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ANTINEUTRINO DIRECTION VIA INVERSE BETA DECAY IN DOUBLE CHOOZ

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Determination of the direction to a source of neutrinos (and antineutrinos) is an important problem for the physics of supernovae and of the Earth. The direction to a source of antineutrinos can be estimated through the reaction of inverse beta decay. We show that the reactor neutrino experiment Double Chooz has unique capabilities to study antineutrino signal from point-like sources. Contemporary experimental data on antineutrino directionality is given. A rigorous mathematical approach for neutrino direction studies has been developed. Exact expressions for the precision of the simple mean estimator of neutrinos' direction for normal and exponential distributions for a finite sample and for the limiting case of many events have been obtained.

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INTRODUCTION

The neutrino has the largest penetrating ability among all particles that can be detected by humankind. This unique property allows neutrinos to carry information from places where no other known methods can throw a glance at. The information about neutrino direction can be of great use for several important fields of physics.

The distribution of radioactive sources in the Earth can be studied using directional information from geoneutrinos [1]. Geological models can be discriminated based on the distribution of uranium and thorium, which emit antineutrinos in their decay chains. Depending on the relative contribution of geoneutrinos from the lower mantle and from the continental crust, measured antineutrino directions should be different [2].

A supernova explosion is a rare and unique physical process, in which large fluxes of neutrinos and antineutrinos are emitted. However, for optical observations most of the supernovae in our Galaxy were hidden by the galactic dust and

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gases. Neutrinos' direction reconstruction can be used to estimate the location of a supernova explosion in our Galaxy in order to:

• know where the supernova occurred;

• take into account the Mikheyev–Smirnov–Wolfenstein effect in the Earth;

• point optical and other telescopes to observe the early times of the supernova explosion if it is possible.

The SuperNova Early Warning System (SNEWS) project includes neutrino detectors Borexino, Daya Bay, KamLAND, IceCube, LVD, and Super-Kamiokande [3]. Its aim is "to provide the astronomical community with a prompt alert" of a supernova. Unfortunately, SNEWS involves no direction information, therefore it is important to implement online direction reconstruction in neutrino experiments. Starting in 2009, Evaluating Gadolinium's Action on Detector Systems (EGADS) was built in the Kamioka mine [4]. It can assist Super-Kamiokande in providing prompt directional information, but it does not involve other detectors yet.

In order to combine directional data from several experiments, one needs to know statistical properties of directional *probability density functions* (pdfs) corresponding to different experimental techniques.

In this paper, we investigate pdfs of estimated directions to a source of antineutrinos that are detected through the reaction of inverse beta decay.

DIRECTION ESTIMATION

Inverse beta decay is one of the most important reactions to detect antineutrinos. This reaction was used in the CHOOZ liquid scintillator experiment to study directions of antineutrinos from nuclear reactors [5]. The CHOOZ collaboration has shown that using a sample of ~ 2500 antineutrino candidates the direction to their source can be defined with the precision of $18°$.

Double Chooz may be considered as a successor of CHOOZ. It has two detectors that register antineutrinos from two nuclear reactors located in France. The reactors are seen at the angle of only $6°$ from the far detector. Since there are only two reactors, much data is collected with one reactor off. These are great advantages of Double Chooz to study a point-like source compared to other experiments, which usually have many reactors with no simple and direct decomposition of signals from them.

The reaction of inverse beta decay

$$\overline{\nu}_e + p \longrightarrow e^+ + n \qquad (1)$$

has the largest cross section compared to other neutrino and antineutrino reactions at supernova energies. It requires free protons, which are contained in liquid scintillators in hydrogen. Another good characteristic of this reaction is that it has low background in experiments, since it can be detected by a technique of delayed coincidence:

1) The positron is almost immediately captured in the scintillator with the emission of its kinetic energy plus 1.022 MeV.

2) The neutron experiences moderation and diffusion and after a mean time of 31.1 μs in Double Chooz is captured by Gd with the emission of 8 MeV.

The positions of the positron and the neutron are reconstructed by software algorithms using a maximum likelihood estimate. More information on Double Chooz and its registration of antineutrinos through the reaction of inverse beta decay can be found in [6].

While positrons are being born in the reaction (1) with almost isotropic momenta, neutrons on average conserve the direction of antineutrinos [7]. After production neutrons experience moderation and diffusion in scintillator. The average length of true displacements of neutrons generated by Monte Carlo is $\simeq 6$ cm, while the average length of reconstructed neutron displacements is $\simeq 22$ cm [8]. The average direction of neutron displacements is preserved during the diffusion. If one takes the angle between the vector of each neutron's reconstructed displacement from positron and the vector of the known antineutrino direction, one can calculate the average cosine of that. For Monte Carlo simulations of Double Chooz the average cosine for the neutrons after production is 0.854, after moderation and diffusion it is 0.233, and after the position reconstruction it is 0.074 [8]. Therefore, in order to have a good direction reconstruction one should have a good position reconstruction or capture the neutron as early as possible. More figures and physical values can be found in the presentation of the talk [9].

As input data for direction reconstruction we have displacement vectors of the neutrons from the positrons for each ith antineutrino candidate:

$$\mathbf{r}_i = \mathbf{r}_{\mathrm{neutron},i} - \mathbf{r}_{\mathrm{positron},i} \in \mathbb{R}^3. \tag{2}$$

CHOOZ considered two estimators of direction:

- simple mean of $\mathbf{r}_i$

$$\hat{\mathbf{r}}_n = \frac{1}{n}\sum_{i=1}^{n} \mathbf{r}_i, \tag{3}$$

- mean of normalized $\mathbf{r}_i$

$$\hat{\mathbf{u}}_n = \frac{1}{n}\sum_{i=1}^{n} \frac{\mathbf{r}_i}{r_i}. \tag{4}$$

The latter method was found by CHOOZ to be about 10% preciser than the simple mean, but its pdf is not completely analytically calculated yet.

The angular precision of a direction reconstruction at a confidence level C.L. (which can be, for example, C.L. $= 68\%$) is defined as the half-aperture of the cone that has the true direction as the axis and contains C.L. fraction of all reconstructed directions for each sample of n events [5]. The angular precision δ at C.L. is a confidence interval in the space of directions. The angular *cumulative distribution function* at $\cos\delta$ is

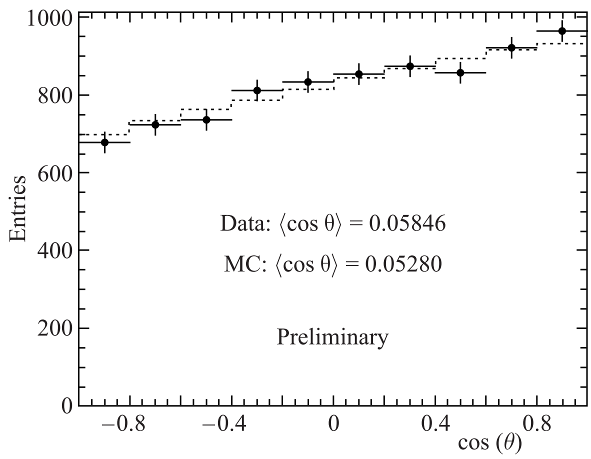
$$\mathrm{CDF}(\cos\delta) = \mathrm{C.L.}, \tag{5}$$

and the precision can be calculated through its inverse

$$\cos\delta = \mathrm{CDF}^{-1}(\mathrm{C.L.}). \tag{6}$$

Directional data are studied by a special field of mathematical statistics, directional statistics [10]. If a distribution of directions on a sphere is narrow, conventional statistics can be used. During a supernova explosion many experiments will detect not a very large number of events, hence one should be able to deal not with a limiting case of very many events, as was investigated previously, but with statistics of a finite sample of directional data. Another distinctive feature of these studies is that the spatial distribution of the displacement vectors may be non-Gaussian, as was assumed by CHOOZ.

The angular distribution for the experimental data of Double Chooz [11] is plotted in the Figure. Double Chooz confirms that determination of antineutrino direction using a large number of events is possible. The mean of normalized vectors estimator (4) was found to have the precision of 6° for 8249 events [11,12],



Cosines of angles between the displacement vectors (2) and the bisector of directions to the reactors [11]. Preliminary experimental data of Double Chooz are shown by points, and Monte Carlo by dashes. Most of the events are along the direction of antineutrinos, but many are backward or of $9.4°$ for 8246 events [13]. The difference between these preliminary results for almost the same data set hints that we need a better theory.

In this paper, we use the simple mean (3) as a direction estimator. All mathematical details can be found in [14].

As spherically symmetric mathematical models of the distribution of one displacement vector $\mathbf{r}_i = (x_i, y_i, z_i)$ (2) shifted by its mean $\langle \mathbf{r}_i \rangle$, we use:

- normal distribution

$$\mathrm{pdf}_{G,1}(x,y,z) = \frac{1}{\left(2\pi\sigma^2\right)^{3/2}} \exp\left[-\frac{\left(x^2+y^2+z^2\right)}{2\sigma^2}\right], \tag{7}$$

- exponential distribution

$$\mathrm{pdf}_{E,1}(x,y,z) = \frac{1}{8\pi l^3} \exp\left[-\frac{\sqrt{x^2+y^2+z^2}}{l}\right]. \tag{8}$$

In order to obtain the angular CDF, we use spherical coordinates

$$\begin{pmatrix} x = r\cos\phi\sin\theta \\ y = r\sin\phi\sin\theta \\ z = r\cos\theta \end{pmatrix}, \quad \theta \in [0,\pi]. \tag{9}$$

Let the true direction be $\cos\theta = 1$. Then the angular CDF is

$$\mathrm{CDF}(\cos\theta) = \int\limits_{\cos\theta}^{1} d\cos\theta' \int\limits_{0}^{\infty} r^2 dr \times \\ \times \int\limits_{0}^{2\pi} d\phi \, \mathrm{pdf}\left((x,y,z-r_0)(r,\phi,\cos\theta')\right). \tag{10}$$

The pdf is shifted by $\mathbf{r}_0 = (0,0,r_0)$. $\mathbf{r}_0$ is the average displacement of the point of neutron capture from the point of positron annihilation.

The sample mean of n normally distributed vectors is distributed normally with σ scaled by $1/\sqrt{n}$. The angular CDF (10) of the offset normal (Gaussian) distribution is [14]

$$\mathrm{CDF}_{G,\hat{\mathbf{r}}_n}(\cos\theta) = \frac{1}{2}\Bigg(1 + \mathrm{erf}\left(\frac{\sqrt{\mathrm{n}}\mathrm{r}_0}{\sqrt{2}\sigma}\right) - \\ - \exp\left[-\frac{nr_0^2}{2\sigma^2}\left(1-\cos^2\theta\right)\right]\left(1+\mathrm{erf}\left(\frac{\sqrt{n}r_0}{\sqrt{2}\sigma}\cos\theta\right)\right)\cos\theta\Bigg), \tag{11}$$

where the error function is defined as $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int\limits_0^x \mathrm{e}^{-t^2} dt$. Note that the CDF has only one parameter, $\sqrt{n}(r_0/\sigma)$.

For $\sqrt{n}(r_0/\sigma) \gg 1$ and $\theta \ll 1$, the angular precision of the simple mean estimator for the normal distribution can be approximated by [14]

$$\theta_G \approx \frac{\sqrt{-2\ln(1-\mathrm{C.L.})}\sigma}{\sqrt{n}r_0}. \tag{12}$$

The sum of exponentially distributed variables (8) is not exponentially distributed. The pdf of a sum of variables corresponds to the convolution of their pdfs. To compute the convolution for a sum of many vectors is numerically impossible. This can be done using the Fourier transform, which transforms the convolution of pdfs into their product. We obtain the pdf of the sum of n exponentially distributed variables by calculating the Fourier transform of (8), rising it to the power of n, and calculating the inverse Fourier transform. The angular CDF (10) of the simple mean estimator (3) for a sample of n shifted exponentially distributed variables (8) is [14]

$$\begin{aligned}\mathrm{CDF}_{E,\hat{\mathbf{r}}_n}(\cos\theta) = \sum_{k=0}^{2n-2} \frac{(4n-4-k)!2^k(k+1)}{(2n-2-k)!}\Bigg(2(k+2)-\\ - \mathrm{e}^{-\frac{n}{l}r_0}\sum_{i=0}^{k+1}(k+2-i)\frac{\left(\frac{n}{l}r_0\right)^i}{i!} - \cos\theta\int\limits_{\frac{n}{l}r_0}^{\infty}\frac{\sum\limits_{i=0}^{k+1}\frac{x^{i+1}}{i!}}{\sqrt{x^2-\frac{n^2}{l^2}r_0^2(1-\cos^2\theta)}}\mathrm{e}^{-x}\,dx-\\ - 2\Theta_{\mathrm{Heaviside}}(\cos\theta)\cos\theta\int\limits_{\frac{n}{l}r_0\sqrt{1-\cos^2\theta}}^{\frac{n}{l}r_0}\frac{\sum\limits_{i=0}^{k+1}\frac{x^{i+1}}{i!}}{\sqrt{x^2-\frac{n^2}{l^2}r_0^2(1-\cos^2\theta)}}\mathrm{e}^{-x}\,dx\Bigg).\end{aligned} \tag{13}$$

For n large the spatial pdf of the exponential simple mean tends near the mode to a Gaussian with $\sigma_{E,n} = (2l)/\sqrt{n}$. In the limit of n large ($\sqrt{n}(r_0/(2l)) \gg 1$ and $\theta \ll 1$) the angular precision can be, using (12), approximated by

$$\theta_E \approx \frac{2\sqrt{-2\ \ln(1-\mathrm{C.L.})}l}{\sqrt{n}r_0}. \tag{14}$$

For usual liquid scintillator detectors $r_0 \sim 1.6$ cm, $\sigma \sim 18$ cm. To have a precise direction reconstruction, $\sqrt{n}(r_0/\sigma)$ should be much more than 1, which corresponds to $n \gg 100$.

CONCLUSIONS

The reaction of inverse beta decay allows reconstruction of the direction to a source of antineutrinos. To have a good direction reconstruction, a detector should have a good position reconstruction. Double Chooz can test methods of direction reconstruction. The angular precision of the estimator (4) used by Double Chooz is less than 10° for ~ 8000 antineutrino candidates.

Two mathematical models of directional data have been proposed in (7) and (8). For the offset normal distribution the exact angular CDF (10) for the estimator (3) is given by (11), and the angular precision in the limiting case of many events is given by (12). For the offset exponential distribution the exact angular CDF for the estimator (3) is (13). In the limiting case of many events the spatial distribution of the estimator (3) for the exponential pdf tends to a Gaussian, and the angular precision of that can be approximated by (14).

The author considers the following as future work:

- find the precision of (4) and compare it with that of (3) experimentally;
- find preciser direction estimators for inverse beta decay;
- investigate direction reconstruction for other experimental techniques;
- create an online supernova monitoring system with a direction reconstruction using existing neutrino detectors.

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