*ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА*
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# CLUSTER CORRELATIONS IN DENSE MATTER AND EQUATION OF STATE

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Correlations in strongly interacting matter at subsaturation densities lead to the formation of clusters and the appearance of phases transitions with a change of thermodynamic properties and chemical composition. These features can be described in a generalized relativistic density functional approach using clusters as explicit degrees of freedom with medium-dependent properties. The model is constructed in order to provide equations of state for astrophysical applications. It can be adapted also to nuclear structure calculations with cluster correlations on the surface of heavy nuclei. The appearance of $\alpha$ particles modifies the neutron skin thickness of neutron-rich nuclei and affects the correlation with the density dependence of the symmetry energy.

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## INTRODUCTION

The equation of state (EoS) of dense matter is an essential ingredient in many astrophysical model calculations. It determines the static properties of neutron stars, affects the dynamical evolution of core-collapse supernovae and sets the conditions for nucleosynthesis. Besides basic thermodynamic properties of matter, EoS models should provide information on the chemical composition of the system. Of course, an application of an EoS in simulations is only reasonable if the timescales of nuclear reactions are much shorter than the timescales of the system evolution and equilibrium conditions (thermal, mechanical, chemical,...) can be assumed. The properties of stellar matter depend on three basic variables: the baryon number density $n_B$, the temperature $T$ and the electron (or proton) fraction $Y_e$ ($Y_p$). These parameters have to cover rather large ranges in practical applications. Hence a global approach to construct EoS tables is needed in order to consider the relevant physical effects and conditions.

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A critical examination of existing EoS models for astrophysical applications suggests to develop an improved EoS in particular with respect to the following features: 1) to extend the set of constituent particles, i.e., in addition to the traditionally considered particles in stellar matter (nucleons, electrons, photons), further baryon species (e.g., hyperons) and a "complete" table of nuclei have to be included; 2) to better constrain the model parameters using up-to-date information from finite nuclei (binding energies, radii, charge form factors, etc.), nuclear matter (saturation properties), heavy-ion collisions (flow, fragment yields) and compact stars (mass–radius relation, maximum mass, cooling); 3) to consider more seriously the effects of correlations, e.g., nucleon–nucleon correlations with the low-density benchmark of the model-independent virial EoS, the medium dependence of composite particles and electromagnetic correlations that are essential for the description of solidification/melting; 4) to treat correctly the transitions between different thermodynamic phases. From these considerations it follows that the construction of a global EoS in a unified model presents a serious challenge to theory.

In this context it is important to distinguish between *nuclear* matter and *stellar* matter with very different thermodynamic properties. In the former system, only strongly interacting particles are considered, electric charges are neglected and the electromagnetic interaction is not taken into account. The balance of repulsion and attraction in the short-range nuclear interaction leads to the feature of saturation, which can be characterized quantitatively by typical nuclear matter parameters such as saturation density, binding energy per nuclon at saturation, incompressibility, symmetry energy and its slope parameter. As a consequence, a noncongruent liquid–gas phase transition is observed [1]. In the latter system, relevant for astrophysics, both hadrons and leptons interacting via the strong and electromagnetic interaction have to be included in the models with the specific condition of total charge neutrality. Typical features of stellar matter are the formation of inhomogeneous matter on different length scales (clusters, "pasta" phases) and the lattice formation at low temperatures and densities with ions immersed in a background of almost uniformly distributed electrons.

In this contribution, the main emphasis is placed on the description of dilute matter, i.e., matter with densities smaller than nuclear saturation density $n_{\mathrm{sat}} \approx 0.15\ \mathrm{fm}^{-3}$ where correlations are decisive for the properties and chemical composition. In general, information on correlations in interacting many-body systems is contained in spectral functions, which often have a complicated structure. A prevalent approximation is the introduction of quasiparticles with self-energies that incorporate the in-medium change of the particle properties, leading to a reduction of residual correlations. The quasiparticles concept is very successful in nuclear physics, e.g., in phenomenological (Skyrme, Gogny, relativistic) mean-field models [2] with nucleons as degrees of freedom or in the treatment of pairing correlations applying a Bogoliubov transformation. There are also different concepts in devising theoretical approaches. In a *physical* picture one starts with mutually interacting nucleons and uses elaborated many-body techniques to describe the formation and dissolution of clusters as many-body correlations. In a *chemical* picture one immediately begins with a mixture of nucleons and nuclei in chemical equilibrium and the main task is to include the effects of the interactions. A unified description in a single model is given by the generalized relativistic density functional (gRDF) approach that is presented in the next section. Dilute matter is not only relevant for the EoS, but it can also be found on the surface of heavy nuclei. There, $\alpha$ particles have to preform as four-nucleon correlations in order to allow for the radioactive decay of unstable heavy nuclei. The adaption of the gRDF model to this situation is discussed in Sec. 2. This contribution closes with conclusions in the final section.

## 1. GENERALIZED RELATIVISTIC DENSITY FUNCTIONAL

In order to model nuclear and stellar matter with the correct limits and explicit cluster degrees of freedom, a generalized relativistic density functional (gRDF) was developed [3–5] by extending well-known relativistic mean-field models with density-dependent meson–nucleon couplings [6]. In a grand canonical approach, a thermodynamic consistent formulation is obtained by introducing the grand canonical potential density $\omega(T, \{\mu_i\})$. It depends on the temperature $T$ and the chemical potentials $\mu_i$ of all particle species $i$, which are related by the condition of chemical equilibrium. In addition to nucleons, heavier baryons (not relevant in the present context) and composite particles such as nuclei (or clusters) are regarded as explicit degrees of freedom. In the present model, four light nuclei ($^2$H, $^3$H, $^3$He, $^4$He) and 16745 heavy nuclei $^{A_i}Z_i$ with mass numbers $A_i > 4$, neutron numbers $N_i \leqslant 184$ and proton numbers $Z_i \leqslant 184$ are considered. If available, experimental binding energies of the nuclei are taken from the 2012 Atomic mass evaluation [7]. The predictions of the DZ10 model [8] are used for all other nuclei between the neutron and protons driplines (determined without the Coulomb contribution to the energy). Nucleon–nucleon correlations, which are important to reproduce the correct low-density limit given by the virial EoS [9], are represented by effective continuum resonances. Antiparticles can be included naturally in the relativistic approach. All particles are treated as quasiparticles with medium-dependent properties including internal thermal excitations for those that are composed of nucleons. In stellar matter, leptons and photons are added without problems and in the crystal phase lattice vibrations can be accounted for in a modified Debye model.

Scalar potentials $S_i$ and vector potentials $V_i$ represent the effective in-medium interaction of all quasiparticles $i$. They receive contributions from Lorentz scalar and vector mesons that are treated as classical fields. The parametrization DD2 [3] is adopted for the density dependence of the nucleon–meson couplings. The characteristic nuclear matter parameters (saturation density $n_{\rm sat} = 0.149$ fm$^{-3}$, energy per nucleon at saturation $E/A|_{\rm sat} = -16.02$ MeV, incompressibility $K = 242.7$ MeV, symmetry energy $J = 31.67$ MeV, symmetry energy slope parameter $L = 55.04$ MeV) of this effective interaction are very reasonable and compatible with most of current constraints. The DD2 neutron matter EoS is located inside the error band given by ab-initio N$^3$LO calculations in chiral effective field theory ($\chi$EFT) [10, 11], see Fig. 1 in [12].

So-called "rearrangement" contributions in the vector potentials $V_i$ ensure the thermodynamic consistency of the model. In stellar matter, there is an effective electromagnetic contribution to $V_i$ for charged particles. It is taken from parametrized results of Monte-Carlo simulations of plasmas [13] and represents Coulomb correlations, which even appear in uniform systems. The scalar potentials $S_i$ contain medium-dependent mass shifts $\Delta m_i$ of composite particles that allow one to describe their formation and dissolution with changing density and temperature. This approach replaces the traditional geometric excluded-volume mechanism in order to suppress the appearance of clusters at high densities. The main contribution to the mass shifts originates from the action of the Pauli exclusion principle that blocks states in the medium from participating in the formation of correlations. As a consequence, the binding energies of clusters are reduced and their dissolution, i.e., the Mott effect, is observed. The mass shifts of light clusters with $A_i \leqslant 4$ have been calculated by solving in-medium few-body Schrödinger equations with realistic nucleon–nucleon potentials. A parametrization of these mass shifts is used in the present gRDF calculations [3]. A different approach is followed for heavy nuclei. Here, fully self-consistent spherical Wigner–Seitz cell calculations with the density functional for nucleons and electrons are performed in an extended Thomas–Fermi approximation [14]. From a comparison with calculations of uniform matter, the change of binding energies of clusters inside the medium can be determined from a comparison with uniform matter calculations. In the present model, only a simplified parametrization of the mass shifts is used. It will be improved with more systematic calculations in the future, which will cover the whole chart of nuclei for various temperatures and medium densities.

The change of the chemical composition of dilute matter can be demonstrated by studying the variation of the mass number fractions with density or temperature. For example, the right panel of Fig. 10 in [14] depicts the mass fractions of nucleons and light clusters as a function of the baryon number density $n_B$ at given temperature and proton fraction (heavy nuclei are already dissolved at these conditions). At low $n_B$, the chemical composition is dominated by nucleons and a few deuteron-like correlations because only two-body correlations are relevant. The abundance of three- and four-nucleon clusters becomes more and more significant with increasing density. All clusters dissolve when the saturation density $n_{\rm sat}$ is approached and pure nucleonic matter remains at higher densities. The temperature dependence of the composition is shown in the left panel of Fig. 2 in [12] in a preliminary calculation for fixed medium density and electron fraction. Stellar matter is mainly composed of nucleons, electrons and very few light clusters at high temperatures. The mass fractions of light clusters increase when the system is cooled down and finally heavy clusters dominate the composition. The mass and charge numbers of the heavy clusters increase substantially with decreasing temperature as shown in the right panel of Fig. 2 in [12]. A phase transition from the gas to the solid crystal phase is expected at very low temperatures. For details, see [12]. The effects of clustering and phase transitions have to be incorporated in models for the EoS to provide a reliable input to astrophysical simulations.

## 2. SYMMETRY ENERGY AND NEUTRON SKINS

The symmetry energy $E_s$ quantifies the variation of the energy per baryon $E/A$ with the isospin asymmetry $\alpha = 1 - 2Y_p$ of strongly interacting matter at constant baryon number density $n_B$. It can be defined as the difference of $E/A$ for pure neutron matter and symmetric nuclear matter. In recent years, there have been many experimental attempts to gain information on the symmetry energy at saturation $J = E_s(n_{\rm sat})$ and its density dependence, measured, e.g., with the slope parameter $L = 3n_{\rm sat}\, dE_s(n_b)/dn_b|_{n_b=n_{\rm sat}}$, see, e.g., [15, 16]. The liquid–gas phase transition in nuclear matter or the appearance of clusters at subsaturation densities modifies the low-density behavior of the symmetry energy [5]. It is well known that the neutron skin thickness $r_{\rm skin}$ of heavy nuclei, i.e., the difference between the neutron and proton root-mean-square radii, is strongly correlated with the stiffness of the neutron matter EoS [17, 18] or, equivalently, the slope parameter $L$ [19]. A measurement of the neutron skin thickness, e.g., using parity violating electron scattering on $^{208}$Pb in the approach of PREX@JLab [20], could help to constrain $L$ and thus the stiffness of the dense matter EoS that is relevant for the structure of neutron stars. However, the $r_{\rm skin} \leftrightarrow L$ correlation is based only on relativistic and nonrelativistic mean-field calculations of nuclei that do not take into account the effect of clustering beyond pairing correlations.

Calculations of heavy nuclei in the medium at finite temperatures in spherical Wigner–Seitz cells using an extended relativistic Thomas–Fermi (RTF) approximation with the gRDF including light clusters show an increased probability of finding light clusters on the surface of nuclei, see Fig. 11 in [14]. With appropriate modifications, the gRDF approach can be applied to the description of heavy nuclei at zero temperature in the vacuum, see [21] for details. It is observed that $\alpha$ clusters appear at the surface of nuclei and the size of the neutron skin is reduced depending on the neutron excess of the nucleus and the amount of $\alpha$ clustering. For the chain of Sn nuclei the effect on the neutron skin thickness is smallest for almost neutron–proton symmetric nuclei and very neutron-rich nuclei. In the former case, $\alpha$ particles can form at the surface, but the neutron skin is practically vanishing. In the latter case, the formation of $\alpha$ clusters is very much suppressed at the surface since it is composed almost entirely of neutrons.

Introducing a series of parametrizations that differ only in the isospin-dependent part of the effective interaction, see Table 1 in [21], the effect of the surface $\alpha$ clustering on the $r_{\rm skin} \leftrightarrow L$ correlation can be studied. In Fig. 5 of [21] the correlation of the neutron skin thickness in $^{208}$Pb with the slope parameter $L$ is depicted without and with $\alpha$ particle correlations at the surface. A reduction of about 0.02 fm or approx. 10% of the neutron thickness is observed when $\alpha$ particles are considered in the fully self-consistent structure calculations. This sizeable shift should be taken into account at least as a systematic error in the $r_{\rm skin} \leftrightarrow L$ correlation if the slope parameter is determined from experimentally measured neutron-skin thicknesses of heavy nuclei. The prediction of $\alpha$-cluster correlations on the surface of heavy nuclei will be tested experimentally with quasi-free $(p, p\alpha)$ reactions in the future at RCNP, Osaka [22].

## CONCLUSIONS

Many-body correlations in nuclear and stellar matter affect the thermodynamic properties and the chemical composition of the system, e.g., through phase transitions or the formation of clusters at subsaturation densities. In a generalized relativistic density functional approach, clusters are included as explicit degrees of freedom with medium-dependent properties. The parameters of this phenomenological model are well constrained with density-dependent meson–nucleon couplings and appropriate parametrizations of the essential mass shifts of composite quasiparticles. The main application of the gRDF approach is the construction of equation-of-state tables for astrophysical simulations such as core-collapse supernovae or for the description of static properties of neutron stars. In the case of dilute matter at subsaturation densities, it is important to include the "full" table of nuclei in the model in order to predict the chemical composition of matter, which strongly depends on temperature, baryon density and isospin asymmetry. The gRDF model can also be used in nuclear structure calculations with appropriate adjustments. As compared to conventional mean-field calculation without cluster correlations, a reduction of the neutron skin thickness of heavy nuclei is found due to the appearance of $\alpha$ particles on the surface. This result affects the correlation of the neutron skin thickness with the symmetry energy slope parameter and in turn with the stiffness of the neutron matter equation of state that is relevant for the structure of neutron stars.

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