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ORIGIN OF THE Q^2-DEPENDENCE OF THE DIS STRUCTURE FUNCTIONS

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We consider in detail Q^2-dependence of the DIS structure functions. Very often this dependence is claimed to be originated by the Q^2-dependence of the QCD coupling. This leads to the small-x asymptotics of the structure functions with Q^2-dependent intercepts. We demonstrate that the DGLAP parametrization $\alpha_s = \alpha_s(Q^2)$ is an approximation valid in the region of large x (where $2pq$ can be approximated by Q^2) only, providing the factorization scale is also large. Outside this region, the DGLAP parametrization fails, so α_s should be replaced by an effective coupling which is independent of Q^2 at small x. As a consequence, intercepts of the structure functions are independent of Q^2. Nevertheless, the small-x asymptotics of the structure functions explicitly depend on Q^2, even when the coupling does not depend on it. We also consider the structure functions at small Q^2 and give a comment on power-Q^2 corrections to the structure functions at large and small Q^2.

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INTRODUCTION

Each of the DIS structure functions has two independent arguments: the invariant energy $w = 2pq$ (with p and q being the hadron target and virtual photon momenta, respectively) and virtuality of the incoming photon $Q^2 = -q^2 > 0$. Early experiments of the 1960s at SLAC proved that the structure functions depended on only one argument $x = Q^2/w$. This phenomenon was called scaling. Later, new experiments proved that the scaling was violated; i.e., in addition to x-dependence, the DIS structure functions also depended on Q^2 straightforwardly. It has since become conventional to regard variables x and Q^2 as independent arguments of the structure functions. However, one has to realize that those arguments (in contrast to the set of really independent arguments w, Q^2) cannot be regarded as independent variables at all times. Indeed, they are independent when Q^2 is fixed while w is scanned, but they are not independent at fixed w and scanned Q^2. The Q^2-dependence of the structure functions is the objective of the DGLAP evolution equations [1] implementing the total resummation of $\ln^k Q^2$. In DGLAP and beyond this approach, the Q^2-dependence of the structure functions is often thought to be induced by the Q^2-dependence of α_s:

$$\alpha_s = \alpha_s(Q^2). \tag{1}$$

As this parametrization takes place in the DGLAP equations [1], throughout the present paper we will address it as the DGLAP parametrization. Parametrization (1), originally deduced for large x, where $2pq$ can be approximated by Q^2, is often exploited to describe DIS in the small-x region and even in the small-x asymptotics of the structure functions, i.e., at $x \to 0$ (see, e.g., [2–6]). As a result, the small-x asymptotics look like

$$f \sim x^{-\Delta(Q^2)}, \tag{2}$$

with the Q^2-dependence of the asymptotics originated by the «intercept» $\Delta(Q^2)$.

In the present paper, we argue against both validity of Eq. (2) and treating Eq. (1) as an exact formula which could be used at any x and Q^2. In what follows, we will show that the parametrization (1) should be used with a certain care: it can be used within the proper DGLAP kinematic region, i.e., at large x and Q^2, and furthermore the factorization scale should also be large. Indeed, show that if any of these requirements is violated, $\alpha_s(Q^2)$ should be replaced by the effective coupling which does not depend on Q^2 at small x as well as at small Q^2. The effective coupling, being independent of Q^2 at small x, cannot bring any Q^2-dependence to the structure functions. In this case the Q^2-dependence of the structure functions is originated by alternative sources. We also argue against considering the approximative parametrization (1) as the basis for the shift $\alpha_s(Q^2) \to \alpha_s(Q^2+\mu^2)$ as a continuation into the small-Q^2 region when the shift is used in order to keep α_s in the perturbative domain. Instead, we suggest an alternative approach to describe the structure functions in the small-Q^2 region.

Our paper is organized as follows: In Sec. 1, we consider the DIS structure functions in general, focusing on their conventional dependence on Q^2. In Sec. 2, we consider the structure functions in the framework of DGLAP and discuss parametrizations of the QCD coupling in different kinematic regions. In Sec. 3, we derive general expressions for the structure functions at small x and a general form of their small-x asymptotics, demonstrating that there is no room for Eq. (2) at $x \to 0$. In Sec. 4, we discuss the role of singular factors conventionally used in DGLAP fits for initial parton distributions. We consider the structure functions at small Q^2 in Sec. 5. Remark on higher twists is given in Sec. 6. The final section is for our concluding remarks.

1. GENERAL STRUCTURE OF THE DIS STRUCTURE FUNCTIONS

All DIS structure functions are given by a convolution of perturbative components $f_r^{(\mathrm{pert})}$ and parton distributions Φ_r. For instance, in collinear factorization and when DGLAP is used, the nonsinglet structure function $f_{\mathrm{NS}}(x,Q^2)$ is represented as the convolution:

$$f_{\mathrm{NS}}(x,Q^2) = C_q(x,x_0) \otimes \Delta q(x_0,Q^2), \tag{3}$$

where C_q is the quark coefficient function, while $\Delta q(Q^2)$ stands for the quark evolved distribution defined at scale Q^2. $C_q(x, x_0)$ controls evolution of this distribution from x_0 to x. The Q^2-evolution of the quark distribution from the initial scale μ^2 to scale Q^2 is controlled by the DGLAP evolution equations [1]. It can be written as

$$\Delta q(x_0, Q^2) = E(Q^2, \mu^2) \otimes \delta q(x_0, \mu^2), \tag{4}$$

where the initial quark distribution $\delta q(x_0, \mu^2)$ is conventionally chosen at $x_0 \sim 1$ and $\mu \sim 1$ GeV. The scale μ^2 is called the factorization scale. Equations (3), (4) can be written as one convolution:

$$f_{\rm NS}(x, Q^2) = [C_q(x, y) \otimes E(Q^2, \mu^2)] \otimes \delta q(\mu^2) \equiv f_{\rm NS}^{(\rm pert)}(x, Q^2) \otimes \delta q(\mu^2), \tag{5}$$

where $f_{\rm NS}^{(\rm pert)}$ denotes the whole perturbative evolution of δq with respect to both x and Q^2. Equation (5) can easily be generalized to the case of the singlet structure functions by adding gluon contributions. So, in basic factorization [7] as well as in collinear and k_T-factorizations, such a convolution can be written as

$$f(x, Q^2) = \sum_r f_r^{(\rm pert)}(q\kappa, Q^2, \kappa^2) \otimes \Phi_r(p\kappa, \kappa^2), \tag{6}$$

where Φ denotes parton distributions in any of the factorizations, the subscript r marks the intermediate partons (quarks and gluons) with momentum κ, and the summation over r takes place when necessary. The notation $\otimes$ means the one-dimensional integration over the longitudinal for collinear factorization and two-dimensional (both the longitudinal and transverse momenta) in the case of k_T-factorization; $\kappa^2 = \mu^2$ in the case of collinear factorization. In Eq. (6), we have dropped a possible dependence on unessential variables like the target spin and mass.

1.1. General Structure of the Perturbative Component. The simplest form of structure functions corresponds to collinear factorization where it can be written in terms of the Mellin integral:

$$f = \int\limits_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} x^{-\omega} \left[f_q^{(\rm pert)}(\omega)\, \delta q(\omega) + f_g^{(\rm pert)}(\omega)\, \delta g(\omega) \right], \tag{7}$$

with δg and $\delta g(\omega)$ standing for the quark and gluon distributions in the ω-space, respectively. In Eq. (7), they are convoluted with the perturbative contributions $f_r^{(\rm pert)}$. These perturbative contributions in any factorization have the following generic structure:

$$f_r^{(\rm pert)} = \int\limits_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} x^{-\omega} C_r(\omega, \alpha_s, \kappa^2)\, {\rm e}^{\widetilde{\Omega}_r(\omega, \alpha_s, Q^2, \kappa^2)}, \tag{8}$$

where the term C_r is a generic notation for the coefficient functions and $\widetilde{\Omega}_r$ is expressed through the matrix of anomalous dimensions. For instance, in LO DGLAP, where, in collinear factorization, $\kappa^2 = \mu^2$, the nonsinglet perturbative component $f_{\rm NS}^{(\rm pert)}(\equiv f_q^{(\rm pert)})$ is expressed in terms of the nonsinglet coefficient function $C_{\rm NS}$ and $\widetilde{\Omega}_{\rm NS}$:

$$C_{\rm NS}^{(\rm LO)} = 1, \quad \Omega_{\rm NS}^{(\rm LO)} = \gamma_{qq}^{(0)}(\omega) \int\limits_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \alpha_s(k_\perp^2) = \gamma_{qq}^{(0)}(\omega) \frac{1}{b} \ln \left[\frac{\ln\left(Q^2/\Lambda^2\right)}{\ln\left(\mu^2/\Lambda^2\right)} \right], \tag{9}$$

with $\gamma_{qq}^{(0)}(\omega)$ being the well-known quark–quark anomalous dimension and $b = (33 - 2n_f)/(12\pi)$. In NLO DGLAP, both $C_{\rm NS}$ and $\Omega_{\rm NS}$ acquire additional terms proportional to α_s. In particular, the structure of the NLO coefficient functions is

$$C_{\rm NS}^{(\rm NLO)} = 1 + \alpha_s(Q^2) \tilde{C}_{\rm NS}(\omega). \tag{10}$$

We have used this simple example in order to demonstrate explicitly that the Q^2-dependence of any structure function f is achieved through the Q^2-dependence of its perturbative components $f_r^{(\rm pert)}$. The Q^2-dependence of $f_r^{(\rm pert)}$ comes from the upper limit Q^2 of integrations over $k_\perp^2$ in expressions for both Ω_r, as explicitly shown in Eq. (9), and in expressions for coefficient functions. The latter is related to the parametrization $\alpha_s = \alpha_s(Q^2)$. This feature is common for all structure functions, although specific expressions for $C_{rr'}$ and $\Omega_{rr'}$ are different for different structure functions.

2. Q^2-DEPENDENCE OF THE QCD COUPLING

In this section, we discuss parametrizations of α_s in DGLAP equations in collinear factorization at different values of x and μ^2. Although for the sake of simplicity we consider the LO DGLAP equations, our conclusions are valid for NLO, NNLO, and so on. In the LO DGLAP evolution equation, the parametrization (1) appears when the integral DGLAP equations for $f^{(\rm pert)}$

$$f_r^{(\rm pert)}(x, Q^2) = \int\limits_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int\limits_x^1 \frac{d\beta}{\beta} \alpha(k_\perp^2) f_{r'}^{(\rm pert)} \left(\frac{x}{\beta}, \frac{Q^2}{k_\perp^2} \right) P_{r'r}(\beta) \tag{11}$$

are reduced to the differential equations

$$\frac{\partial f_r^{(\rm pert)}(x, Q^2)}{\partial \ln Q^2} = \alpha(Q^2) \int\limits_x^1 \frac{d\beta}{\beta} f_{r'}^{(\rm pert)} \left(\frac{x}{\beta}, Q^2 \right) P_{r'r}(\beta). \tag{12}$$

Equations (11), (12) demonstrate explicitly that Eq. (1) holds when (i) the upper limit of integration over $k_\perp^2$ in Eqs. (11) is Q^2 which corresponds to $x \sim 1$; (ii) the parametrization

$$\alpha_s = \alpha_s(k_\perp^2) \tag{13}$$

is used in the involved Feynman graphs. Throughout the present paper, we will refer to the parametrization of Eq. (13) as the standard parametrization. According to the results of [8], one can use the standard parametrization (13) in expressions for $f^{(\mathrm{pert})}$ in collinear factorization only when the following two conditions are fulfilled: First, values of x should be large:

$$x \sim 1 \tag{14}$$

to ensure the use of Q^2 instead of $2pq$. Second, the factorization point μ^2 should obey the strong inequality

$$\mu^2 \gg e^\pi \Lambda^2 \approx 23\Lambda^2. \tag{15}$$

When one of the two requirements in Eqs. (14), (15) is violated, the coupling $\alpha_s(k_\perp^2)$ in the integral DGLAP evolutions equations for $f^{(\mathrm{pert})}$ should be replaced by the effective coupling α_{eff}, as shown in [8]. This coupling can be represented by different approximative expressions, depending on the kinematics. In Subsecs. 2.1 and 2.2, we consider the cases of violating either (14) or (15) and consider violation of both of them in Subsec. 2.3.

2.1. Case A. When Eq. (15) is violated but x is so close to 1 that essential β in Eq. (11) are ~ 1, the coupling $\alpha_s(k_\perp^2)$ in Eq. (11) should be replaced by

$$\alpha_{\mathrm{eff}}(k_\perp^2) = \frac{1}{b}\frac{l_0}{(l_0^2+\pi^2)} - \frac{1}{\pi b}\arctan\left(\frac{\pi}{l_0}\right) + \frac{1}{\pi b}\arctan\left(\frac{\pi}{\widetilde{l}(k_\perp^2)}\right), \tag{16}$$

where we have denoted

$$l_0 = \ln(\mu^2/\Lambda^2), \quad \widetilde{l}(k_\perp^2) = \ln(k_\perp^2/\Lambda^2). \tag{17}$$

It leads to replacement of the integration of $\alpha_s(k_\perp^2)$ in Eq. (9) by the integration of $\alpha_{\mathrm{eff}}(k_\perp^2)$ over the same interval:

$$\int\limits_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\ \alpha_s(k_\perp^2) \to \int\limits_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\ \alpha_{\mathrm{eff}}(k_\perp^2). \tag{18}$$

Let us notice that when μ obeys Eq. (15), the first and second terms in Eq. (16) cancel each other, while the third term can be approximated by $\alpha_s(k_\perp^2)$. This reduces the integral in Eq. (18) to the DGLAP expression in Eq. (9).

2.2. Case B. Let us consider the opposite situation when μ satisfies Eq. (15) but x is small. In this case, $\alpha_s(k_\perp^2)$ in Eq. (11) should be replaced by $\alpha_s(k_\perp^2/\beta)$ and the upper limit of integration over $k_\perp^2$ should be changed. The point is that integration over $k_\perp^2$ always runs from some starting point μ^2 to the total invariant energy

$$s = (p+q)^2 \approx w(1-x), \tag{19}$$

with $w = 2pq$. In the DGLAP framework, where x is not far from 1, $s \approx Q^2$, so in the DGLAP equation (11) the upper limit is Q^2. On the other hand, $s \approx w$ at small x. This leads to replacement of Eq. (11) by the following equation:

$$f_r^{(\mathrm{pert})}(x,Q^2) = \int\limits_{\mu^2}^{w} \frac{dk_\perp^2}{k_\perp^2} \int\limits_{\beta_0}^{1} \frac{d\beta}{\beta} \alpha_s\left(\frac{k_\perp^2}{\beta}\right) f_{r'}^{(\mathrm{pert})}\left(\frac{x}{\beta}, \frac{Q^2}{k_\perp^2}\right) P_{r'r}(\beta). \tag{20}$$

with

$$\beta_0 = x + k_\perp^2/w. \tag{21}$$

Obviously, $\beta_0 \approx x$ at large x. In addition, one can neglect the β-dependence of $\alpha_s(k_\perp^2/\beta)$ at large x, arriving at the integral DGLAP Eq. (11).

2.3. Case C. Finally, when μ does not satisfy Eq. (15) and, in addition, $x \ll 1$, the coupling $\alpha_s(k_\perp^2/\beta)$ in Eq. (20) should be replaced by $\alpha_{\mathrm{eff}}(k_\perp^2/\beta)$:

$$\alpha_{\mathrm{eff}}(k_\perp^2/\beta) = \frac{1}{b}\frac{l_0}{(l_0^2+\pi^2)} - \frac{1}{\pi b}\arctan\left(\frac{\pi}{l_0}\right) + \frac{1}{\pi b}\arctan\left(\frac{\pi}{l}\right), \tag{22}$$

with $l = \ln\left(k_\perp^2/(\beta\Lambda^2)\right)$. It converts Eq. (20) into a new equation:

$$f_r^{(\mathrm{pert})}(x,Q^2) = \int\limits_{\mu^2}^{w} \frac{dk_\perp^2}{k_\perp^2} \int\limits_{\beta_0}^{1} \frac{d\beta}{\beta} \alpha_s\left(\frac{k_\perp^2}{\beta}\right) f_{r'}^{(\mathrm{pert})}\left(\frac{x}{\beta}, \frac{Q^2}{k_\perp^2}\right) P_{r'r}(\beta). \tag{23}$$

Equations (20), (23) explicitly demonstrate that there is no factorization between the integrations over β and $k_\perp^2$ at small x. In other words, the factorization between the longitudinal and transverse spaces taking place in DGLAP vanish in the small-x kinematics. Equations (20), (23) also show that structure functions at small x depend on Q^2 through the integration limit β_0, while the effective couplings at $x \ll 1$ differ a lot from the standard parametrization given by Eq. (1). Equation (23) corresponds to the case when both Eqs. (14) and (15) are violated, while the expressions for the couplings and structure functions in Subsecs. 2.1 and 2.2 correspond to violation of either Eq. (14) or (15) and can easily be obtained from Eqs. (22), (23). Nevertheless, we accentuate that Eq. (23) is also approximation obtained in [8] in order to factorize the coupling. This expression should be used only in the context of evolution equations. A more general treatment of the QCD coupling, where the factorization

was not required, was done in [9]. We discuss it briefly in the next section. To conclude, we would like to notice that Eq. (23) was obtained with constructing dispersion relations for forward scattering amplitudes. This approach is similar to the approach of [10] but differs from the alternative approaches (see, e.g., [11, 12]) where the dispersion relations were constructed for α_s by itself.

Obviously, logarithms of x are large in the small-x region, so their resummation to all orders in α_s is important. This is beyond the reach of DGLAP, where only logarithms of Q^2 are resummed. A generalization of the DGLAP equations to the small-x region was done in [13] (see also overview [14]) for the spin structure function g_1 and nonsinglet component of F_1: there the leading, double logarithms of both x and Q^2 were resummed and, at the same time, the QCD coupling in each rung of the involved Feynman graphs was running. Applying the saddle-point method to the structure functions obtained in [13] led to their Regge behavior at $x \to 0$, with the intercepts being just numbers independent of external parameters (e.g., α_s, etc.). The intercept of the nonsinglet F_1 proved to be 0.38, the intercept of the nonsinglet g_1 was 0.42, and the intercept of the singlet g_1 was 0.86.

3. GENERAL STRUCTURE OF THE EVOLUTION EQUATIONS FOR DIS STRUCTURE FUNCTIONS

In this section, we consider general expressions for the structure functions, skipping unessential details. Throughout the present section we imply that collinear factorization is used, though a generalization to k_T-factorization is easy to do. As equations for the structure functions involve convolutions (see, e.g., Eq. (5)), it is convenient to represent the perturbative contribution $f_r^{(\mathrm{pert})}$ in terms of the Mellin transform:

$$f_r^{(\mathrm{pert})} = \int\limits_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} x^{-\omega} F_r(\omega, y), \tag{24}$$

where we have denoted $y = \ln(Q^2/\mu^2)$ and introduced the Mellin amplitude F_r. Focusing on the t-channel intermediate state with two partons and neglecting other contributions (see [14] for detail) allow us to compose the following system of equations for F_r:

$$\left[\omega + \frac{\partial}{\partial y}\right] F_r(\omega, y) = F_{r'}(\omega, y) H_{r'r}(\omega). \tag{25}$$

Solving these equations, we arrive at

$$f(x,Q^2)=\sum_r \int\limits_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} x^{-\omega} F_r(\omega,y)\delta r(\omega) = \\ = \sum_r \int\limits_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi i} x^{-\omega} C_r(\omega)\, \mathrm{e}^{y\Omega_r(\omega)}\, \delta r(\omega), \quad (26)$$

where $r=q,g$, so that $\delta r=\delta q,\delta g$, and Ω_r is made of new matrix of anomalous dimensions $H_{r'r}$. Both C_r and $H_{r'r}$ account for both the total resummation of logarithms of x and running α_s effects. As α_s depends on the longitudinal Sudakov variables, it participates in the Mellin transform (see [9] for detail). As a result, α_s with the time-like argument is replaced by

$$A(\omega)=\frac{1}{b}\left[\frac{\eta}{\eta^2+\pi^2}-\int\limits_0^\infty \frac{d\rho\,\mathrm{e}^{-\omega\rho}}{(\rho+\eta)^2+\pi^2}\right], \quad (27)$$

where $\eta=\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})$ and μ is the IR cut-off. When the argument of the coupling is space-like, the π^2 terms are absent.

Comparison of Eqs. (26) and (8) shows that the coefficient functions C_r do not depend on Q^2 at all. The Q^2-dependence of $f_r(x,Q^2)$ is located in the exponent of (26) and it is controlled by the anomalous dimensions. The small-x asymptotics of f_r can be obtained by applying the saddle-point method to Eq. (26). The stationary phase is obtained as a solution to the following equation:

$$\frac{d}{d\omega}\left[\omega\xi+\ln C_r(\omega)+y\Omega_r(\omega)\right]=\xi+\frac{1}{C_r}\frac{dC_r}{d\omega}+y\frac{d\Omega_r}{d\omega}=0, \quad (28)$$

where $\xi=\ln(1/x)$. This equation should be solved at $\xi\to\infty$ and fixed y, and the solution, ω_0, is called the stationary point. The rightmost root of Eq. (28) stipulates the small-x asymptotic behavior of f_r, with

$$\Delta\equiv\max[\omega_0] \quad (29)$$

being the intercept of the structure function. Equation (28) implies that the large factor ξ must be equated by another large factor. As y is fixed, such a factor can be any of the following options:

(i)
$$C_r(\omega)\to -0, \quad (30)$$

(ii)
$$\frac{dC_r(\omega)}{d\omega}\to-\infty, \quad (31)$$

(iii)

$$\frac{d\Omega_r(\omega)}{d\omega} \to -\infty, \tag{32}$$

when $\omega \to \omega_0$. Accounting for logarithmic contributions $\sim \ln^n x$ to $C_r(\omega)$ and Ω_r means that they acquire contributions $\sim c_n/\omega^{1+n}$ and $\sim c'_n/\omega^n$, respectively, and, therefore, any of them and $dC_r/d\omega$, which is $\sim c_n/\omega^{2+n}$, are singular at $\omega \to \omega_0 = 0$. Obviously, fulfilment of Eq. (30) can be achieved only when a part of c_k is negative, which is in contrast calculations made in DGLAP and to expressions for the nonsinglet structure functions obtained in [13]. Then, let us note that, as contributions to $dC_r/d\omega$ are more singular than contributions to C_r or Ω_r, the case (ii) is the most important compared to (i) and (iii). Indeed, ω should be small in order to prevent oscillations of the factor $\exp(\xi\omega)$ in Eq. (26) at large ξ. The series of the pole contributions $\sim 1/\omega^k$ can be summed up. For example, the coefficient function $C_{\rm NS}$ and anomalous dimension $H_{\rm NS}$ for the nonsinglet structure function $F_1^{\rm NS}$ (for this case $\Omega_r(\omega)$ is replaced by $H_{\rm NS}(\omega)$) proved to be

$$C_{\rm NS} = \frac{\omega}{\omega - H_{\rm NS}}, \quad H_{\rm NS} = (1/2)\Big[\omega - \sqrt{\omega^2 - B_{\rm NS}(\omega)}\Big]. \tag{33}$$

When α_s is fixed, $B_{\rm NS} = 2\alpha_s C_F/\pi$, otherwise it is given by a much more complicated expression (see [13] for detail). It is easy to see that all coefficients in the expansion of $C_{\rm NS}$ into series in $1/\omega^n$ are positive. This also excludes Eq. (30) and demonstrates that Eq. (31) corresponds to the most important case. So, we arrive at the asymptotics of f in the form of the following contributions of the Regge type: at $x \to 0$

$$f(x, Q^2) \sim x^{-\Delta} \left(\frac{Q^2}{\mu^2}\right)^{\Omega(\Delta)} \delta r. \tag{34}$$

Equations (26), (34) demonstrate explicitly that the dependence of the structure functions on Q^2 has nothing to do with the Q^2-dependence of the coupling.

4. REMARK ON THE DGLAP FITS FOR STRUCTURE FUNCTIONS

The asymptotics (34) is of the Regge type. The Regge behavior is generated by total resummations of leading logarithms of x and it is unrelated to behavior of the initial parton densities $\delta q(x)$ and $\delta g(x)$. In particular, both $\delta q(x)$ and $\delta g(x)$ are not supposed to reveal a power behavior $\sim x^{-a}$ at small x. On the contrary, the standard DGLAP fits

$$\delta q, \delta g = N x^{-a}(1-x)^b(1+cx)^d, \tag{35}$$

with $a, b, c, d, N > 0$, conventionally include such factors. The singular factors x^{-a} are incorporated in the fits ad hoc in order to provide f with a steep growth

at small x and thereby meet experimental data. Indeed, in the ω-space the factor x^{-a} corresponds to the pole contribution

$$\delta q_p = \frac{1}{\omega - a}. \tag{36}$$

Being substituted in Eq. (7), the pole contribution (36) straightforwardly leads to the Regge asymptotics

$$f \sim x^{-a} \left[\frac{\ln\left(Q^2/\Lambda^2\right)}{\ln\left(\mu^2/\Lambda^2\right)} \right]^{\gamma(a)/b}, \tag{37}$$

which differs a lot from both Eq. (34) and the well-known LO DGLAP asymptotics

$$f \sim \exp \sqrt{\frac{C}{2\pi b} \ln\left(1/x\right) \ln\left[\frac{\ln\left(Q^2/\Lambda^2\right)}{\ln\left(\mu^2/\Lambda^2\right)} \right]}. \tag{38}$$

Let us note that Eqs. (34), (38) involve nonsingular fits. Confronting (34) to (37) proves that the only role of the singular factors x^{-a} is, actually, to mimic the total resummation of $\ln^n x$. When the resummation is accounted for, these factors can be dropped and, therefore, the number of parameters in the fits can be reduced. The qualitative difference between the asymptotics (34) and (37) includes two items:

(i) The intercept Δ in Eq. (34) is calculated, while a in (37) is fixed from experiment.

(ii) The factor $x^{-\Delta}$ in Eq. (34) is present in the asymptotic expressions, i.e., at $x \to 0$, only. It never appears in expressions for the structure functions like Eq. (7) at finite x. On the contrary, as soon as the fit (35) is used, the factor x^{-a} is present in Eq. (7) at any x, including large $x \sim 1$. It means that, in a sense, the x-dependence of f is always controlled by its asymptotics, when singular fits are used. This contradicts the observation made in [17]: the small-x asymptotics represent the structure functions reliable at very small x ($x < 10^{-8}$) only, while at $x > 10^{-8}$ the structure function is much greater than its asymptotics. The latter leads to the quantitative difference between a and Δ: In order to force Eq. (37) to represent f at presently available x, i.e., at $x > 10^{-8}$, one has to increase a, which leads to the following relation:

$$a > \Delta. \tag{39}$$

For this reason, we will name the fictitious intercepts a as pseudo-intercepts. As their values are determined from experiment at available x, they always exceed genuine intercepts. So, the situation looks like that: On the one hand, the use of the pseudo-intercepts a allows one to approximate Eq. (7) by its asymptotics which is given by simple Regge-like expressions (37). On the other hand, the

power in the x-dependence in Eq. (37) has nothing to do with impact of genuine reggeons introduced in the theory of Regge poles. The wide-spread tactics is to use pseudo-reggeons, instead of resummation of $\ln x$, in order to solve immediate practical tasks. This leads to serious theoretical problems, especially important for the singlet structure function F_1 and the singlet parton distributions: there are no available expressions for those objects besides their Regge asymptotics $x^{-\Delta_P}$, with Δ_P being the pomeron intercept. In order to fit such asymptotics to explanation of experimental data at available energies, they use one or several pomerons with large pseudo-intercepts Δ_P violating the Froissart bound. Finally, let us note that the necessity of including singular factors in the fits is a clear indication that essential logarithms of x are not accounted for.

5. STRUCTURE FUNCTIONS AT SMALL Q^2

The description of the structure functions at small Q^2 is important because this kinematics has been investigated experimentally. For instance, the spin-dependent structure functions at small Q^2 are investigated by the COMPASS collaboration (see, e.g., [15]). On the other hand, this region is absolutely beyond the reach of DGLAP. Despite this, sometimes in the literature (see, e.g., [2, 3], recent paper [5] and references therein), the DGLAP parametrization $\alpha_s = \alpha_s(Q^2)$ is treated as an exact expression where Q^2 can acquire arbitrarily small values and, therefore, the shift

$$\alpha_s(Q^2) \to \alpha_s(Q^2 + \mu^2) \tag{40}$$

is needed at small Q^2 and especially at $Q^2 \to 0$ in order to keep α_s within the perturbative domain. However, in the present paper we have shown that the DGLAP parametrization $\alpha_s = \alpha_s(Q^2)$ is an approximation valid for large Q^2 only, where the shift (40) is totally unnecessary and even cannot be seen at $\mu^2 \ll Q^2$. Actually, the DGLAP parametrization fails at small Q^2 or at small x, so $\alpha_s(Q^2)$ should be replaced by the effective coupling $\alpha_{\rm eff}$. The coupling $\alpha_{\rm eff}$ does not depend on Q^2 at small Q^2 or small x at all, which makes the shift (40) absolutely unnecessary. Let us stress that interpretation of the approximation $\alpha_s = \alpha_s(Q^2)$ and the shift (40) as exact expressions has led to various misconceptions abundant in the literature. On the other hand, the small-Q^2 kinematics has been investigated experimentally by the COMPASS collaboration (see, e.g., [15]), so it is important to describe the structure functions in this region. In [16], we proved that the expressions in [13] for the structure function g_1 at small x and large Q^2 can be extended to small Q^2 by shifting

$$k_\perp^2 \to k_\perp^2 + \mu^2 \tag{41}$$

in propagators of the soft quarks and gluons. The shift (41), where μ is a cut-off, is necessary to regulate infrared singularities in involved Feynman graphs. This

shift eventually leads to the shift

$$Q^2 \to Q^2 + \mu^2 \tag{42}$$

in logarithmic contributions to the structure functions. Our estimate for μ obtained using the principle of minimal sensitivity [18] was $\mu \approx 10\Lambda_{\rm QCD}$ (more details can be found in [19]). The shift (42) can be neglected at $Q^2 \gg \mu^2$ but becomes essential at small Q^2, making possible description of the kinematic region where Q^2 are small. For example, DIS at such a kinematics has been studied experimentally by the COMPASS collaboration. Therefore, it is the shift (41) that brings theoretical grounds for description of the small-Q^2 region. Let us stress that this shift should not be applied to α_s and it has nothing to do with the baseless modification (40) of the coupling. On the other hand, the the shift (41) together with the shift $\alpha_s(k_\perp^2) \to \alpha_s(k_\perp^2 + \mu^2)$ can be used when k are the momenta of the t-channel virtual gluons connecting the perturbative part of the structure function to the parton distributions in the factorization convolutions (see [19] for detail).

6. REMARK ON HIGHER TWISTS

It is interesting to notice that the shift (40) can be used to clarify the problem of higher twists and, in addition, it explains the puzzle of the power-Q^2 corrections. Namely, the contributions $\sim 1/(Q^2)^n$ to DIS structure functions, usually attributed to higher twists, are known to be present at large Q^2 but enigmatically disappear at small Q^2 where they could be extremely impactive. A simple and natural solution of this puzzle was found in [16] (see also overview [14]). In brief, it can be reduced to the following: Among different contributions to the structure functions, there are contributions

$$T_n \sim \ln^n\left(\frac{Q^2+\mu^2}{\mu^2}\right). \tag{43}$$

In the region of large Q^2, where $Q^2 > \mu^2$, such contributions can be expanded into power series in the following way:

$$T_n \sim \ln^n\left(\frac{Q^2}{\mu^2}\right) + \sum\left(\frac{\mu^2}{Q^2}\right)^k. \tag{44}$$

The logarithmic term in Eq. (44) is included into the leading twist contributions, whereas the power terms are identical to the power terms attributed to higher twists. Such terms are conventionally supposed to have nonperturbative origin. However, Eq. (44) explicitly demonstrates that there are power terms of purely perturbative nature. Surely, they should be accounted for in the first time and only the rest should be attributed to higher twists.

On the other hand, the expansion (44) holds at large Q^2 only. In the small-Q^2 region, the power expansion of Eq. (43) looks different:

$$T_n \sim 1 + \sum \left(\frac{Q^2}{\mu^2}\right)^k. \tag{45}$$

The first term in the r.h.s. of Eq. (45) is included into the leading twist contributions, while the other terms are again the power-Q^2 corrections, though with Q^2 in the nominator. Equation (45) proves that the power corrections $\sim 1/(Q^2)^n$ can never become singular at small Q^2. Comparing Eq. (44) to (45) reveals that the corrections $\sim 1/(Q^2)^n$ are about vanishing at

$$Q^2 \sim \mu^2, \tag{46}$$

where none of expansions (44), (45) can be used. As we have mentioned, we fixed μ, using the principle of minimal sensitivity [18] and arrived at $\mu \approx 1$ GeV for the nonsinglet contributions to the structure functions F_1 and g_1. Our prediction of vanishing power corrections $\sim 1/(Q^2)^n$ at Q^2 approaching 1 GeV^2 perfectly agrees with estimates obtained by analysis of experimental data.

CONCLUSION

In the present paper, we have analyzed the Q^2-dependence of the structure functions, considering separately this dependence in the kinematic region of large and small x. Our consideration embraced both large and small values of Q^2. We argued against the conventional point of view that the Q^2-dependence at any Q^2 and x is originated by the DGLAP parametrization $\alpha_s = \alpha_s(Q^2)$ leading to Q^2-dependence of the intercepts of the structure functions. We showed that this parametrization holds exceptionally in the region of large x and Q^2, providing that in this region the factorization scale should be large. Outside this region, α_s should be replaced by the effective coupling $\alpha_{\rm eff}$. At small x, coupling $\alpha_{\rm eff}$ depends on both transverse and longitudinal momenta, which destroys the factorization of the phase space into the longitudinal and transverse spaces taking place in DGLAP. In addition, the integration limits do not involve Q^2. As a result, the effective coupling does not depend on Q^2 in the small-x region. This leads to the Regge small-x asymptotics of the structure functions, with intercepts independent of Q^2. We also argued against the shift $\alpha_s(Q^2) \to \alpha_s(Q^2+\mu^2)$ used in the literature to keep α_s in perturbative domain at small Q^2. In this regard, we reminded that the parametrization $\alpha_s(Q^2)$ as well as DGLAP in whole should be used at large Q^2 only. Superficial use of the shift $\alpha_s(Q^2) \to \alpha_s(Q^2+\mu^2)$ at small Q^2 has led to various misconceptions known in the literature. In contrast, we advocated the use of shift $Q^2 \to Q^2 + \mu^2$ in such evolution equations, derivation

of which is not based on assuming Q^2 large. We stressed that the shift did not involve parametrization of the QCD coupling. Eventually, we have considered theoretical grounds for such a shift and discussed its application to the power-Q^2 contributions to the structure functions.

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