bound was obtained from the analysis of the data of the reactor CHOOZ experiment [74].

In the CHOOZ experiment the detector (5 t of Gd-loaded liquid scintillator) was at a distance of about 1 km from each of the two reactors of the CHOOZ power station (8.5 GWth). The detector had 300 m water equivalent of rock overburden which reduced the cosmic muon flux. The antineutrinos were detected through the observation of the reaction

$$\bar{\nu}_e + p \to e^+ + n. \tag{205}$$

For the ratio R of the total number of detected $\bar{\nu}_e$ events to the number of the expected events it was found

$$R = 1.01 \pm 2.8\%\,(\text{stat.}) \pm 2.7\%\,(\text{syst.}). \tag{206}$$

The data of the experiment was analyzed in the framework of two-neutrino oscillations with the $\bar{\nu}_e$-survival probability given by the expression

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{23} L}{2E}\right). \tag{207}$$

From the data of the CHOOZ experiment the following upper bound

$$\sin^2 2\theta_{13} \leqslant 0.16 \tag{208}$$

was obtained.

Recently among indications in a new long-baseline T2K neutrino experiment [75], an indication in favor of nonzero θ_{13} was obtained. In this experiment muon neutrinos produced at the J-PARC accelerator in Japan are detected at a distance of 295 km in the water-Cherenkov Super-Kamiokande detector. The T2K experiment is the first off-axis neutrino experiment: the angle between the direction to the detector and the flight direction of the parent π^+'s is equal to 2°. This allows one to obtain a narrow-band neutrino beam with a maximal intensity at the energy $E \simeq 0.6$ GeV which corresponds at the distance of $L = 295$ km to the first oscillation maximum ($E_0 = (2.54/\pi)\Delta m^2_{23} L$).

At a distance of about 280 m from the target there are several near detectors which are used for the measurement of the neutrino spectrum and flux and for the measurement of cross sections of different CC and NC processes.

The initial beam (from decays of pions and kaons) is a beam of ν_μ's with a small (about 0.4%) admixture of ν_e's. The search for electrons in the Super-Kamiokande detector due to $\nu_\mu \to \nu_e$ transitions was performed. Six ν_e events were observed in the experiment. The expected number of electron events (without neutrino oscillations) is equal to 1.5 ± 0.3. From the analysis of the data for the normal neutrino mass spectrum it was found:

$$0.03 < \sin^2 2\theta_{13} < 0.28 \ (90\% \ \text{CL}) \quad \text{best fit:} \quad \sin^2 2\theta_{13} = 0.11. \tag{209}$$

For the inverted neutrino mass spectrum it was obtained:

$$0.04 < \sin^2 2\theta_{13} < 0.34 \ (90\% \ \text{CL}) \quad \text{best fit:} \quad \sin^2 2\theta_{13} = 0.14. \tag{210}$$

A similar experiment was performed by the MINOS collaboration. In this experiment for the normal (inverted) neutrino mass spectrum the following best fit value was found:

$$2\sin^2(\theta_{23})\sin^2(2\theta_{13}) = 0.041^{+0.047}_{-0.031} \ (0.079^{+0.071}_{-0.053}). \tag{211}$$

The Double CHOOZ collaboration published the first indication in favor of reactor $\bar{\nu}_e$'s disappearence [76]. For the ratio of the observed and predicted $\bar{\nu}_e$ events the value $0.944 \pm 0.016 \pm 0.040$ was found. At 90% CL it was obtained $0.015 < \sin^2 2\theta_{13} < 0.16$.

Recently, the angle θ_{13} was measured by two reactor experiments. In the Daya Bay experiment [77] antineutrinos from six reactors (the thermal power of each reactor is 2.9 GW) were detected by three 20 t Gd-loaded liquid scintillator near detectors (flux-weighted distances 470 and 570 m) and three far detectors (1648 m). All detectors are identical and $\bar{\nu}_e$'s are detected via observation of the standard reaction

$$\bar{\nu}_e + p \to e^+ + n. \tag{212}$$

During 55 days of the data taking, 10 416 (80 376) candidate-events were observed in far (near) detectors. The number of $\bar{\nu}_e$ events in the far detector can be predicted on the basis of measurements performed in the near detectors (assuming that there are no neutrino oscillations). Significant deficit of antineutrino events in the far detectors was observed in the Daya Bay experiment. For the ratio of the observed and predicted events it was found the value:

$$R = 0.940 \pm 0.011 \pm 0.004. \tag{213}$$

From χ^2 analysis of the data it was found the following value of the parameter $\sin^2 2\theta_{13}$:

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005. \tag{214}$$

Thus, zero value of the parameter $\sin^2 2\theta_{13}$ is excluded at the level of $5.2\,\sigma$.

In the reactor RENO experiment [78], $\bar{\nu}_e$'s from six reactors with total thermal power 16.5 GW were detected by near and far Gd-loaded 16 t liquid scintillator detectors (294 and 1383 m from the center of the reactor array). During 229 days in the far and near detectors, 17 102 and 154 088 candidate-events were observed, respectively. For the ratio of the observed and predicted antineutrino events in the far detector it was found the value:

$$R = 0.920 \pm 0.009 \pm 0.014. \tag{215}$$

From analysis of the data, it was obtained:

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019 \tag{216}$$

($4.9\,\sigma$ effect).

Summarizing, the discovery of neutrino oscillations, envisaged by B. Pontecorvo in 1958, was a result of efforts of many physicists for many years. It required to build very large neutrino detectors (like Super-Kamiokande, SNO, KamLAND and others) and to overcome severe background problems. Nevertheless, there were several «lucky circumstances» which made it possible to discover and to investigate this phenomenon in some detail.

In the case of tree-neutrino mixing there are two independent mass-squared differences Δm^2_{23} and Δm^2_{12}. It was a «lucky circumstance» that both mass-squared differences could be reached in neutrino experiments: the first one in the atmospheric Super-Kamiokande experiment and long-baseline accelerator experiments (K2K, MINOS, and T2K) and the second one in the long baseline KamLAND reactor experiment.

The second «lucky circumstance» was the fact that the neutrino mixing angles θ_{23} and θ_{12} are large. As a result, *effects of neutrino oscillations in the Super-Kamiokande, KamLAND, K2K, and MINOS experiments were large.* This, of course, «simplified» the observation of neutrino oscillations in these experiments.

On the other hand, the smallness of the angle θ_{13} puts challenging problems to experimental neutrino physics. Effects of neutrino oscillations induced by this angle are so small that special accelerator and reactor experiments with near and far detectors are necessary for that. Luckily, it occurred that the angle θ_{13} was not very small and this important neutrino oscillation parameter was measured with $\sim 20\%$ accuracy in reactor neutrino experiments. The value of the angle θ_{13} is crucial for the investigation of such fundamental problems of neutrino masses and mixing as CP violation in the lepton sector and the character of the neutrino mass spectrum.

CONCLUSION

Neutrinos are exceptional unique particles.

The neutrino history, which we partially followed in this paper, is very interesting and instructive. There were many wrong experiments in the history of the neutrino (like β-decay experiments on electron–neutrino correlation which favored S, T couplings in the fifties, first experiment on the search for $\pi \to e\nu$ decay, experiments from which the existence of a heavy neutrino with a mass of 17 keV followed in the beginning of the nineties, etc.) and wrong common opinions lasting for many years (like the general opinion that the neutrino is an undetectable particle in the thirties and forties, the general opinion that the neutrino is a massless particle in the fifties and sixties, etc.).

There are two unique properties of neutrinos which determine their importance and their problems:

1. Neutrinos have only weak interaction.
2. Neutrinos have very small masses.

Since neutrinos have only weak interaction, cross sections of interaction of neutrinos with nucleons are extremely small. This means that it is necessary to develop special methods of neutrino detection (large detectors which often are situated in underground laboratories in order to prevent cosmic-ray background, etc.). However, when methods of neutrino detection were developed, neutrinos became a unique instrument in the study of the sun (solar neutrino experiments allow us to obtain information about the central invisible region of the sun in which solar energy is produced in thermonuclear reactions), in the investigation of a mechanism of the Supernova explosion* (99% of the energy produced in a Supernova explosion is emitted in neutrinos), in establishing the quark structure of a nucleon (through the study of the deep inelastic processes $\nu_\mu(\bar{\nu}_\mu) + N \to \mu^-(\mu^+ + X)$, etc.

In the fifties, the majority of physicists believed that the neutrino was a massless particle. This was an important, constructive assumption. The theory of the two-component neutrino, which was based on this assumption, inspired the creation of the phenomenological V–A theory and later became part of the Standard Model of the electroweak interaction.

Neutrino masses are very small and it is very difficult to observe *effects of neutrino masses in the β decay and in other weak processes.* However, small neutrino masses and, correspondingly, mass-squared differences make possible the production (and detection) of the *coherent flavor neutrino states* ν_e, ν_μ, ν_τ and quantum-mechanical periodical transitions between different flavor neutrino states (neutrino oscillations). The observation of neutrino oscillations at large (macroscopic) distances allowed one to resolve small neutrino mass-squared differences.

The discovery of neutrino oscillations signifies a new era in neutrino physics, the era of investigation of neutrino properties. From the analysis of the

*On February 23, 1987 for the first time antineutrinos from Supernova SN1987A in the Large Magellanic Cloud were detected by Kamiokande, IMB, and Baksan detectors.

existing neutrino oscillation data, two mass-squared differences Δm^2_{23} and Δm^2_{12} and two mixing parameters $\sin^2\theta_{23}$ and $\tan^2\theta_{12}$ are determined with accuracies in the range 3–12%. The results of the first measurement of the parameter $\sin^2 2\theta_{13}$ was recently announced by the Daya Bay and RENO collaboration.

One of the most urgent problems which will be addressed in the next neutrino oscillation experiments are

1. CP violation in the lepton sector;

2. the character of the neutrino mass spectrum (normal or inverted?).

The «large» value of the angle θ_{13} obtained in the Daya Bay and RENO experiments will open the way for the investigation of these problems in the near years.

One of the most important problems of the physics of massive and mixed neutrinos is the problem of the nature of neutrinos with definite masses ν_i.

Are neutrinos with defined masses Dirac particles possessing conserved lepton number or truly neutral Majorana particles? The answer to this fundamental question can be obtained in experiments on the search for neutrinoless double β decay ($0\nu\beta\beta$ decay) of some even–even nuclei

$$(A, Z) \to (A, Z+2) + e^- + e^-. \tag{217}$$

This process is allowed only if the total lepton number is violated. If massive neutrinos are Majorana particles, $0\nu\beta\beta$ decay is the second-order process in G_F process with the exchange of the virtual neutrinos between neutron–proton–electron vertices. The matrix element of the process is proportional to the effective Majorana mass

$$m_{\beta\beta} = \sum_i U^2_{ei} m_i. \tag{218}$$

Many experiments on the search for the $0\nu\beta\beta$ decay of different nuclei were performed. No compelling evidence in favor of the process was found. The stringent lower bound for the half-life of the process was obtained in the Heidelberg–Moscow experiment [79] on the search for the decay

$$^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + e^- + e^-.$$

In this experiment the following lower bound was obtained:

$$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 1.9 \cdot 10^{25}\ \mathrm{y}.$$

Taking into account different calculations of the nuclear matrix element from this bound, it can be found

$$|m_{\beta\beta}| < 0.20\text{–}0.32\ \mathrm{eV}.$$

Let us notice that some participants of the Heidelberg–Moscow experiment claim the observation of the $0\nu\beta\beta$ decay of ^{76}Ge with half-life in the range

$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) = (1.30 - 3.55) \cdot 10^{25}$ y [80]. The estimated value of the effective Majorana mass is $|m_{\beta\beta}| \simeq 0.17{-}0.45$ eV. This result will be checked by the running ^{76}Ge GERDA experiment [81].

Future experiments on the search for the $0\nu\beta\beta$ decay will be sensitive to the value

$$|m_{\beta\beta}| \simeq \text{(a few) } 10^{-2} \text{ eV}$$

and can probe the Majorana nature of ν_i in the case of the inverted hierarchy of the neutrino masses

$$m_3 \ll m_1 < m_2. \tag{219}$$

Another fundamental question of the neutrino physics is

What is the value of the neutrino mass? From the data of neutrino oscillation experiments only the mass-squared differences can be determined. The absolute value of the neutrino mass m_β can be inferred from the investigation of β spectra. From the data of the Mainz [82] and Troitsk [83] tritium experiments, the following bound was obtained:

$$m_\beta < 2.3 \text{ eV},$$

where $m_\beta = \sqrt{\sum_i |U_{e1}|^2 m_i^2}$ is the «average» neutrino mass. The future tritium experiment KATRIN [84] will be sensitive to

$$m_\beta < 0.2 \text{ eV}.$$

Precision modern cosmology became an important source of information about absolute values of neutrino masses. Different cosmological observables (Large Scale Structure of the Universe, Gravitational Lensing of Galaxies, Primordial Cosmic Microwave Background, etc.) are sensitive to the sum of the neutrino masses $\sum_i m_i$. From the existing data the following bounds were obtained [85]:

$$\sum_i m_i < 0.2{-}1.3 \text{ eV}. \tag{220}$$

It is expected that future cosmological observables will be sensitive to the sum of neutrino masses in the range [86]

$$\sum_i m_i \simeq 0.05{-}0.6 \text{ eV}. \tag{221}$$

These future measurements (219), apparently, will probe the inverted neutrino mass hierarchy $\left(\sum_i m_i \simeq 0.1 \text{ eV}\right)$ and even the normal neutrino mass hierarchy

$$m_1 < m_2 \ll m_3. \tag{222}$$

The next question which needs to be answered

How many neutrinos with definite masses exist in nature? We considered the minimal scheme with three flavor neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ and, correspondingly, three massive neutrinos (ν_1, ν_2, ν_3). However, the number of massive light neutrinos can be more than three. In this case flavor neutrinos could oscillate into sterile states ν_s, which do not have the standard weak interaction.

For many years there existed an indication in favor of more than three light neutrinos with definite masses obtained in a short-baseline LSND experiment [87]. In this experiment, the $\bar{\nu}_\mu \to \bar{\nu}_e$ transition driven by $\Delta m^2 \simeq 1$ eV^2, which is much larger than the atmospheric mass-squared difference, was observed. Some indications in favor of more than three massive neutrinos were also obtained in the MiniBooNE and reactor experiments (see [88]). New short-baseline accelerator and reactor experiments are urgently needed. Such experiments are now at preparation. There are other questions connected with neutrinos which are now being actively discussed in the literature (neutrino magnetic moments, nonstandard interaction of neutrinos, etc.).

An explanation of small neutrino masses requires a new, beyond the Standard Model (Higgs) mechanism of neutrino mass generation. But what mechanism, what kind of new physics is required to explain small neutrino masses and peculiar neutrino mixing? This is at the moment an open question. Several mechanisms of neutrino mass generation were proposed in the literature. Apparently, the most plausible mechanism is the seesaw mechanism of the neutrino mass generation [66].

The seesaw mechanism is based on the assumption that the total lepton number L is violated at a large scale $M \simeq 10^{15}$ GeV. If this mechanism is realized, in this case neutrino masses are given in the form of products of electroweak masses and a very small factor v/M, which is the ratio of the electroweak scale $v \simeq 250$ GeV and a new scale M which characterizes the violation of L.

From the seesaw mechanism the following general consequences follow:

1. Neutrinos with definite masses ν_i are Majorana particles.

2. Neutrino masses are given by the seesaw formula

$$m_i \simeq y_i \frac{v^2}{M}.$$

The suppression factor v/M ensures the smallness of neutrino masses with respect to the masses of quarks and leptons.

3. Heavy Majorana particles, the seesaw partners of neutrinos, with masses which are characterized by M must exist. CP-violating decays of these particles in the early Universe are considered as a plausible source of the baryon asymmetry of the Universe (see [89]).

In order to reveal the true nature of neutrino masses and mixing, many new investigations must be performed. The history of neutrino is continuing. There are no doubts that new surprises, discoveries (and, possibly, Nobel Prizes) are ahead.

Bruno Pontecorvo is one of the fathers of neutrino physics. He made important, pioneer contributions to different aspects of physics of neutrino:

1. B. Pontecorvo proposed the first method of neutrino detection. Pontecorvo's radiochemical Cl–Ar method allowed one to discover and detect solar neutrinos.

2. B. Pontecorvo was the first who proposed idea of $\mu{-}e$ universality of the weak interqaction.

3. B. Pontecorvo proposed accelerator neutrino experiment which allowed one to discover muon neutrino.

4. B. Pontecorvo was a pioneer of the idea of neutrino oscillations. He proposed reactor neutrino oscillation experiments and predicted «solar neutrino puzzle». Together with collaborators he considered all possible schemes of neutrino mixing and proposed different experiments on the search for neutrino oscillations.

The discovery of neutrino oscillations was triumph of Bruno Pontecorvo who proposed neutrino oscillations and pursued the idea of oscillations for many years at the time when the general opinion favored massless nonoscillating neutrinos.

Bruno Pontecorvo was a great physicist. His ingenious intuition and ability to understand complicated problems in a clear and simple way were gifts of God. He devoted all his resources and great intellect to science. His main stimulus was *search for the truth*.

More than ten last years were for Bruno Pontecorvo years of courageous struggle against Parkinson illness. His love to physics and to neutrino helped him to overcome difficult problems of the illness. He never stopped to work, to think about neutrinos and to continue active life.

It is a pleasure for me to thank Walter Potzel for careful reading of the paper and numerous remarks and suggestions.

Appendix A
POSSIBLE SCHEMES OF NEUTRINO MIXING

In papers [55,57,61,62] (see also [63]) by B. Pontecorvo and his collaborators all possible schemes of neutrino mixing were developed. The scheme of neutrino mixing is determined by *a neutrino mass term*. There are three possible neutrino mass terms.

A.1. Dirac Mass Term. Let us introduce columns of left-handed flavor fields ν_{lL} and right-handed fields ν_{lR} $(l = e, \mu, \tau)$

$$\nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu'_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}. \tag{A.1}$$

A mass term is a Lorentz-invariant product of left-handed and right-handed fields. The Dirac neutrino mass term has the form

$$\mathcal{L}^{D}(x) = -\bar{\nu}'_L(x) M^D \nu'_R(x) + \text{h.c.} \tag{A.2}$$

Here M^D is nondiagonal complex 3×3 matrix.

An arbitrary nonsingular matrix M can be diagonalized by a biunitary transformation

$$M = U^\dagger m V. \tag{A.3}$$

Here U and V are unitary matrices and $m_{ik} = m_i \delta_{ik}$, $m_i > 0$ *.

From (A.2) and (A.3) for the neutrino mass term we find

$$\mathcal{L}^{\rm D}(x) = -\bar{\nu}_L(x)\, m \nu_R(x) + \text{h.c.} = -\bar{\nu}(x)\, m \nu(x) = -\sum_i m_i\, \bar{\nu}_i(x)\, \nu_i(x). \tag{A.4}$$

Here

$$\nu_L = U^\dagger \nu'_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}, \quad \nu_R = V^\dagger \nu'_R = \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix} \tag{A.5}$$

and $\nu = \nu_L + \nu_R$.

After the diagonalization of the matrix $M^{\rm D}$ we come to the standard expression for the neutrino mass term. From (A.4) and (A.5) it follows that $\nu_i(x)$ is the field of neutrinos with mass m_i and flavor neutrino fields $\nu_{lL}(x)$ are «mixtures» of left-handed components of fields of neutrinos with definite masses:

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x). \tag{A.6}$$

The fields $\nu_i(x)$ are complex (non-Hermitian) fields. There are no any constraints on them. The total Lagrangian is invariant under the following *phase transformation*:

$$\nu_i(x) \to {\rm e}^{i\Lambda}\, \nu_i(x), \quad l(x) \to {\rm e}^{i\Lambda}\, l(x), \tag{A.7}$$

where Λ is an arbitrary constant.

From invariance under the global gauge transformation (A.7) it follows that the total lepton number L, the same for all charged leptons e, μ, τ and all neutrinos ν_i, is conserved and that neutrinos ν_i and antineutrinos $\bar{\nu}_i$ are different

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*We will present here a simple derivation of this relation. Let us consider the matrix $MM^\dagger$. Taking into account that this is Hermitian matrix with positive eigenvalues, we have $MM^\dagger = Um^2U^\dagger$, where U is unitary matrix and m^2 is a diagonal matrix with positive diagonal elements. Obviously, we have $M = UmV^\dagger$, where $V^\dagger = m^{-1}U^\dagger M$. It is easy to prove that V is a unitary matrix. In fact, we have $V^\dagger V = m^{-1}U^\dagger MM^\dagger U m^{-1} = 1$.

particles which differ by the values of the conserved lepton number. We can choose

$$L(\nu_i) = -L(\bar{\nu}_i) = 1, \quad L(l^-) = -L(l^+) = 1. \tag{A.8}$$

Thus, in the case of the mass term (A.2) *neutrinos with definite masses are Dirac particles*. This is the reason why the mass term (A.2) is called the Dirac mass term.

Summarizing this subsection, we stress the following:

1. In order to build the Dirac mass term we need flavor left-handed fields ν_{lL} and (sterile) right-handed fields ν_{lR}.

2. The fields ν_{lL} and ν_{lR} are connected, correspondingly. with ν_{iL} and ν_{iR} by (different) unitary transformations.

3. Transitions in vacuum active to sterile neutrinos ($\nu_{lL} \to \bar{\nu}_{l'L}$) are forbidden by the conservation of the total lepton number L.

A.2. Majorana Mass Term. It is easy to prove that $(\nu_{lL})^c = C\bar{\nu}_{lL}^T$ and $(\nu_{lR})^c = C\bar{\nu}_{lR}^T$ (C is the matrix of the charge conjugation which satisfies the relations $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha$, $C^T = -C$) are right-handed and left-handed fields, correspondingly. In fact, $\nu_{L,R}$ are determined by the conditions

$$\gamma_5 \nu_{L,R} = \mp \nu_{L,R}. \tag{A.9}$$

We have

$$\gamma_5 (\nu_{L,R})^c = C(\bar{\nu}_{L,R}\gamma_5)^T = -C\overline{(\gamma_5 \nu_{L,R})}^T = \pm(\nu_{L,R})^c. \tag{A.10}$$

The neutrino mass term (a product of left-handed and right-handed fields) can have the following form:

$$\mathcal{L}^M(x) = -\frac{1}{2}\bar{\nu}'_L(x)\, M^M (\nu'_L(x))^c + \text{h.c.}, \tag{A.11}$$

where M^M is a complex nondiagonal matrix. We will prove now that M^M is a symmetrical matrix. In fact, we have*

$$\begin{aligned} \bar{\nu}'_L M^M (\nu'_L)^c = (\bar{\nu}'_L M^M C (\bar{\nu}'_L)^T)^T = \\ = -\bar{\nu}'_L C^T (M^M)^T (\bar{\nu}'_L)^T = \bar{\nu}'_L (M^M)^T C (\bar{\nu}'_L)^T. \end{aligned} \tag{A.12}$$

Thus, we have

$$M^M = (M^M)^T. \tag{A.13}$$

A complex symmetrical matrix can be diagonalized with the help of one unitary matrix. We have

$$M^M = U\ m\ U^T, \tag{A.14}$$

where $U^\dagger U = 1$ and $m_{ik} = m_i \delta_{ik}$, $m_i > 0$.

*Let us notice that in (A.12) we take into account the Fermi–Dirac statistics of fermion fields ν'_L.

From (A.11) and (A.14) for the neutrino mass term we obtain the following expression:

$$\mathcal{L}^M = -\frac{1}{2}\,\overline{(U^\dagger \nu'_L)}\, m\, (U^\dagger \nu'_L)^c + \text{h.c.} = -\frac{1}{2}\bar{\nu}^m m \nu^m. \tag{A.15}$$

Here

$$\nu^m = U^\dagger \nu'_L + (U^\dagger \nu'_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3. \end{pmatrix}. \tag{A.16}$$

From (A.15) and (A.16) we have

$$\mathcal{L}^M = -\frac{1}{2}\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i. \tag{A.17}$$

Thus, $\nu_i(x)$ is neutrino field with mass m_i. From (A.16) it follows that the field $\nu_i(x)$ satisfies the condition

$$\nu_i(x) = \nu_i^c(x) = C\bar{\nu}_i^T(x) \tag{A.18}$$

and is *the Majorana field*.

From (A.16) we conclude that

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x). \tag{A.19}$$

Thus, in the case of the neutrino mass term (A.11) flavor fields $\nu_{lL}(x)$ are mixtures of left-handed components of Majorana fields with definite masses.

If neutrino mass term (A.11) enters into the Lagrangian, there is no invariance under the global gauge transformation

$$\nu_{lL}(x) \to \mathrm{e}^{i\Lambda}\, \nu_{lL}(x), \quad l(x) \to \mathrm{e}^{i\Lambda}\, l(x). \tag{A.20}$$

This means that there is no conserved lepton number in this case. This is the reason why after the diagonalization of the mass term (A.11), we came to the fields of Majorana neutrinos with definite masses. If neutrino field satisfies the condition (A.18), in this case there is no notion of neutrino and antineutrino: neutrino and antineutrino are identical.

The mass term (A.11) is called the Majorana mass term. It provides *the minimal schemes of neutrino mixing*: only flavor neutrino fields $\nu_{lL}(x)$ enter into the Lagrangian. Let us notice that Majorana mass term is generated by the standard seesaw mechanism [66], which explains smallness of the neutrino masses.

A.3. Dirac and Majorana Mass Term. If we assume that in the mass term enter flavor left-handed fields $\nu_{lL}(x)$, right-handed fields $\nu_{lR}(x)$, and the lepton number L is not conserved, we come to the most general neutrino mass term

$$\mathcal{L}^{D+M} = -\frac{1}{2}\bar{\nu}'_L M_L^M (\nu'_L)^c - \bar{\nu}'_L\, M^D \nu'_R - \frac{1}{2}\,\overline{(\nu'_R)^c}\, M_R^M \nu'_R + \text{h.c.}, \qquad (A.21)$$

where M_L^M, M_R^M, and M^D are complex nondiagonal $3{\times}3$ matrices and columns ν'_L and ν'_R are given by (A.1).

From Fermi–Dirac statistics of neutrino fields $\nu'_L(x)$ and $\nu'_R(x)$ it follows that

$$M_L^M = (M_L^M)^T, \quad M_R^M = (M_R^M)^T. \qquad (A.22)$$

The first term of (A.21) is the left-handed Majorana mass term, the second term is the Dirac mass term and the third term is the right-handed Majorana mass term. It is obvious that if the mass term (A.21) enters into the total Lagrangian, in this case the total lepton number L is not conserved.

Let us consider the Dirac mass term. We have

$$\bar{\nu}'_L M^D \nu'_R = (\nu'_R)^T\, C^{-1} (M^D)^T\, C (\bar{\nu}'_L)^T = \overline{(\nu'_R)^c}\, (M^D)^T (\nu'_L)^c. \qquad (A.23)$$

Taking into account this relation, we can right down the Dirac and Majorana mass term (A.21) in the form

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\bar{\nu}_L M^{\mathrm{M+D}} (\nu_L)^c + \text{h.c.} \qquad (A.24)$$

Here

$$\nu_L = \begin{pmatrix} \nu'_L \\ (\nu'_R)^c \end{pmatrix}, \qquad (A.25)$$

and

$$M^{M+D} = \begin{pmatrix} M_L^M & M^D \\ (M^D)^T & M_R^M \end{pmatrix}. \qquad (A.26)$$

From (A.22) and (A.26) it follows that $M^{\mathrm{M+D}}$ is a symmetrical 6×6 matrix. We have

$$M^{M+D} = U m U^T, \qquad (A.27)$$

where U is a unitary 6×6 matrix and $m_{ik} = m_i \delta_{ik}$. From (A.27) for the Dirac and Majorana mass term we obtain the following expression:

$$\mathcal{L}^{\mathrm{D+M}}(x) = -\frac{1}{2}\bar{\nu}^m(x)\, m \nu^m(x) = -\frac{1}{2}\sum_{i=1}^{6} m_i \bar{\nu}_i(x)\, \nu_i(x), \qquad (A.28)$$

where

$$\nu^m(x) = U^\dagger \nu_L(x) + (U^\dagger \nu_L(x))^c = \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \\ \cdot \\ \cdot \\ \cdot \\ \nu_6(x). \end{pmatrix}. \tag{A.29}$$

From (A.29) it is obvious that the field $\nu_i(x)$ satisfies the Majorana condition

$$\nu_i(x) = \nu_i^c(x) = C\bar{\nu}_i^T(x). \tag{A.30}$$

Thus the field $\nu_i(x)$ is the field of the Majorana neutrino with mass m_i.

From the relation (A.30) we have

$$\nu_L(x) = U\nu_L^m(x). \tag{A.31}$$

From this relation it follows that in the case of the Dirac and Majorana mass term we have the following generalized neutrino mixing relations:

$$\nu_{lL}(x) = \sum_{i=1}^{6} U_{li}\nu_{iL}(x), \quad (\nu_{lR}(x))^c = \sum_{i=1}^{6} U_{\bar{l}i}\nu_{iL}(x). \tag{A.32}$$

Thus, in general, flavor fields $\nu_{lL}(x)$ are combinations of left-handed components of six Majorana fields. These six components are connected by a unitary transformations with sterile fields $(\nu_{lR}(x))^c$.

The famous seesaw mechanism of neutrino mass generation [66] is based on the Dirac and Majorana mass term. In the seesaw case, in the mass spectrum of the Majorana particles there are three light masses m_i and three very heavy masses M_i ($M_i \gg m_i$, $i = 1, 2, 3$).

If all neutrino masses are small, in this case transitions of flavor neutrinos ν_e, ν_μ, ν_τ into sterile states ν_s are possible. Let us notice that at the moment there exist experimental indications ([87] and others) in favor of such transitions. These indications will be checked in future experiments.

Appendix B
ON THE CALCULATION OF THE VACUUM TRANSITION PROBABILITY

In this Appendix, we will present a simple method of the calculation of the probability of transition between different neutrinos in vacuum.

B.1. Standard Expression for the Vacuum Neutrino Transition Probability. We will derive first the standard expression for the probability of the neutrino transition in vacuum.

The probability of the transition $\nu_\alpha \to \nu_{\alpha'}$ during the time t is given by the following general expression ($\alpha, \alpha' = e, \mu, \tau, s_1, \ldots$, the index p is fixed).

The probability of the transition $\nu_\alpha \to \nu_{\alpha'}$ for the time t is given by the following general expression:

$$P(\nu_\alpha \to \nu_{\alpha'}) = \Big| \sum_i U_{\alpha' i}\, \mathrm{e}^{-iE_i t}\, U^*_{\alpha i} \Big|^2. \tag{B.1}$$

Here ν_α and ν'_α could be flavor or sterile neutrinos ($\alpha, \alpha' = e, \mu, \tau, s_1, \ldots$). From this expression we obviously have*

$$P(\nu_\alpha \to \nu_{\alpha'}) = \sum_i |U_{\alpha' i}|^2 |U_{\alpha i}|^2 + 2\,\mathrm{Re} \sum_{i>k} U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k}\, \mathrm{e}^{-i(E_i - E_k)t}. \tag{B.2}$$

We can rewrite this expression in a different form. From the unitarity of the matrix U, we have

$$\sum_i U_{\alpha' i} U^*_{\alpha i} = \delta_{\alpha' \alpha}. \tag{B.3}$$

From (B.3) we obtain the following relation:

$$\sum_i |U_{\alpha' i}|^2 |U_{\alpha i}|^2 + 2\,\mathrm{Re} \sum_{i>k} U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k} = \delta_{\alpha' \alpha}. \tag{B.4}$$

For the ultrarelativistic neutrinos we have

$$E_i - E_k \simeq \frac{\Delta m^2_{ki} L}{2E}, \tag{B.5}$$

where $\Delta m^2_{ki} = m_i^2 - m_k^2$ and $L \simeq t$ is the source-detector distance. From (B.2), (B.4), and (B.2) we find the following expression for the transition probability:

$$P(\nu_\alpha \to \nu_{\alpha'}) = \delta_{\alpha' \alpha} - 2\,\mathrm{Re} \sum_{i>k} U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k} \left[1 - \exp\left(-i \frac{\Delta m^2_{ki} L}{2E} \right) \right]. \tag{B.6}$$

*We used the following relation:

$$\Big| \sum_i a_i b_i \Big|^2 = \sum_{i,k} a_i a_k^* b_i b_k^* = \sum_i |a_i|^2 |b_i|^2 + 2\,\mathrm{Re} \sum_{i>k} a_i a_k^* b_i b_k^*.$$

In order to obtain $\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}$ transition probability we must perform in (B.6) the following change: $U_{\alpha i} \to U^*_{\alpha i}$. Finally we have

$$P\left(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_{\alpha'}\right) = \delta_{\alpha'\alpha} - 2\sum_{i>k} \operatorname{Re} U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k} \left(1 - \cos\frac{\Delta m^2_{ki} L}{2E}\right) \pm$$
$$\pm 2\sum_{i>k} \operatorname{Im} U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k} \sin\frac{\Delta m^2_{ki} L}{2E}. \quad \text{(B.7)}$$

Let us notice that in order to obtain from (B.7), for example, tree-neutrino transition probabilities, additional relations, based on the unitarity of the mixing matrix, must be used. We will consider here more simple and direct way of the derivation of the neutrino (antineutrino) transition probabilities. In relations we will derive, the unitarity of the mixing matrix will be fully implored. No additional relations will be required. Dependence on character of the neutrino mass spectrum will be clearly visible.

B.2. Alternative Expression for the Transition Probability. The expression (B.1) for the transition probability can be written in the form

$$P(\nu_\alpha \to \nu_{\alpha'}) = \left|\sum_i U_{\alpha' i}\, \mathrm{e}^{-i(E_i - E_p)t}\, U^*_{\alpha i}\right|^2, \quad \text{(B.8)}$$

where p is arbitrary fixed index. Further, taking into account the unitarity relation (B.3), we can rewrite the transition probability in the form

$$P(\nu_\alpha \to \nu_{\alpha'}) = \left|\delta_{\alpha'\alpha} + \sum_i U_{\alpha' i} \left[\exp\left(-i\frac{\Delta m^2_{pi} L}{2E}\right) - 1\right] U^*_{\alpha i}\right|^2. \quad \text{(B.9)}$$

It is obvious that due to the factor $\left[\exp\left(-i\frac{\Delta m^2_{pi} L}{2E}\right) - 1\right]$, the index i runs over values $i \neq p$.

From (B.9) we find

$$P(\nu_\alpha \to \nu_{\alpha'}) = \delta_{\alpha'\alpha} - 2\sum_i |U_{\alpha i}|^2 (\delta_{\alpha'\alpha} = |U_{\alpha' i}|^2)(1 - \cos 2\Delta_{pi}) +$$
$$+ 2\sum_{i>k} \operatorname{Re} U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k} (1 + \cos 2(\Delta_{pi} - \Delta_{pk}) - \cos 2\Delta_{pi} - \cos 2\Delta_{pk}) +$$
$$+ 2\sum_{i>k} \operatorname{Im} U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k} (\sin 2(\Delta_{pi} - \Delta_{pk}) - \sin 2\Delta_{pi} + \sin 2\Delta_{pk}),$$
$$\text{(B.10)}$$

where

$$\Delta_{pi} = \frac{\Delta m^2_{pi} L}{4E}. \tag{B.11}$$

The expression (B.10) can be easily simplified if we use the following relations:

$$1 - \cos 2(a-b) - \cos 2a - \cos 2b = \\ = 2\cos(a-b)(\cos(a-b) - \cos(a+b)) = 4\cos(a-b)\sin a \sin b \tag{B.12}$$

and

$$\sin 2(a-b) - \sin 2a + \sin 2b = \\ = 2\sin(a-b)(\cos(a-b) - \cos(a+b)) = 4\sin(a-b)\sin a \sin b, \tag{B.13}$$

where a and b are arbitrary angles.

Finally, for probability of the transition $\nu_\alpha \to \nu_{\alpha'}$ ($\bar\nu_\alpha \to \bar\nu_{\alpha'}$), we find the following general expression:

$$P\left(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_{\alpha'}\right) = \delta_{\alpha'\alpha} - 4\sum_i |U_{\alpha i}|^2 (\delta_{\alpha'\alpha} - |U_{\alpha' i}|^2)\ \sin^2 \Delta_{pi} + \\ + 8\sum_{i>k} \mathrm{Re}\, U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k} \cos(\Delta_{pi} - \Delta_{pk}) \sin\Delta_{pi} \sin\Delta_{pk} \pm \\ \pm 8\sum_{i>k} \mathrm{Im}\, U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k} \sin(\Delta_{pi} - \Delta_{pk}) \sin\Delta_{pi} \sin\Delta_{pk}. \tag{B.14}$$

B.3. Two-Neutrino Mixing. Let us consider the simplest case of the two-neutrino mixing

$$\nu_\alpha = \sum_{i-1,2} U_{\alpha i} \nu_{iL}, \tag{B.15}$$

where the matrix U has the form

$$U = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix}, \tag{B.16}$$

where θ is the mixing angle.

Let us label neutrino masses in such a way that $m_1 < m_2$ and choose $p = 1$. In this case, indices i, k take the value 2 and there are no $i > k$ terms in (B.14). We have

$$P(\nu_\alpha \to \nu_{\alpha'}) = P(\bar\nu_\alpha \to \bar\nu_{\alpha'}) = \\ = \delta_{\alpha'\alpha} - 2|U_{\alpha 2}|^2 (\delta_{\alpha'\alpha} - |U_{\alpha' 2}|^2) \left(1 - \cos\frac{\Delta m^2_{12} L}{2E}\right). \tag{B.17}$$

For $\alpha' \neq \alpha$, we find the following transition probability:

$$P(\nu_\alpha \to \nu_{\alpha'}) = P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) = 2|U_{\alpha 2}|^2 |U_{\alpha' 2}|^2 \left(1 - \cos\frac{\Delta m_{12}^2 L}{2E}\right) = \\ = \frac{1}{2}\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m_{12}^2 L}{2E}\right). \quad \text{(B.18)}$$

From this relation it follows that in the two-neutrino case the CP violation in the lepton sector cannot be revealed.

For the probability of ν_α ($\bar{\nu}_\alpha$) to survive from (B.17), we find

$$P(\nu_\alpha \to \nu_\alpha) = P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) = 1 - 2|U_{\alpha 2}|^2 (1 - |U_{\alpha 2}|^2)\left(1 - \cos\frac{\Delta m_{12}^2 L}{2E}\right) = \\ = 1 - \frac{1}{2}\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m_{12}^2 L}{2E}\right). \quad \text{(B.19)}$$

From the unitarity of the mixing matrix U, it follows that

$$|U_{\alpha 2}|^2 = 1 - |U_{\alpha' 2}|^2 \quad (\alpha \neq \alpha'). \quad \text{(B.20)}$$

We conclude from (B.18) and (B.19) that in the two-neutrino case the following relations hold:

$$P\left(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha\right) = P\left(\overset{(-)}{\nu}_{\alpha'} \to \overset{(-)}{\nu}_{\alpha'}\right) \quad (\alpha \neq \alpha'). \quad \text{(B.21)}$$

B.4. Three-Neutrino Mixing. If the numbers of the flavor and massive neutrinos are equal, in this case

$$\nu_\alpha = \sum_{i-1}^{3} U_{\alpha i}\nu_{iL}, \quad \alpha = e, \mu, \tau, \quad \text{(B.22)}$$

where U is the PMNS mixing matrix.

In the case of the three-neutrino mixing, two neutrino mass spectra are possible:

1. Normal spectrum (NS)

$$m_1 < m_2 < m_3, \quad \Delta m_{12}^2 \ll \Delta m_{23}^2. \quad \text{(B.23)}$$

2. Inverted spectrum (IS)

$$m_3 < m_1 < m_2, \quad \Delta m_{12}^2 \ll |\Delta m_{13}^2|. \quad \text{(B.24)}$$

We will introduce the atmospheric and solar mass-squared differences in the following way:

$$\Delta m_{23}^2(\text{NS}) = |\Delta m_{13}^2|\,(\text{IS}) = \Delta m_A^2, \quad \Delta m_{12}^2\,(\text{NS}) = \Delta m_{12}^2\,(\text{IS}) = \Delta m_S^2. \quad \text{(B.25)}$$

Let us consider the normal spectrum. We will choose $p = 2$. Thus, indices i, k can take values $1, 3$. In the last two terms of (B.14) we have $i > k$. The only possibility for these terms is $k = 1$, $i = 3$. Thus, from (B.14) for the $\nu_\alpha \to \nu_{\alpha'}(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'})$ transition probabilities we find the following expressions:

$$P^{\rm NS}\left(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_{\alpha'}\right) = \delta_{\alpha'\alpha} - 4|U_{\alpha 3}|^2(\delta_{\alpha'\alpha} - |U_{\alpha' 3}|^2)\sin^2\Delta_A - \\ - 4|U_{\alpha 1}|^2(\delta_{\alpha'\alpha} - |U_{\alpha' 1}|^2)\sin^2\Delta_S - \\ - 8\,{\rm Re}\,U_{\alpha' 3}U^*_{\alpha 3}U^*_{\alpha' 1}U_{\alpha 1}\cos(\Delta_A + \Delta_S)\sin\Delta_A\sin\Delta_S \mp \\ \mp 8\,{\rm Im}\,U_{\alpha' 3}U^*_{\alpha 3}U^*_{\alpha' 1}U_{\alpha 1}\sin(\Delta_A + \Delta_S)\sin\Delta_A\sin\Delta_S, \quad \text{(B.26)}$$

where

$$\Delta_A = \frac{\Delta m_A^2 L}{4E}, \quad \Delta_S = \frac{\Delta m_S^2 L}{4E}. \quad \text{(B.27)}$$

In the case of the inverted neutrino mass spectrum we choose $p = 1$. Indices i, k take values $2, 3$ and for the last two terms of (B.14) we have $i = 3, k = 2$. The transition probabilities are given by the following expressions:

$$P^{\rm IS}\left(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_{\alpha'}\right) = \delta_{\alpha'\alpha} - 4|U_{\alpha 3}|^2(\delta_{\alpha'\alpha} - |U_{\alpha' 3}|^2)\sin^2\Delta_A - \\ - 4|U_{\alpha 2}|^2(\delta_{\alpha'\alpha} - |U_{\alpha' 2}|^2)\sin^2\Delta_S - \\ - 8\,{\rm Re}\,U_{\alpha' 3}U^*_{\alpha 3}U^*_{\alpha' 2}U_{\alpha 2}\cos(\Delta_A + \Delta_S)\sin\Delta_A\sin\Delta_S \pm \\ \pm 8\,{\rm Im}\,U_{\alpha' 3}U^*_{\alpha 3}U^*_{\alpha' 2}U_{\alpha 2}\sin(\Delta_A + \Delta_S)\sin\Delta_A\sin\Delta_S. \quad \text{(B.28)}$$

In the standard parameterization, the PMNS mixing matrix U has the form

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}\,{\rm e}^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}\,{\rm e}^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}\,{\rm e}^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}\,{\rm e}^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}\,{\rm e}^{i\delta} & c_{13}c_{23} \end{pmatrix}, \quad \text{(B.29)}$$

where $c_{12} = \cos\theta_{12}$, $s_{12} = \sin\theta_{12}$, etc.

We will consider now transition probabilities in the leading approximation.

B.4.1. Leading Approximation. There are two small neutrino oscillation parameters:

$$\frac{\Delta m_S^2}{\Delta m_A^2} \simeq 3 \cdot 10^{-2}, \quad \sin^2\theta_{13} \simeq 2.4 \cdot 10^{-2}. \quad \text{(B.30)}$$

In atmospheric region of the parameter $\dfrac{L}{E}$ $\left(\dfrac{\Delta m_A^2 L}{2E} \gtrsim 1\right)$ effects of neutrino oscillations are large. In the first approximation we can neglect contributions of Δm_S^2 and $\sin^2\theta_{13}$ into neutrino transition probabilities. From (B.26), (B.28), and (B.29) for the probability of ν_μ ($\bar{\nu}_\mu$) to survive (for both neutrino mass spectra), we obtain the following expression:

$$P(\nu_\mu \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_\mu) \simeq 1 - 2|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)\left(1 - \cos\frac{\Delta m_A^2 L}{2E}\right) \simeq$$
$$\simeq 1 - \frac{1}{2}\sin^2 2\theta_{23}\left(1 - \cos\frac{\Delta m_A^2 L}{2E}\right). \quad \text{(B.31)}$$

For $\nu_\mu \to \nu_\tau$ ($\bar{\nu}_\mu \to \bar{\nu}_\tau$) appearance probability in the leading approximation, we have

$$P(\nu_\mu \to \nu_\tau) = P(\bar{\nu}_\mu \to \bar{\nu}_\tau) \simeq 2|U_{\mu 3}|^2|U_{\tau 3}|^2)\left(1 - \cos\frac{\Delta m_A^2 L}{2E}\right) \simeq$$
$$\simeq \frac{1}{2}\sin^2 2\theta_{23}\left(1 - \cos\frac{\Delta m_A^2 L}{2E}\right). \quad \text{(B.32)}$$

In this approximation we have $P(\nu_\mu \to \nu_e) \simeq 0$ and

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - P(\nu_\mu \to \nu_e). \quad \text{(B.33)}$$

Thus, in the atmospheric region, predominantly two-neutrino $\nu_\mu \rightleftarrows \nu_\tau$ oscillations take place.

Let us consider now $\bar{\nu}_e \to \bar{\nu}_e$ transition in the reactor KamLAND region $\left(\frac{\Delta m_S^2 L}{2E} \gtrsim 1\right)$. Neglecting contribution of $\sin^2\theta_{13}$, we have

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - 2|U_{e1}|^2(1 - |U_{e1}|^2)\left(1 - \cos\frac{\Delta m_S^2 L}{2E}\right) =$$
$$= 1 - \frac{1}{2}\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m_S^2 L}{2E}\right). \quad \text{(B.34)}$$

For appearance probabilities we find

$$P(\bar{\nu}_e \to \bar{\nu}_\mu) \simeq 2|U_{e1}|^2|U_{\mu 1}|^2)\left(1 - \cos\frac{\Delta m_S^2 L}{2E}\right) \simeq$$
$$\simeq \frac{1}{2}\sin^2 2\theta_{12}\cos^2\theta_{23}\left(1 - \cos\frac{\Delta m_S^2 L}{2E}\right) \quad \text{(B.35)}$$

and

$$P(\bar{\nu}_e \to \bar{\nu}_\tau) \simeq 2|U_{e1}|^2|U_{\tau 1}|^2)\left(1 - \cos\frac{\Delta m_S^2 L}{2E}\right) \simeq$$
$$\simeq \frac{1}{2}\sin^2 2\theta_{12}\sin^2\theta_{23}\left(1 - \cos\frac{\Delta m_S^2 L}{2E}\right). \quad \text{(B.36)}$$

We have

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - P(\bar{\nu}_e \to \bar{\nu}_\mu) - P(\bar{\nu}_e \to \bar{\nu}_\tau) \tag{B.37}$$

and

$$\frac{P(\bar{\nu}_e \to \bar{\nu}_\tau)}{P(\bar{\nu}_e \to \bar{\nu}_\mu)} \simeq \tan^2\theta_{23} \simeq 1. \tag{B.38}$$

Thus, in the reactor Kamland region $\bar{\nu}_e \rightleftarrows \bar{\nu}_\tau$ and $\bar{\nu}_e \rightleftarrows \bar{\nu}_\mu$ oscillations take place.

B.4.2. $\bar{\nu}_e \to \bar{\nu}_e$ Survival Probability. From (B.26) we will obtain $\bar{\nu}_e \to \bar{\nu}_e$ survival probabilities for both neutrino mass spectra. For the normal and inverted neutrino mass spectrum we have, correspondingly,

$$P^{\rm NS}(\bar{\nu}_e \to \bar{\nu}_e) = 1-4|U_{e3}|^2(1-|U_{e3}|^2)\sin^2\Delta_A - 4|U_{e1}|^2(1-|U_{e1}|^2)\sin^2\Delta_S - \\ - 8|U_{e3}|^2|U_{e1}|^2\cos(\Delta_A+\Delta_S)\sin\Delta_A\sin\Delta_S \tag{B.39}$$

and

$$P^{\rm IS}(\bar{\nu}_e \to \bar{\nu}_e) = 1-4|U_{e3}|^2(1-|U_{e3}|^2)\sin^2\Delta_A - 4|U_{e2}|^2(1-|U_{e2}|^2)\sin^2\Delta_S - \\ - 8|U_{e3}|^2|U_{e2}|^2\cos(\Delta_A+\Delta_S)\sin\Delta_A\sin\Delta_S. \tag{B.40}$$

In the standard parameterization of the PMNS mixing matrix we obtain the following expressions:

$$P^{\rm NS}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1-\cos\frac{\Delta m_A^2 L}{2E}\right) - \\ - \frac{1}{2}\cos^2\theta_{13}(\sin^2 2\theta_{12} + 4\sin^2\theta_{13}\cos^4\theta_{12})\left(1-\cos\frac{\Delta m_S^2 L}{2E}\right) - \\ - 2\sin^2 2\theta_{13}\cos^2\theta_{12}\cos\frac{(\Delta m_A^2+\Delta m_S^2)L}{4E}\sin\frac{\Delta m_A^2 L}{4E}\sin\frac{\Delta m_S^2 L}{4E} \tag{B.41}$$

and

$$P^{\rm IS}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1-\cos\frac{\Delta m_A^2 L}{2E}\right) - \\ - \frac{1}{2}\cos^2\theta_{13}(\sin^2 2\theta_{12} + 4\sin^2\theta_{13}\sin^4\theta_{12})\left(1-\cos\frac{\Delta m_S^2 L}{2E}\right) - \\ - 2\sin^2 2\theta_{13}\sin^2\theta_{12}\cos\frac{(\Delta m_A^2+\Delta m_S^2)L}{4E}\sin\frac{\Delta m_A^2 L}{4E}\sin\frac{\Delta m_S^2 L}{4E}. \tag{B.42}$$

B.4.3. $\nu_\mu \to \nu_e$ ($\bar{\nu}_\mu \to \bar{\nu}_e$) Transition Probabilities. In experiments with accelerator neutrinos (T2K and others) appearance of electron neutrinos (antineutrinos) is searched for. In this subsection, we will obtain $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ vacuum transition probabilities. From (B.26) and (B.28) we find

$$P^{\rm NS}\left(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e\right) = 4|U_{e3}|^2|U_{\mu3}|^2)\sin^2\Delta_A + 4|U_{e1}|^2|U_{\mu1}|^2)\sin^2\Delta_S - \\ - 8\,{\rm Re}\,U_{e3}U^*_{\mu3}U^*_{e1}U_{\mu1}\cos(\Delta_A+\Delta_S)\sin\Delta_A\sin\Delta_S \mp \\ \mp 8\,{\rm Im}\,U_{e3}U^*_{\mu3}U^*_{e1}U_{\mu1}\sin(\Delta_A+\Delta_S)\sin\Delta_A\sin\Delta_S \quad \text{(B.43)}$$

and

$$P^{\rm IS}\left(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e\right) = 4|U_{e3}|^2|U_{\mu3}|^2)\sin^2\Delta_A + 4|U_{e2}|^2|U_{\mu2}|^2)\sin^2\Delta_S - \\ - 8\,{\rm Re}\,U_{e3}U^*_{\mu3}U^*_{e2}U_{\mu2}\cos(\Delta_A+\Delta_S)\sin\Delta_A\sin\Delta_S \pm \\ \pm 8\,{\rm Im}\;U_{e3}U^*_{\mu3}U^*_{e2}U_{\mu2}\sin(\Delta_A+\Delta_S)\sin\Delta_A\sin\Delta_S. \quad \text{(B.44)}$$

Taking into account the standard parameterization of the PMNS mixing matrix, we have

$$P^{\rm NS}\left(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e\right) = \sin^2 2\theta_{13}s^2_{23}\sin^2\frac{\Delta m^2_A L}{4E} + \\ + (\sin^2 2\theta_{12}c^2_{13}c^2_{23} + \sin^2 2\theta_{13}c^4_{12}s^2_{23} + Kc^2_{12}\cos\delta)\sin^2\frac{\Delta m^2_S L}{4E} + \\ + (2\sin^2 2\theta_{13}s^2_{23}c^2_{12} + K\cos\delta)\cos\frac{(\Delta m^2_A + \Delta m^2_S)L}{4E}\sin\frac{\Delta m^2_A L}{4E}\sin\frac{\Delta m^2_S L}{4E} \mp \\ \mp K\sin\delta\sin\frac{(\Delta m^2_A + \Delta m^2_S)L}{4E}\sin\frac{\Delta m^2_A L}{4E}\sin\frac{\Delta m^2_S L}{4E}. \quad \text{(B.45)}$$

Here

$$K = \sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}c_{13}. \quad \text{(B.46)}$$

In the case of the inverted neutrino mass spectrum we find the following expressions for the transition probabilities:

$$P^{\rm IS}\left(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e\right) = \sin^2 2\theta_{13}s^2_{23}\sin^2\frac{\Delta m^2_A L}{4E} + \\ + (\sin^2 2\theta_{12}c^2_{13}c^2_{23} + \sin^2 2\theta_{13}s^4_{12}s^2_{23} - Ks^2_{12}\cos\delta)\sin^2\frac{\Delta m^2_S L}{4E} + \\ + (2\sin^2 2\theta_{13}s^2_{23}s^2_{12} - K\cos\delta)\cos\frac{(\Delta m^2_A + \Delta m^2_S)L}{4E}\sin\frac{\Delta m^2_A L}{4E}\sin\frac{\Delta m^2_S L}{4E} \mp \\ \mp K\sin\delta\sin\frac{(\Delta m^2_A + \Delta m^2_S)L}{4E}\sin\frac{\Delta m^2_A L}{4E}\sin\frac{\Delta m^2_S L}{4E}. \quad \text{(B.47)}$$

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