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THE NUCLEAR SYMMETRY ENERGY IN HEAVY-ION COLLISIONS

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In this article we discuss the nuclear symmetry energy in the regime of hadronic degrees of freedom. The density dependence of the symmetry energy is important from very low densities in supernova explosions, to the structure of neutron-rich nuclei around saturation density, and to several times saturation density in neutron stars. Heavy-ion collisions are the only means to study this density dependence in the laboratory. Transport theories are used to extract the symmetry energy from heavy-ion collisions. We finally study some examples, which relate particularly to the high density symmetry energy, which is of particular interest today. We point out the status and open problems in the theoretical analyses of HIC.

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INTRODUCTION

The nuclear Equation of State (EoS) specifies the energy of nuclear matter as a function of density, temperature and asymmetry. For zero temperature it can be written as $E(\rho,\delta) = E_{nm}(\rho) + E_{\rm sym}(\rho)\delta^2 + \ldots$, where ρ is the total density of the system and $\delta = (\rho_n - \rho_p)/\rho$ the asymmetry with the neutron and proton densities. The part proportional to δ^2 is the symmetry energy. For saturation density it is related to the symmetry energy term in empirical mass formula. However, the dependence on density is of great importance in nuclei away from stability and in astrophysics in core-collapse supernovae and neutron stars, which have a large neutron excess and where a large range of densities from very low (in supernovae) to several times saturation density (in neutron stars) is involved. Predictions of mircroscopic many-body calculations of the symmetry energy differ widely, especially above saturation density [1]. The reason is the short-range isovector repulsion, which is poorly known [2]. Thus, there are extensive efforts to determine the symmetry energy from nuclear structure, heavy-ion collisions and astrophysical observations.

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In nuclear structure one explores the symmetry energy around saturation density ρ_0, using an expansion of the form

$$E_{\rm sym} = S_0 + \frac{L}{3}\,\frac{\rho - \rho_0}{\rho} + \ldots$$

Another parametrization is to separate the symmetry energy into a kinetic contribution taken in the form of a free Fermi gas, and to parametrize the potential part by a power law

$$E_{\rm sym} = \frac{1}{3}\epsilon_F\Big(\frac{\rho}{\rho_0}\Big)^{2/3} + C\Big(\frac{\rho}{\rho_0}\Big)^{\gamma}.$$

The slope of the symmetry energy L, or more specifically the correlation between the value and the slope, S_0 vs. L, has been extensively investigated using various observables, like nuclear masses, Giant and Pygmy dipole resonances, dipole polarizabilities, neutron skin radii (difference between neutron and proton radii), and isobaric analog state energies [3, 4]. Neutron star observations, on the other hand, also provide access to the nuclear EoS. A given EoS determines uniquely the mass–radius relation of neutron stars. The simultaneous measurement of these two properties is difficult, but significant progress has been made in the last years, using sophisticated modelling of neutron star atmospheres and statistical analyses [5]. However, detailed conclusions are still controversial.

In this situation heavy-ion collisions (HIC) from Fermi energies up to intermediate energies of several GeV per particle provide a means to investigate the nuclear EoS in the laboratory. In a collision, nuclear matter is first compressed, and then expands in the final state, and thus different regions of density are explored. The advantage of using heavy-ion collisions is that one may vary the energy and the impact parameter (and thus the compression), and the asymmetry of the colliding system in limits, which will be extended in the future with new rare isotope facilities. The difficulty, on the other hand, is that HIC is fundamentally a nonequilibrium process, and thus the detailed evolution has to be modelled using transport theory, about which are complicated and on which there are still open questions. Also one has to identify observables which are especially sensitive to the symmetry energy in the presence of uncertainities of the much larger contribution of symmetric nuclear matter and the above-mentioned uncertainities of transport theory.

In this article we aim to give a brief overview of the present status of the investigation of the symmetry energy using HIC. Certainly, this is not possible in any exhaustive sense in the short space of this article. Thus, we will select specific examples, which also point to the still open problems in these investigations. Recently there have been extensive reviews on the symmetry energy in the form of volumes collecting articles of experts [6, 7], review articles [8, 9], and feature articles [3, 4], just to mention the more recent ones. These sources, of course, contain much more information.

1. THEORETICAL CONSIDERATIONS

The main method to interpret HIC is transport theories. They describe the temporal evolution of the one-body phase-space distribution function $f(\mathbf{r},\mathbf{p},t)$ under the action of a mean-field potential $U(\mathbf{r},\mathbf{p})$, possibly momentum-dependent, and 2-body collisions with the in-medium cross section $\sigma(\Omega)$:

$$\frac{df_i}{dt} = \frac{\partial f_i}{\partial t} + \frac{\mathbf{p}_i}{m}\nabla^{(r)} f_i - \nabla^{(r)} U_i(\mathbf{r},\mathbf{p})\nabla^{(p)} f_i - \nabla^{(p)} U_i(\mathbf{r},\mathbf{p})\nabla^{(r)} f_i = \\ = \sum_{j,i',j'} \int d\mathbf{p}_j d\mathbf{p}_{i'} d\mathbf{p}_{j'} v_{ij} \sigma_{i,j\to i',j'}(\Omega)\delta(\mathbf{p}_i + \mathbf{p}_j - \mathbf{p}_{i'} - \mathbf{p}_{j'})\times \\ \times [(1-f_i)(1-f_j)f_{i'}f_{j'} - f_i f_j (1-f_{i'})(1-f_{j'})]. \quad (1)$$

Here the indices (i,j,i',j') run over neutrons and protons, such that these are coupled equations via the collision term and indirectly via the potentials. If the production of other particles is considered, like Δ's, nucleon resonances, or π and K mesons, these have their own dynamical equations coupled through the corresponding inelastic cross sections. The mean-field potentials U_i can be derived from an energy functional. Mean fields and cross sections can be related through a theory for the in-medium effective interaction, like, e.g., Brueckner theory, even though this is not necessarily done in many applications. The isospin effects enter via the differences in neutron and proton potentials and the isospin-dependent cross sections, but they are always small relative to the dominant isoscalar effects. Thus, one often resorts to differences or ratios of observables between isospin partners, in order to eliminate as much as possible the uncertainties in the isoscalar part.

2. ISOSPIN OBSERVABLES

We first briefly discuss the effect of the symmetry energy in the different density regimes. In central reactions at Fermi energies densities somewhat above saturation are reached. Recently the expansion phase of such reactions has been studied in detail, where very low densities of about 1/10 to 1/1000 of ρ_0 are attained. From isotope ratios (so-called isoscaling) the symmetry energy at very low densities has been determined which is important for the simulation for supernova explosions. In this density regime few-body clustering effects become important. A theoretical investigation has shown that the symmetry energy, in fact, is finite at very low densities in qualitative agreement with experiment [10].

At Fermi energy collisions one observes the phenomenon of multifragmentation. The distribution of the isospin to the different fragments ("isospin fractionation") [8] and the isospin transport through the low-density neck ("isospin diffusion") in more peripheral collisions have been very useful to constrain the symmetry energy below ρ_0 [11].

At intermediate energies in the initial phase of the collision, one observes pre-equilibrium emission of high-energy particles and light fragments. The yield ratios of isotopic partners, like n/p or ${}^3\mathrm{He}/t$, contain information on the relative strengths of the neutron and proton potentials. In the compression phase the pressure determines the momentum distribution of the emitted particles, generally called "flow", in-plane (directed) and out-of-plane (elliptic). Neutron–proton difference flow observables have been an important means to extract information on the high density symmetry energy. On the other hand, inelastic NN collisions lead to the production of Δ resonances, which may decay into pions or lead to the production of strangeness.

Let us briefly note that also at higher energies the influence of the symmetry energy has been discussed, suggesting that the deconfinement transition may be substantially influenced by the difference in the symmetry energy between the hadronic and partonic phase, and may, in fact, occur at lower density in asymmetric systems [12].

2.1. Pre-Equilibrium Emission. The neutron-to-proton ratio of emitted particles has first been measured at MSU for $\mathrm{Sn} + \mathrm{Sn}$ systems at $50A$ MeV, and a systematic analysis of several observables has yielded rather good limits on the γ exponent around $\gamma \approx 0.6$ [11]. For the pre-equilibrium emission at higher energies the momentum dependence of the symmetry potential, i.e., the proton–neutron effective mass splitting, becomes important as first pointed out in [13, 14]. We have systematically studied this effect for nucleons and light clusters for INDRA data of different $\mathrm{Xe} + \mathrm{Sn}$ reactions at energies between 32 and $150A$ MeV [15]. A result from these calculations is shown in Fig. 1 for central collisions at $150A$ MeV with different stiffnesses of the symmetry energy and different effective mass splittings. In Fig. 1, *a* the n/p ratio is shown as a function of the transverse energy of the emitted particles, in Fig. 1, *b* the corresponding result for $t/{}^3\mathrm{He}$. One observes a clear pattern, namely that the stiffness of the symmetry energy governs the lower part of the transverse energy spectrum, such that the softer symmetry energy yields a larger n/p ratio. On the other hand, the higher part of the spectrum is dominated by the effective mass ordering, such that a smaller neutron effective mass favors the emission of neutrons and increases the ratio. A similar result has been obtained by Zhang et al. [16]. Thus, this observable should serve as a promising probe to disentangle the density and momentum dependences of the symmetry potential at higher density. The ratio $t/{}^3\mathrm{He}$ in the panel *b* shows a very similar pattern. A qualitative comparison with the data (not shown) favors a stiff symmetry energy with $m_n^* > m_p^*$.

2.2. Flow. The momentum distribution of the particles and fragments emitted in the final stage of HIC are characterized via a Fourier series expansion of the azimuthal distribution as

$$N(\Theta; p_t, y) = N_0(1 - v_1(p_t, y)\cos\Theta + v_2(p_t, y)\cos 2\Theta + \ldots$$

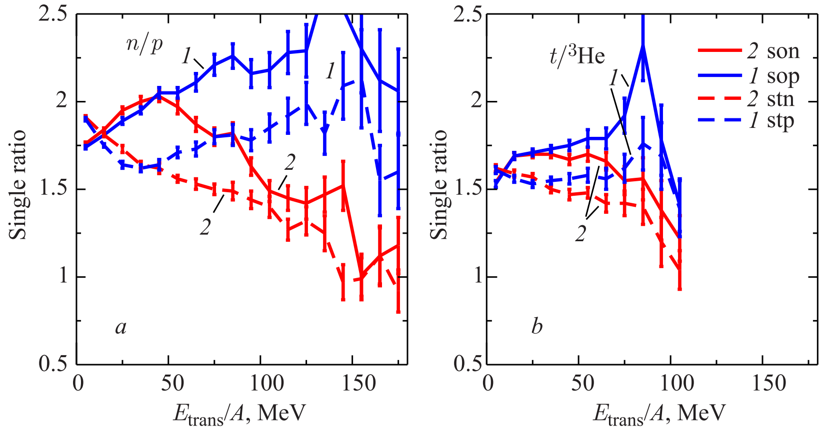

Fig. 1 (color online). *a*) The neutron–proton ratio in ^{136}Xe+^{124}Sn collisions at 150A MeV for different choices of the symmery energy (solid — asy-soft, dashed — asy-stiff) and orderings of the effective masses (blue (*1*), $m_n^* < m_p^*$, red (*2*), $m_n^* > m_p^*$). *b*) The corresponding ratio of tritium over ^{3}He

The first two Fourier coefficients, depending on the transverse momentum p_t and the longitudinal rapidity y, are called directed and elliptic flow, respectively. Differences of flow parameters between isospin partners directly reflect the isospin-dependent potentials and thus the symmetry energy. Results from a new experiment, ASY-EOS, by the FOPI collaboration are shown in Fig. 2 for Au + Au collisions at 400A MeV [17]. Data for the ratio of the elliptic flow of neutrons relative to hydrogen are shown together with calculations for a soft and a stiff symmetry energy. A best fit yields a coefficient γ of about 0.75, i.e.,

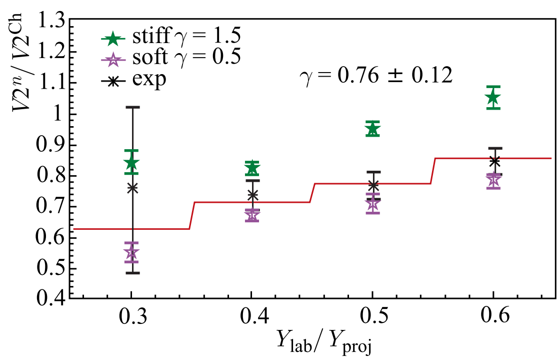

Fig. 2. Ratio of elliptic flow of neutrons over hydrogen for forward rapidities in Au + Au collisions at 400A MeV. Data of [17] and calculations with a symmetry energy characterized by exponent γ

a moderately soft symmetry energy. This is an important advance in trying to constrain the symmetry energy at supersaturation densities.

2.3. Particle Production. The n/p asymmetry of the compressed system also influences the ratio of newly produced particles, which thus can serve as indicators of the symmetry energy in the high density phase. In particular, pions are produced predominantly via the Δ resonances, $NN \rightarrow N\Delta$, and in the subsequent decay $\Delta \rightarrow N\pi$. The ratio of the isospin partners π^-/π^+ can thus serve as a probe of the high density symmetry energy. As analyzed in [18], there are competing effects on the Δ and pion production from the isospin-dependent mean fields and threshold conditions.

In Fig. 3, *a* we have collected results from different recent theoretical analyses of this ratio using different program codes and different symmetry energies [19], which are compared to the FOPI data [20]. In Fig. 3, *b* we show the corresponding density dependence. For each model the results for two SEs of different stiffness are shown (stiffer — blue (*1*), softer — red (*2*)). As is seen, the results of the different models are not only very different quantitatively but even the trend with the asy-stiffness is not consistent. A reason may lie in different modelling of the Δ dynamics, and also in the competing mean field and the threshold effects, where slightly different treatments might lead to large differences. This issue needs clarification in view of the sensitivity of the pion observables and the excellent data situation.

It has also been suggested that the ratio of the antistrange kaon isospin partners, K^0/K^+, could be a useful observable for the symmetry energy [21].

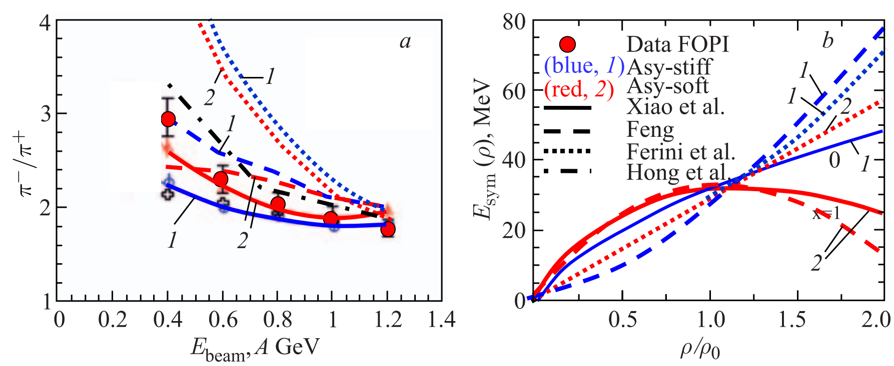

Fig. 3 (color online). *a*) The π^-/π^+ ratio in $\mathrm{Au}+\mathrm{Au}$ collisons as a function of incident energy as measured by the FOPI collaboration and as calculated by different groups, as indicated in the legend and discussed in the text. *b*) The corresponding models for the symmetry energy

Indeed, kaon production has been one of the most useful observables to determine the EoS of symmetric nuclear matter. The antistrange kaons weakly interact with nuclear matter and are thus a direct probe of the dense matter where they are produced. Theoretical analyses show similar if not larger sensitivity to the symmetry energy compared to pion ratios.

3. DISCUSSION AND CONCLUSIONS

In Fig. 4 we give a summary of the present information on the density dependence symmetry energy $E_{\rm sym}(\rho)$ or equivalently $S(\rho)$ from HIC. In Fig. 4, *a* the region below saturation density ρ_0 is shown. The blue hatched area (*1*) is the result from the investigation of $\rm Sn + Sn$ collisions at 50 MeV from MSU [3] using various observables from isospin transport between different Sn isotopes. The isolated symbols represent information from the fits to nuclear masses or GDR energies, which are plotted at about $0.6\rho_0$, which is an average density of nuclei where different models of the EoS converge. The blue-bordered areas (*2*) are derived from an analysis of isobaric analog states which give information also on the lower densities in the surface [22]. When the analysis is combined with the information on the neutron skin radius of Pb, the constraint is still sharpened. The points in the lower left corner, labelled "cluster", come from an analysis of the very low density matter in the expansion phase of low-energy heavy-ion collisions, mentioned above [23]. Here the matter is not any more homogeneous,

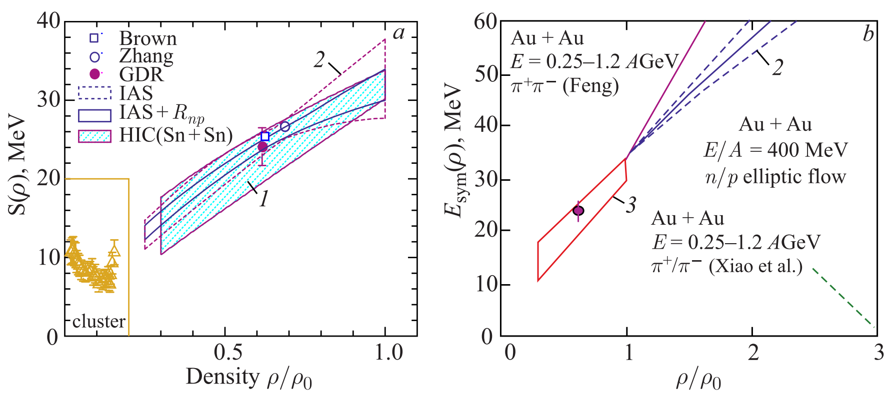

Fig. 4 (color online). *a*) Summary of information on the density dependence of symmetry energy for below (*a*) and above (*b*) saturation density. In the panel *a* the information is collected from nuclear structure and low-energy HIC [4], while in the panel *b* the still not consistent information from higher-energy collisions. The figure is discussed in the text but cluster correlations become important, which have the effect of making the symmetry energy finite at very low densitiy. Altogether the various sources of information on the symmetry energy in this density range seem to converge, and they also converge with the theoretical many-body results.

The information on the symmetry energy above saturation is shown in Fig. 4, *b* (where the low densitiy results from the panel *b* are shown again by the red-bordered area (*3*)). We show the results from the neutron/hydrogen flow analysis from Subsec. 2.3, and two results from the analysis of the pion ratios from Subsec. 2.4, one favoring a very soft symmetry energy and the other a rather stiff one [19]. Here the information is still controversial and further analyses are necessary. Microscopic many-body results in the region of up to $2\rho_0$ favor a behavior more similar to the flow experiment.

We have attempted to give a brief overview of the determination of the nuclear symmetry energy in HICs. HICs are interpreted with transport theories and we have discussed some of the challenges in such descriptions. Generally, today a picture emerges where the information on the symmetry energy from HICs, nuclear structure, and neutron stars increasingly converges. However, here we have put more emphasis on current questions in the determination of the symmetry energy in heavy-ion collisions, where a more thorough understanding of the mechanism is needed. In the end it is desirable to obtain a consistent picture of many observables in heavy-ion collisions.

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