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BELOKUROV–USYUKINA LOOP REDUCTION IN NONINTEGER DIMENSION

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Belokurov–Usyukina loop reduction method was proposed in 1983 to reduce the number of rungs in triangle ladder-like diagram by one. The disadvantage of the method is that it works in $d = 4$ dimensions only and it cannot be used for calculation of amplitudes in field theory in which we are required to put all the incoming and outgoing momenta on shell. We generalize the Belokurov–Usyukina loop reduction technique to noninteger $d = 4 - 2\varepsilon$ dimensions. In this paper, we show how a two-loop triangle diagram with particular values of indices of scalar propagators in the position space can be reduced to a combination of three one-loop scalar diagrams. It is known that any one-loop massless momentum integral can be presented in terms of Appell's function F_4. This means that particular diagram considered in the present paper can be represented in terms of Appell's function F_4 too. Such a generalization of Belokurov–Usyukina loop reduction technique allows us to calculate that diagram by this method exactly without decomposition in terms of the parameter ε.

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INTRODUCTION

The main part of the results of multiloop calculus in high-energy physics has been done as an expansion in terms of ε that is the parameter of dimensional regularization [1, 2]. However, one-loop massless diagrams can be calculated in all order in ε and can be represented in terms of Appell's hypergeometric function [3, 4]. To calculate any loop (in momentum space) integral, it would be good to have a technique that reduces the number of loops by one in a recursive manner. Such a method exists in $d = 4$ space-time dimensions for triangle ladder diagrams. It was discovered in the early eighties by Belokurov and Usyukina [5–7]. We refer to that construction as to Belokurov–Usyukina loop reduction technique. To our knowledge, there was no analog of this loop reduction procedure in $d = 4 - 2\varepsilon$ dimensions. The result of calculation of the triangle ladder diagrams in $d = 4$ is UD functions [8–10]. Their properties, in particular the invariance with respect to Fourier transform, have been studied in [11–14] and their MB transforms have been studied in [14, 15].

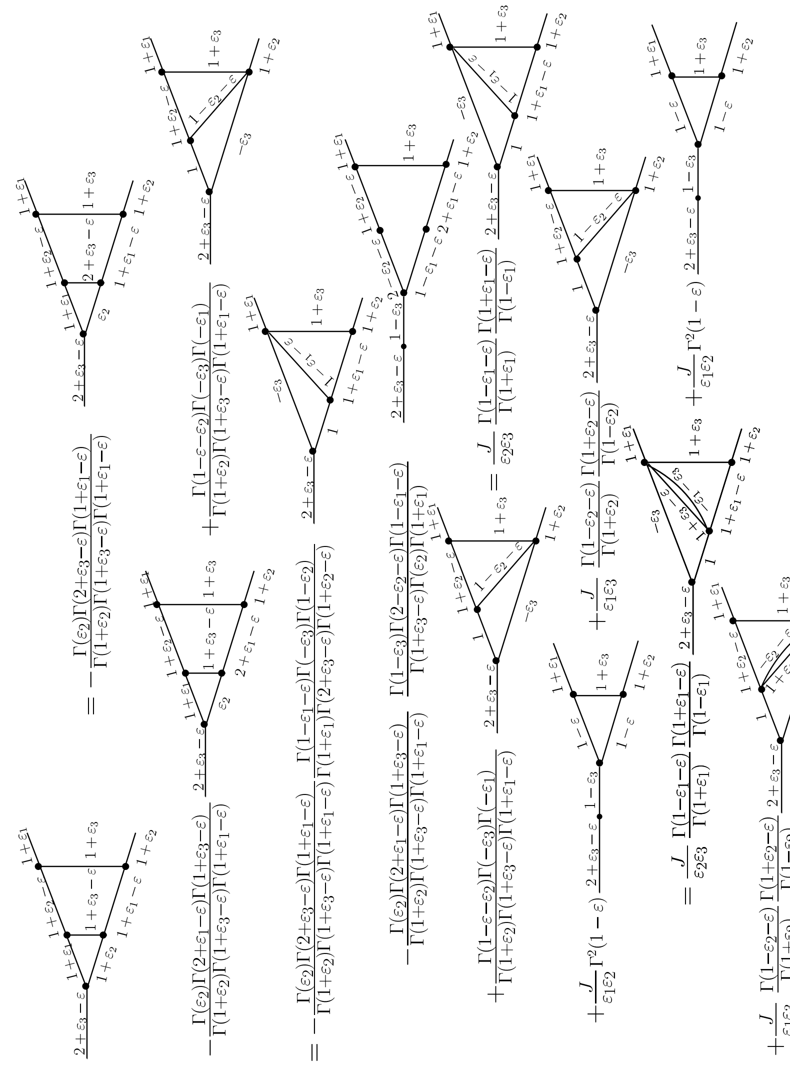

Fig. 1. Reduction of two-loop diagram

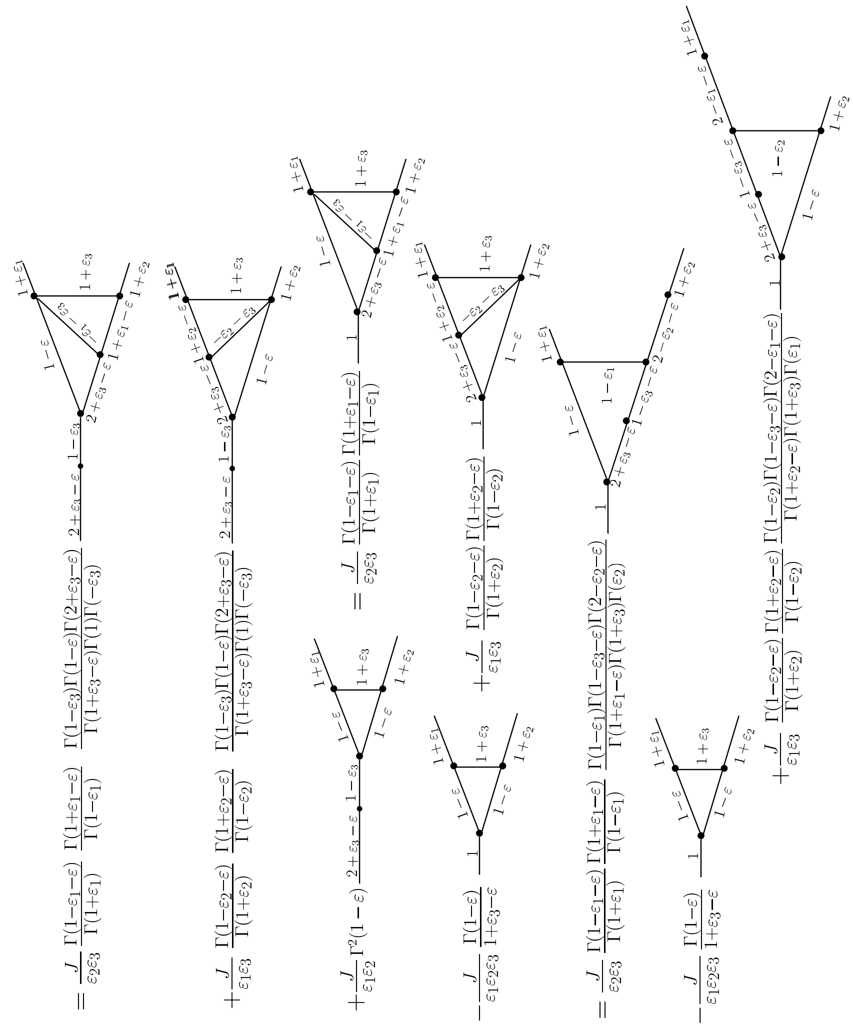

Fig. 2. Reduction of two-loop diagram. Continuation of Fig. 1

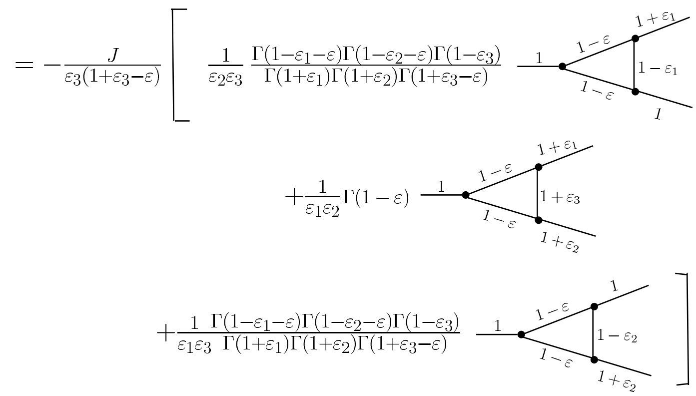

Fig. 3. Final result

In this paper, we propose generalization of the Belokurov–Usyukina loop reduction technique to noninteger dimensions. In particular, we consider a two-loop triangle diagram in which the propagator indices in the position space are $1-\varepsilon$ or 1. We use the uniqueness method and method of integration by parts [16–19]. The detailed step-by-step construction for $d = 4$ space-time dimensions is presented in [15]. Here we construct analogs of Figs. 2 and 3 of [15] with slightly modified indices of line in order to apply the uniqueness technique to the case of triangle ladder diagram in noninteger number of dimensions.

LOOP REDUCTION IN $d = 4 - 2\varepsilon$ DIMENSIONS

The result of the reduction is presented in three figures, Fig. 2 is continuation of Fig. 1, and Fig. 3 is continuation of Fig. 2. The transformations depicted in the diagrams are integration by parts, triangle–star and star–triangle relations (for review of these relations, see [19]). As we can see, the final result depicted in Fig. 3 is a sum of one-loop diagrams. Each of the diagrams in the r.h.s. of Fig. 3 can be transformed to the momentum space in which the result for each one of them is a combination of Appell's functions [4]. To our knowledge, it is the first known case where two-loop diagram in noninteger number of dimensions can be reduced to Appell's function F_4 in all order in the regularization parameter ε for arbitrary kinematic region in the momentum space.

The figures are self-explaining. A new d-dimensional measure $Dx \equiv \pi^{-d/2}d^d x$ introduced in [20] is assumed in the position space to avoid powers of π in figures. The factor J that appears in Figs. 1–3 is

$$J = \frac{\Gamma(1-\varepsilon_1)\Gamma(1-\varepsilon_2)\Gamma(1-\varepsilon_3)}{\Gamma(1+\varepsilon_1-\varepsilon)\Gamma(1+\varepsilon_2-\varepsilon)\Gamma(1+\varepsilon_3-\varepsilon)}.$$

This generalizes the corresponding factor J of [15]. The condition for auxiliary parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$ remains the same as in [5, 7, 15],

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0.$$

At the end of the calculation, we have to take the limit of vanishing these ε-terms.

CONCLUSION

We have shown that the loop reduction in noninteger number of dimensions apparently exists. The two-loop diagram in $d = 4 - 2\varepsilon$ dimensions has been represented as one-loop diagrams in the same kinematic region in the momentum space. We have considered an arbitrary kinematic region, and even on-shell external momenta can be taken. In that case the result remains finite and regularized dimensionally in terms of poles in regularization parameter ε. However, not all of the indices in the position space are $1 - \varepsilon$. This index in the position space means index 1 in the momentum space, which corresponds to the physical case of momentum propagator in the regularized $(4 - 2\varepsilon)$-dimensional theory.

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