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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

A MODEL OF A TRANSITION NEUTRAL PION FORM FACTOR MEASURED IN ANNIHILATION AND SCATTERING CHANNELS AT HIGH MOMENTUM TRANSFER

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We consider an alternative explanation of newly found growth of neutral pion transition form factor with high virtuality of one of photons. It is based on Sudakov suppression of quark–photon vertex. Some applications to scattering and annihilation channels are considered including the relevant experiments with lepton–proton scattering.

Рассматривается альтернативное объяснение недавно обнаруженного роста формфактора нейтрального пиона с одним реальным и одним виртуальным фотоном. В данной статье подход основан на судаковском подавлении кварк-фотонной вершины. Представлены возможные приложения к эксперименту в каналах рассеяния и аннигиляции.

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INTRODUCTION

A lot of attention was paid to the problem of describing the transition form factor of neutral pion [1–3]. It is the information about the wave function of neutral pion; namely, the distribution on the energy fractions of u, d quarks inside a neutral pion is the motivation of numerous theoretical approaches to describe the transition form factor. Recently some experimental information about its behavior was obtained in the process $e^+e^- \to e^+e^-\pi_0$. The kinematics when one of the photons is almost real and the other is highly virtual was considered [4, 5]. The result was presented by the authors of the experiment as some nondecreasing function of the module of square of momentum of a virtual photon. Such a type of behavior is in clear contradiction with the predictions of factorization theorem applied to this process (see [6–9] and references therein).

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Below we consider another reason to explain such a type of behavior, using the well-known expression of a virtual photon–quark vertex (so-called Sudakov form factor [10, 11]) which is entered in the triangle Feynman diagram, describing the conversion of two photons to the neutral pseudoscalar meson. It is the motivation of this paper. Here we consider the regime of sufficiently large virtualities of one of the photons.

Both channels of pseudoscalar meson production in elastic electron–positron collisions — the scattering and annihilation ones — are considered. The second one $e^+e^- \to \pi_0 l^+ l^-$ can be the subject of experimental investigation.

1. SCATTERING CHANNEL

In the scattering type of experiments

$$e^+ (p_+) + e^- (p_-) \to e^+(p'_+) + e^-(p'_-) + \pi^0 (q_\pi) \tag{1}$$

($p_\pm^2 = p_\pm'^2 = m_e^2$, $q_\pi^2 = M^2$) neutral pion is created by two photons with momenta $q = p_+ - p'_+$ and $q_1 = p_- - p'_-$ that are involved in lepton interaction as presented in Fig. 1. Due to Weizsäcker–Williams (WW) kinematics of this process (the scattered electron assumed to move close to beam direction, i.e., the BaBar facility kinematics), one of the photons is almost real $\left|q_1^2\right| \ll M^2$ and the other is off mass shell $Q^2 = -q^2 \gg M^2$.

The matrix element has the form (for details, see Appendix)

$$\begin{aligned} M &= \frac{2s(4\pi\alpha)^2}{q^2 q_1^2} \left[\mathbf{q} \times \mathbf{q_1}\right]_z V(Q^2) N_+ N_-, \\ V(Q^2) &= \frac{M_q^2}{2\pi^2 F_\pi Q^2} F\left(\frac{Q^2}{M_q^2}\right), \\ \left|N_\pm\right|^2 &= \frac{1}{s^2} \mathrm{Tr}\left[p_- p_+ p_- p_+\right] = 2, \end{aligned} \tag{2}$$

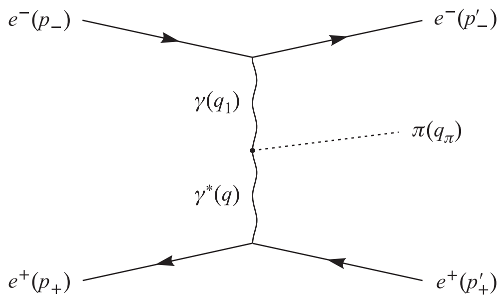

Fig. 1. Scattering process of pion production

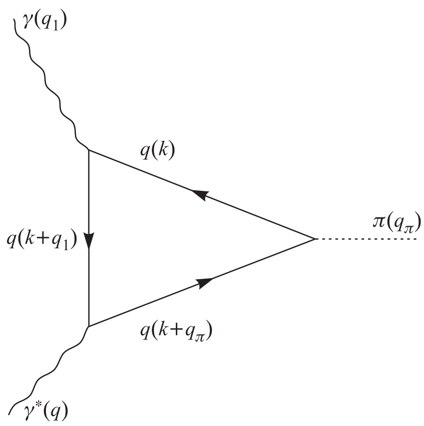

Fig. 2. Triangle vertex for $\gamma\gamma^* \to \pi_0$ process

where $s = (p_+ + p_-)^2 \gg Q^2$ and $V(Q^2)$ is the transition form factor of pion, $F_\pi = 93$ MeV is the decay constant of pion, $M_q = M_u = M_d = 280$ MeV is the quark mass, $\mathbf{q}$, $\mathbf{q}_1$ are components of photon momenta transversal to the beam axis (z) direction. The quantity $F(Q^2/M_q^2)$ has the form

$$F\frac{Q^2}{M_q^2} = -\int \frac{d^4k}{i\pi^2} \frac{Q^2 V_S(Q^2, p_1^2, p_2^2)}{(k^2 - M_q^2 + i0)(p_1^2 - M_q^2 + i0)(p_2^2 - M_q^2 + i0)}, \tag{3}$$

where $p_1 = k + q_1$ and $p_2 = k + q_\pi$, and the Sudakov vertex function V_S [12, 13] is

$$V_S(Q^2, p_1^2, p_2^2) = \exp\left(-\frac{\alpha_s C_F}{2\pi} \ln\frac{Q^2}{|p_1^2|} \ln\frac{Q^2}{|p_2^2|}\right), \tag{4}$$

where $Q^2 \gg |p_{1,2}^2| \gg M_q^2$ and $C_F = \left(N^2 - 1\right) / (2N) = 4/3$. We use here the Goldberger–Treiman relation on the quark level $F_\pi = M_q / g_{q\bar{q}\pi} = 93$ MeV.

The cross section of the process (1) takes the form

$$d\sigma = \frac{1}{8s} |M|^2 \, d\Gamma_3. \tag{5}$$

The phase volume of the final state $d\Gamma_3$ can be expressed through the Sudakov parametrization of the photon's momenta which turns out to be convenient:

$$q_1 = \alpha_1 \tilde{p}_+ + \beta_1 \tilde{p}_- + q_{1\perp}, \quad q = \alpha \tilde{p}_+ + \beta \tilde{p}_- + q_\perp, \quad a_\perp p_\pm = 0, \quad q_{1\perp}^2 = -\mathbf{q}_{1\perp}^2, \tag{6}$$

and we imply the 4-vectors $\tilde{p}_\pm$ to be light-like, $2\tilde{p}_+\tilde{p}_- = s$. Therefore (for details, see Appendix),

$$d\Gamma_3 = (2\pi)^{-5} \delta^4 \left(p_+ + p_- - p'_+ - p'_- - q_\pi\right) \frac{d^3p'_+ d^3p'_- q^3 q_\pi}{2E'_+ 2E'_- 2E_\pi} = \\ = (2\pi)^{-5} \frac{1}{4s} \frac{d\beta_1}{\beta_1 (1 - \beta_1)} d^2q_1 d^2q, \tag{7}$$

$$\frac{Q^2 + M^2}{s} < \beta_1 < 1.$$

Using the expression for the square of momentum of «almost» real photon

$$q_1^2 = -\frac{1}{1 - \beta_1} \left[\mathbf{q}_{1\perp}^2 + m_e^2 \beta_1^2\right], \quad \mathbf{q}_{1\perp}^2 \ll Q^2 \tag{8}$$

and performing the integration on the parameters of scattered electron $(\beta_1, \mathbf{q}_1)$, moving close to z axis, we obtain for the cross section

$$\frac{d\sigma}{dQ^2} = \frac{\alpha^4}{4Q^2} V^2(Q^2) J(Q^2), \\ J(Q^2) = \frac{1}{2} L_s^2 + L_s(L_e - 1) - (L_e + 1), \tag{9}$$

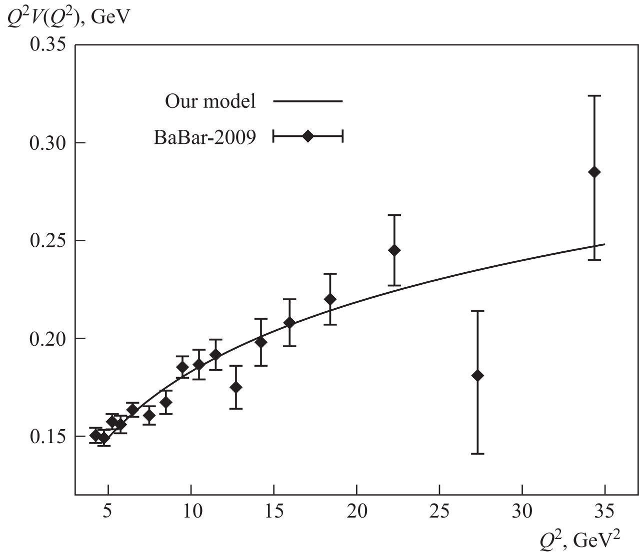

Fig. 3. A fit of our approach (Eq. (12)) with resultant fitting parameters $A = 0.49$, $B = 0.23$ for π^0 production and the comparison with the experimental data of BaBar facility [5]

where

$$L_s = \ln \frac{s}{Q^2 + M^2}, \quad L_e = \ln \frac{Q^2}{m_e^2}. \tag{10}$$

There are several approaches to infer the value $V(Q^2)$, which is named the pion transition form factor.

One of them is based on QCD collinear factorization theorem [2]:

$$V^{\rm BL}(Q^2) = \frac{2F_\pi}{3} \int\limits_0^1 \frac{dx}{xQ^2} \phi_\pi(x, s), \tag{11}$$

and in papers [6, 9] different forms of pion wave function $\phi_\pi(x, s)$ were used. Another possible mechanism of the effect was given in [8, 14].

Also, in papers [7, 15] it was pointed out that the pion form factor in the framework of the constituent quark model has the double logarithmic asymptotic at large momentum transfer.

The approach used here is based on the Sudakov form of the vertex function which describes interaction of a photon with large four-momentum square $|q^2|$ with two quarks of an anomalous three-angle quark diagram describing the conversion of two photons to the neutral pion.

The three-angle quark-loop diagram itself at large Q^2 has the double-logarithm asymptotic [15], and insertion of the Sudakov form of the vertex function which includes also QCD-inspired corrections gives the asymptotic $\ln\left(\ln\left(Q^2/m^2\right)\right)$ behavior.

The final expression is (details are in Appendix)

$$V(Q^2) = A \frac{M_q^2}{2\pi F_\pi \alpha_s C_F} \Phi(z_B), \tag{12}$$

where

$$\Phi(z_B) = \int\limits_0^1 \frac{dx}{x}(1 - \mathrm{e}^{-z_B x(1-x)}), \quad z_B = \frac{C_F \alpha_s}{2\pi} \ln^2 \frac{Q^2}{BM_q^2},$$

where A, B can be considered as positive fitting parameters of order of unity. We find it through BaBar data fitting. Function $Q^2 V(Q^2)$ is presented in Fig. 3, where the experimental data are also presented.

2. ANNIHILATION CHANNEL

Let us consider now the annihilation channel depicted in Fig. 4:

$$e^+(p_+) + e^-(p_-) \to \gamma^*(q) \to \pi^0(q_\pi)\, l^+(q_+)\, l^-(q_-)\,, \tag{13}$$

where $l = e, \mu$ and $p_\pm^2 = 0$, $q_\pm^2 = m_l^2$, $q_\pi^2 = M^2$, $s = q^2 = (p_+ + p_-)^2$, $s_1 = q_1^2 = (q_+ + q_-)^2$. We put the matrix element of this process in the form

$$\begin{gathered} M = \frac{(4\pi\alpha)^2}{q_1^2 q^2} J^\mu J^{\nu(l)} V(s)\, \epsilon_{\mu\nu\alpha\beta} q^\alpha q_1^\beta, \\ J^\mu = \bar{v}(p_+)\gamma_\mu u(p_-)\,, \quad J^{\nu(l)} = \bar{v}_\mu(q_+)\gamma_\nu u_\mu(q_-)\,, \end{gathered} \tag{14}$$

where quantity $V(s)$ describing conversion of two off-mass-shell photons to the neutral pion (pion transition form factor) is defined in (2).

The phase volume of the final state

$$d\Gamma_3 = (2\pi)^{-5} \delta^4(p_+ + p_- - q_+ - q_- - q_\pi) \frac{d^3q_+\, d^3q_- q^3 q_\pi}{2E_+ 2E_- 2E_\pi} \tag{15}$$

could be written in the form

$$d\Gamma_3 = (2\pi)^{-5} \pi d\Gamma_{q_1} dq_1^2 \frac{\Lambda^{1/2}(s, q_1^2, M_\pi^2)}{2s}, \tag{16}$$

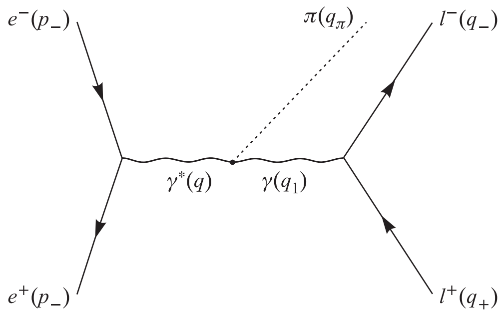

Fig. 4. The annihilation channel of the process $e^+e^- \to \pi^0 l^+ l^-$

where $\Lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$ and

$$d\Gamma_{q_1} = \int \frac{d^3q_+\, d^3q_-}{2E_+ 2E_-} \delta^4 \left(q_1 - q_+ - q_-\right). \tag{17}$$

Performing the integration on the invariant mass square of the lepton pair (we use the approximation $s_1 \ll s$ and $s_1 < M^2$), the relevant cross section has the form (details are in Appendix)

$$\sigma^{e\bar{e}\to\pi_0 l\bar{l}} = \frac{\pi\alpha^4 V(s)^2}{6} \left(1 - \frac{M^2}{s}\right)^3 \left[\ln \frac{s}{m_l^2} - \frac{5}{3}\right]. \tag{18}$$

CONCLUSION

From our point of view, the QCD corrections connected with the vertex of interaction of the highly virtual photon with quarks are essential and pion can be considered as a point particle. So at rather large values of Q^2 the details of pion wave function become irrelevant.

For heavy s quarks $M_s = 400$ MeV involved in the heavy pseudoscalar mesons η' the effect of Sudakov form factor becomes weaker.

In literature there are alternative explanations (see the end of Sec. 1) of the BaBar experimental data [5].

In Fig. 3 we represent a numerical estimation fit of the BaBar data. We obtain the qualitative logaritm-logarithmic growth (see Eq. (27)) of the transitional form factor (12). On the plot we put the best fitting of BaBar data with two adjustable parameters A and B.

Similar phenomena can take place for the case of scalar meson production as well.

We remind also the possibility to measure the transition pion form factor in electron–proton scattering $ep \to e\pi_0 p$. The relevant cross section will be

$$\frac{d\sigma^{ep\to e\pi_0 p}}{dQ^2} = \left(\frac{\alpha g_{\rho qq} g_{\rho NN}}{8\pi(Q^2 + M_\rho^2)}\right)^2 \frac{V^2(Q^2)}{Q^2} \left[F_1^2(Q^2) + \frac{Q^2}{4M_p^2} F_2^2(Q^2)\right] J(Q^2), \tag{19}$$

where F_1, F_2 are Dirac and Pauli proton form factors. Here instead of virtual photon the virtual vector meson takes place; $g_{\rho qq}$, $g_{\rho NN}$ are the ρ-meson couplings with quarks and nucleons, correspondingly. In this case a problem with background ($ep \to e\Delta^+ \to e\pi^0 p$) must be overcomed.

We consider Sudakov form factor for time-like transfer momentum. With ordinary particles in the loop we must take into account the imaginary part of relevant amplitude. For quarks inside a loop the imaginary part is absent.

Taking into account the nonleading terms in expression of Sudakov exponent results in modification of quark mass and a general shift of normalization:

$$F\left(q^2, p_1^2, p_2^2\right) \to AF\left(q^2, p_1^2, p_2^2\right), \quad Q^2\alpha\beta > BM_Q^2, \tag{20}$$

where $A \sim B \sim 1$ can be considered as fitting parameters $A, B > 0$.

In our opinion, the fitting quark mass $M_q \approx 135$ MeV (which corresponds to the value of fitting parameter $B = 0.23$) is quite realistic value which takes into account the effect of decreasing of quark mass at high virtualities.

In conclusion we should emphasize once again that, applying Sudakov approximation to quark vertex function, we imply a rather large value of virtualities of one of the photons (i.e., $\left|q^2\right| \geqslant 5$ GeV^2). Thus, our approach differs from the ones based on pion wave function modification [8] as well as ones based on instanton model [7, 16–18] which impose some restriction in loop momentum integration.

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Appendix

DETAILS OF CALCULATION

Transformation of the phase volume of 2-gamma creation process $2 \to 3$ consists in introduction of two transferred vectors q_1, q:

$$d\Gamma_3 = (2\pi)^{-5} d^4q_1\, d^4q\, d^4q_\pi\, d^4p'_+\, d^4p'_- \times \\ \times \delta^4\left(p_- - q_1 - p'_-\right) \delta^4\left(p_+ - q - p'_+\right) \delta^4\left(q + q_1 - q_\pi\right) \delta\left(\left(q_1 - p_-\right)^2 - m_e^2\right) \times \\ \times \delta\left(\left(q - p_+\right)^2 - m_e^2\right) \delta\left(\left(q_1 + q\right)^2 - M^2\right),$$

and using the Sudakov parameterization for the scattering channel (6), we put it in the form

$$d\Gamma_3 = (2\pi)^{-5} \frac{s}{2} d\alpha_1\, d\beta_1\, d^2\mathbf{q}_1 \frac{s}{2} d\alpha\, d\beta\, d^2\mathbf{q} \times \\ \times \delta\left(s\alpha\beta - \mathbf{q}^2 - s\beta - m_e^2\alpha\right) \delta\left(s\alpha_1\beta_1 - \mathbf{q}_1^2 - s\alpha_1 - m_e^2\beta_1\right) \times \\ \times \delta\left(s\alpha\beta_1 - (\mathbf{q} + \mathbf{q}_1)^2 - M^2\right). \quad (21)$$

Performing the integrations over α_1, α, β, we obtain the result given above (see Eq. (7)).

Expression for scalar loop integral with 3 denominators and Sudakov vertex inserted has the form

$$F\left(\frac{Q^2}{M_q^2}\right) = -\int \frac{d^4k}{i\pi^2} \frac{Q^2 V_S(Q^2, p_1^2, p_2^2)}{(k)(1)(2)},$$
$$(k) = k^2 - M_q^2 + i0,$$
$$(1) = (k + q_1)^2 - M_q^2 + i0,$$
$$(2) = (k + q_\pi)^2 - M_q^2 + i0,$$

where $V_S(Q^2, p_1^2, p_2^2)$ was defined in (4).

To perform the integration, we use Sudakov parameterization of the loop momentum:

$$k = \alpha n_1 + \beta n_2 + k_\perp, \quad (22)$$

with $n_{1,2}$ being light-like 4-vectors $n_i^2 = 0$, transversal to $k_\perp$, $n_i k_\perp = 0$, built from the 4-vectors q_π, q_1 such that $2n_1n_2 = Q^2$. In such a parameterization

$$d^4k = \frac{Q^2}{2} d\alpha\, d\beta\, d^2k_\perp. \quad (23)$$

Expressing the denominators of quark Green functions

$$(k) = Q^2\alpha\beta - \mathbf{k}^2 - M_q^2 + i0, \quad (1) \approx Q^2\alpha, \quad (2) \approx Q^2\beta$$

and performing the integration over $k_\perp^2 = -\mathbf{k}^2$ as (we imply that the principal part does no contribute)

$$\int \frac{\pi d\mathbf{k}^2}{(k)} = -i\pi^2 \int d\mathbf{k}^2 \delta(-\mathbf{k}^2 + Q^2\alpha\beta - M_q^2) = -i\pi^2\theta[Q^2\alpha\beta - M_q^2], \tag{24}$$

we obtain

$$F\left(\frac{Q^2}{M_q^2}\right) = \frac{1}{2}\int\limits_{M_q^2/Q^2}^{1} \frac{d\alpha}{\alpha} \int\limits_{M_q^2/Q^2}^{1} \frac{d\beta}{\beta}\theta\left(Q^2\alpha\beta - M_q^2\right)\exp\left(-\frac{\alpha_s C_F}{2\pi}\ln\frac{1}{\alpha}\ln\frac{1}{\beta}\right). \tag{25}$$

Performing one integration, we obtain

$$F\left(\frac{Q^2}{M_q^2}\right) = \frac{\pi}{\alpha_s C_F}\Phi(z), \tag{26}$$

where

$$\Phi(z) = \int\limits_0^1 \frac{dx}{x}\left(1 - \mathrm{e}^{-zx(1-x)}\right), \quad z = \frac{\alpha_s C_F}{2\pi}L^2, \quad L = \ln\frac{Q^2}{M_q^2}.$$

For large values of Q^2 we obtain

$$F\left(\frac{Q^2}{M_q^2}\right) = \frac{\pi}{\alpha_s C_F}\left(\ln(z) + c\right), \tag{27}$$

where

$$c = \int\limits_0^1 \frac{dx}{x}(1 - \mathrm{e}^{-x}) - \int\limits_1^\infty \frac{dx}{x\mathrm{e}^x} \approx 0.57. \tag{28}$$

In reality the quantity $L \sim 5$ for light consistent quarks u, d which are present in the neutral pion and $L \sim 1{-}2$ for s quark in η' meson.

When considering the integration on the pair phase volume in annihilation channel, we use the relation (consequence of gauge invariance)

$$\begin{gathered}\sum_{\mathrm{pol}} \int d\Gamma_{q1} J^{(\mu(l))}(J^{(\nu(l))})^* = -\frac{1}{3}\left(g_{\mu\nu} - \frac{q_{1\mu}q_{1\nu}}{q_1^2}\right)\left(q_1^2 + 2m_l^2\right)\frac{\pi\beta_-}{2}, \\ \beta_- = \sqrt{1 - \frac{4m_l^2}{q_1^2}}.\end{gathered} \tag{29}$$

The differential cross section is

$$d\sigma^{e\bar{e}\to\pi^0 l\bar{l}} = \frac{\alpha^4 M_q^4}{24\pi^3 F_\pi^2 s^5}\Lambda^{3/2}\left(s, q_1^2, M^2\right)\frac{dx}{x}\left(1 + \frac{1}{2x}\right)\sqrt{1 - \frac{1}{x}}, \tag{30}$$

where $x = q_1^2/\left(4m_l^2\right) > 1$.

REFERENCES

1. *Chernyak V. L.* Asymptotic Behavior of Hadron Form Factors in Quark Model // JETP Lett. 1977. V. 25. P. 510 (in Russian).
2. *Lepage G.P.* Exclusive Processes in Quantum Chromodynamics: Evolution Equations for Hadronic Wave Functions and the Form Factors of Mesons // Phys. Lett. B. 1979. V. 87. P. 359–365.
3. *Efremov A. V.* Factorization and Asymptotical Behavior of Pion Form Factor in QCD // Phys. Lett. B. 1980. V. 94. P. 245–250.
4. *Druzhinin V. P.* Recent Results on Two-Photon Physics at BaBar. hep-ex/0909.3148. 2009.
5. *Aubert B.* Measurement of the $\gamma\gamma^* \to \pi^0$ Transition Form Factor // Phys. Rev. D. 2009. V. 80. P. 052002.
6. *Mikhailov S. V.* Transition Form Factors of the Pion in Light-Cone QCD Sum Rules with Next-to-Next-to-Leading Order Contributions // Nucl. Phys. B. 2009. V. 821. P. 291–326.
7. *Dorokhov A. E.* Rare Decay $\pi^0 \to e^+e^-$ as a Test of Standard Model // Intern. School-Seminar on New Physics and Quantum Chromodynamics at External Conditions (NPQCD 2009), Dnipropetrovsk, Ukraine, 2009.
8. *Radyushkin A. V.* Shape of Pion Distribution Amplitude // Phys. Rev. D. 2009. V. 80. P. 094009.
9. *Polyakov M. V.* On the Pion Distribution Amplitude Shape // JETP Lett. 2009. V. 90. P. 228–231.
10. *Sudakov V. V.* Vertex Parts at Very High Energies in Quantum Electrodynamics // Sov. Phys. JETP. 1956. V. 3. P. 65–71.
11. *Kuraev E. A.* Double Logarithmical Asymptotic of Fermion Inelastic Form Factor in Non-Abelian Gauge Theory // Yad. Fiz. 1978. V. 27. P. 1107–1111.
12. *Carazzone J. J.* A Calculation of Asymptotic Behavior of Form Factors in Non-Abelian Gauge Theories // Phys. Rev. D. 1975. V. 11. P. 2286–2297.
13. *Cornwall J. M.* Infrared Behavior of Non-Abelian Gauge Theories // Phys. Rev. D. 1976. V. 13. P. 3370.
14. *Li H.-N.* Pion Transition Form Factor in k_T Factorization // Phys. Rev. D. 2009. V. 80. P. 074024.
15. *Ametller L. et al.* The Quark Triangle: Application to Pion and Eta Decays // Nucl. Phys. B. 1983. V. 228. P. 301.
16. *Dorokhov A. E.* How the Recent BaBar Data for $P \to \gamma\gamma^*$ Affect the Standard Model Predictions for the Rare Decays $P \to l^+l^-$ // JETP Lett. 2010. V. 91. P. 163–169.
17. *Dorokhov A. E.* Muon g-2, Rare Decays $P \to l^+l^-$ and Transition Form Factors $P \to \gamma\gamma^*$ // Nucl. Phys. Proc. Suppl. 2010. V. 198. P. 190–193.
18. *Dorokhov A. E.* Photon–Pion Transition Form Factor: BaBar Puzzle is Cracked. hep-ph/1003.4693. 2010.

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