

THE SAME KEY TO DIFFERENT DOORS — TEMPERATURE PUZZLES

*L. Turko**

University of Wrocław, Institute of Theoretical Physics, Wrocław, Poland

The notion of temperature in many-body elementary particle processes has been in common use for decades. Thermal models have become simple and universal effective tools to describe particle production not only in high-energy heavy-ion collisions but also in high-energy elementary particle collisions. We perform a critical analysis of the temperature concepts in such processes. Although the temperature concept is a very useful tool, nevertheless, it should be used with care, taking into account that usually it is just a model-dependent fitted parameter.

PACS: 07.20.Dt

INTRODUCTION

There is the famous article [1] by E.P. Wigner based on the lecture he delivered at New York University in May 1959. You can read there about *“that uncanny usefulness of mathematical concepts that raises the question of the uniqueness of our physical theories”* and that *“we are in a position similar to that of a man who was provided with a bunch of keys and who, having to open several doors in succession, always hit on the right key on the first or second trial. He became skeptical concerning the uniqueness of the coordination between keys and door.”*

That was about mathematics. But some suspicions concerning universal keys can be raised not only to mathematical concepts. We are in possession of some universal key whose usefulness becomes more and more universal up to the point when another question arises — is this a real key?

*E-mail: ludwik.turko@ift.uni.wroc.pl

1. FITTING THE KEYS

Discovery of pions in 1948, new light strongly interacting particles, copiously produced and observed in cosmic ray processes, created a new situation in the theoretical description of production processes. There was a need to deal with the global description of these processes, leaving temporally aside microscopic subtleties of interactions. The natural tool available at that time was some analogy to concepts used in many-body classical physics. There were attempts to treat mesonic cloud as a hydrodynamical medium [5] or as a kind of atmospheric pion gas or fluid surrounding nucleons. This pion fluid would be set in a kind of turbulent motion in the course of a high-energy collision of two nucleons. This turbulence would govern the distribution of energy among different excited states.

The concept of temperature was introduced in elementary particle physics, in a more or less systematic way, many times. It seems, however, that the first, who applied there this notion to particle physics was H. Koppe [2, 3]. He treated a nucleus as a “black body” with regard to mesonic radiation. This made it possible to calculate the probability for emission of a meson by statistical methods.

Then, two years later, famous Fermi Model [4] has appeared, where temperature was introduced in a more systematic way. The concept of the statistical equilibrium was then used to describe a high-energy collision. Fermi wrote there:

“When two nucleons collide with very great energy in their center-of-mass system this energy will be suddenly released in a small volume surrounding the two nucleons. (...) all the portion of space occupied by the nucleons and by their surrounding pion field will be suddenly loaded with a very great amount of energy. Since the interactions of the pion field are strong we may expect that rapidly this energy will be distributed among the various degrees of freedom present in this volume according to statistical laws. One can then compute statistically the probability that in this tiny volume a certain number of pions will be created with a given energy distribution. It is then assumed that the concentration of energy will rapidly dissolve and that the particles into which the energy has been converted will fly out in all directions. (...) First of all, there are conservation laws of charge and of momentum that evidently must be fulfilled.”

The main idea of statistical model, as sketched in by Fermi [4], remains still valid. Let us consider the probability $\mathcal{P}_n(i \rightarrow f)$ to produce n -particle state:

$$\mathcal{P}_n(i \rightarrow f) = \int \prod_{f=1}^n \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} |\langle p'_1, \dots, p'_n | \mathcal{S} | i \rangle|^2 \delta \left(P_i - \sum_f p_f \right). \quad (1)$$

One can clearly separate here the dynamical part

$$|\langle p'_1, \dots, p'_n | \mathcal{S} | i \rangle|^2$$

and the kinematical part of the process

$$\delta \left(P_i - \sum_f p_f \right) \prod_{f=1}^n \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f}.$$

With the increasing number of final particles the number of degrees of freedom increases much more. We are not able to measure all of them and there is also no need to do it. Only some global quantities are measured. We are in the situation when

- measurable quantities are much less detailed than $\langle p'_1, \dots, p'_n | \mathcal{S} | i \rangle$;
- with the integration over a large region of the phase space the dynamical details are averaged and only a few parameters remain;
- restricted knowledge of $\langle p'_1, \dots, p'_n | \mathcal{S} | i \rangle$ is not needed.

Then, there is a place for statistical physics.

Our probability (1) can be written as

$$P_n = \bar{S}_n \mathcal{R}_n,$$

with the constant averaged value \bar{S}_n of the S matrix element and with the exact kinematical part

$$\mathcal{R}_n = \int \prod_{f=1}^n \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \delta \left(P_i - \sum_f p_f \right). \quad (2)$$

The crucial point here is that these arguments work only if the thermodynamic equilibrium is reached.

Rolf Hagedorn was the first who systematically analyzed high-energy phenomena using all tools of statistical physics [6,7]. He also introduced the concept of *the limiting temperature* based then on the statistical bootstrap model. This made possible the introduction of the possibility of the phase transition and phase structure of the hadronic matter.

The spirit and the philosophy of the statistical approach remain the same as in the standard approach but ingredients of statistical models used in high-energy problems are different. The main difference is that a number of particles is no longer conserved, so we have no chemical potentials related to that quantity. The only nontrivial chemical potentials are those related to conserved charges, so the role of internal symmetries is a crucial one. As was stated in [7] “*Symmetries, not material particles are fundamental.*”

For the simplest case of an ideal hadron gas in thermal and chemical equilibrium, which consists of l species of particles, energy density ϵ , baryon number density n_B , strangeness density n_S , and electric charge density n_Q (read

$\hbar = c = 1$ always), one gets the equations

$$\epsilon = \frac{1}{2\pi^2} \sum_{i=1}^l (2s_i + 1) \int_0^{\infty} dp \frac{p^2 E_i}{\exp\left\{\frac{E_i - \mu_i}{T}\right\} + g_i}, \quad (3a)$$

$$n_B = \frac{1}{2\pi^2} \sum_{i=1}^l (2s_i + 1) \int_0^{\infty} dp \frac{p^2 B_i}{\exp\left\{\frac{E_i - \mu_i}{T}\right\} + g_i}, \quad (3b)$$

$$n_S = \frac{1}{2\pi^2} \sum_{i=1}^l (2s_i + 1) \int_0^{\infty} dp \frac{p^2 S_i}{\exp\left\{\frac{E_i - \mu_i}{T}\right\} + g_i}, \quad (3c)$$

$$n_Q = \frac{1}{2\pi^2} \sum_{i=1}^l (2s_i + 1) \int_0^{\infty} dp \frac{p^2 Q_i}{\exp\left\{\frac{E_i - \mu_i}{T}\right\} + g_i}, \quad (3d)$$

where $E_i = (m_i^2 + p^2)^{1/2}$; m_i , B_i , S_i , μ_i , s_i , and g_i are the mass, baryon number, strangeness, chemical potential, spin, and a statistical factor of specie i , respectively (we treat an antiparticle as a different specie).

And $\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$, where μ_B , μ_S , and μ_Q are overall baryon number and strangeness chemical potentials, respectively.

To get particle yields, one should also consider entropy density $n(s)$:

$$s = \frac{1}{6\pi^2 T^2} \sum_{i=1}^l (2s_i + 1) \int_0^{\infty} dp \frac{p^4}{E_i} \frac{(E_i - \mu_i) \exp\left\{\frac{E_i - \mu_i}{T}\right\}}{\left(\exp\left\{\frac{E_i - \mu_i}{T}\right\} + g_i\right)^2}. \quad (4)$$

These equations, enriched by unstable particles effects, form a basis for successful calculations [8] of relativistic heavy-ion production processes concerning particle yields and rates.

2. TEMPERATURE, WHICH TEMPERATURE?

Although temperature appeared a quite successful tool to characterize high-energy hadronic collision, there are still discussions related to the physical interpretation of this concept. The temperature, as was introduced in classical physics, based on the direct contact of the measuring device — thermometer — with the given object. It was also tacitly assumed that the thermometer would be small

enough not to change the thermodynamic characteristics of the object, and the result would be obtained in the state of the thermal equilibrium between the object and the thermometer. This quantity, measured by the direct contact, is called the “physical temperature”.

No-thermometer measurements of the quantity called “temperature” are based on given model assumptions. Their relations to the physical temperature depend on the validity of the assumed model and on the very existence of the physical temperature of the system. The situation becomes even more complicated in the case of many-particle quantum systems, e.g., multiproduction processes where you deal with nontrivial (mixed states) density matrix. Because of impossibility of the full microscopic description, one uses *relevant* entropy $S_{\mathbb{X}}(A_1, A_2, \dots)$ with the entropy maximized with respect to the set $\mathbb{X} = \{A_1, A_2, \dots\}$ of relevant macrovariables. The relevant entropy is maximized under constraints $\langle \hat{A}_i \rangle = A_i$:

$$S_{\mathbb{X}}(A_1, A_2, \dots) = -\max_{\rho} \text{Tr } \rho \ln \rho, \quad \text{Tr } \rho \hat{A}_i = A_i. \quad (5)$$

The maximum is set over all possible distributions ρ satisfying constraints $\text{Tr } \rho \hat{A}_i = A_i$.

This relevant entropy takes into account only information connected with relevant variables. If one of the relevant variables is taken energy, then temperature is defined as

$$\frac{1}{T} = \frac{\partial S_{\mathbb{X}}}{\partial E}.$$

This is in fact just the temperature widely used in thermal hadronic models. It is obvious that the entropy is unique for the given set of relevant variables. For different choice of relevant variables, the temperatures would be different but still consistent with the scheme of statistical physics.

CONCLUSIONS

There is a lot of discussion about the temperature concept in hadronic physics. One should have in mind, however, that temperature here is not a self-consistent quantity. A little more careful analysis shows that this is just a model-dependent parameter fitted to experimental data. Within the given class of models, such thermal models, based on similar assumptions, had the same philosophy. One gets similar temperatures when models are applied to explain similar data. There are rather no expectations to build a universal hadronic thermometer which would give the “real” temperature of the hadronic medium. We have a set of different keys fitted to different doors. They are constructed according to the same principles but without hope to create a universal passkey.

Acknowledgements. This work was supported by the Polish National Science Center under grant No. DEC-2013/10/A/ST2/00106.

REFERENCES

1. *Wigner E. P.* The Unreasonable Effectiveness of Mathematics in the Natural Sciences // Commun. in Pure and Appl. Math. 1960. V. 13. P. 1–14.
2. *Koppe H.* Die Mesonenausbeute beim Beschuß von leichtem Kernen mit α -Teilchen // Zeits. f. Naturforschung. 1948. V. 3(a). P. 251.
3. *Koppe H.* On the Production of Mesons // Phys. Rev. 1949. V. 76. P. 688.
4. *Fermi E.* High-Energy Nuclear Events // Prog. Theor. Phys. 1950. V. 5. P. 570.
5. *Heisenberg W.* Production of Meson Showers // Nature. 1949. V. 164. P. 65.
6. *Hagedorn R.* Statistical Thermodynamics of Strong Interactions at High Energies // Nuovo Cim. Suppl. 1965. V. 3. P. 147.
7. *Hagedorn R.* Remarks on the Thermodynamical Model of Strong Interactions // Nucl. Phys. B. 1970. V. 24. P. 93.
8. For a review see, e.g., *Braun-Munzinger P., Redlich K., Stachel J.* Quark Gluon Plasma 3 / Eds. R. C. Hwa and X. N. Wang. Singapore: World Sci., 2004. P. 491–599.