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ONE NEEDS POSITIVE SIGNATURES FOR DETECTION OF DARK MATTER

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One believes there is a huge amount of dark matter particles in our Galaxy which manifest themselves only gravitationally. There is a big challenge to prove their existence in a laboratory experiment. To this end, it is not sufficient to fight only for the best exclusion curve, one has to see an annual recoil spectrum modulation — the only available positive direct dark matter detection signature. A necessity to measure the recoil spectra is stressed.

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Galactic Dark Matter (DM) particles do not emit (or reflect) any detectable electromagnetic radiation and manifest themselves only gravitationally by affecting other astrophysical objects. According to the estimates based on a detailed model of our Galaxy [1], the local density of DM (nearby the Solar System) amounts to about $\rho^{\rm DM}_{\rm local} \simeq 0.3$ GeV/cm$^3 \simeq 5 \cdot 10^{-25}$ g/cm^3 (see also recent reviews [2, 3]). The local flux of DM particles χ is expected to be $\Phi^{\rm DM}_{\rm local} \simeq \dfrac{100\ {\rm GeV}}{m_\chi} \cdot 10^5\ {\rm cm}^{-2} \cdot {\rm s}^{-1}$, where m_χ is the DM particle mass. This value is often considered as a promising basis for direct laboratory dark matter search experiments.

The problem of the DM in the Universe is a challenge for modern physics and experimental technology. To solve the problem, i.e., *at least* to detect the DM particles, one simultaneously needs to apply the front-end knowledge of modern particle physics, astrophysics, cosmology and nuclear physics and to develop and use over long time extremely high-sensitive experimental setups and complex data analysis methods (see, for example, recent discussion in [4]).

Weakly Interacting Massive Particles (WIMPs) are among the most popular candidates for the relic DM. These particles are nonbaryonic and there is no room for them in the Standard Model of particle physics (SM). The Lightest Supersymmetric (SUSY) Particle (LSP), neutralino (being massive, neutral and stable), is currently often assumed to be a favorite WIMP dark matter particle.

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The nuclear recoil energy due to elastic WIMP–nucleus scattering is the main quantity to be measured by a terrestrial detector in direct DM detection laboratory experiments [5]. Detection of the very rare events of such WIMP interactions is a quite complicated task because of very weak WIMP coupling with ordinary matter. The rates expected in the SUSY models range from 10 to 10^{-7} events per kilogram detector material a day [6–13]. Moreover, for WIMP masses between a few GeV/c^2 and 1 TeV/c^2, the energy deposited by the recoil nucleus is less than 100 keV. Therefore, in order to be able to detect a WIMP, an experiment with a low-energy threshold and an extremely low radioactive background is required. Furthermore, to certainly detect a WIMP, one has to unambiguously register some positive signature of WIMP–nucleus interactions (directional recoil or annual signal modulation) [7, 14]. This means one has to perform a stable measurement with a detector of large target mass during 3–5 years under extremely low radioactive background conditions. There are also some other complications discussed recently in [2, 4].

Till now, only the DAMA (DArk MAtter) Collaboration [4, 15, 16] has certainly observed the first evidence for the DM signal due to model-independent registration of the predicted annual modulation of specific shape and amplitude due to the combined motions of the Earth and the Sun around the galactic center [14]. This experiment has released a total exposure of 1.17 $\mathrm{t} \cdot \mathrm{y}$ over 13 annual cycles, obtaining positive model-independent evidence for the presence of DM particles in the galactic halo at 8.9σ CL [4, 15, 16].

Although there are other experiments like EDELWEISS, CDMS, XENON, CRESST, etc., which give sensitive exclusion curves, no one of them at present has the sensitivity to look for the modulation effect. Due to the relatively small target masses and short running times, these experiments are unable to see a positive annual modulation signature of the WIMP interactions. Unfortunately, some other experiments with targets of much larger mass (mostly NaI) were also unable to register the positive signature due to not good enough background conditions [17–19].

Despite the strong and reliable belief of the DAMA Collaboration in the observation of the annual modulation signature, it is obvious that such a serious claim should be verified by at least another one completely independent experiment.

If one wants to confirm (more important, if one wants to reject) the DAMA result, one should perform a new experiment which would have the same or better sensitivity to the annual modulation signature (and also it would be reasonable to locate this new setup in another low-background underground laboratory). In particular, search for the modulation could be carried out by new-generation experiments with high-purity germanium detectors of large enough mass, perhaps, with both spin ^{73}Ge and spinless natural Ge [20]. It is interesting that recently the CoGeNT experiment with a germanium detector has reported some preliminary positive indication of the annual modulation [21].

Together with necessary figthing against backgrounds, the main direction in development of new-generation DM detectors concerns remarkable enlargement of the target mass to allow observing these positive signatures and thus detecting DM and proving or disproving the DAMA claim. In particular, an enlarged version of the EDELWEISS setup with 40 kg bolometric Ge detectors [22] together with, perhaps, SuperCDMS [23,24], as well as the enlarged ZEPLIN [25] or KIMS [26] experiments might become sensitive to the annual modulation in the future.

To estimate the expected direct detection rate for these WIMPs (in particular, neutralinos) any SUSY-like model or some measured data, for example, from the DAMA experiment [27], can be used. On this basis, the WIMP–proton and WIMP–neutron spin $\sigma_{\rm SD}^{p,n}(0)$ and scalar $\sigma_{\rm SI}^{p,n}(0)$ cross sections at zero-momentum transfer can be calculated (see Appendix). These calculations are usually compared with measurements, which (with the only exception of the DAMA result) are presented in the form of exclusion curves — upper limits of the cross section as functions of the WIMP mass. In the case of nonobservation of any DM signal, the exclusion curve simply reflects the sensitivity of a given direct DM search experiment and potentially allows one to constrain some version of the SUSY-like theory if the curve is sensitive enough. Therefore, the best exclusion curve is currently a clear aim of almost all dark matter search experiments (DAMA/LIBRA and CoGeNT are the only exceptions). The main competition between the experiments is in the field of these exclusion curves.

Before 2000, all exclusion curves were evaluated mainly in the one-coupling dominance approach (when only one cross-section limit was defined from measurements for fixed WIMP mass), which gave slightly pessimistic (for spin-nonzero target experiments), but universal limits for all experiments. One would say that the competition between the DM experiments was honest. The predictions from SUSY-like models were, in general, far from being reached by the data.

Mainly, after the paper [28] was published in 2000 (and as well after the DAMA evidence [15]), a new kind of exclusion curves appeared. In particular, for the first time these curves were obtained for the spin-dependent WIMP–nucleon cross-section limits when nonzero subdominant spin WIMP–nucleon contributions were also taken into account [29, 30]. This procedure obviously improved the quality of the exclusion curves. Therefore, a direct comparison of the old-fashioned exclusion curve with the new one could, in principle, bring one to a wrong conclusion about better sensitivity of more recent experiments. There is generally possible incorrectness in the direct comparison of the exclusion curves for the WIMP-proton (neutron) spin-dependent cross section obtained with and without the nonzero WIMP-neutron (proton) spin-dependent contribution. Furthermore, the above-mentioned incorrectness concerns, to a great extent, the direct comparison of the spin-dependent exclusion curves obtained with and without nonzero spin-independent contributions [15, 31]. Taking into account both spin couplings a_p and a_n but ignoring the scalar coupling c_0 (see Appendix for definitions), one can easily arrive at a misleading conclusion especially for not very light target nuclei when it is not obvious that (both) spin couplings dominate over the scalar one. To be consistent, one has to use the mixed spin–scalar coupling approach as was first proposed by the DAMA Collaboration [15, 31, 32].

This approach was used in [33] to demonstrate, by the example of the HDMS experiments with natural Ge and with the neutron-odd group high-spin isotope ^{73}Ge [34, 35], how one can strongly improve the exclusion curves. The approach allowed both upper limits for the spin-dependent $\sigma_{\rm SD}^{n(p)}$ and spin-independent $\sigma_{\rm SI}$ cross sections of the WIMP–nucleon interaction to be *simultaneously* determined from the experimental data. In this way visible (one order of magnitude) improvement in the form of the exclusion curves was achieved [33] relative to the traditional one-coupling dominance scheme used previously for the same setup [36].

As a by-product of the approach, there are correlations (first mentioned in [37]) between the mesured upper limits $\sigma_{\rm SD}^{n}$ and $\sigma_{\rm SI}$, which can be considered as a new requirement — for any fixed WIMP mass m_χ one should have $\sigma_{\rm SI}\,(\rm theor.) \leqslant \sigma_{\rm SI}\,(\rm exp.)$ and $\sigma_{\rm SD}^{n}\,(\rm theor.) \leqslant \sigma_{\rm SD}^{n}\,(\rm exp.)$ simultaneously, provided that $\sigma_{\rm SD(SI)}^{n(p)}\,(\rm theor.)$ are calculated in some underlying SUSY-like theory.

It is important to note that without proper knowledge of the nuclear and nucleon structure it is not possible to extract reliable and useful information (at least in the form of these $\sigma_{\rm SD}^{n}$ and $\sigma_{\rm SI}$ cross sections) from direct DM search experiments. However, astrophysical uncertainties, in particular, the DM distribution in the vicinity of the Earth [38–43], make it far more difficult to interpret the results of the DM search experiments. At the moment, to have a chance to compare sensitivities of different experiments, people adopted a common truncated Maxwellian DM particle distribution, but nobody can prove its correctness. In the case of undoubted direct DM detection, one can make some conclusions about the real DM particle distribution in the vicinity of the Earth.

Furthermore, almost by definition (from the very beginning), a modern experiment aiming at the best exclusion curve is doomed to nonobservation of the DM signal. This is due to the fact that a typical expected DM signal spectrum exponentially drops with recoil energy, and it is practically impossible to single it out from the background non-WIMP spectrum of a typical (semiconductor) detector.

In fact, one needs a clear, the so-called «positive» signature of interactions between WIMPs and target nuclei. Only exclusion curves are not enough. Ideally, this signature should be a unique feature of such an interaction [44].

There are some typical characteristics of WIMPs interactions with a nuclear target which can potentially play the role of these positive WIMP signatures [45]. First of all, WIMPs produce nuclear recoils, whereas most radioactive backgrounds produce electron recoils. Nevertheless, for example, neutrons (and any other heavy neutral particle) can also produce nuclear recoils. There are also proposals which rely on WIMP detection via electron recoils [46, 47].

Due to the extremely rare event rate of the WIMP–nucleus interactions (the mean free path of a WIMP in matter is of the order of a light year), one can expect two features. One is that the probability of two consecutive interactions in a single detector or two closely located detectors is completely negligible. Multiple interactions of photons, gamma rays or neutrons under the same conditions are much more common. Therefore, only nonmultiple interaction events can claim to be from WIMPs. The other feature is a uniform distribution of the WIMP-induced events throughout a detector. This feature can also be used in the future to identify background events (from photons, neutrons, beta and alpha particles) in rather large-volume position-sensitive detectors.

The shape of the WIMP-induced recoil energy spectrum can be predicted rather accurately (for given WIMP mass, fixed nuclear structure functions, and astrophysical parameters). The observed energy spectrum, claiming to be from WIMPs, must be consistent with the expectation. However, this shape is exponential, right as it is the case for many background sources.

Unfortunately, the nuclear-recoil feature, the nonmultiple interaction, the uniform event distribution throughout a detector, and the shape of the recoil energy spectrum could not be the clear «positive signature» of the WIMP interactions. It is believed that the following three features of WIMP–nucleus interaction can serve as a clear «positive signature».

The currently most promising, technically feasible and already used (by the DAMA Collaboration) «positive signature» is the annual modulation signature (see Appendix). The WIMP flux and its average kinetic energy vary annually due to the combined motions of the Earth and the Sun relative to the galactic center. The impact WIMP energy increases (decreases) when the Earth velocity is added to (subtracted from) the velocity of the Sun. The amplitude of the annual modulation depends on many factors — details of the halo model, mass of the WIMP, the year-averaged rate (or total WIMP–nucleus cross sections), etc. In general, the expected modulation amplitude is rather small [7, 14, 15, 31] and to observe it, one needs huge (at best tonne scale) detectors which can continuously operate for 5–7 years. Of course, to reliably use this signature, one should prove the absence of annually modulated backgrounds.

Another potentially promising positive WIMP signature is connected with the possibility of measuring the direction of the recoil nuclei induced by a WIMP. In these directional recoil experiments it is planned to measure the correlation of the event rate with the Sun motion [47–49]. Unfortunately, the task is extremely complicated [50–54].

The third well-known potentially useful positive WIMP signature is connected with the coherence of the WIMP–nucleus spin-independent interaction. Due to a rather low momentum transfer, a WIMP coherently scatters by the whole target nucleus and the elastic cross section of this interaction should be proportional to A^2, where A is the atomic number of the target nucleus. Contrary to the A^2 behavior, the cross section of neutron scattering by nuclei (due to the strong nature of this interaction) is proportional to the geometrical cross section of the target nucleus ($A^{2/3}$ dependence). To reliably use this A^2 signature, one has to satisfy at least two conditions. First, one should be sure that the spin-independent WIMP–nucleus interaction really dominates over the relevant spin-dependent interaction. This is far from being obvious [33, 55–57]. Second, one should, at least for two targets with a different atomic number A, rather accurately measure the recoil spectra (in the worst case integrated event rates) under the same background conditions. Currently, this goal looks far from being achievable. Developing further the idea of this third signature, one can also consider as a possible extra WIMP signature an observation of the similarity (or coherent behavior) of measured spectra at different (also nonzero spin) nuclear targets. This possibility relies on rather accurate spin structure functions for the experimentally interesting nuclei [58, 59].

Ideally, in order to be convincing, an eventual DM signal should combine more than one of these positive DM signatures [44, 45].

In the case of currently very promising event-by-event active background reduction techniques (like in the CDMS, EDELWEISS, and XENON experiments), one inevitably needs clear positive WIMP signature(s). Without these signatures one can hardly convince anybody that the final spectrum is saturated by WIMPs. Furthermore, with the help of these extra signatures and on the basis of measured recoil spectra, one can estimate the WIMP mass [60, 61].

It is known (see, for example, discussions in [62, 63]) that a proof of the observation of a DM signal is an extremely complicated problem. As pointed out above, on this way an interpretation of measurements in the form of exclusion curves helps almost nothing. Of course, an exclusion curve is at least something from nothing observed. It allows a sensitivity comparison of different experiments and therefore allows deciding who at the moment is the best «excluder». But, for example, supersymmetric theory is, in general, very flexible, it has a lot of parameters, and one hardly believes that an exclusion curve can ever impose any decisive constraint on it. Furthremore, almost all experimental groups presenting their exclusion curves try to compare them with some SUSY predictions. It is clear from this comparison that there are some domains of the SUSY parameter space, which are now already excluded by these exclusion curves. What is remarkable, however, is that nobody yet has seriously considered these constraints for SUSY.

The situation is much worse due to the already mentioned famous nuclear and astrophysical uncertainties involved in the evaluation of the exclusion curves [64–71]. This is why, it does not look very decisive (or wise) to use very refined data and methods (nuclear, astrophysical, numerical, statistical [72], etc.)

and spend big resources fighting only for the best exclusion curve. This fighting could only be accepted when one tries to strongly improve the sensitivity of a small detector with a view of using many copies of it in a huge detector array with a total tonne-scale mass [20].

There are remarks concerning comparison of results from DM search experiments with passive (off-line) background reduction (like DAMA) and from experiments with active (on-line) background reduction (like CDMS, XENON, ZEPLIN, etc.). First, it was demostrated [28–30, 33] that any extra positively defined background-like contribution to the spectra improves the extracted (upper limit) values of the cross section. Next, within the passive background reduction scheme, the measured spectrum is not affected by hardware or software influence during the data taking. Further background reduction can be done off-line on the basis of careful investigation of the spectrum itself or, for example, with the help of the pulse shape analysis. In this case, the extracted background contribution is under control and well defined. On the other side, within the active background reduction approach the measured spectrum already contains results of this active reduction influence on the data taking process. In this case, it is not simple to hold under control the real level of extracted on-line background contribution which can easily be overestimated (see, for example, the relevant discussion in [4]). Therefore, due to this obvious difference, a direct comparison of exclusion curves from experiments with passive and active background reductions could be, in principle, rather misleading.

Finally, it seems that at the level of our present knowledge, the DM problem could not be solved independently of other related problems (proof of SUSY, astrophysical dark matter properties, etc.). Furthermore, due to the huge complexity of the DM search (technical, physical, astrophysical, necessity for positive signatures, etc.), one should deal with the DM problem boldly using a reliable model-dependent framework — for example, the framework of SUSY, where the same LSP neutralino should be seen coherently or lead to effects in all available experiments (direct and indirect DM searches, rare decays, high-energy searches at LHC, etc.). Only if such a SUSY framework leads to a specific and decisive positive WIMP signature, this could mean a proof of SUSY and simultaneous solution of the dark matter problem. In some sense, this SUSY framework can serve as a specific and very decisive positive WIMP signature.

SUMMARY

A physical reason to *improve* an exclusion curve is usually an attempt to constrain a SUSY-like model. Unfortunately, this is almost hopeless due to the huge flexibility of these models and the inevitable necessity of having extra information from other SUSY-sensitive observables (for example, from LHC). At the present and foreseeable level of experimental accuracy, simple fighting

for the best exclusion curve is almost useless either for real DM detection or for substantial restrictions for SUSY.

One should inevitably go beyond an exclusion curve. New generations of DM experiments right from their beginning *should aim at detection* of the DM particles. This will require development of new setups, which will be able to register *positive signatures* of the DM particle interactions with nuclear targets.

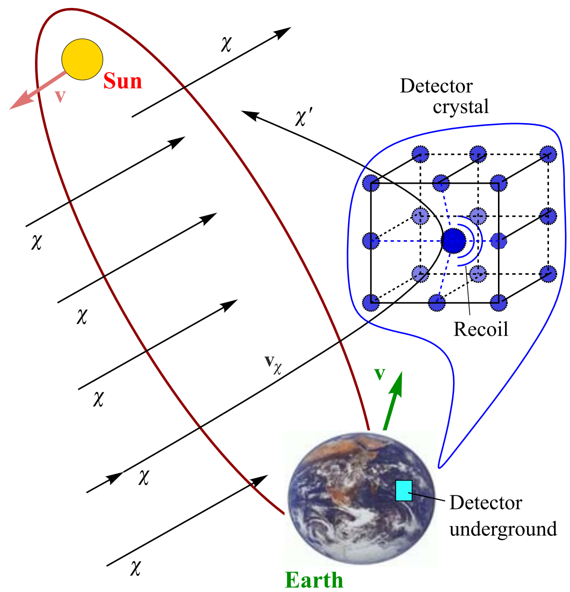
One should try to obtain a reliable *recoil energy spectrum*. First, very accurate off-line investigation of the measured spectrum allows one to single out different non-WIMP background sources and to perform controllable background subtractions. Second, the spectrum allows one to look for the annual modulation effect, the only currently available positive signature of DM particle interactions with terrestrial nuclei. This effect is not simply a possibility (among many others) of rejecting background (as claimed again recently in [3]), but it is a unique signature which reflects the inner physical properties of the DM interaction with matter. It is a very decisive and eagerly welcomed feature, which is inevitable for the laboratory proof of the DM exsitence.

This letter was written in connection with Prof. D. I. Kazakov's 60th birthday and contains updated key messages from the extended review «Direct Search for Dark Matter — Striking the Balance — and the Future» [20].

APPENDIX

The nuclear recoil energy E_R is measured by a proper detector deep underground (Figure). The differential event rate in respect to the recoil energy (the spectrum) is the subject of the measurements. The recoil spectrum produced from WIMP–nucleus scattering in a target detector is expected to show the annual modulation effect due to the Earth motion around the Sun [14]. The velocity of the Earth relative to the Galaxy is $v_E(t) = v_S + v_O \cos\gamma \cos\omega(t - t_0)$, where v_S is the Sun velocity relative to the Galaxy ($v_S = 232$ km/s); v_O is the Earth orbital velocity around the Sun ($v_O = 30$ km/s), and γ is the angle of inclination of the plane of the Earth orbit relative to the galactic plane ($\gamma \cong 60°$). One has $\omega = 2\pi/T$ ($T = 1$ y) and the maximum velocity occurs at day $t_0 = 155.2$ (June 2). The change in the Earth velocity relative to the incident WIMPs leads to a yearly modulation of the scattering event rates of about 7%. It is convenient to introduce a dimensionless variable $\eta = v_E/v_0$, then $\eta(t) = \eta_0 + \triangle\eta \cos\omega(t - t_0)$, where the amplitude of the modulated part ($\Delta\eta \simeq 0.07$) is small compared to the annual average $\eta_0 \simeq 1.05$. Within this framework, the expected count rate of WIMP interactions can be written as

$$S[\eta(t)] \simeq S[\eta_0] + \left.\frac{\partial S}{\partial \eta}\right|_{\eta_0} \triangle\eta \cos\omega(t - t_0) = S_0 + S_m \cos\omega(t - t_0), \qquad (1)$$



Detection of dark matter (WIMPs) by elastic scattering from target nuclei in the detector. Due to the expected annual modulation signature of the event rate (2), the Sun–Earth system is a particularly proper setup for successful direct DM detection. From [20]

where S_0 is the constant part, and S_m is the amplitude of the modulated signal. Both parts of the event rate S_0 and S_m depend on the target nucleus (A, Z), WIMP (or neutralino χ) mass m_χ, density $\rho^{\rm DM}_{\rm local}$, velocity distribution of the WIMPs in the solar vicinity $f(v)$, and cross section of WIMP–nucleus elastic scattering (see, for example, [6, 7, 73, 74]).

The differential event rate per unit mass of the target material has the form

$$S(t) \equiv \frac{dR}{dE_R} = N_T \frac{\rho_\chi}{m_\chi} \int\limits_{v_{\rm min}}^{v_{\rm max}} dv\, f(v)\, v \frac{d\sigma^A}{dq^2}(v, q^2). \qquad (2)$$

Assuming that WIMPs are the dominant component of the DM halo of our Galaxy, one has $\rho_\chi = \rho^{\rm DM}_{\rm local}$. The nuclear recoil energy $E_R = q^2/(2M_A)$ is typically about $10^{-6} m_\chi$; N_T is the number density of target nuclei with mass M_A, $v_{\rm max} = v_{\rm esc} \approx 600$ km/s, and $v_{\rm min} = (M_A E_R / 2\mu_A^2)^{1/2}$ is the minimal WIMP velocity which still can produce the recoil energy E_R. The WIMP–nucleus differential elastic scattering cross section for spin-nonzero ($J \neq 0$) nuclei contains coherent (spin-independent, or SI) and axial (spin-dependent, or SD) terms [75, 76]

$$\frac{d\sigma^A}{dq^2}(v,q^2) = \frac{S^A_{\rm SD}(q^2)}{v^2(2J+1)} + \frac{S^A_{\rm SI}(q^2)}{v^2(2J+1)} = \frac{\sigma^A_{\rm SD}(0)}{4\mu_A^2 v^2}F^2_{\rm SD}(q^2) + \frac{\sigma^A_{\rm SI}(0)}{4\mu_A^2 v^2}F^2_{\rm SI}(q^2). \tag{3}$$

The normalized ($F^2_{\rm SD,SI}(0) = 1$) finite-momentum-transfer nuclear form factors $F^2_{\rm SD,SI}(q^2) = \dfrac{S^A_{\rm SD,SI}(q^2)}{S^A_{\rm SD,SI}(0)}$ can be expressed in terms of the nuclear structure functions as follows [75, 76]:

$$\begin{aligned} S^A_{\rm SI}(q) &= \sum_{L\,\rm even} |\langle J||\mathcal{C}_L(q)||J\rangle|^2 \simeq |\langle J||\mathcal{C}_0(q)||J\rangle|^2, \\ S^A_{\rm SD}(q) &= \sum_{L\,\rm odd} \left(|\langle N||\mathcal{T}^{el5}_L(q)||N\rangle|^2 + |\langle N||\mathcal{L}^5_L(q)||N\rangle|^2\right). \end{aligned} \tag{4}$$

The explicit form of the transverse electric $\mathcal{T}^{el5}(q)$ and longitudinal $\mathcal{L}^5(q)$ multipole projections of the axial vector current operator and the scalar function $\mathcal{C}_L(q)$ can be found in [58, 59, 75, 76]. For $q = 0$, the nuclear SD and SI cross sections can be represented as

$$\sigma^A_{\rm SI}(0) = \frac{4\mu_A^2\ S_{\rm SI}(0)}{(2J+1)} = \frac{\mu_A^2}{\mu_p^2}A^2\sigma^p_{\rm SI}(0), \tag{5}$$

$$\sigma^A_{\rm SD}(0) = \frac{4\mu_A^2 S_{\rm SD}(0)}{(2J+1)} = \frac{4\mu_A^2}{\pi}\frac{(J+1)}{J}\left\{a_p\langle\mathbf{S}^A_p\rangle + a_n\langle\mathbf{S}^A_n\rangle\right\}^2, \tag{6}$$

$$= \frac{\mu_A^2}{\mu_p^2}\frac{4}{3}\frac{J+1}{J}\sigma^{pn}_{\rm SD}(0)\left\{\langle\mathbf{S}^A_p\rangle\cos\theta + \langle\mathbf{S}^A_n\rangle\sin\theta\right\}^2. \tag{7}$$

Following Bernabei et al. [15,37], the effective spin WIMP–nucleon cross section $\sigma^{pn}_{\rm SD}(0)$ and the coupling mixing angle θ were introduced,

$$\sigma^{pn}_{\rm SD}(0) = \frac{\mu_p^2}{\pi}\frac{4}{3}\Big[a_p^2 + a_n^2\Big], \quad \tan\theta = \frac{a_n}{a_p}, \tag{8}$$

$$\sigma^p_{\rm SD} = \sigma^{pn}_{\rm SD}\cdot\cos^2\theta, \quad \sigma^n_{\rm SD} = \sigma^{pn}_{\rm SD}\cdot\sin^2\theta. \tag{9}$$

Here, $\mu_A = \dfrac{m_\chi M_A}{m_\chi + M_A}$ is the reduced mass of the neutralino and the nucleus, and it is assumed that $\mu_n^2 = \mu_p^2$. The dependence on effective WIMP–quark (in SUSY neutralino–quark) couplings $\mathcal{C}_q$ and $\mathcal{A}_q$ in the underlying theory

$$\mathcal{L}_{\rm eff} = \sum_q(\mathcal{A}_q\cdot\bar\chi\gamma_\mu\gamma_5\chi\cdot\bar q\gamma^\mu\gamma_5 q + \mathcal{C}_q\cdot\bar\chi\chi\cdot\bar q q) + \ldots \tag{10}$$

and on the spin ($\Delta_q^{(p,n)}$) and the mass or scalar ($f_q^{(p)} \approx f_q^{(n)}$) structure of the proton and neutron enter into these formulas via the zero-momentum-transfer WIMP–proton and WIMP–neutron SI and SD cross sections

$$\sigma_{\rm SI}^{p}(0) = 4\frac{\mu_p^2}{\pi}c_0^2, \quad c_0 = c_0^{p,n} = \sum_q \mathcal{C}_q f_q^{(p,n)}, \tag{11}$$

$$\sigma_{\rm SD}^{p,n}(0) = 12\frac{\mu_{p,n}^2}{\pi}a_{p,n}^2 \quad a_p = \sum_q \mathcal{A}_q \Delta_q^{(p)}, \quad a_n = \sum_q \mathcal{A}_q \Delta_q^{(n)}. \tag{12}$$

The factors $\Delta_q^{(p,n)}$, which parameterize the quark spin content of the nucleon, are defined as $2\Delta_q^{(n,p)} s^\mu \equiv \langle p,s|\bar{\psi}_q \gamma^\mu \gamma_5 \psi_q|p,s\rangle_{(p,n)}$. The quantity $\langle \mathbf{S}^A_{p(n)}\rangle$ denotes the total spin of protons (neutrons) averaged over all A nucleons of the nucleus (A,Z)

$$\langle \mathbf{S}^A_{p(n)}\rangle \equiv \langle A|\mathbf{S}^A_{p(n)}|A\rangle = \langle A|\sum_i^A \mathbf{s}^i_{p(n)}|A\rangle. \tag{13}$$

The mean velocity $\langle v\rangle$ of the relic neutralinos of our Galaxy is about 300 km/s = $10^{-3}c$. Assuming $q_{\rm max}R \ll 1$, where R is the nuclear radius and $q_{\rm max} = 2\mu_A v$ is the maximum of the momentum transfer in the process of the χA scattering, the spin-dependent matrix element takes a simple form (*zero-momentum-transfer limit*) [77, 78]

$$\mathcal{M} = C\langle A|a_p \mathbf{S}_p + a_n \mathbf{S}_n|A\rangle \cdot \mathbf{s}_\chi = C\Lambda\langle A|\mathbf{J}|A\rangle \cdot \mathbf{s}_\chi. \tag{14}$$

Here, $\mathbf{s}_\chi$ denotes the spin of the neutralino, and

$$\Lambda = \frac{\langle N|a_p\mathbf{S}_p + a_n\mathbf{S}_n|N\rangle}{\langle N|\mathbf{J}|N\rangle} = \frac{\langle N|(a_p\mathbf{S}_p + a_n\mathbf{S}_n)\cdot\mathbf{J}|N\rangle}{J(J+1)} = \frac{a_p\langle\mathbf{S}_p\rangle}{J} + \frac{a_n\langle\mathbf{S}_n\rangle}{J}. \tag{15}$$

The normalization factor C involves the coupling constants, the masses of the exchanged bosons, and the mixing parameters relevant to the LSP, i.e., it is not related to the associated nuclear matrix elements [79]. In the limit of zero momentum transfer $q = 0$, the spin structure function in (4) reduces to the form

$$S^A(0) = \frac{1}{4\pi}\left|\langle A|\left|\sum_i \frac{1}{2}(a_0 + a_1\tau_3^i)\sigma_i\right||A\rangle\right|^2 = \frac{2J+1}{\pi}J(J+1)\Lambda^2.$$

For the most interesting isotopes either $\langle \mathbf{S}^A_p\rangle$ or $\langle \mathbf{S}^A_n\rangle$ dominate ($\langle \mathbf{S}^A_{n(p)}\rangle \ll \langle \mathbf{S}^A_{p(n)}\rangle$).

The differential event rate (2) can be also given in the form [15, 55]

$$\frac{dR(E_R)}{dE_R} = \kappa_{\rm SI}(E_R, m_\chi)\,\sigma_{\rm SI} + \kappa_{\rm SD}(E_R, m_\chi)\,\sigma_{\rm SD},$$
$$\kappa_{\rm SI}(E_R, m_\chi) = N_T \frac{\rho_\chi M_A}{2m_\chi \mu_p^2} B_{\rm SI}(E_R)[M_A^2], \qquad (16)$$

$$\kappa_{\rm SD}(E_R, m_\chi) = N_T \frac{\rho_\chi M_A}{2m_\chi \mu_p^2} B_{\rm SD}(E_R) \left[\frac{4}{3}\frac{J+1}{J}\left(\langle \mathbf{S}_p\rangle \cos\theta + \langle \mathbf{S}_n\rangle \sin\theta\right)^2\right],$$
$$B_{\rm SI,SD}(E_R) = \frac{\langle v\rangle}{\langle v^2\rangle} F^2_{\rm SI,SD}(E_R) I(E_R). \qquad (17)$$

The dimensionless integral $I(E_R)$ is a dark-matter-particle velocity distribution correction

$$I(E_R) = \frac{\langle v^2\rangle}{\langle v\rangle} \int_{x_{\min}} \frac{f(x)}{v} dx =$$
$$= \frac{\sqrt{\pi}}{2} \frac{3 + 2\eta^2}{\sqrt{\pi}(1+2\eta^2)\,{\rm erf}\,(\eta) + 2\eta\,{\rm e}^{-\eta^2}} [{\rm erf}\,(x_{\min}+\eta) - {\rm erf}\,(x_{\min}-\eta)], \qquad (18)$$

where WIMPs in the rest frame of our Galaxy are assumed to have a Maxwell–Boltzmann velocity distribution; the dimensionless Earth speed with respect to the halo η is used, and $x^2_{\min} = \dfrac{3}{4}\dfrac{M_A E_R}{\mu_A^2 \bar{v}^2}$ [7, 14]. The error function is ${\rm erf}(x) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^x dt\,{\rm e}^{-t^2}$. The velocity variable is the dispersion $\bar{v} \simeq 270$ km/s. The mean WIMP velocity $\langle v\rangle = \sqrt{\dfrac{5}{3}}\,\bar{v}$. Integrating the differential rate (2) from the recoil energy threshold ϵ to some maximal energy ε, one obtains the total detection rate $R(\epsilon, \varepsilon)$ as a sum of the SD and SI terms

$$R(\epsilon, \varepsilon) = R_{\rm SI}(\epsilon, \varepsilon) + R_{\rm SD}(\epsilon, \varepsilon) =$$
$$= \int_\epsilon^\varepsilon dE_R \kappa_{\rm SI}(E_R, m_\chi)\,\sigma_{\rm SI} + \int_\epsilon^\varepsilon dE_R \kappa_{\rm SD}(E_R, m_\chi)\,\sigma_{\rm SD}. \qquad (19)$$

To accurately estimate the event rate $R(\epsilon, \varepsilon)$, one needs to know a number of quite uncertain astrophysical and nuclear structure parameters as well as the very specific characteristics of the experimental setup [4].

As m_χ increases, the product qR becomes non-negligible and *the finite-momentum-transfer limit* must be considered [58,59,75,76,78]. With the isoscalar spin coupling constant $a_0 = a_n + a_p$ and the isovector spin coupling constant $a_1 = a_p - a_n$, one can split the nuclear structure function $S^A(q)$ into a pure isoscalar term, $S^A_{00}(q)$, a pure isovector term, $S^A_{11}(q)$, and an interference term, $S^A_{01}(q)$, in the following way:

$$S^A(q) = a_0^2 S^A_{00}(q) + a_1^2 S^A_{11}(q) + a_0 a_1 S^A_{01}(q). \tag{20}$$

The relations $S^A_{00}(0) = C(J)(\langle \mathbf{S}_p\rangle + \langle \mathbf{S}_n\rangle)^2$, $S^A_{11}(0) = C(J)(\langle \mathbf{S}_p\rangle - \langle \mathbf{S}_n\rangle)^2$, and $S^A_{01}(0) = 2C(J)(\langle \mathbf{S}^2_p\rangle - \langle \mathbf{S}^2_n\rangle)$ with $C(J) = \dfrac{2J+1}{4\pi}\dfrac{J+1}{J}$ connect the nuclear spin structure function $S^A(q=0)$ with the proton $\langle \mathbf{S}_p\rangle$ and neutron $\langle \mathbf{S}_n\rangle$ spin contributions averaged over the nucleus [58].

To analyze modern data in the finite-momentum-transfer approximation, it seems reasonable to use the formulas for the differential event rate (2) as schematically given below

$$\begin{gathered}
\frac{dR(\epsilon,\varepsilon)}{dE_R} = \mathcal{N}(\epsilon,\varepsilon,E_R,m_\chi)\left[\eta_{\rm SI}(E_R,m_\chi)\,\sigma^p_{\rm SI} + \eta'_{\rm SD}(E_R,m_\chi,\omega)\,a_0^2\right],\\
\mathcal{N}(\epsilon,\varepsilon,E_R,m_\chi) = \left[N_T\frac{c\rho_\chi}{2m_\chi}\frac{M_A}{\mu_p^2}\right]\frac{4\mu_A^2}{\langle q^2_{\rm max}\rangle}\left\langle\frac{v}{c}\right\rangle I(E_R)\theta(E_R-\epsilon)\theta(\varepsilon-E_R),\\
\eta_{\rm SI}(E_R,m_\chi) = \left\{A^2F^2_{\rm SI}(E_R)\right\},\\
\eta'_{\rm SD}(E_R,m_\chi,\omega) = \mu_p^2\left\{\frac{4}{2J+1}\left(S_{00}(q) + \omega^2\,S_{11}(q) + \omega\,S_{01}(q)\right)\right\}.
\end{gathered} \tag{21}$$

Here the isovector-to-isoscalar nucleon coupling ratio is $\omega = a_1/a_0$. The detector threshold recoil energy ϵ and the maximal available recoil energy ε ($\epsilon \leqslant E_R \leqslant \varepsilon$) have been introduced in (19). In practice, with an ionization or scintillation signal, one has to take into account the quenching of the recoil energy, when the visible recoil energy is smaller than the real recoil energy transmitted by the WIMP to the target nucleus.

Formulas (21) allow experimental recoil spectra to be directly described in terms of only *three* [80] (it is rather reasonable to assume $\sigma^p_{\rm SI}(0) \approx \sigma^n_{\rm SI}(0)$) independent parameters ($\sigma^p_{\rm SI}$, a_0^2, and ω) for any fixed WIMP mass m_χ and any neutralino composition. Comparing this formula with the observed recoil spectra for different targets (Ge, Xe, F, NaI, etc.), one can directly and simultaneously restrict both isoscalar c_0 (via $\sigma^p_{\rm SI}$) and isovector neutralino–nucleon effective couplings $a_{0,1}$. These constraints, based on the nuclear spin structure functions for finite q, will impose *the most model-independent and most accurate restrictions* on any SUSY parameter space. Contrary to some other possibilities (see, for example, [15, 28]), this procedure is direct and uses as much as possible the results of the accurate nuclear spin structure calculations.

It is seen from (9) and (21) that the SD cross sections $\sigma^p_{\rm SD}$ and $\sigma^n_{\rm SD}$ (or equivalently a_0^2 and $\omega = a_1/a_0$) are the only two WIMP–nucleon spin variables which can be constrained (or extracted) from DM measurements. Therefore, there is no sense in extracting effective WIMP–nucleon couplings a_p and a_n from the data (with «artificial» twofold ambiguity).

Finally, to estimate the expected direct DM detection rates (with formulas (2), (19), or (21)), one should calculate the cross sections $\sigma_{\rm SI}$ and $\sigma_{\rm SD}$ (or WIMP–nucleon couplings c_0 and $a_{p,n}$) within a SUSY-based model or take them from experimental data (if it is possible).

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