ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2015. Т. 46. ВЫП. 5

DEFECTS IN THE AdS/CFT CORRESPONDENCE

J. Jankowski *

Institute of Physics, Jagiellonian University, Cracow, Poland

We present initial results of the studies of localized line-like sources for a scalar field in anti-de Sitter space. Such configurations reveal some subtleties and require novel numerical analysis. Dual field theory configurations involve simple models of lattices and defects.

PACS: 11.10.-z

INTRODUCTION

Most of phenomena in condensed matter physics can be understood as a consequence of the Schrödinger equation. However, for some systems, like graphene, electronic excitations are described by the Dirac Hamiltonian and for other, which include high-T_c superconductors, the description is still unclear. Since it is suspected that part of the relevant physics in these cases is governed by the quantum critical point, with the effective scale invariance, the descriptions involve strongly coupled conformal field theories (CFT). Widely used tool to analyze certain CFT systems is anti-de Sitter conformal field theory (AdS/CFT) correspondence [1]. In this setup a nongravitational system is mapped to a theory with gravity in higher dimensional space. Over the past few years, it has been applied to many interesting phenomena, including superconductivity and superfluidity, Fermi surfaces and non-Fermi liquids — see [2] for a review. One of the main directions in the current research is to incorporate the key element of condensed matter systems: the lattice. In the initial studies minimal model has been adopted: a periodic configuration of a neutral scalar field was put on the top equilibrium configuration with finite temperature and charge density [3]. This, in turn, caused momentum dissipation of charge carriers and had a direct impact on frequency-dependent electric conductivity, revealing mysterious power-law behaviour in a certain range of frequencies. The aim of this research is to develop more realistic models for holographic lattices and defects and to get better understanding of resulting physics, in particular, striking regularities in electric transport.

*E-mail: jakubj@th.if.uj.edu.pl

1. HOLOGRAPHIC MODEL

Solid state phenomena, which are of interest for us, happen in two-dimensional layers, so the relevant field theory will be three-dimensional. The minimal model of holographic lattice consists of Einstein-scalar field theory in $d = 3 + 1$ dimensions with negative cosmological constant $\Lambda = -3$. The action includes minimal coupling of scalar to gravity [3]

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[R + 6 - \frac{1}{2} \nabla_a \phi \nabla^a \phi - \frac{1}{2} m^2 \phi^2 \right], \tag{1}$$

where G_N is the Newton constant. In the Poincare coordinates (z, t, x, y) the conformal boundary, where the field theory lives, is at $z = 0$. Vacuum state of the field theory is mapped to the pure AdS_4 solution on the gravity side

$$ds^2 = \frac{1}{z^2} \left(dz^2 - dt^2 + dx^2 + dy^2 \right), \tag{2}$$

with radius set to one. Equilibrium configuration in field theory is mapped to a black hole solution to action (1) with Hawking temperature interpreted as a temperature of the dual theory. For the scalar we choose $m^2 = -2$ since the near boundary behaviour is then particularly simple:

$$\phi(x, z) \sim \phi_1(x) z + \phi_2(x) z^2 + \ldots \tag{3}$$

According to the standard holographic dictionary, $\phi_1(x)$ is a source of an operator $\mathcal{O}(x)$ of conformal dimension $\Delta = 2$ and the subleading term is related to the corresponding expectation value $\phi_2(x) = \langle \mathcal{O}(x) \rangle$. In [3] the lattice was imposed as a single Fourier mode in the source term $\phi_1(x) = A_0 \sin(kx)$. Our approach is to consider opposite limit of all Fourier modes entering at the same time in the form of a Dirac delta function located at point $x = 0$, namely, $\phi_1(x) = A_0 \delta(x)$, and possibly periodic set of delta functions located at $x = n$, where $n \in \mathbb{Z}$. This is a dual realization Kronig–Penney model [4], where atomic lattice is imposed as a periodic set of delta functions in a potential in the Schrödinger equation. In contrast to the holographic setup, this model is semi-analytically solvable.

1.1. Linearized Solution. As a first step of the analysis, it is instructive to find linearized scalar profile around empty AdS_4 with neglected backreaction on the geometry. The solution which has the correct boundary conditions for a delta function located at a line $x = 0$ is found analytically to be

$$\phi_0(x, z) = \frac{A_0 z^2}{\pi (x^2 + z^2)} \ . \tag{4}$$

On the dual theory side this corresponds to the shift of the original Lagrangian

$$\mathcal{L} = \mathcal{L}_{CFT_3} + \xi\delta(x)\mathcal{O}(x), \tag{5}$$

with the coupling ξ, which induces the vacuum expectation value $\langle\mathcal{O}(x)\rangle \sim 1/x^2$. This deformation breaks translational invariance of the original theory. Full conformal symmetry is broken to the $(1+1)$ conformal symmetry along the defect, where the operator is sourced. This approximation is valid for $\xi \ll 1$. Periodic solution is obtained by an infinite summation

$$\phi_P(x,z) = \sum_{n\in\mathbb{Z}} \phi_0(x+n,z) = \frac{A_0 z \sinh(2\pi z)}{\cosh(2\pi z) - \cos(2\pi x)}, \tag{6}$$

and can be interpreted as a model for periodic lattice. Because $\phi_P(x,z) \sim z$ as $z \to \infty$ backreaction on geometry in this case cannot be consistently neglected.

2. LINE DEFECT: FULL SOLUTION

In order to solve the full set of Einstein-scalar equation, we adopt the slicing coordinates with the following ansatz:

$$ds^2 = \frac{1}{A(\alpha)^2}\left(\frac{d\alpha^2}{p^2} + \frac{dr^2 - dt^2 + dy^2}{r^2}\right), \tag{7}$$

where $0 \leqslant \alpha \leqslant \pi/2$, $r > 0$ and $p \leqslant 1$ is a constant determined from equations of motion. This particular choice is motivated by symmetries of the linearized solution, in particular, conformal $(1+1)$-dimensional symmetry is reflected in the AdS_3 slicing of geometry. In this coordinate system AdS space with linearized scalar profile (4) is

$$A(\alpha) = \cos(\alpha), \quad p = 1, \quad \phi_0(\alpha) = \frac{A_0}{\pi}\cos(\alpha)^2. \tag{8}$$

The conformal boundary consists of two parts $\alpha_0 = \pm\pi/2$ joined together along the defect. Transformation to the Poincare coordinates is in that case $z = r\cos(\alpha)$ and $x = r\sin(\alpha)$ and will get changed in the full solution. Due to the symmetry of the problem we assume only α dependence of the relevant functions. The coupled set of Einstein-scalar equations reads

$$\phi(\alpha)^2 + 6p^2 A(\alpha)A''(\alpha) - 6p^2 A'(\alpha)^2 - p^2 A(\alpha)^2\phi'(\alpha)^2 + 6 = 0, \tag{9}$$

$$\frac{\phi(\alpha)^2}{2} + p^2 A(\alpha)A''(\alpha) - 3p^2 A'(\alpha)^2 - 2A(\alpha)^2 + 3 = 0, \tag{10}$$

$$2\phi(\alpha) + p^2 A(\alpha)\left(A(\alpha)\phi''(\alpha) - 2\phi'(\alpha)A'(\alpha)\right) = 0. \tag{11}$$

In the case of a line defect, solution can be found in a systematic expansion

$$A(\alpha) = \sum_{n=0}^{\infty} A_n(\alpha)\epsilon^{2n}, \quad \phi(\alpha) = \sum_{n=0}^{\infty} f_n(\alpha)\epsilon^{2n+1}, \quad p = \sum_{n=0}^{\infty} p_n \epsilon^{2n}, \tag{12}$$

where the lowest order is AdS solution with scalar profile (8), with $\epsilon = A_0/\pi$ being the expansion parameter. This solution is similar in spirit to the Janus configuration [5] where dilatonic profile interpolating between two values at the boundary has been considered in the AdS_5 space.

We demand the following boundary conditions for equations (9–11):

$$\partial_\alpha \phi(0) = \partial_\alpha A(0) = 0, \quad A(\pi/2) = 0, \quad \phi(0) = \text{fixed}. \tag{13}$$

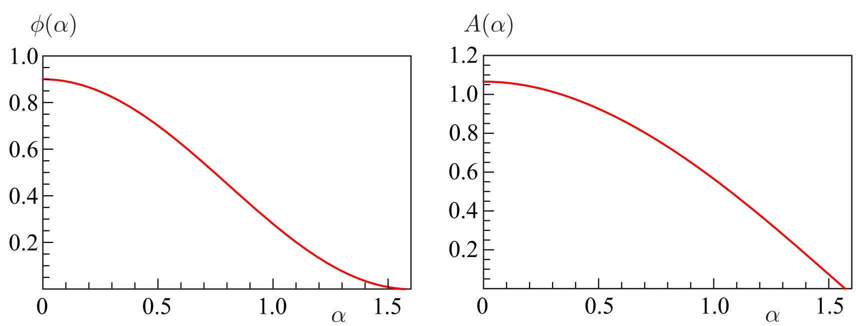
To determine constant d, we need another condition, which we take in the form

$$p \cdot \partial_\alpha A(\pi/2) = -1, \tag{14}$$

in order for the geometry to have the correct AdS_4 boundary behaviour. For numerical simulations we used standard spectral collocation method using expansion Chebyshev polynomials to account for the α dependence and solving resulting nonlinear algebraic equations by the Newton method. Curves on the plots in figure were calculated with $N = 47$ spectral points.

We observe that generic configuration has $\partial_\alpha \phi(\pi/2) \neq 0$, which means that a dynamically generated source for the scalar appears in the full solution. This strongly suggests that dynamically generated energy scale breaks conformal invariance. Possibly, there is no reduction to a system of ordinary differential equations while keeping only pure delta function boundary condition for the scalar. The r-dependent piece may contain logarithmic terms as a consequence of scale generation. Clarification of this point is a subject of our current research.

It is interesting to note that exactly the same configuration for the scalar has been considered in the M-theory setup in [6], where an analytic solution has been found. However, due to the supersymmetry of that system, there was no dynamical source for the scalar field and whence no dynamical scale generation.



Metric and scalar field for $\phi(0) = 0.9$ with $N = 47$ spectral points

3. EPILOGUE: OPTICAL CONDUCTIVITY

Optical conductivity calculated in holography [3] reveals two remarkable surprises. First, at low frequencies, conductivity follows Drude-like behaviour

$$\sigma(\omega) = \frac{K}{1 - i\omega\tau}, \tag{15}$$

where the constant K and relaxation time τ are ω-independent. This is a bit unexpected, because such a behaviour is related to weakly coupled quasiparticle, with K being proportional to particle density. Such a description was not a part of the model assumptions. A nice explanation of this relation in terms of gravitational Higgs mechanism was given in [7]. In this picture graviton acquires radial-dependent mass term $M(z)$ due to the interaction with a scalar field. Relaxation time is determined by the graviton mass evaluated at the horizon, which in turn is determined by the scalar

$$\tau^{-1} \sim M^2(z_h) \sim \int\limits_0^{2\pi} dx\, (\partial_x \phi(x, z_h))^2 \,, \tag{16}$$

where z_h is radial position of the event horizon. In this way Drude behaviour is related to the low-energy physics of small perturbations around the black hole horizon.

Second, in the range of frequencies $2 < \omega\tau < 8$, the optical conductivity exhibits power-law behaviour

$$|\sigma(\omega)| = \frac{B}{(\omega\tau)^{2/3}} + C, \tag{17}$$

where B and C are temperature-independent constant parameters. In subsequent studies [8], exponent in this power-like behaviour was found to be nonuniversal and depending on the definition of the intermediate range of frequencies. For high frequencies, conductivity tends to a constant value, which is a generic property of CFTs in $2 + 1$ dimensions. Similar behaviour (with $C = 0$) is experimentally observed in some cuprates [9]. For the moment, there is no deep understanding of why gravitational models predictions are in agreement with results seen in real materials.

Acknowledgements. This research was supported by the National Science Center (NCN) within a post-doctoral internship grant No. DEC-2013/08/S/ST2/00547.

REFERENCES

1. *Maldacena J. M.* The Large N Limit of Superconformal Field Theories and Supergravity // Adv. Theor. Math. Phys. 1998. V. 2. P. 231.
2. *Hartnoll S. A.* Lectures on Holographic Methods for Condensed Matter Physics // Class. Quant. Grav. 2009. V. 26. P. 224002.

3. *Horowitz G. T., Santos J. E., Tong D.* Optical Conductivity with Holographic Lattices // JHEP. 2012. V. 1207. P. 168.
4. *Kronig R. de L., Penney W. G.* Quantum Mechanics of Electrons in Crystal Lattices // Proc. Roy. Soc. A. 1931. V. 130. P. 499.
5. *Bak D., Gutperle M., Hirano S.* A Dilatonic Deformation of AdS(5) and Its Field Theory Dual // JHEP. 2003. V. 0305. P. 072.
6. *D'Hoker E. et al.* Janus Solutions in M-Theory // JHEP. 2009. V. 0906. P. 018.
7. *Blake M., Tong D., Vegh D.* Holographic Lattices Give the Graviton a Mass // Phys. Rev. Lett. 2014. V. 112. P. 071602.
8. *Ling Y. et al.* Holographic Lattice in Einstein–Maxwell–Dilaton Gravity // JHEP. 2013. V. 1311. P. 006.
9. *van der Marel D. et al.* Powerlaw Optical Conductivity with a Constant Phase Angle in High-T_c Superconductors // Nature. 2003. V. 425. P. 271.