

FINITE UNIFIED THEORIES AND THEIR PREDICTIONS

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All-loop Finite Unified Theories (FUTs) are very interesting $N = 1$ supersymmetric Grand Unified Theories (GUTs) realizing an old field theory dream, and moreover have a remarkable predictive power due to the required reduction of couplings. The reduction of the dimensionless couplings in $N = 1$ GUTs is achieved by searching for renormalization group invariant (RGI) relations among them holding beyond the unification scale. Finiteness results from the fact that there exist RGI relations among dimensional couplings that guarantee the vanishing of all beta functions in certain $N = 1$ GUTs even to all orders. Additional developments in the soft supersymmetry breaking sector of $N = 1$ GUTs and FUTs lead to exact RGI relations, i.e., reduction of couplings, in this dimensional sector of the theory, too. Based on the above theoretical framework, phenomenologically consistent FUTs have been constructed. Here we review two FUT models based on the $SU(5)$ gauge group. Confronting their predictions with the top and bottom quark masses and other experimental constraints, a light Higgs-boson mass in the range $M_H \sim 121-126$ GeV has been predicted, in striking agreement with the recent experimental results from ATLAS and CMS. Furthermore, naturally a relatively heavy s-spectrum emerged with colored supersymmetric particles above ~ 1.5 TeV in agreement with the nonobservation of those particles at the LHC. Restricting further the parameter space of the best version of the $SU(5)$ FUT according to the reported accuracy of the Higgs-boson mass and B -physics observables, we find predictions for the rest of the Higgs masses and the s-spectrum.

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INTRODUCTION

A large and sustained effort has been made in recent years aiming to achieve a unified description of all interactions. Out of this endeavor two main directions have emerged as the most promising to attack the problem, namely, the super-

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string theories and noncommutative geometry. The two approaches, although at a different stage of development, have common unification targets and share similar hopes for exhibiting improved renormalization properties in the ultraviolet (UV) as compared to ordinary field theories. Moreover, the two frameworks came closer by the observation that a natural realization of noncommutativity of space appears in the string theory context of D-branes in the presence of a constant background antisymmetric field [1]. Among the numerous important developments in both frameworks, it is worth noting two conjectures of utmost importance that signal the developments in certain directions in string theory and not only, related to the main theme of the present review. The conjectures refer to (i) the duality among the 4-dimensional $N = 4$ supersymmetric Yang–Mills theory and the type IIB string theory on $AdS_5 \times S^5$ [2]; the former being the maximal $N = 4$ supersymmetric Yang–Mills theory is known to be UV all-loop finite theory [3,4], (ii) the possibility of «miraculous» UV divergence cancellations in 4-dimensional maximal $N = 8$ supergravity leading to a finite theory, as has recently been confirmed in a remarkable 4-loop calculation [5–9]. However, despite the importance of having frameworks to discuss quantum gravity in a self-consistent way and possibly to construct there finite theories, it is also very interesting to search for the minimal realistic framework in which finiteness can take place. After all, the history of our field teaches us that if a new idea works, it does that in its simplest form. In addition, the main goal expected from a unified description of interactions by the particle physics community is to understand the present-day large number of free parameters of the Standard Model (SM) in terms of a few fundamental ones. In other words, to achieve *reduction of couplings* at a more fundamental level.

To reduce the number of free parameters of a theory and thus render it more predictive, one is usually led to introduce a symmetry. Grand Unified Theories (GUTs) are very good examples of such a procedure [10–14]. For instance, in the case of minimal $SU(5)$, because of (approximate) gauge coupling unification, it was possible to reduce the gauge couplings by one and give a prediction for one of them. In fact, LEP data [15] seem to suggest that a further symmetry, namely $N = 1$ global supersymmetry (SUSY) [16,17] should also be required to make the prediction viable. GUTs can also relate the Yukawa couplings among themselves, again $SU(5)$ provided an example of this by predicting the ratio M_τ/M_b [18] in the SM. Unfortunately, requiring more gauge symmetry does not seem to help, since additional complications are introduced due to new degrees of freedom, in the ways and channels of breaking the symmetry, and so on.

A natural extension of the GUT idea is to find a way to relate the gauge and Yukawa sectors of a theory, that is to achieve Gauge–Yukawa Unification (GYU) [19–21]. A symmetry which naturally relates the two sectors is supersymmetry, in particular $N = 2$ SUSY [22]. It turns out, however, that $N = 2$ supersymmetric theories have serious phenomenological problems due to light

mirror fermions. Also in superstring theories and in composite models there exist relations among the gauge and Yukawa couplings, but both kinds of theories have phenomenological problems, which we are not going to address here.

In our studies [19–21, 23–28], we have developed a complementary strategy in searching for a more fundamental theory possibly at the Planck scale, whose basic ingredients are GUTs and supersymmetry, but its consequences certainly go beyond the known ones. Our method consists of hunting for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. This programme, called Gauge–Yukawa unification scheme, applied in the dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, had already noticeable successes by predicting correctly, among others, the top quark mass in the finite and in the minimal $N = 1$ supersymmetric $SU(5)$ GUTs [23–25]. An impressive aspect of the RGI relations is that one can guarantee their validity to all orders in perturbation theory by studying the uniqueness of the resulting relations at one loop, as was proven in the early days of the programme of *reduction of couplings* [29–32]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all orders in perturbation theory [33–37].

It is worth noting that the above principles have only been applied in supersymmetric GUTs for reasons that will be transparent in the following sections. We should also stress that our conjecture for GYU is by no means in conflict with the interesting proposals mentioned before (see also [38–40]), but it rather uses all of them, hopefully in a more successful perspective. For instance, the use of SUSY GUTs comprises the demand of the cancellation of quadratic divergences in the SM. Similarly, the very interesting conjectures about the infrared fixed points are generalized in our proposal, since searching for RGI relations among various couplings corresponds to searching for *fixed points* of the coupled differential equations obeyed by the various couplings of a theory.

Although SUSY seems to be an essential feature for a successful realization of the above programme, its breaking has to be understood too, since it has the ambition to supply the SM with predictions for several of its free parameters. Indeed, the search for RGI relations has been extended to the soft SUSY breaking sector (SSB) of these theories [28, 41], which involves parameters of dimension one and two. Then a very interesting progress has been made [42–48] concerning the renormalization properties of the SSB parameters based conceptually and technically on the work of [49]: the powerful supergraph method [50–53] for studying supersymmetric theories has been applied to the softly broken ones by using the «spurion» external space-time independent superfields [54]. In the latter method, a softly broken supersymmetric gauge theory is considered as a supersymmetric one in which the various parameters such as couplings and masses have been promoted to external superfields that acquire «vacuum expectation values». Based on this method, the relations among the soft term renormalization

and that of an unbroken supersymmetric theory have been derived. In particular, the β functions of the parameters of the softly broken theory are expressed in terms of partial differential operators involving the dimensionless parameters of the unbroken theory. The key point in the strategy of [45–48] in solving the set of coupled differential equations so as to be able to express all parameters in an RGI way, was to transform the partial differential operators involved to total derivative operators. This is indeed possible to be done on the RGI surface which is defined by the solution of the reduction equations.

On the phenomenological side there exist some serious developments, too. Previously an appealing «universal» set of soft scalar masses was assumed in the SSB sector of supersymmetric theories, given that apart from economy and simplicity (1) they are part of the constraints that preserve finiteness up to two loops [55,56], (2) they are RGI up to two loops in more general supersymmetric gauge theories, subject to the condition known as $P = 1/3Q$ [41], and (3) they appear in the attractive dilaton dominated SUSY breaking superstring scenarios [57–59]. However, further studies have exhibited a number of problems, all due to the restrictive nature of the «universality» assumption for the soft scalar masses. For instance, (a) in finite unified theories the universality predicts that the lightest supersymmetric particle is a charged particle, namely the superpartner of the τ lepton $\tilde{\tau}$, (b) the Minimal Supersymmetric Standard Model (MSSM) with universal soft scalar masses is inconsistent with the attractive radiative electroweak symmetry breaking [59], and (c) which is the worst of all, the universal soft scalar masses lead to charge and/or color breaking minima deeper than the standard vacuum [60]. Therefore, there have been attempts to relax this constraint without losing its attractive features. First, an interesting observation was made that in $N = 1$ Gauge–Yukawa unified theories there exists an RGI sum rule for the soft scalar masses at lower orders; at one loop for the nonfinite case [61] and at two loops for the finite case [62]. The sum rule manages to overcome the above unpleasant phenomenological consequences. Moreover, it was proven [48] that the sum rule for the soft scalar masses is RGI to all orders for both the general and the finite case. Finally, the exact β function for the soft scalar masses in the Novikov–Shifman–Vainshtein–Zakharov (NSVZ) scheme [63–65] for the softly broken supersymmetric QCD has been obtained [48]. Armed with the above tools and results, we are in a position to study and predict the spectrum of the full finite models in terms of few input parameters. In particular, a prediction for the lightest MSSM Higgs boson can be obtained. It turned out that the prediction is naturally in very good agreement with the discovery of a Higgs-like particle at the LHC [66,67] at around ~ 126 GeV. Identifying the lightest Higgs boson with the newly discovered state, one can restrict the allowed parameter space of the model. We review how this reduction of parameter space impacts the prediction of the SUSY spectrum and the discovery potential of the LHC and future e^+e^- colliders.

1. UNIFICATION OF COUPLINGS BY THE RGI METHOD

Let us next briefly outline the idea of reduction of couplings. Any RGI relation among couplings (which does not depend on the renormalization scale μ explicitly) can be expressed, in the implicit form $\Phi(g_1, \dots, g_A) = \text{const}$, which has to satisfy the partial differential equation (PDE)

$$\mu \frac{d\Phi}{d\mu} = \nabla \cdot \beta = \sum_{a=1}^A \beta_a \frac{\partial \Phi}{\partial g_a} = 0, \quad (1)$$

where β_a is the β function of g_a . This PDE is equivalent to a set of ordinary differential equations, the so-called reduction equations (REs) [29, 30, 68],

$$\beta_g \frac{dg_a}{dg} = \beta_a, \quad a = 1, \dots, A, \quad (2)$$

where g and β_g are the primary coupling and its β function, and the counting on a does not include g . Since maximally $(A - 1)$ independent RGI «constraints» in the A -dimensional space of couplings can be imposed by the Φ_a 's, one could in principle express all the couplings in terms of a single coupling g . The strongest requirement is to demand power series solutions to the REs,

$$g_a = \sum_n \rho_a^{(n)} g^{2n+1}, \quad (3)$$

which formally preserve perturbative renormalizability. Remarkably, the uniqueness of such power series solutions can be decided already at the one-loop level [29, 30, 68]. To illustrate this, let us assume that the β functions have the form

$$\begin{aligned} \beta_a &= \frac{1}{16\pi^2} \left[\sum_{b,c,d \neq g} \beta_a^{(1)bcd} g_b g_c g_d + \sum_{b \neq g} \beta_a^{(1)b} g_b g^2 \right] + \dots, \\ \beta_g &= \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \dots, \end{aligned} \quad (4)$$

where \dots stands for higher order terms, and $\beta_a^{(1)bcd}$'s are symmetric in b, c, d . We then assume that the $\rho_a^{(n)}$'s with $n \leq r$ have been uniquely determined. To obtain $\rho_a^{(r+1)}$'s, we insert the power series (3) into the REs (2) and collect terms of $\mathcal{O}(g^{2r+3})$ and find

$$\sum_{d \neq g} M(r)_a^d \rho_d^{(r+1)} = \text{lower order quantities},$$

where the r.h.s. is known by assumption, and

$$M(r)_a^d = 3 \sum_{b,c \neq g} \beta_a^{(1)bcd} \rho_b^{(1)} \rho_c^{(1)} + \beta_a^{(1)d} - (2r+1)\beta_g^{(1)} \delta_a^d, \quad (5)$$

$$0 = \sum_{b,c,d \neq g} \beta_a^{(1)bcd} \rho_b^{(1)} \rho_c^{(1)} \rho_d^{(1)} + \sum_{d \neq g} \beta_a^{(1)d} \rho_d^{(1)} - \beta_g^{(1)} \rho_a^{(1)}, \quad (6)$$

Therefore, the $\rho_a^{(n)}$'s for all $n > 1$ for a given set of $\rho_a^{(1)}$'s can be uniquely determined if $\det M(n)_a^d \neq 0$ for all $n \geq 0$.

As will be clear later by examining specific examples, the various couplings in supersymmetric theories have easily the same asymptotic behavior. Therefore, searching for a power series solution of the form (3) to the REs (2) is justified. This is not the case in nonsupersymmetric theories, although the deeper reason for this fact is not fully understood.

The possibility of coupling unification described in this section is without any doubt attractive because the «completely reduced» theory contains only one independent coupling, but it can be unrealistic. Therefore, one often would like to impose fewer RGI constraints, and this is the idea of partial reduction [69,70].

2. REDUCTION OF DIMENSIONFUL PARAMETERS

The reduction of couplings was originally formulated for massless theories on the basis of the Callan–Symanzik equation [29, 30, 68]. The extension to theories with massive parameters is not straightforward if one wants to keep the generality and the rigor on the same level as for the massless case; one has to fulfill a set of requirements coming from the renormalization group equations, the Callan–Symanzik equations, etc., along with the normalization conditions imposed on irreducible Green's functions [71]. See [72] for interesting results in this direction. Here, to simplify the situation, we would like to assume that a mass-independent renormalization scheme has been employed so that all the RG functions have only trivial dependencies of dimensional parameters.

To be general, we consider a renormalizable theory which contains a set of $(N+1)$ dimension-zero couplings, $\{\hat{g}_0, \hat{g}_1, \dots, \hat{g}_N\}$, a set of L parameters with dimension one, $\{\hat{h}_1, \dots, \hat{h}_L\}$, and a set of M parameters with dimension two, $\{\hat{m}_1^2, \dots, \hat{m}_M^2\}$. The renormalized irreducible vertex function satisfies the RG equation

$$0 = \mathcal{D}\Gamma[\Phi' s; \hat{g}_0, \hat{g}_1, \dots, \hat{g}_N; \hat{h}_1, \dots, \hat{h}_L; \hat{m}_1^2, \dots, \hat{m}_M^2; \mu], \quad (7)$$

$$\mathcal{D} = \mu \frac{\partial}{\partial \mu} + \sum_{i=0}^N \beta_i \frac{\partial}{\partial \hat{g}_i} + \sum_{a=1}^L \gamma_a^h \frac{\partial}{\partial \hat{h}_a} + \sum_{\alpha=1}^M \gamma_\alpha^{m^2} \frac{\partial}{\partial \hat{m}_\alpha^2} + \sum_J \Phi_I \gamma_J^{\phi I} \frac{\delta}{\delta \Phi_J}.$$

Since we assume a mass-independent renormalization scheme, the γ 's have the form

$$\begin{aligned}\gamma_a^h &= \sum_{b=1}^L \gamma_a^{h,b}(g_0, \dots, g_N) \hat{h}_b, \\ \gamma_\alpha^{m^2} &= \sum_{\beta=1}^M \gamma_\alpha^{m^2,\beta}(g_0, \dots, g_N) \hat{m}_\beta^2 + \sum_{a,b=1}^L \gamma_\alpha^{m^2,ab}(g_0, \dots, g_N) \hat{h}_a \hat{h}_b,\end{aligned}\quad (8)$$

where $\gamma_a^{h,b}$, $\gamma_\alpha^{m^2,\beta}$ and $\gamma_\alpha^{m^2,ab}$ are power series of the dimension-zero couplings g 's in perturbation theory.

As in the massless case, we then look for conditions under which the reduction of parameters,

$$\hat{g}_i = \hat{g}_i(g) \quad (i = 1, \dots, N), \quad (9)$$

$$\hat{h}_a = \sum_{b=1}^P f_a^b(g) h_b \quad (a = P+1, \dots, L), \quad (10)$$

$$\hat{m}_\alpha^2 = \sum_{\beta=1}^Q e_\alpha^\beta(g) m_\beta^2 + \sum_{a,b=1}^P k_\alpha^{ab}(g) h_a h_b \quad (\alpha = Q+1, \dots, M), \quad (11)$$

is consistent with the RG equation (1), where we assume that $g \equiv g_0$, $h_a \equiv \hat{h}_a$ ($1 \leq a \leq P$) and $m_\alpha^2 \equiv \hat{m}_\alpha^2$ ($1 \leq \alpha \leq Q$) are independent parameters of the reduced theory. We find that the following set of equations has to be satisfied:

$$\beta_g \frac{\partial \hat{g}_i}{\partial g} = \beta_i \quad (i = 1, \dots, N), \quad (12)$$

$$\beta_g \frac{\partial \hat{h}_a}{\partial g} + \sum_{b=1}^P \gamma_b^h \frac{\partial \hat{h}_a}{\partial h_b} = \gamma_a^h \quad (a = P+1, \dots, L), \quad (13)$$

$$\beta_g \frac{\partial \hat{m}_\alpha^2}{\partial g} + \sum_{a=1}^P \gamma_a^h \frac{\partial \hat{m}_\alpha^2}{\partial h_a} + \sum_{\beta=1}^Q \gamma_\beta^{m^2} \frac{\partial \hat{m}_\alpha^2}{\partial m_\beta^2} = \gamma_\alpha^{m^2} \quad (\alpha = Q+1, \dots, M). \quad (14)$$

Using Eq. (7) for γ 's, one finds that Eqs. (12)–(14) reduce to

$$\begin{aligned}\beta_g \frac{df_a^b}{dg} + \sum_{c=1}^P f_a^c \left[\gamma_c^{h,b} + \sum_{d=P+1}^L \gamma_c^{h,d} f_d^b \right] - \gamma_a^{h,b} - \sum_{d=P+1}^L \gamma_a^{h,d} f_d^b &= 0 \\ (a = P+1, \dots, L; \quad b = 1, \dots, P),\end{aligned}\quad (15)$$

$$\beta_g \frac{de_\alpha^\beta}{dg} + \sum_{\gamma=1}^Q e_\alpha^\gamma \left[\gamma_\gamma^{m^2, \beta} + \sum_{\delta=Q+1}^M \gamma_\gamma^{m^2, \delta} e_\delta^\beta \right] - \gamma_\alpha^{m^2, \beta} - \sum_{\delta=Q+1}^M \gamma_\alpha^{m^2, \delta} e_\delta^\beta = 0$$

$$(\alpha = Q+1, \dots, M; \quad \beta = 1, \dots, Q), \quad (16)$$

$$\beta_g \frac{dk_\alpha^{ab}}{dg} + 2 \sum_{c=1}^P \left(\gamma_c^{h, a} + \sum_{d=P+1}^L \gamma_c^{h, d} f_d^a \right) k_\alpha^{cb} +$$

$$+ \sum_{\beta=1}^Q e_\alpha^\beta \left[\gamma_\beta^{m^2, ab} + \sum_{c, d=P+1}^L \gamma_\beta^{m^2, cd} f_c^a f_d^b +$$

$$+ 2 \sum_{c=P+1}^L \gamma_\beta^{m^2, cb} f_c^a + \sum_{\delta=Q+1}^M \gamma_\beta^{m^2, \delta} k_\delta^{ab} \right] - \left[\gamma_\alpha^{m^2, ab} + \sum_{c, d=P+1}^L \gamma_\alpha^{m^2, cd} f_c^a f_d^b +$$

$$+ 2 \sum_{c=P+1}^L \gamma_\alpha^{m^2, cb} f_c^a + \sum_{\delta=Q+1}^M \gamma_\alpha^{m^2, \delta} k_\delta^{ab} \right] = 0$$

$$(\alpha = Q+1, \dots, M; \quad a, b = 1, \dots, P). \quad (17)$$

If these equations are satisfied, the irreducible vertex function of the reduced theory

$$\Gamma_R[\Phi' s; g; h_1, \dots, h_P; m_1^2, \dots, \hat{m}_Q^2; \mu] \equiv$$

$$\equiv \Gamma[\Phi' s; g, \hat{g}_1(g), \dots, \hat{g}_N(g); h_1, \dots, h_P, \hat{h}_{P+1}(g, h), \dots, \hat{h}_L(g, h);$$

$$m_1^2, \dots, \hat{m}_Q^2, \hat{m}_{Q+1}^2(g, h, m^2), \dots, \hat{m}_M^2(g, h, m^2); \mu] \quad (18)$$

has the same renormalization group flow as the original one.

The requirement for the reduced theory to be perturbative renormalizable means that the functions $\hat{g}_i, f_a^b, e_\alpha^\beta$ and k_α^{ab} , defined in Eqs. (9)–(11), should have a power series expansion in the primary coupling g :

$$\hat{g}_i = g \sum_{n=0}^{\infty} \rho_i^{(n)} g^n, \quad f_a^b = g \sum_{n=0}^{\infty} \eta_a^{b(n)} g^n,$$

$$e_\alpha^\beta = \sum_{n=0}^{\infty} \xi_\alpha^{\beta(n)} g^n, \quad k_\alpha^{ab} = \sum_{n=0}^{\infty} \chi_\alpha^{ab(n)} g^n. \quad (19)$$

To obtain the expansion coefficients, we insert the power series ansatz above into Eqs. (12), (15)–(17) and require that the equations are satisfied at each order in g . Note that the existence of a unique power series solution is a nontrivial matter: It depends on the theory as well as on the choice of the set of independent parameters.

3. FINITENESS IN $N = 1$ SUPERSYMMETRIC GAUGE THEORIES

Let us consider a chiral, anomaly-free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g . The superpotential of the theory is given by

$$W = \frac{1}{2}m_{ij}\phi_i\phi_j + \frac{1}{6}C_{ijk}\phi_i\phi_j\phi_k, \quad (20)$$

where m_{ij} and C_{ijk} are gauge-invariant tensors and the matter field ϕ_i transforms according to the irreducible representation R_i of the gauge group G . The renormalization constants associated with the superpotential (20), assuming that SUSY is preserved, are

$$\phi_i^0 = (Z_i^j)^{(1/2)}\phi_j, \quad (21)$$

$$m_{ij}^0 = Z_{ij}^{i'j'} m_{i'j'}, \quad (22)$$

$$C_{ijk}^0 = Z_{ijk}^{i'j'k'} C_{i'j'k'}. \quad (23)$$

The $N = 1$ nonrenormalization theorem [52,73,74] ensures that there are no mass and cubic-interaction-term infinities and therefore

$$\begin{aligned} Z_{ijk}^{i'j'k'} Z_{i'}^{1/2i''} Z_{j'}^{1/2j''} Z_{k'}^{1/2k''} &= \delta_{(i}^{i''} \delta_j^{j''} \delta_{k)}^{k''}, \\ Z_{ij}^{i'j'} Z_{i'}^{1/2i''} Z_{j'}^{1/2j''} &= \delta_{(i}^{i''} \delta_j^{j'')}. \end{aligned} \quad (24)$$

As a result, the only surviving possible infinities are the wave-function renormalization constants Z_i^j , i.e., one infinity for each field. The one-loop β function of the gauge coupling g is given by [75]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2} \left[\sum_i l(R_i) - 3C_2(G) \right], \quad (25)$$

where $l(R_i)$ is the Dynkin index of R_i and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G . The β functions of C_{ijk} , by virtue of the nonrenormalization theorem, are related to the anomalous dimension matrix γ_{ij} of the matter fields ϕ_i as

$$\beta_{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma_k^l + C_{ikl}\gamma_j^l + C_{jkl}\gamma_i^l. \quad (26)$$

At one-loop level, γ_{ij} is [75]

$$\gamma_j^{i(1)} = \frac{1}{32\pi^2} [C^{ikl}C_{jkl} - 2g^2C_2(R_i)\delta_j^i], \quad (27)$$

where $C_2(R_i)$ is the quadratic Casimir of the representation R_i , and $C^{ijk} = C_{ijk}^*$. Since dimensional coupling parameters such as masses and couplings of cubic scalar field terms do not influence the asymptotic properties of a theory in which we are interested here, it is sufficient to take into account only the dimensionless supersymmetric couplings such as g and C_{ijk} . So we neglect the existence of dimensional parameters and assume furthermore that C_{ijk} are real so that C_{ijk}^2 always are positive numbers.

As one can see from Eqs. (25) and (27), all the one-loop β functions of the theory vanish if $\beta_g^{(1)}$ and $\gamma_{ij}^{(1)}$ vanish, i.e.,

$$\sum_i \ell(R_i) = 3C_2(G), \quad (28)$$

$$C^{ikl} C_{jkl} = 2\delta_j^i g^2 C_2(R_i). \quad (29)$$

The conditions for finiteness for $N = 1$ field theories with $SU(N)$ gauge symmetry are discussed in [76], and the analysis of the anomaly-free and no-charge renormalization requirements for these theories can be found in [77]. A very interesting result is that the conditions (28), (29) are necessary and sufficient for finiteness at the two-loop level [75, 78–81].

In case SUSY is broken by soft terms, the requirement of finiteness in the one-loop soft breaking terms imposes further constraints among themselves [55]. In addition, the same set of conditions that are sufficient for one-loop finiteness of the soft breaking terms render the soft sector of the theory two-loop finite [55].

The one- and two-loop finiteness conditions (28), (29) restrict considerably the possible choices of the irreps. R_i for a given group G as well as the Yukawa couplings in the superpotential (20). Note in particular that the finiteness conditions cannot be applied to the minimal supersymmetric standard model (MSSM), since the presence of a $U(1)$ gauge group is incompatible with the condition (28), due to $C_2[U(1)] = 0$. This naturally leads to the expectation that finiteness should be attained at the grand unified level only, the MSSM being just the corresponding, low-energy, effective theory.

Another important consequence of one- and two-loop finiteness is that SUSY (most probably) can only be broken due to the soft breaking terms. Indeed, due to the unacceptability of gauge singlets, F-type spontaneous symmetry breaking [82] terms are incompatible with finiteness, as well as D-type [83] spontaneous breaking which requires the existence of a $U(1)$ gauge group.

A natural question to ask is what happens at higher loop orders. The answer is contained in a theorem [33, 34] which states the necessary and sufficient conditions to achieve finiteness at all orders. Before we discuss the theorem, let us make some introductory remarks. The finiteness conditions impose relations between gauge and Yukawa couplings. To require such relations which render the couplings mutually dependent at a given renormalization point is trivial. What

is not trivial is to guarantee that relations leading to a reduction of the couplings hold at any renormalization point. As we have seen, the necessary, and also sufficient, condition for this to happen is to require that such relations are solutions to the REs

$$\beta_g \frac{dC_{ijk}}{dg} = \beta_{ijk} \quad (30)$$

and hold at all orders. Remarkably, the existence of all-order power series solutions to (30) can be decided at one-loop level, as already mentioned.

Let us now turn to the all-order finiteness theorem [33, 34], which states that if an $N = 1$ supersymmetric gauge theory can become finite to all orders in the sense of vanishing β functions, that is of physical scale invariance. It is based on (a) the structure of the supercurrent in $N = 1$ supersymmetric gauge theory [84–86], and on (b) the nonrenormalization properties of $N = 1$ chiral anomalies [33, 34, 87–89]. Details on the proof can be found in [33, 34] and further discussion in [35, 87–90]. Here, following mostly [90], we present a comprehensible sketch of the proof.

Consider an $N = 1$ supersymmetric gauge theory, with simple Lie group G . The content of this theory is given at the classical level by the matter supermultiplets S_i , which contain a scalar field ϕ_i and a Weyl spinor ψ_{ia} , and the vector supermultiplet V_a , which contains a gauge vector field A_μ^a and a gaugino Weyl spinor λ_α^a .

Let us first recall certain facts about the theory:

(1) A massless $N = 1$ supersymmetric theory is invariant under a $U(1)$ chiral transformation R under which the various fields transform as follows:

$$\begin{aligned} A'_\mu &= A_\mu, & \lambda'_\alpha &= \exp(-i\theta)\lambda_\alpha, \\ \phi' &= \exp\left(-i\frac{2}{3}\theta\right)\phi, & \psi'_\alpha &= \exp\left(-i\frac{1}{3}\theta\right)\psi_\alpha, \dots \end{aligned} \quad (31)$$

The corresponding axial Noether current $J_R^\mu(x)$,

$$J_R^\mu(x) = \bar{\lambda}\gamma^\mu\gamma^5\lambda + \dots, \quad (32)$$

is conserved classically, while in the quantum case is violated by the axial anomaly

$$\partial_\mu J_R^\mu = r(\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho} + \dots). \quad (33)$$

From its known topological origin in ordinary gauge theories [91–93], one would expect that the axial vector current J_R^μ satisfies the Adler–Bardeen theorem and receives corrections only at the one-loop level. Indeed, it has been shown that the same nonrenormalization theorem holds also in supersymmetric theories [87–89]. Therefore,

$$r = \hbar\beta_g^{(1)}. \quad (34)$$

(2) The massless theory we consider is scale-invariant at the classical level and, in general, there is a scale anomaly due to radiative corrections. The scale anomaly appears in the trace of the energy-momentum tensor $T_{\mu\nu}$, which is traceless classically. It has the form

$$T_{\mu}^{\mu} = \beta_g F^{\mu\nu} F_{\mu\nu} + \dots \quad (35)$$

(3) Massless, $N = 1$ supersymmetric gauge theories are classically invariant under the supersymmetric extension of the conformal group — the superconformal group. By examining the superconformal algebra, it can be seen that the subset of superconformal transformations consisting of translations, SUSY transformations, and axial R transformations is closed under SUSY; i.e., these transformations form a representation of SUSY. It follows that the conserved currents corresponding to these transformations make up a supermultiplet represented by an axial vector superfield called supercurrent J ,

$$J \equiv \{J_R^{\mu}, Q_{\alpha}^{\mu}, T_{\nu}^{\mu}, \dots\}, \quad (36)$$

where J_R^{μ} is the current associated to R invariance, Q_{α}^{μ} is the one associated to SUSY invariance, and T_{ν}^{μ} the one associated to translational invariance (energy-momentum tensor).

The anomalies of the R current J_R^{μ} , the trace anomalies of the SUSY current, and the energy-momentum tensor, form also a second supermultiplet, called the supertrace anomaly

$$S = \{\text{Re } S, \text{Im } S, S_{\alpha}\} = \{T_{\mu}^{\mu}, \partial_{\mu} J_R^{\mu}, \sigma_{\alpha\beta}^{\mu} \bar{Q}_{\mu}^{\dot{\beta}} + \dots\}, \quad (37)$$

where T_{μ}^{μ} was given in Eq.(35) and

$$\partial_{\mu} J_R^{\mu} = \beta_g \epsilon^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho} + \dots, \quad (38)$$

$$\sigma_{\alpha\beta}^{\mu} \bar{Q}_{\mu}^{\dot{\beta}} = \beta_g \lambda^{\beta} \sigma_{\alpha\beta}^{\mu\nu} F_{\mu\nu} + \dots \quad (39)$$

(4) It is very important to note that the Noether current defined in (32) is not the same as the current associated to R invariance that appears in the supercurrent J in (36), but they coincide in the tree approximation. So, starting from a unique classical Noether current $J_{R(\text{class})}^{\mu}$, the Noether current J_R^{μ} is defined as the quantum extension of $J_{R(\text{class})}^{\mu}$, which allows for the validity of the nonrenormalization theorem. On the other hand, J_R^{μ} is defined to belong to the supercurrent J , together with the energy-momentum tensor. The two requirements cannot be fulfilled by a single-current operator at the same time.

Although the Noether current J_R^{μ} which obeys (33) and the current J_R^{μ} belonging to the supercurrent multiplet J are not the same, there is a rela-

tion [33,34] between the quantities associated with them

$$r = \beta_g(1 + x_g) + \beta_{ijk}x^{ijk} - \gamma_A r^A, \quad (40)$$

where r was given in Eq.(34). The r^A are the nonrenormalized coefficients of the anomalies of the Noether currents associated to the chiral invariances of the superpotential, and, like r , are strictly one-loop quantities. The γ_A 's are linear combinations of the anomalous dimensions of the matter fields, and x_g and x^{ijk} are radiative correction quantities. The structure of equality (40) is independent of the renormalization scheme.

One-loop finiteness, i.e., vanishing of the β functions at one loop, implies that the Yukawa couplings λ_{ijk} must be functions of the gauge coupling g . To find a similar condition to all orders, it is necessary and sufficient for the Yukawa couplings to be a formal power series in g , which is solution of the REs (30).

We can now state the theorem for all-order vanishing β functions.

Theorem: Consider an $N = 1$ supersymmetric Yang–Mills theory, with simple gauge group. If the following conditions are satisfied:

1. There is no gauge anomaly;
2. The gauge β function vanishes at one loop

$$\beta_g^{(1)} = 0 = \sum_i l(R_i) - 3C_2(G); \quad (41)$$

3. There exist solutions of the form

$$C_{ijk} = \rho_{ijk}g, \quad \rho_{ijk} \in \mathbb{C} \quad (42)$$

to the conditions of vanishing one-loop matter fields anomalous dimensions

$$\begin{aligned} \gamma_j^{i(1)} &= 0 \\ &= \frac{1}{32\pi^2} [C^{ikl}C_{jkl} - 2g^2 C_2(R_i)\delta_{ij}]; \end{aligned} \quad (43)$$

4. These solutions are isolated and nondegenerate when considered as solutions of vanishing one-loop Yukawa β functions:

$$\beta_{ijk} = 0, \quad (44)$$

then each of the solutions (42) can be uniquely extended to a formal power series in g , and the associated super Yang–Mills models depend on the single coupling constant g with a β function which vanishes at all orders.

It is important to note a few things: The requirement of isolated and nondegenerate solutions guarantees the existence of a unique formal power series

solution to the reduction equations. The vanishing of the gauge β function at one loop, $\beta_g^{(1)}$, is equivalent to the vanishing of the R current anomaly (33). The vanishing of the anomalous dimensions at one loop implies the vanishing of the Yukawa couplings β functions at that order. It also implies the vanishing of the chiral anomaly coefficients r^A . This last property is a necessary condition for having β functions vanishing at all orders*.

Proof: Insert β_{ijk} as given by the REs into the relationship (40) between the axial anomalies coefficients and the β functions. Since these chiral anomalies vanish, we get for β_g a homogeneous equation of the form

$$0 = \beta_g(1 + O(\hbar)). \quad (45)$$

The solution of this equation in the sense of a formal power series in \hbar is $\beta_g = 0$, order by order. Therefore, due to the REs (30), $\beta_{ijk} = 0$, too.

Thus, we see that finiteness and reduction of couplings are intimately related. Since an equation like Eq. (40) is lacking in nonsupersymmetric theories, one cannot extend the validity of a similar theorem in such theories.

4. SUM RULE FOR SB TERMS IN $N = 1$ SUPERSYMMETRIC AND FINITE THEORIES: ALL-LOOP RESULTS

The method of reducing the dimensionless couplings has been extended [28], to the soft SUSY breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. In addition, it was found [61] that RGI SSB scalar masses in Gauge–Yukawa unified models satisfy a universal sum rule. Here we will describe first how the use of the available two-loop RG functions and the requirement of finiteness of the SSB parameters up to this order leads to the soft scalar-mass sum rule [62].

Consider the superpotential given by (20) along with the Lagrangian for SSB terms:

$$-\mathcal{L}_{\text{SB}} = \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}(m^2)_i^j\phi^{*i}\phi_j + \frac{1}{2}M\lambda\lambda + \text{h.c.}, \quad (46)$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Since we would like to consider only finite theories here, we assume that the gauge group is a simple group and the one-loop β function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the form

$$C^{ijk} = g \sum_n \rho_{(n)}^{ijk} g^{2n}. \quad (47)$$

*There is an alternative way to find finite theories [94].

According to the finiteness theorem of [33,34], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved by [56]

$$h^{ijk} = -MC^{ijk} + \dots = -M\rho_{(0)}^{ijk}g + O(g^5), \quad (48)$$

where \dots stand for higher-order terms.

Now, to obtain the two-loop sum rule for soft scalar masses, we assume that the lowest-order coefficients $\rho_{(0)}^{ijk}$ and also $(m^2)_j^i$ satisfy the diagonality relations

$$\rho_{ipq(0)}\rho_{(0)}^{j pq} \propto \delta_i^j \text{ for all } p \text{ and } q \text{ and } (m^2)_j^i = m_j^2\delta_j^i, \quad (49)$$

respectively. Then we find the following soft scalar-mass sum rule [21,62,95]:

$$\frac{m_i^2 + m_j^2 + m_k^2}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4) \quad (50)$$

for i, j, k with $\rho_{(0)}^{ijk} \neq 0$, where $\Delta^{(2)}$ is the two-loop correction

$$\Delta^{(2)} = -2 \sum_l \left[\frac{m_l^2}{MM^\dagger} - \frac{1}{3} \right] T(R_l), \quad (51)$$

which vanishes for the universal choice in accordance with the previous findings of [56].

If we know higher-loop β functions explicitly, we can follow the same procedure and find higher-loop RGI relations among SSB terms. However, the β functions of the soft scalar masses are explicitly known only up to two loops. In order to obtain higher-loop results, some relations among β functions are needed.

Making use of the spurion technique [50–54], it is possible to find the following all-loop relations among SSB β functions [42–47]:

$$\beta_M = 2\mathcal{O} \left(\frac{\beta_g}{g} \right), \quad (52)$$

$$\beta_h^{ijk} = \gamma^i_l h^{ljk} + \gamma^j_l h^{ilk} + \gamma^k_l h^{ijl} - 2\gamma_{1l}^i C^{ljk} - 2\gamma_{1l}^j C^{ilk} - 2\gamma_{1l}^k C^{ijl}, \quad (53)$$

$$(\beta_{m^2})^i_j = \left[\Delta + X \frac{\partial}{\partial g} \right] \gamma^i_j, \quad (54)$$

$$\mathcal{O} = \left(Mg^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial C^{lmn}} \right), \quad (55)$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2 \frac{\partial}{\partial g^2} + \tilde{C}_{lmn} \frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn} \frac{\partial}{\partial C^{lmn}}, \quad (56)$$

where $(\gamma_1)^i_j = \mathcal{O}\gamma^i_j$, $C_{lmn} = (C^{lmn})^*$, and

$$\tilde{C}^{ijk} = (m^2)^i_l C^{ljk} + (m^2)^j_l C^{ilk} + (m^2)^k_l C^{ijl}. \quad (57)$$

It was also found [43] that the relation

$$h^{ijk} = -M(C^{ijk})' \equiv -M \frac{dC^{ijk}(g)}{d \ln g} \quad (58)$$

among couplings is all-loop RGI. Furthermore, using the all-loop gauge β function of Novikov et al. [63–65] given by

$$\beta_g^{\text{NSVZ}} = \frac{g^3}{16\pi^2} \left[\frac{\sum_l T(R_l)(1 - \gamma_l/2) - 3C(G)}{1 - g^2 C(G)/8\pi^2} \right], \quad (59)$$

the all-loop RGI sum rule [48] was found:

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \left[\frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d \ln C^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln C^{ijk}}{d(\ln g)^2} \right] + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d \ln C^{ijk}}{d \ln g}. \quad (60)$$

In addition, the exact β function for m^2 in the NSVZ scheme has been obtained [48] for the first time and it is given by

$$\beta_{m_i^2}^{\text{NSVZ}} = \left\{ |M|^2 \left[\frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d}{d \ln g} + \frac{1}{2} \frac{d^2}{d(\ln g)^2} \right] + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d}{d \ln g} \right\} \gamma_i^{\text{NSVZ}}. \quad (61)$$

Surprisingly enough, the all-loop result (60) coincides with the superstring result for the finite case in a certain class of orbifold models [62] if $d \ln C^{ijk}/d \ln g = 1$.

5. FINITE $SU(5)$ UNIFIED THEORIES

Finite Unified Theories (FUTs) have always attracted interest for their intriguing mathematical properties and their predictive power. One very important result is that the one-loop finiteness conditions (26), (27) are sufficient to guarantee two-loop finiteness [75]. A classification of possible one-loop finite models

was done by two groups [96–98]. The first one- and two-loop finite $SU(5)$ model was presented in [99], and shortly afterwards the conditions for finiteness in the soft SUSY-breaking sector at one loop [80] were given. In [100], a one- and two-loop finite $SU(5)$ model was presented where the rotation of the Higgs sector was proposed as a way of making it realistic. The first all-loop finite theory was studied in [23,24], without taking into account the soft breaking terms. Finite soft breaking terms and the proof that one-loop finiteness in the soft terms also implies two-loop finiteness was done in [56]. The inclusion of soft breaking terms in a realistic model was done in [101] and their finiteness to all loops studied in [46], although the universality of the soft breaking terms leads to a charged LSP. This fact was also noticed in [102], where the inclusion of an extra parameter in the boundary condition of the Higgs mixing mass parameter was introduced to alleviate it. The derivation of the sum rule in the soft SUSY-breaking sector and the proof that it can be made all-loop finite were done in [62] and [48] respectively, thus allowing for the construction of all-loop finite realistic models.

From the classification of theories with vanishing one-loop gauge β function [96], one can easily see that there exist only two candidate possibilities to construct $SU(5)$ GUTs with three generations. These possibilities require that the theory should contain as matter fields the chiral supermultiplets $\mathbf{5}, \bar{\mathbf{5}}, \mathbf{10}, \bar{\mathbf{5}}, \mathbf{24}$ with the multiplicities $(6, 9, 4, 1, 0)$ or $(4, 7, 3, 0, 1)$, respectively. Only the second one contains a $\mathbf{24}$ -plet which can be used to provide the spontaneous symmetry breaking (SB) of $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. For the first model, one has to incorporate another way, such as the Wilson flux breaking mechanism to achieve the desired SB of $SU(5)$ [23,24]. Therefore, for a self-consistent field theory discussion, we would like to concentrate only on the second possibility.

The particle content of the models we will study consists of the following supermultiplets: three $(\bar{\mathbf{5}} + \mathbf{10})$, needed for each of the three generations of quarks and leptons, four $(\bar{\mathbf{5}} + \mathbf{5})$ and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken, the theory is no longer finite, and we will assume that we are left with the MSSM.

Therefore, a predictive Gauge–Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements already mentioned, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)j} \propto \delta_i^j$.
2. The three fermion generations, in the irreducible representations $\bar{\mathbf{5}}_i, \mathbf{10}_i$ ($i = 1, 2, 3$), should not couple to the adjoint $\mathbf{24}$.
3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and antiqintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model: the model of [23,24], which will be labeled **A**, and a slight variation of this model (labeled **B**), which can also be obtained from the class of the models sug-

gested in [44, 45] with a modification to suppress nondiagonal anomalous dimensions [62].

The superpotential which describes the two models before the reduction of couplings takes places is of the form [23, 24, 62, 99, 100]

$$W = \sum_{i=1}^3 \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \bar{\mathbf{5}}_i \bar{H}_i \right] + g_{23}^u \mathbf{10}_2 \mathbf{10}_3 H_4 + g_{23}^d \mathbf{10}_2 \bar{\mathbf{5}}_3 \bar{H}_4 + g_{32}^d \mathbf{10}_3 \bar{\mathbf{5}}_2 \bar{H}_4 + \sum_{a=1}^4 g_a^f H_a \mathbf{24} \bar{H}_a + \frac{g^\lambda}{3} (\mathbf{24})^3, \quad (62)$$

where H_a and \bar{H}_a ($a = 1, \dots, 4$) stand for the Higgs quintets and anti-quintets.

The main difference between model **A** and model **B** is that two pairs of Higgs quintets and anti-quintets couple to the $\mathbf{24}$ in **B**, so that it is not necessary to mix them with H_4 and \bar{H}_4 in order to achieve the triplet–doublet splitting after the symmetry breaking of $SU(5)$ [62]. Thus, although the particle content is the same, the solutions to Eqs. (26), (27) and the sum rules are different, which will reflect in the phenomenology, as we will see.

5.1. FUTA. After the reduction of couplings the symmetry of the superpotential W (62) is enhanced. For model **A**, one finds that the superpotential has the $Z_7 \times Z_3 \times Z_2$ discrete symmetry with the charge assignment as shown in Table 1, and with the following superpotential:

$$W_A = \sum_{i=1}^3 \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \bar{\mathbf{5}}_i \bar{H}_i \right] + g_4^f H_4 \mathbf{24} \bar{H}_4 + \frac{g^\lambda}{3} (\mathbf{24})^3. \quad (63)$$

Table 1. Charges of the $Z_7 \times Z_3 \times Z_2$ symmetry for model FUTA

	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\bar{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	H_1	H_2	H_3	H_4	\bar{H}_1	\bar{H}_2	\bar{H}_3	\bar{H}_4	$\mathbf{24}$
Z_7	4	1	2	1	2	4	5	3	6	-5	-3	-6	0	0	0
Z_3	0	0	0	1	2	0	1	2	0	-1	-2	0	0	0	0
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0

The nondegenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for model **FUTA**, which are the boundary conditions for the Yukawa couplings at the GUT scale, are

$$\begin{aligned} (g_1^u)^2 &= \frac{8}{5} g^2, & (g_1^d)^2 &= \frac{6}{5} g^2, & (g_2^u)^2 &= (g_3^u)^2 = \frac{8}{5} g^2, \\ (g^\lambda)^2 &= \frac{15}{7} g^2, & (g_2^d)^2 &= (g_3^d)^2 = \frac{6}{5} g^2, & (g_4^f)^2 &= g^2, \\ (g_{23}^u)^2 &= (g_{23}^d)^2 = (g_{32}^d)^2 &= (g_2^f)^2 &= (g_3^f)^2 = (g_1^f)^2 &= 0. \end{aligned} \quad (64)$$

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale for this model [62]:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = m_{H_d}^2 + m_{\overline{\mathbf{5}}}^2 + m_{\mathbf{10}}^2 = M^2, \quad (65)$$

and thus we are left with only three free parameters, namely, $m_{\overline{\mathbf{5}}} \equiv m_{\overline{\mathbf{5}}_3}$, $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ and M .

5.2. FUTB. Also in the case of **FUTB** the symmetry is enhanced after the reduction of couplings. The superpotential has now a $Z_4 \times Z_4 \times Z_4$ symmetry with charges as shown in Table 2 and with the following superpotential:

$$W_B = \sum_{i=1}^3 \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \overline{\mathbf{5}}_i \overline{H}_i \right] + g_{23}^u \mathbf{10}_2 \mathbf{10}_3 H_4 + g_{23}^d \mathbf{10}_2 \overline{\mathbf{5}}_3 \overline{H}_4 + g_2^f H_2 \mathbf{24} \overline{H}_2 + g_3^f H_3 \mathbf{24} \overline{H}_3 + \frac{g^\lambda}{3} (\mathbf{24})^3. \quad (66)$$

Table 2. Charges of the $Z_4 \times Z_4 \times Z_4$ symmetry for model FUTB

	$\overline{\mathbf{5}}_1$	$\overline{\mathbf{5}}_2$	$\overline{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	H_1	H_2	H_3	H_4	\overline{H}_1	\overline{H}_2	\overline{H}_3	\overline{H}_4	$\mathbf{24}$
Z_4	1	0	0	1	0	0	2	0	0	0	-2	0	0	0	0
Z_4	0	1	0	0	1	0	0	2	0	3	0	-2	0	-3	0
Z_4	0	0	1	0	0	1	0	0	2	3	0	0	-2	-3	0

For this model the nondegenerate and isolated solutions to $\gamma_i^{(1)} = 0$ give us

$$\begin{aligned} (g_1^u)^2 &= \frac{8}{5} g^2, & (g_1^d)^2 &= \frac{6}{5} g^2, & (g_2^u)^2 &= (g_3^u)^2 = (g_{23}^u)^2 = \frac{4}{5} g^2, \\ (g_2^d)^2 &= (g_3^d)^2 = (g_{23}^d)^2 &= (g_{32}^d)^2 &= \frac{3}{5} g^2, & & \\ (g^\lambda)^2 &= \frac{15}{7} g^2, & (g_2^f)^2 &= (g_3^f)^2 = \frac{1}{2} g^2, & (g_1^f)^2 &= (g_4^f)^2 = 0, \end{aligned} \quad (67)$$

and from the sum rule we obtain [62]

$$\begin{aligned} m_{H_u}^2 + 2m_{\mathbf{10}}^2 &= M^2, & m_{H_d}^2 - 2m_{\mathbf{10}}^2 &= -\frac{M^2}{3}, \\ m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 &= \frac{4M^2}{3}, \end{aligned} \quad (68)$$

i.e., in this case we have only two free parameters $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ and M for the dimensionful sector.

As already mentioned, after the $SU(5)$ gauge symmetry breaking we assume we have the MSSM, i.e., only two Higgs doublets. This can be achieved by introducing appropriate mass terms that allow one to perform a rotation of the Higgs sector [23, 24, 99, 100, 103], in such a way that only one pair of Higgs doublets, coupled mostly to the third family, remains light and acquires vacuum expectation values. To avoid fast proton decay, the usual fine tuning to achieve doublet–triplet splitting is performed. Notice that, although similar, the mechanism is not identical to minimal $SU(5)$, since we have an extended Higgs sector.

Thus, after the gauge symmetry of the GUT theory is broken, we are left with the MSSM, with the boundary conditions for the third family given by the finiteness conditions, while the other two families are basically decoupled.

We will now examine the phenomenology of such all-loop Finite Unified Theories with $SU(5)$ gauge group and, for the reasons expressed above, we will concentrate only on the third generation of quarks and leptons. An extension to three families, and the generation of quark mixing angles and masses in Finite Unified Theories has been addressed in [104], where several examples are given. These extensions are not considered here.

5.3. Restrictions from Low-Energy Observables. Since the gauge symmetry is spontaneously broken below M_{GUT} , the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (64) or (67), the $h = -MC$ relation (48), and the soft scalar-mass sum rule (50) at M_{GUT} , as applied in the two models. Thus, we examine the evolution of these parameters according to their RGEs up to two loops for dimensionless parameters and at one loop for dimensionful ones with the relevant boundary conditions. Below M_{GUT} their evolution is assumed to be governed by the MSSM. We further assume a unique SUSY-breaking scale M_{SUSY} (which we define as the geometrical average of the stop masses) and therefore below that scale the effective theory is just the SM. This allows one to evaluate observables at or below the electroweak scale.

In the following, we briefly describe the low-energy observables used in our analysis. We discuss the current precision of the experimental results and the theoretical predictions. We also give relevant details of the higher-order perturbative corrections that we include. We do not discuss theoretical uncertainties from the RG running between the high-scale parameters and the weak scale. At present, these uncertainties are expected to be less important than the experimental and theoretical uncertainties of the precision observables.

As precision observables we first discuss the 3rd generation quark masses that are leading to the strongest constraints on the models under investigation. Next we apply B physics and Higgs-boson mass constraints. We also briefly discuss the anomalous magnetic moment of the muon.

5.4. Predictions. We now present the comparison of the predictions of the four models with the experimental data, see [105] for more details, starting with

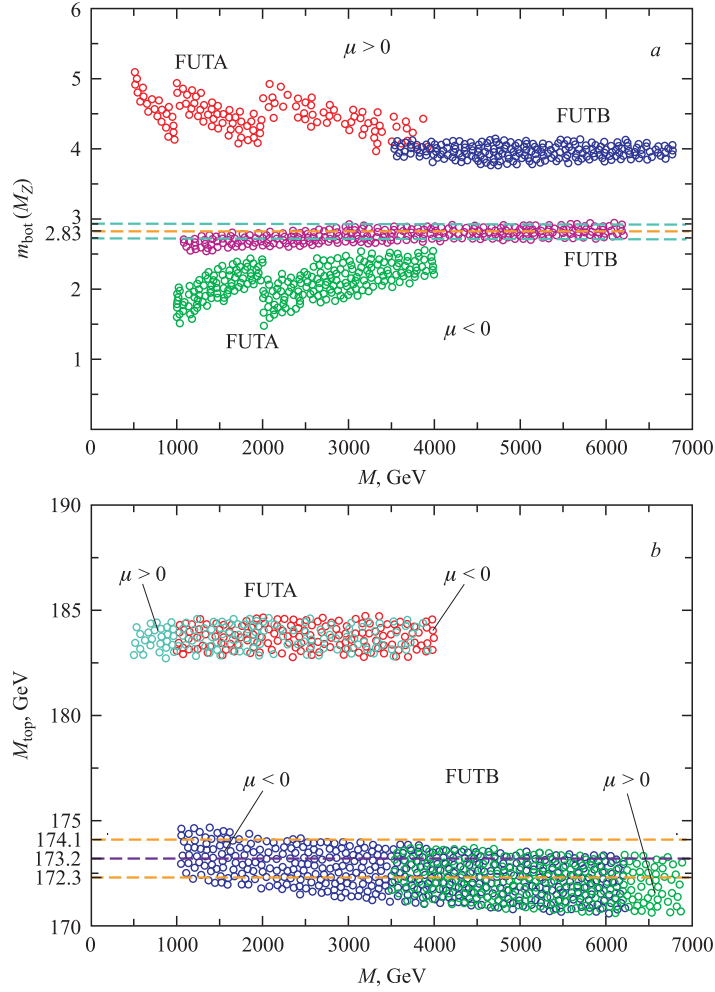


Fig. 1. (Color online) The bottom-quark mass at the Z boson scale (a) and top-quark pole mass (b) are shown as a function of M for both models

the heavy quark masses. In Fig. 1, we show the **FUTA** and **FUTB** predictions for the top pole mass, M_{top} , and the running bottom mass at the scale M_Z , $m_{\text{bot}}(M_Z)$, as a function of the unified gaugino mass M , for the two cases $\mu < 0$ and $\mu > 0$. The running bottom mass is used to avoid the large QCD uncertainties inherent for the pole mass. In the evaluation of the bottom mass m_{bot} , we have included the corrections coming from bottom squark–gluino loops and top squark–chargino loops [106]. We compare the predictions for the running

bottom-quark mass with the experimental value [107]

$$m_b(M_Z) = (2.83 \pm 0.10) \text{ GeV}. \quad (69)$$

One can see that the value of m_b depends strongly on the sign of μ due to the above-mentioned radiative corrections involving SUSY particles. For both models, **A** and **B**, the values for $\mu > 0$ are above the central experimental value, with $m_b(M_Z) \sim 4.0\text{--}5.0$ GeV. For $\mu < 0$, on the other hand, model **B** shows overlap with the experimentally measured values, $m_b(M_Z) \sim 2.5\text{--}2.8$ GeV. For model **A** we find $m_b(M_Z) \sim 1.5\text{--}2.6$ GeV, and there is only a small region of allowed parameter space at large M where we find agreement with the experimental value at the two σ level. Therefore, the experimental determination of $m_b(M_Z)$ clearly selects the negative sign of μ .

Now we turn to the top-quark mass. The predictions for the top-quark mass m_t are ~ 183 and ~ 172 GeV in the models **A** and **B**, respectively, as shown in the lower plot of Fig. 1. Comparing these predictions with the experimental value [108]

$$m_t^{\text{exp}} = (173.2 \pm 0.9) \text{ GeV} \quad (70)$$

and recalling that the theoretical values for m_t may suffer from a correction of $\sim 4\%$ [20, 95, 109], we see that clearly model **B** is singled out. In addition, the value of $\tan\beta$ is found to be $\tan\beta \sim 54$ and ~ 48 for models **A** and **B**, respectively. Thus, from the comparison of the predictions of the two models with experimental data, only **FUTB** with $\mu < 0$ survives.

We now analyze the impact of further low-energy observables on the model **FUTB** with $\mu < 0$. As additional constraints we consider the following observables: the rare b decays $\text{BR}(b \rightarrow s\gamma)$ and $\text{BR}(B_s \rightarrow \mu^+\mu^-)$. More details and a complete set of references can be found in [105].

For the branching ratio $\text{BR}(b \rightarrow s\gamma)$, we take the experimental value estimated by the Heavy Flavor Averaging Group (HFAG) [110–112]:

$$\text{BR}(b \rightarrow s\gamma) = (3.55 \pm 0.24_{-0.10}^{+0.09} \pm 0.03) \cdot 10^{-4}, \quad (71)$$

where the first error is the combined statistical and uncorrelated systematic uncertainty, the latter two errors are correlated systematic theoretical uncertainties and corrections, respectively. For the branching ratio $\text{BR}(B_s \rightarrow \mu^+\mu^-)$, the SM prediction is at the level of $3 \cdot 10^{-9}$, while the present experimental upper limit from the Tevatron is

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) = 4.5 \cdot 10^{-9} \quad (72)$$

at the 95% C.L. [113].

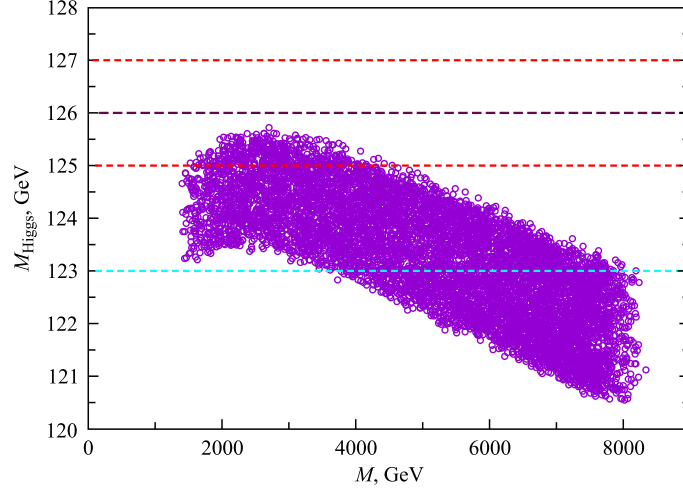


Fig. 2. (Color online) The lightest Higgs mass, M_h , as a function of M for the model **FUTB** with $\mu < 0$, see text

The prediction of the lightest Higgs boson mass, as obtained with FeynHiggs [114–117], as a function of M is shown in Fig. 2, where the B -physics constraints are already taken into account. One can see that the lightest Higgs boson mass range is

$$M_h \sim 121\text{--}126 \text{ GeV}, \quad (73)$$

where the uncertainty comes from variations of the soft scalar masses, and from finite (i.e., not logarithmically divergent) corrections in changing renormalization scheme. To this value one has to add ± 2 GeV coming from unknown higher-order corrections [116]. We have also included a small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions. Interpreting the light Higgs boson as the Higgs-like state discovered at the LHC [66, 67], we can impose a constraint on our results to the Higgs mass of

$$M_h \sim (126.0 \pm 1 \pm 2) \text{ GeV}, \quad (74)$$

where ± 1 comes from the experimental error and ± 2 corresponds to the theoretical error, and see how this affects the SUSY spectrum. Constraining the allowed values of the Higgs mass this way puts a limit on the allowed values of the unified gaugino mass, as can be seen from Fig. 2. The red lines correspond to the pure experimental uncertainty and restrict $2 \lesssim M \lesssim 5$ TeV. The blue line includes the additional theory uncertainty of ± 2 GeV. Taking this uncertainty into account, no bound on M can be placed. However, a substantial part of the

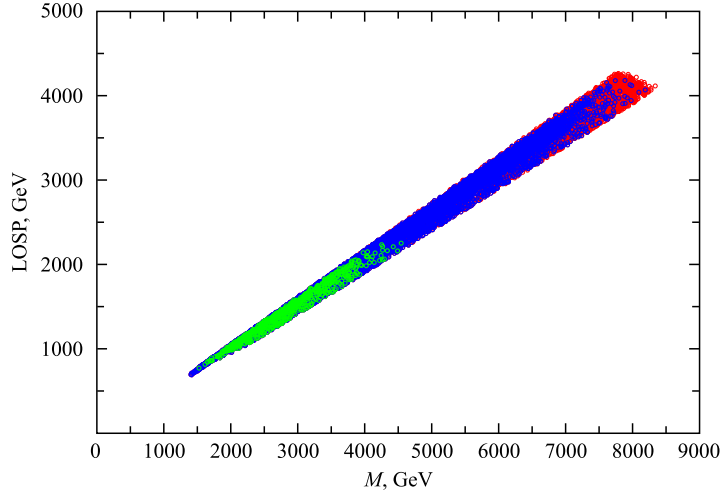


Fig. 3. (Color online) The LOSP in model **FUTB** with $\mu < 0$, see text

formerly allowed parameter points are now excluded. This in turn restricts the lightest observable SUSY particle (LOSP), which turns out to be the light scalar tau. In Fig. 3, the effects on the mass of the LOSP are demonstrated. Without any Higgs mass constraint all colored points are allowed. Imposing $M_h (126 \pm 1)$ GeV only the green (light shaded) points are allowed, restricting the mass to be between

Table 3. A representative spectrum of a light FUTB, $\mu < 0$ spectrum, compliant with the B-physics constraints

$M_{\text{bot}}(M_Z)$	2.72	M_{top}	172.3
M_h	123.2	M_A	1179
M_H	1177	M_{H^\pm}	1181
Stop ₁	2126	Stop ₂	2414
Sbot ₁	2015	Sbot ₂	2387
M_{stau_1}	723	M_{stau_2}	899
Char ₁	1218	Char ₂	1828
Neu ₁	660	Neu ₂	1219
Neu ₃	1824	Neu ₄	1828
M_1	661	M_2	1222
M_{gluino}	3092		

about 700 and 2500 GeV. The lower values might be experimentally accessible at the ILC with 1000 GeV center-of-mass energy or at CLIC with an energy up to ~ 3 TeV. Taking into account the theory uncertainty on M_h also the blue (dark shaded) points are allowed, permitting the LOSP mass up to ~ 4 TeV. If the upper end of the parameter space were realized, the light scalar tau would remain unobservable even at CLIC.

In the same way the whole SUSY particle spectrum can be derived. The resulting SUSY masses for **FUTB** with $\mu < 0$ are rather large. The lightest SUSY particle starts around

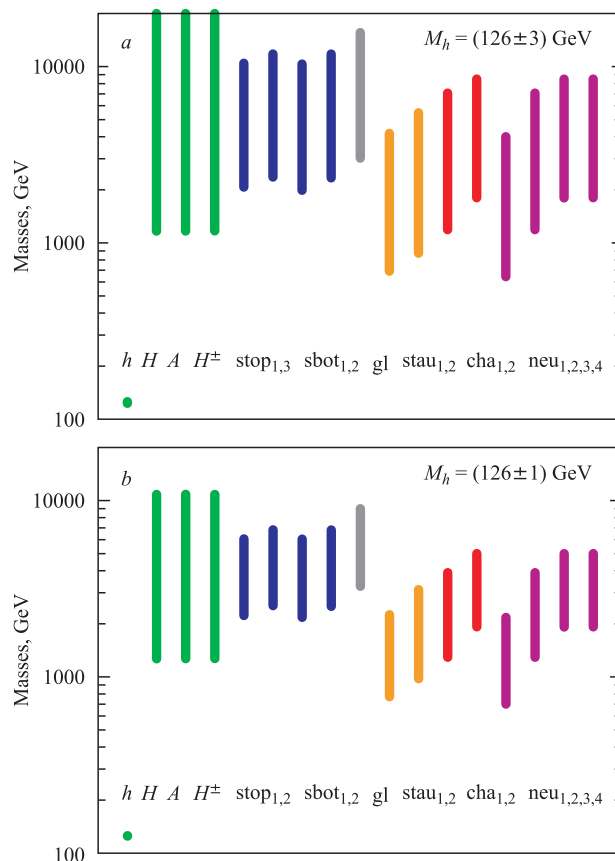


Fig. 4. (Color online) The particle spectrum of model **FUTB** with $\mu < 0$, where the points shown are in agreement with the quark mass constraints and the B -physics observables. The light (green) points on the left are the various Higgs boson masses. The dark (blue) points following are the two scalar top and bottom masses, followed by the lighter (gray) gluino mass. Next come the lighter (orange) scalar tau masses. The darker (red) points to the right are the two chargino masses followed by the lighter shaded (pink) points indicating the neutralino masses

600 GeV, with the rest of the spectrum being very heavy. A numerical example of the lighter part of the spectrum is shown in Table 3.

The full particle spectrum of model **FUTB** with $\mu < 0$, again compliant with quark mass constraints and the B -physics observables, is shown in Fig. 4. Including the Higgs mass constraint favors the lower parts of the parameter space. However, even neglecting the theory uncertainty on M_h (Fig. 4, *a*) permits

SUSY masses which would remain unobservable at the LHC, the ILC or CLIC. Including the theory uncertainties, even higher masses are permitted, further weakening the discovery potential of the LHC and future e^+e^- colliders. The colored supersymmetric particles are above ~ 1.8 TeV in agreement with the nonobservation of those particles at the LHC [118–120].

We note that with such a heavy SUSY spectrum the anomalous magnetic moment of the muon, $(g-2)_\mu$ (with $a_\mu \equiv (g-2)_\mu/2$), gives only a negligible correction to the SM prediction. The comparison of the experimental result and the SM value (based on the latest combination using e^+e^- data) [121]

$$a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (28.7 \pm 8.0) \cdot 10^{-10} \quad (75)$$

would disfavor **FUTB** with $\mu < 0$ [122, 123]. However, since the results would be very close to the SM result, we cannot exclude the model on this fact alone.

CONCLUSIONS

A number of proposals and ideas have matured with time and have survived after careful theoretical studies and confrontation with experimental data. These include part of the original GUTs ideas, mainly the unification of gauge couplings and, separately, the unification of the Yukawa couplings, a version of fixed point behavior of couplings, and certainly the necessity of SUSY as a way to take care of the technical part of the hierarchy problem. On the other hand, a very serious theoretical problem, namely, the presence of divergencies in Quantum Field Theories (QFT), although challenged by the founders of QFT [124–126], was mostly forgotten in the course of developments of the field partly due to the spectacular successes of renormalizable field theories, in particular of the SM. However, as was already mentioned in the Introduction, fundamental developments in Theoretical Particle Physics are based on reconsiderations of the problem of divergencies and serious attempts to solve it. These include the motivation and construction of string and noncommutative theories, as well as $N = 4$ supersymmetric field theories [3, 4], $N = 8$ supergravity [5–9] and the AdS/CFT correspondence [2]. It is a thoroughly fascinating fact that many interesting ideas that have survived various theoretical and phenomenological tests, as well as the solution to the UV divergencies problem, find a common ground in the framework of $N = 1$ Finite Unified Theories, which we have described in the previous sections. From the theoretical side, they solve the problem of UV divergencies in a minimal way. On the phenomenological side, since they are based on the principle of reduction of couplings (expressed via RGI relations among couplings and masses), they provide strict selection rules in choosing realistic models which lead to testable predictions. The celebrated success of predicting the top-quark mass [23–28] is now extended to the predictions of the Higgs masses and the supersymmetric

spectrum of the MSSM [105, 127]. The predicted mass of the lightest Higgs boson turns out to be naturally in agreement with the discovery of a Higgs-like state at the LHC. Identifying the lightest Higgs boson with the newly discovered state, we restrict the allowed parameter space of the surviving model (**FUTB** with $\mu < 0$) by the other low-energy constraints. We reviewed how this reduction of parameter space impacts the prediction of the SUSY spectrum.

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