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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

UNIVERSE AS A REPRESENTATION OF AFFINE AND CONFORMAL SYMMETRIES

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The evolution of the Universe is considered by means of a nonlinear realization of affine and conformal symmetries via Maurer–Cartan forms. Conformal symmetry is realized by the geometry of similarity with the Dirac scalar dilaton. We provide preliminary quantitative evidence that the zeroth harmonic of the Dirac scalar dilaton may lead to observationally viable cosmology, where the type Ia supernova luminosity distances – redshift relation can be explained by vacuum dilaton dark energy. The diffeo-invariance of spin connection coefficients of the affine formulation leaves only one degree of freedom of strong gravitation waves. We discuss that the dark matter effect in spiral galaxies can be described by the gravitation waves expressed through the spin connection coefficients of the affine formulation. We show that the evolution equations of the affine gravitons with respect to the dilaton zeroth mode coincide with the equations of «squeezed oscillator». The list of theoretical and observational arguments is given in favor of that the origin of the Universe can be described by quantum vacuum creation of these squeezed oscillators.

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INTRODUCTION

The Blue Band predictions of the light Higgs particle masses [4] $114.4 < m_h < 134$ GeV are based on the belief that the minimal Standard Model of the electroweak interactions can describe amplitudes of all elementary particle processes. This minimal Standard Model does not contain heavy fermions, SUSY generalization, and other modifications that accompany the accepted cosmological ΛCold Dark Matter Model [5].

The Hamlet question of the modern physics is «To be Λ and Cold Dark Matter, or not to be?» Large Hadron Collider (LHC) is needed to reply to this question.

In the present paper, we show that a part of functions of LHC can be covered by the present-day cosmological data described in the framework of an alternative cosmological model compatible with the physical content of the minimal Blue Band Standard Model. This model can be formulated on the way of generalization of the Poincaré group as the basis of all relativistic physics beginning with the relativistic transformations of the Cosmic Microwave Background and galaxy superclusters and finishing with the Standard Model (SM) of elementary particles and the QFT axiomatics [3]. It is sufficient to add to the Poincaré group only ten proper affine transformations, and the affine group of all linear transformations of four-dimensional space-time arises as the basis of the gravitation theory, as was shown by V. I. Ogievetsky [1, 2]. In the joint nonlinear realization [6, 7] of the affine and conformal groups, the new ten affine transformation parameters are identified with ten gravitons as the Bogoliubov–Goldstone modes accompanying the spontaneous affine and conformal symmetry breaking.

However, the affine–conformal gravitation theory differs from the Einstein one. The new fact is the conformal symmetry broken by the nonsingular initial data. The conformal symmetry can be included into affine theory by the Dirac version of the geometry of similarity [8]. The latter means that measurable quantities are identified with the conformal ones for which the cosmological scale factor is identified with the zeroth harmonics of the dilaton scalar field that changes masses but not distances. In this case, introduction of the Λ term can be completely avoided [9–11]. In this context we emphasize that SNeIa data reveal that observational distances $R_{\rm SNeIa}$ are greater than the distances predicted by the accepted cosmology with the massive matter dominance $R_{\Omega_M \simeq 1}$. There are two possibilities to increase these distances. First of them is to suppose the Λ-term dominance $R_{\rm SNeIa} = R_{\Omega_{\Lambda:p=-\rho}=0.7, \Omega_{M:p=0}=0.3}$, and the second is the Dirac geometry of similarity, where the real measurable distance becomes greater because it acquires an additional z-factor. In this case, the SNeIa data require the dominance of the kinetic energy of an additional sterile scalar field Q with $R_{\rm SNeIa} = R_{\Omega_{\rm p=+\rho}=0.85\pm0.10}(1+z)$ [9].

Thus, the affine gravitation theory differs from the initial Einstein theory by the dilaton geometry of similarity and the Maurer–Cartan forms as objects of the Lorentz transformations in the tangent Minkowskian space-time. In the present paper, within this affine gravitation theory we will attempt to provide a novel view on the topical problems of the modern cosmology (see [5]) including dark matter, dark energy, and the Planck epoch at the beginning of the Universe.

The content of the paper is the following. Section 1 is devoted to a version of General Relativity obtained in [2] as joint nonlinear realization of affine and conformal symmetries and adapted to the Dirac geometry of similarity [8]. Section 2 considers the frame of reference defined as the Dirac–ADM foliation [12, 13]. Section 3 is devoted to the cosmology as a representation of the Poincaré group. In Sec. 4, we study the problems of energy and the Hamiltonian evolution in GR. The strong graviton is considered in Sec. 5. In Sec. 6, we consider the evolution equations of gravitons and their quantum vacuum creation. In Sec. 7, we consider a possible unification of the GR and SM, so that the Higgs scalar field evolution with respect to the dilaton zeroth mode is consistent with the squeezed oscillator dynamics. In Appendix A, we briefly recapitulate the standard Hamiltonian approach in terms of the Cartan forms in order to compare it with our approach. Appendix B is devoted to the dilaton cosmological perturbation theory.

1. REALIZATION OF AFFINE & CONFORMAL SYMMETRIES

The nonlinear realization of affine $A(4)$ and conformal C symmetries in the factor space $A(4) \otimes C/L$ with the Lorentz subgroup L of the stable vacuum was constructed in [1].

According to a general theory of the nonlinear realization [6, 7] of an affine group G, the basic elements are the subgroup of the vacuum stability L and finite transformations $G = \mathrm{e}^{iP\cdot x}\mathrm{e}^{iR\cdot h}$ in the factor space $A(4)/L$. Here, these transformations deal with four coordinates x_μ and ten Goldstone fields, $h_{(\alpha)(\beta)}$, gravitons, multiplied by the operators of shift P and proper affine transformations R, respectively [1]. Further, the infinitesimal transformation for the affine group is

$$GdG^{-1} = i[P_{(\alpha)} \cdot \omega^P_{(\alpha)} + R_{(\alpha\beta)} \cdot \omega^R_{(\alpha\beta)} + L_{(\alpha\beta)} \cdot \omega^L_{(\alpha\beta)}],$$

which yields the Maurer–Cartan forms (introduced in the GR by Fock and Cartan [14, 15])

$$\omega^P_{(\alpha)}(d) = e_{(\alpha)\mu}dx^\mu, \tag{1.1}$$

$$\omega^R_{(\alpha\beta)}(d) = \frac{1}{2}\left(e^\mu_{(\alpha)}de_{(\beta)\mu} + e^\mu_{(\beta)}de_{(\alpha)\mu}\right), \tag{1.2}$$

$$\omega^L_{(\alpha\beta)}(d) = \frac{1}{2}\left(e^\mu_{(\alpha)}de_{(\beta)\mu} - e^\mu_{(\beta)}de_{(\alpha)\mu}\right). \tag{1.3}$$

Note that there are two types of indexes: one belongs to the subgroup L and the other (bracket-indexes) to the coset $A(4)/L$. In this approach the Maurer–Cartan forms with the coset indexes are main objects of the Poincaré and Lorentz transformations.

In general, to construct a gravitational theory, one needs to consider the covariant differentiation of a set of the fields Ψ transformed by the representations with the Lorentz group generators $L^\Psi_{\mu\nu}$

$$D_{(\lambda)}\Psi = D\Psi/\omega^P_{(\lambda)} = \left[\partial_{(\lambda)} + \frac{1}{2}iv_{(\mu)(\nu),(\lambda)}L^\Psi_{(\mu)(\nu)}\right]\Psi, \tag{1.4}$$

where $\partial_{(\lambda)} = (e^{-1})_{\sigma(\lambda)}\partial_\sigma$. The action of the Goldstone fields, $h_{(\alpha)(\beta)}$, can be obtained with the aid of the commutator of the covariant differentiations of a set of the fields Ψ [16]

$$\left[D_{(\lambda)}D_{(\gamma)} - D_{(\gamma)}D_{(\lambda)}\right]\Psi = iR_{\alpha\beta,\lambda\gamma}L^\Psi_{(\alpha)(\beta)}\Psi/2, \tag{1.5}$$

where the Riemannian tensor

$$R_{(\mu)(\nu),(\lambda)(\rho)} = \partial_{(\lambda)}v_{(\mu)(\nu),(\rho)} + v_{(\mu)(\nu),(\gamma)}v_{(\rho)(\gamma),(\lambda)} + v_{(\mu)(\gamma),(\rho)}v_{(\nu)(\gamma),(\lambda)} - ((\lambda) \leftrightarrow (\rho)) \tag{1.6}$$

is constructed with the help of Maurer–Cartan forms (1.2) and (1.3)

$$v_{(\mu)(\nu),(\gamma)} = [\omega^L_{(\mu\nu)}(\partial_{(\gamma)}) + \omega^R_{(\mu\gamma)}(\partial_{(\nu)}) - \omega^R_{(\nu\gamma)}(\partial_{(\mu)})]. \tag{1.7}$$

Thus, these elements enable us to formulate the Einstein–Hilbert action

$$W_{\text{Hilbert}} = -\int d^4x|-e|\frac{1}{6}R(e) \tag{1.8}$$

and the interval

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = \omega^P_{(\alpha)}(d) \otimes \omega^P_{(\beta)}(d)\eta_{(\alpha)(\beta)} \tag{1.9}$$

via the diffeo-invariant simplex components (1.1) and Maurer–Cartan forms (1.2) and (1.3) [14–17] in the local tangent Minkowskian space with the metric $\eta_{(\alpha)(\beta)} = \mathrm{Diag}\,(1,-1,-1,-1)$. We recall that, in contrast to the standard approaches (where the main relativistic covariant objects are the coordinate parameters x_μ), in our approach the objects of relativistic transformations are the Maurer–Cartan forms.

Following Dirac [8], in order to introduce the conformal symmetry of the GR as the nonlinear realization of a joint affine and conformal symmetries, we employ the dilaton scalar field D. This nonlinear realization contains the dilaton D as the Bogoliubov–Goldstone particle accompanying the spontaneous conformal symmetry breaking.

Exploiting the conformal symmetry, one can see that this nonlinear realization [1] coincides with the simplest scalar version of geometry of similarity [18] proposed by Dirac [8]. We recall that in the geometry of similarity one can measure only a ratio of two spatial intervals at the same instance. In other words, measured physical quantities are identified with the conformal ones

$$\widetilde{F}^{(n)} = \mathrm{e}^{nD} F^{(n)}, \quad \widetilde{ds}^2 = \widetilde{g}_{\mu\nu}\, dx^\mu\, dx^\nu = \mathrm{e}^{2D} ds^2 \tag{1.10}$$

obtained from the standard ones $F^{(n)}$ by the conformal transformation, where (n) is the conformal weight. Here, the dilaton D is fixed by the unit spatial metric determinant $|\widetilde{g}^{(3)}| = 1$ [19], so that

$$D = -\frac{1}{6}\log|g^{(3)}_{ij}|. \tag{1.11}$$

Thus, the Hilbert action (1.8) as the nonlinear realization of a joint affine and conformal symmetries takes the form

$$W_{\mathrm{Hilbert}} = -\int d^4x \left[\frac{|-\widetilde{e}|\mathrm{e}^{-2D}}{6} R(\widetilde{e}) - \mathrm{e}^{-D}\partial_\mu\left(|-\widetilde{e}|\widetilde{g}^{\mu\nu}\partial_\nu \mathrm{e}^{-D}\right)\right]. \tag{1.12}$$

Hereafter, one uses the units $M^2_{\mathrm{Pl}}3/(8\pi) = 1$. Below, with the aid of the action (1.12), we formulate our approach to the classical and quantum version of the GR.

2. THE DIRAC–ADM FOLIATION

We start our approach from the formulation of the Hamiltonain dynamics based on the Dirac–ADM $4 = 1 + 3$ foliation [12, 13]. In particular, in the interval

$$\widetilde{ds}^2 = \mathrm{e}^{2D} ds^2 = \widetilde{\omega}_{(0)} \otimes \widetilde{\omega}_{(0)} - \widetilde{\omega}_{(b)} \otimes \widetilde{\omega}_{(b)} \tag{2.1}$$

the Maurer–Cartan forms in view of the dilaton D may be written as

$$\widetilde{\omega}_{(0)} = \mathrm{e}^{-2D} N\, dx^0 = \mathrm{e}^{-2D}\mathcal{N} N_0\, dx^0 \equiv \mathrm{e}^{-2D}\mathcal{N}\, d\tau,$$

$$\widetilde{\omega}_{(b)} = \mathbf{e}_{(b)i}\, dx^i + \mathbf{e}_{(b)j} N^j\, dx^0 = \overline{\omega}_{(b)} + \mathcal{N}_{(b)}\, d\tau, \tag{2.2}$$

$$\overline{\omega}_{(b)} = \mathbf{e}_{(b)i}\, dx^i, \tag{2.3}$$

$$\mathbf{e}_{(b)j} N^j\, dx^0 = \mathcal{N}_{(b)}\, d\tau. \tag{2.4}$$

Here $d\tau = N_0\,dx^0$ is the dilaton proper time-interval defined by the constraint $\langle N^{-1}\rangle = N_0^{-1}$, the shift vector components $N^j\mathbf{e}_{j(b)} = N_{(b)} = N^{\|}_{(b)} + N^{\perp}_{(b)}$ $(\partial_{(b)}N^{\perp}_{(b)} = 0)$ and the Dirac lapse function $N(x^0, x^j) = N_0(x^0)\mathcal{N}(\tau, x)$ are the nondynamical potentials, $\widetilde{\omega}_{(b)}$ are the linear forms defined via the triads $\mathbf{e}_{(b)i}$ with unit spatial metric determinant $|\widetilde{g}^{(3)}_{ij}| \equiv |\mathbf{e}_{(b)j}\mathbf{e}_{(b)i}| = 1$. Triads $\mathbf{e}_{(a)i}$ form the spatial curvature [17]:

$$R^{(3)} = R^{(3)}(\mathbf{e}) - \frac{4}{3}\mathrm{e}^{7D/2}\triangle\mathrm{e}^{-D/2}, \tag{2.5}$$

$$R^{(3)}(\mathbf{e}) = -2\partial_i\,[\mathbf{e}^i_{(b)}\sigma_{(c)|(bc)}] - \sigma_{(c)|(bc)}\sigma_{(a)|(ba)} + \sigma_{(c)|(df)}\sigma_{(f)|(d)(c)}; \tag{2.6}$$

$$\sigma_{(c)|(ab)} = [\omega^L_{(ab)}(\partial_{(c)}) + \omega^R_{(ac)}(\partial_{(b)}) - \omega^R_{(bc)}(\partial_{(a)})],$$

$$\omega^R_{(ab)}(\partial_{(c)}) = \frac{1}{2}[\mathbf{e}^j_{(a)}\partial_{(c)}\mathbf{e}^j_{(b)} + \mathbf{e}^i_{(b)}\partial_{(c)}\mathbf{e}^i_{(a)}], \tag{2.7}$$

$$\omega^L_{(ab)}(\partial_{(c)}) = \frac{1}{2}[\mathbf{e}^j_{(a)}\partial_{(c)}\mathbf{e}^j_{(b)} - \mathbf{e}^i_{(b)}\partial_{(c)}\mathbf{e}^i_{(a)}]. \tag{2.8}$$

Here, $\triangle = \partial_i[\mathbf{e}^i_{(a)}\mathbf{e}^j_{(a)}\partial_j]$ is the Laplace operator.

According to a general wisdom, in the Hamiltonian approach, the general coordinate transformations are reduced into the kinemetric subgroup [20]

$$x^0 \to \widetilde{x}^0 = \widetilde{x}^0(x^0), \tag{2.9}$$

$$x^k \to \widetilde{x}^k = \widetilde{x}^k(x^0, x^1, x^2, x^3). \tag{2.10}$$

In the case of the reparametrization invariance (2.9), one of the variables (3.2) plays the role of the evolution parameter and its momentum — the measurable energy[1]. Following the DeWitt analogy with special relativity [22], we employ the zeroth dilaton harmonic $\langle D\rangle$ as the evolution parameter.

As a result, the Hilbert action (1.12) can be represented as a sum

$$W_{\text{Hilbert}} = W_z + \widetilde{W}_g \tag{2.11}$$

of the zeroth dilaton harmonic term

$$W_z = -\int d^3x \int dx^0\,\frac{\left(\partial_0\langle D\rangle\right)^2}{N_0} \equiv -\int d^3x \int\limits_{\tau_I}^{\tau_0} d\tau\left(\partial_\tau\langle D\rangle\right)^2 \tag{2.12}$$

and the graviton term $\widetilde{W}_g$

$$\widetilde{W}_g = \int dx^0\left[N_0\,\mathrm{e}^{-2\langle D\rangle}\mathsf{L}_g\right], \tag{2.13}$$

$$\mathsf{L}_g = \mathrm{e}^{2\langle D\rangle}\int d^3x\mathcal{N}\left[-(v_{\overline{D}})^2 + \frac{v^2_{(ab)}}{6} - \mathrm{e}^{-4D}\frac{R^{(3)}}{6}\right]. \tag{2.14}$$

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[1]Therefore, the coordinate evolution parameter x^0 as an object of reparametrizations cannot be considered as measurable quantities. And the corresponding Einstein equation (3.9) cannot be treated as a zero value of measurable energy. This treatment is used in the ADM approach [13] as an argument in favor of a nonlocal energy. Therefore, the acceptable introduction of a nonlocal energy proposed in [13,21] has no substantial foundation.

Here

$$v_{\overline{D}} = \frac{1}{N}\left[(\partial_0 - N^l\partial_l)\overline{D} + \partial_l N^l/3\right], \tag{2.15}$$

$$v_{(ab)} = \frac{1}{N}\left[\omega^R_{(ab)}(\partial_0 - N^l\partial_l) + \partial_{(a)}N^{\perp}_{(b)} + \partial_{(a)}N^{\perp}_{(b)}\right]$$

are velocities of the dilaton and of the triad components.

Following Dirac [12, 21], one can define such a coordinate system, where the covariant velocity of the local volume element $v_{\overline{D}}$ and its momentum become zero:

$$P_{\overline{D}} = 2v_{\overline{D}} = \frac{2}{N}\left[(\partial_0 - N^l\partial_l)\overline{D} + \partial_l N^l/3\right] = 0. \tag{2.16}$$

Thus, the dilaton deviation $\overline{D}$ can be treated as a static potential (see Appendix B).

3. RELATIVISTIC-INVARIANT EVOLUTION OF UNIVERSE

We recall that the accepted description of the cosmological evolution of the Universe is based on the hypothesis of homogeneity of Einstein equations [23], which considers the interval in the form

$$ds^2_{\rm hom} = (dt)^2 - a^2(t)(dx^j)^2 = a^2(t)[(d\eta)^2 - (dx^j)^2] = a^2(t)\widetilde{ds}^2_{\rm hom}. \tag{3.1}$$

Here, $a = (1+z)^{-1}$ is the homogeneous cosmological scale factor (or z-factor); dt is the world time interval; $d\eta = dt/a(t)$ is the conformal (proper) time interval of the photon, and spatial coordinates x^j are associated with the coordinate distance $r = \sqrt{(x^j)^2}$.

The Dirac dilaton variation principle [8] enables us to introduce the cosmological scale factor into the Hilbert action as the zeroth harmonic of the dilaton

$$-\log a = \log(1+z) = \langle D\rangle = V_0^{-1}\int\limits_{V_0} d^3x D(x). \tag{3.2}$$

Note that this definition is consistent with Einstein's cosmological principle of averaging of all scalar fields of the theory over the finite volume $V_0 = \int d^3x$ [24]. In this case, one obtains that $ds^2_{\rm hom} = \mathrm{e}^{-2\langle D\rangle}\widetilde{ds}^2_{\rm hom}$ (see Eq. (3.1)). The homogeneous interval (3.1) is expressed in our approach to the GR by means of the zeroth harmonic of the dilaton in the Hilbert action.

In the standard approach to the GR the interval (3.1) is treated as the frame-independent one. Using the transformation laws of simplex components (1.1) in the tangent Minkowskian space-time, we introduce the relativistic-invariant time interval derivative

$$\int \frac{\omega^P_{(1)}\wedge\omega^P_{(2)}\wedge\omega^P_{(3)}}{\omega^P_{(0)}}\langle D\rangle \equiv \int\limits_{V_0} d^3x\frac{d}{N(x^0,x^j)\,dx^0}\langle D\rangle = V_0\frac{d}{N_0\,dx^0}\langle D\rangle \equiv V_0\frac{d}{d\tau}\langle D\rangle. \tag{3.3}$$

The construction (2.2), (3.1), and (3.2), $dx^0N_0(x^0) = d\tau = a^{-2}d\eta$ gives the proper vacuum time interval and the perturbation series over deviations $\mathcal{N} = 1 + \overline{\delta}\ldots$ with the consistent constraint $\int d^3x\overline{\delta} = 0$. The derivative provides the frame-independent cosmic evolution in the tangent Minkowskian space-time with the metric $\eta_{(\alpha)(\beta)} = \mathrm{Diag}\,(1,-1,-1,-1)$ and interval (1.9), where objects of relativistic transformation are the simplex components $\omega^P_{(\alpha)}$.

In accordance with the principle of relativistic invariance of the cosmological motion, the time derivative in (3.3)

$$\frac{d}{d\tau}\langle D\rangle = H_0 = \mathrm{const} \tag{3.4}$$

is a constant of motion treated as the present-day value of the Hubble parameter. The definitions $d\tau = a^{-2}d\eta$, $a = (1+z)^{-1}$, and the condition $\partial_\tau\langle D\rangle = H_0$ determine the conformal Hubble parameter H_c

$$\partial_\eta\langle D\rangle = \frac{1}{a}\frac{da}{d\eta} = H_0 a^{-2} = H_0(1+z)^2 = H_c. \tag{3.5}$$

This condition fixes the single cosmological vacuum regime $a\dfrac{da}{d\eta} = H_0$ of the rigid state, where the pressure is equal to the energy density $p = \rho$ [23]. Indeed, the dominance of this regime, determined by the dilaton zeroth mode energy, describes the SNeIa data [25] in the framework of the Dirac geometry of similarity [9].

The vacuum approximation of the Hilbert action keeps only the zeroth dilaton harmonic term W_v

$$W_{\mathrm{Hilbert}} \simeq W_v = -V_0 \int dx^0 \left[\frac{(\partial_0\langle D\rangle)^2}{N_0} + N_0\rho_{\mathrm{cr}}\right], \tag{3.6}$$

which repeats the action of a relativistic particle at rest. Here $\rho_{\mathrm{cr}} = H_0^2 M_{\mathrm{Pl}}^2 3/(8\pi) = H_0^2$ is the critical density in the units (1.12) considered as a motion integral. It corresponds to the relativistic-invariant vacuum state (3.5) in the Empty Universe. Further we will show that this approximation of the Empty Universe in our picture dominates during all evolution epoch, while other contributions (matter, radiation, etc.) are small.

The canonical momentum $P_{\langle D\rangle}$

$$P_{\langle D\rangle} = \frac{\partial W_v}{\partial_0\langle D\rangle} = -2V_0\partial_\tau\langle D\rangle \tag{3.7}$$

gives us the well-defined Hamiltonian formulation

$$W_v = V_0 \int dx^0 \left[-P_{\langle D\rangle}\partial_0\langle D\rangle + N_0\frac{P^2_{\langle D\rangle} - \mathsf{E}_v^2}{4V_0}\right], \tag{3.8}$$

where $\mathsf{E}_v^2 = 4V_0^2\rho_{\mathrm{cr}}$.

The DeWitt analogy with the special relativity [22] determines the measurable energy of the Universe in the field space of events as the solution of the energy constraint

$$-N\frac{\delta W_v}{\delta N} = 0 \to P^2_{\langle D\rangle} - \mathsf{E}_v^2 = 0 \tag{3.9}$$

with respect to the zeroth dilaton momentum (3.7).

The elimination of the canonical momentum $P_{\langle D\rangle}$ from the system of Eqs. (3.7), (3.9) provides the solution

$$\tau_0 - \tau_I = \pm 2V_0 \int\limits_{\langle D\rangle_I}^{\langle D\rangle_0} \frac{d\langle D\rangle}{\mathsf{E}_v(\langle D\rangle)}, \tag{3.10}$$

where the cosmological energy $\mathsf{E}_v(\langle D\rangle)$ is determined as a positive constraint-shell value of the zeroth mode momentum. This equation can be rewritten as the redshift — coordinate distance relation, using Eqs. (3.2) and $d\tau = a^2 d\eta$:

$$r/c = \eta_0 - \eta = H_0^{-1} \int\limits_{1}^{a=(1+z)^{-1}} a\, da. \tag{3.11}$$

The Hilbert action in the vacuum approximation admits the ordinary quantization. It leads to the Universe wave function

$$\Psi_U = \frac{1}{\sqrt{2\mathsf{E}_v}} \left[A^+ \exp\{iW_+\}\,\theta[\langle D\rangle_0 - \langle D\rangle_I] + A^- \exp\{iW_-\}\,\theta[\langle D\rangle_I - \langle D\rangle_0]\right], \tag{3.12}$$

where $W_\pm = \pm \int\limits_{\langle D\rangle_I}^{\langle D\rangle_0} d\langle D\rangle P_{\langle D\rangle}$ is the constraint-shell action, $\theta[x>0]=1, \theta[x<0]=0$ are the theta functions. The coefficients $A^\pm$ are treated as the creation $(+)$ and annihilation $(-)$ operators. It solves the vacuum problem and gives the arrow of the time $\tau_0 > \tau_I$ in Eq. (3.10). One can see that in the wave function (3.12) there are two changes of sign of both energy and time intervals. Thus, the arrow of the time $\tau_0 > \tau_I$ and beginning of the time at $\tau \to \infty$ are the quantum anomalies as a consequence of the vacuum postulate.

In the rigid regime (3.11) defined by the horizon $r = [2H_0(1+z)^2]^{-1}$ using

$$M_{\rm Pl} = 1.2211 \cdot 10^{19}\ \text{GeV}, \tag{3.13}$$

$$H_0 = 100\ \text{km} \cdot \text{s}^{-1} \cdot h = 2.1332 \cdot 10^{-42}\ \text{GeV} \cdot h, \tag{3.14}$$

one can obtain the value of the z-factor at the Planck epoch $M_{\rm Pl} = [1+z_{\rm Pl}]^4 4H_0$, or

$$1 + z_{\rm Pl} = \left(\frac{M_{\rm Pl}}{4H_0}\right)^{1/4} \simeq 1.2 \cdot 10^{15}. \tag{3.15}$$

It is just the instance where a value of the action is equal to unit. Recall that a value of the gravitation action in the four-dimensional space-time is proportional to the product of the squired Planck mass and the four-dimensional space-time horizon volume $[2H_0(1+z)^2]^{-4}$.

The acceptable naive relation $M_{\rm Pl} \sim [1+z_{\rm Pl}]_{\rm inflat} H_0$ leads to the tremendous value $[1+z_{\rm Pl}]_{\rm inflat} \sim 10^{61}$.

4. DILATON VERSION OF GR

Let us consider the dilaton version of the GR supplemented by a scalar field Q and photon. The action is given by Eq. (A.1) in Appendix A, where the standard Dirac Hamiltonian approach is adapted to the affine gravitation theory.

The difference of our approach to the theory (A.1) from the standard Dirac one given in Appendix A is the operation of the separation of the zeroth and nonzero harmonics

$$D = \langle D \rangle + \overline{D}, \quad Q = \langle Q \rangle + \overline{Q} \tag{4.1}$$

with the strong constraint $\int\limits_{V_0} \overline{D} d^3x = 0$ in the action (A.1). This action takes the form of the sum of the zeroth modes and nonzero ones

$$W = W_z + \widetilde{W}, \tag{4.2}$$

where

$$W_z = \int dx^0 \left[(\partial_0 \langle Q \rangle)^2 - (\partial_0 \langle D \rangle)^2 \right] \int d^3x N^{-1}(x^0, \mathbf{x}) \tag{4.3}$$

is the zeroth mode action, where the integral $\int d^3x N^{-1}(x^0, \mathbf{x}) = V_0 N_0^{-1}$ $N^{-1}(x^0, \mathbf{x}) = V_0 N_0^{-1}$ rewritten as the average

$$V_0^{-1} \int d^3x N^{-1}(x^0, \mathbf{x}) \equiv \langle N^{-1} \rangle = N_0^{-1} \to \langle N_0 N^{-1} \rangle \equiv \langle \mathcal{N}^{-1} \rangle = 1 \tag{4.4}$$

determines the diffeo-invariant time interval $dx^0 N_0 = d\tau$ given by Eq. (3.3); while $\widetilde{W}$ repeats action (A.1) for nonzero harmonics associated with local excitations.

One can see that the separation (4.1) does not commute with the operation of the variation of the action.

The variation of action (4.2) with respect to the lapse function N leads to the energy constraint

$$N \frac{\delta W}{\delta N} = 0 \to \frac{[\partial_0 \langle D \rangle]^2 - [\partial_0 \langle Q \rangle]^2}{N} - N\mathcal{H} = 0, \tag{4.5}$$

where $\mathcal{H}$ is defined by Eqs. (A.13) and (A.14), while the Dirac energy constraint takes the form $\mathcal{H} = 0$. This fact reveals a crucial role of zeroth harmonics (3.2) in both the classical Hamiltonian evolution and the quantum one.

The energy constraint (4.5) is resolved by the averaging over the volume V_0. Using Eq. (4.4), we obtain the solutions as the global constraint

$$[\partial_\tau \langle D \rangle]^2 - [\partial_\tau \langle Q \rangle]^2 = \langle \mathcal{N}\mathcal{H} \rangle = \left\langle \sqrt{\mathcal{H}} \right\rangle^2 \tag{4.6}$$

and the local one

$$\mathcal{N} = \frac{N}{N_0} = \frac{\left\langle \sqrt{\mathcal{H}} \right\rangle}{\sqrt{\mathcal{H}}}. \tag{4.7}$$

These constraints determine the diffeo-invariant lapse function through the energy density $\mathcal{H}$ given by Eq. (A.13) and fix the diffeo-invariant interval (3.3) $d\tau = N_0\, dx^0$.

In terms of the zero mode momenta

$$P_{\langle D\rangle} = 2V_0\partial_\tau\langle D\rangle, \quad P_{\langle Q\rangle} = 2V_0\partial_\tau\langle Q\rangle \tag{4.8}$$

the energy constraint (4.6) takes the form (3.9)

$$P^2_{\langle D\rangle} - \mathsf{E}^2 = 0, \tag{4.9}$$

where

$$\mathsf{E}^2 = P^2_{\langle Q\rangle} + 4V_0\mathsf{H}_{\rm Dirac}, \tag{4.10}$$

$$\mathsf{H}_{\rm Dirac} = \int d^3x\, \mathcal{N}\mathcal{H}, \tag{4.11}$$

$\mathcal{H}$ is the conventional field Hamiltonian given by Eq. (A.13) and $P_{\langle Q\rangle} = \mathsf{E}_v = 2V_0H_0\sqrt{\Omega_v}$ is an integral of motion $\partial_\tau P_{\langle Q\rangle} = 0$ determined by fitting with observational data (3.4).

The global energy constraint (4.9) has two solutions: positive and negative

$$\mathsf{E}_\pm = \pm\sqrt{P^2_{\langle Q\rangle} + 4V_0\mathsf{H}_{\rm Dirac}} = \pm\left[P_{\langle Q\rangle} + \frac{2V_0}{P_{\langle Q\rangle}}\mathsf{H}_{\rm Dirac} + \ldots\right] \simeq \pm\left[\mathsf{E}_v + \frac{\mathsf{H}_{\rm Dirac}}{H_0}\right]. \tag{4.12}$$

In the context of the stability of the physical theory, one can recall that the vacuum is the state with the minimal energy defined as the positive solution of the energy constraint with respect to the canonical momentum of the zeroth dilaton harmonic. The negative energy is removed by the quantization procedure, where the creation of particles with negative energy is replaced by the annihilation of particles with positive energy. This change leads to the classical symmetry breaking as quantum anomalies [26, 27]. One of these anomalies is the vacuum creation of the Universe. What does Quantum Gravity mean? There are a lot of problems on the way of consistent quantization of the metric components and field variables in GR. However, in the relativistic physics one can quantize the phase space of initial data as the motion integrals obtained by the Bogoliubov transformations [28], in the spirit of the Blokhintsev statistical ensembles of states in Quantum Mechanics [29]. Therefore, on the stage of construction of irreducible unitary representations, we propose the priority of quantum principles of the type of the vacuum postulate, spectrality, and uncertainty. The priority of quantum principle leads to new physical consequences, because the separation of quantum zeroth modes is fulfilled before the variation of the action.

Here is the point, where one can introduce the concept of the evolution operator

$$\mathsf{U}_+(I,0) = T_D \exp\left\{-i\int\limits_{D_I}^{D_0} d\langle D\rangle \mathsf{E}\right\}. \tag{4.13}$$

If homogeneous vacuum energy $P^2_{\langle Q\rangle} = \mathsf{E}^2_v$ dominates in (4.12), so that $\mathsf{E} \gg \mathsf{E} - \mathsf{E}_v$, the evolution operator is factorized

$$\mathsf{U}_+(I,0) \simeq \mathrm{e}^{-i\mathsf{E}_v(D_0-D_I)}T_\tau \exp\left\{-i\int\limits_{\tau_I}^{\tau_0} d\tau\, \mathsf{H}_{\rm Dirac}\right\}, \tag{4.14}$$

where $\mathsf{H}_{\rm Dirac} = \mathsf{H}_0 + \mathsf{H}_{\rm int}$ is given by Eq. (4.11). In the light of the Dirac Hamiltonian approach this evolution operator is the cosmological generalization of the Faddeev–Popov expression for the evolution operator in GR of the local graviton excitations considered in [21]. The evolution operator (4.14) includes both their Newton-like interactions and cosmological creation of gravitons. Finally, we got the method of calculation of the energy budget of the Universe using the definition of the present-day state of the Universe as the evolution of the Bogoliubov vacuum state and the standard interaction representation of QFT [3] in terms of the Bogoliubov field operators $U_+(I,0)|I\rangle = \mathsf{S}_b(\tau_I,\tau_0)|\mathrm{in}\rangle$, where

$$\mathsf{S}_b(\tau_I,\tau_0) = T_\tau \exp\left\{-i\int\limits_{\tau_I}^{\tau_0} d\tau\, \mathsf{H}_{\rm int}(b^+,b^-)\right\}. \tag{4.15}$$

In this case, the energy budget of the Universe is described as the Bogoliubov vacuum expectation value. It is given by the formula

$$(\partial_\tau\langle D\rangle)^2 = \rho_v + \langle\mathrm{out}|\mathsf{S}_b^{-1}(\tau_I,\tau_0){V_0}^{-1}\mathsf{H}_{\rm Dirac}\mathsf{S}_b(\tau_I,\tau_0)|\mathrm{in}\rangle =$$
$$= \rho_v + \langle\mathrm{out}|V_0^{-1}\left\{\mathsf{H}_0 + i\int\limits_{\tau_I}^{\tau_0} d\tau\,[\mathsf{H}_0(\tau_0),\mathsf{H}_{\rm int}(\tau)] + \ldots\right\}|\mathrm{in}\rangle. \tag{4.16}$$

5. STRONG GRAVITON

The key point of our approach is to express the Hamiltonian approach to the GR action directly in terms of the Maurer–Cartan forms. Note, that in the spin formulation of the GR considered by Schwinger [17], the equations of motion were derived in terms of the simplex components of an unconstrained system. We express the simplex components by means of the spin connection coefficients constrained by the condition of diffeo-invariance. The dependence of the linear forms

$$\overline{\omega}_{(b)}(d) = \mathbf{e}_{(b)i}\,dx^i = dX_{(b)} - X_{(c)}\mathbf{e}^i_{(c)}\,d\mathbf{e}_{(b)i} = dX_{(b)} - X_{(c)}[\omega^R_{(cb)} + \omega^L_{(cb)}] \tag{5.1}$$

on the tangent space coordinates $X_{(b)} \equiv \int dx^i\mathbf{e}_{(b)i} = x^i\mathbf{e}_{(b)i}$ via the spin connection coefficients can be obtained by means of the Leibniz rule $AdB = d(AB) - (AB)d\log(A)$ (in particular, $d[x^i]\mathbf{e}^T_{\underline{b}i} = d[x^i\mathbf{e}^T_{\underline{b}i}] - x^i d[\mathbf{e}^T_{\underline{b}i}]$).

The difference between this approach to gravitational waves and the accepted one is that the symmetry with respect to diffeomorphisms is imposed on spin connection coefficients. This difference leads to the novel result that follows from the theorem [30]: *any arbitrary two-dimensional space metric $dl^2 = h_{AB}\,dx^A\,dx^B$; $A,B = 1,2$ can be presented by diffeomorphisms $x^A \to \widetilde{x}^A = \widetilde{x}^A(x^1,x^2)$ in a diagonal form.* The result consists in the fact that a kinemetric-invariant nonlinear plane wave moving along a direction $\mathbf{k}$ with the unit determinant $det\ h\ = 1$ contains only a single metric component. Using two photon-like polarization vectors $\varepsilon^{(\alpha)}_{(a)}(\mathbf{k})$ and the condition $\sum\limits_{\alpha=1,2}\varepsilon^{(\alpha)}_{(a)}(\mathbf{k})\varepsilon^{(\alpha)}_{(b)}(\mathbf{k}) = \delta_{(a)(b)} - \dfrac{\mathbf{k}_{(a)}\mathbf{k}_{(b)}}{\mathbf{k}^{(2)}}$, one can express the linear graviton form with the aid of these vectors:

$$\omega^R_{(ab)}(\partial_{(c)}) = \sum_{\mathbf{k}^2\neq 0}\frac{\mathrm{e}^{i\mathbf{kX}}}{\sqrt{2\omega_\mathbf{k}}}\mathbf{k}_{(c)}[\varepsilon^R_{(ab)}(\mathbf{k})g^+_\mathbf{k}(\eta) + \varepsilon^R_{(ab)}(-\mathbf{k})g^-_\mathbf{k}(\eta)], \tag{5.2}$$

where $\varepsilon^{L}_{(ab)} = 0$ and $\varepsilon^{R}_{(ab)}(\mathbf{k}) = \mathrm{diag}\,[1,-1,0]$ in the orthogonal basis of spatial vectors $[\boldsymbol{\varepsilon}^{\,(1)}(\mathbf{k}), \boldsymbol{\varepsilon}^{\,(2)}(\mathbf{k}), \mathbf{k}]$. Here, $g^{\pm}$ are the holomorphic variables of the single degree of freedom, $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2}$ is the graviton energy normalized (like a photon in QED) on the units of volume and time

$$\overline{g}^{\pm}_{\mathbf{k}} = \frac{\sqrt{8\pi}}{M_{\mathrm{Pl}} V_0^{1/2}} g^{\pm}_{\mathbf{k}}. \tag{5.3}$$

Thus, in the class of plane wave functions, the kinemetric-invariant section of the nonlinear Hamiltonian of the GR coincides with the bilinear theory of a scalar-like field, if one neglects Newtonian interactions (2.5).

Let us compare the commonly accepted (see, e.g., [31, 32]) interval

$$ds_h^2 = a^2(\eta)\left[\left(dx^0\right)^2 - dx^i dx^j \left(\delta_{ij} + 2h^{TT}_{ij} + 2h^{TT}_{il} h^{TT}_{lj} + \ldots\right)\right] \tag{5.4}$$

with the diffeo-invariant one in the mean field approximation $N(x^0, x^j) = N_0(x^0)$, $N^j = 0$, $\overline{D} = 0$

$$ds_g^2 = a^2(\eta)[(d\eta)^2 - (dX_{(b)} - X_{(c)}\omega^R_{(cb)})^2], \tag{5.5}$$

where $a = \mathrm{e}^{-\langle D\rangle}$ is the cosmological scale factor. Here only the independent dynamical part $\omega^R_{(cb)}$ is taken into account in the interval (5.1).

In the standard case, graviton moves in the direction of vector $\mathbf{k}$, its wave amplitude $\cos\{\omega_{\mathbf{k}} x_{(k)}\}$ depends on the scalar product $x_{(k)} = (\mathbf{k}\cdot\mathbf{x})/\omega_{\mathbf{k}}$. One can see that the graviton changes the squared test particle velocity $\left(\frac{ds}{d\eta}\right)^2 \sim \frac{dx^i\, dx^j}{d\eta\, d\eta}\varepsilon^{\alpha}_{ij}$ in the plane orthogonal to the motion direction. All these effects are produced by the first order of the series (5.4)

$$dl_h^2 = 2dx^i\, dx^j h^{TT}_{ij}(t, \mathbf{x}) = dx^i\, dx^j \varepsilon^{\alpha}_{ij}\sqrt{6}\cos\{\omega_{\mathbf{k}} x_{(k)}\}\left(\frac{H_0}{\omega_{\mathbf{k}}}\right)\Omega^{1/2}_{\mathbf{k}h}, \tag{5.6}$$

where $\Omega_{\mathbf{k}h} = \omega_{\mathbf{k}} N_{\mathbf{k}h}/[V_0 \rho_{\mathrm{cr}}]$ is the energy density of the gravitons in units of the cosmological critical energy density $\rho_{\mathrm{cr}} = H_0^2 M_{\mathrm{Pl}}^2 3/(8\pi)$. One observes that in the accepted perturbation theory the contribution of a single gravitational wave into the geometrical intervals in units of the critical density is suppressed by the factor $H_0/\omega_{\mathbf{k}}$.

In the diffeo-invariant version (5.5)

$$dl_g^2 = 2dX_{(b)} X_{(c)}\omega^R_{(cb)} = dX_{(b)} X_{(c)}\varepsilon^{\alpha}_{(cb)}\sqrt{6}\cos\{\omega_{\mathbf{k}} X_{(k)}\} H_0 \Omega^{1/2}_{\mathbf{k}h} \tag{5.7}$$

graviton contains the additional factor. To find this factor, we take the ratio of the above intervals (5.6) and (5.7):

$$\left|\frac{dl_h^2}{dl_g^2}\right| = \left|\frac{dx^i\, dx^j\, h^{TT}_{ij}}{dX_{(b)} X_{(c)}\omega^R_{(cb)}}\right| \simeq \frac{1}{r_\perp\, \omega_{\mathbf{k}}}. \tag{5.8}$$

Here, $r_\perp = \sqrt{|\mathbf{X}_\perp|^2}$ is the coordinate distance between two test particles in the plane perpendicular to the wave motion direction. This factor yields a spatial amplification of the velocities of the test particles in the plane orthogonal to the graviton motion direction, i.e., rotation in this plane. If so, we can suggest a *hypothesis* claiming that rotation of galaxies could have been initiated by primordial strong gravitational waves passed through clouds of gas and dust.

Moreover, one can try to estimate the corresponding effect, if the center of a rotated spiral galaxy can be considered as a source of the graviton emission along the rotation axis. This effect leads to the additional velocity of a test particle with coordinate $X_{(b)} = (r_\perp, 0, 0)$ ($r_\perp$ now is counted from the galaxy center). As a result, the total velocity of a particle in the plane perpendicular to the wave propagation includes the «Newtonian» (N), the «graviton» (g), and the «Hubble» (H) terms [26]:

$$|\mathbf{v}|^2 = \left[\mathbf{n}_N \sqrt{\frac{r_g}{2R_\perp}} + \mathbf{n}_g \sqrt{R_\perp H_0}\sqrt{\Omega_g} + \mathbf{n}_H H_0 R_\perp \gamma\right]^2, \tag{5.9}$$

where the unit velocities vectors read

$$\begin{cases} \mathbf{n}_N = (0, 1, 0), \\ \mathbf{n}_g = (+1/\sqrt{2}, -1/\sqrt{2}, 0), \\ \mathbf{n}_H = (1, 0, 0). \end{cases} \tag{5.10}$$

The scalar products of vectors of velocities are $\mathbf{n}_N \cdot \mathbf{n}_g \neq 0$, $\mathbf{n}_N \cdot \mathbf{n}_H = 0$, $H_0 = a'/a$ is the Hubble parameter, $R_\perp = r_\perp a(\eta)$, factor γ is defined by the cosmological density [26]: $\gamma_{\mathrm{CC:p}=+\rho} = \sqrt{2}$, or $\gamma_{\Lambda\mathrm{CDM}} = \sqrt{2 - (3/2)\Omega_{M:p=0} - 3\Omega_{\Omega:p=-\rho}}$, and Ω_g is the energy density of the graviton in units of the cosmological critical energy density ρ_{cr}[1]. One observes that the interference of the Newtonian and the graviton-induced velocities in (5.9) $v_{N-g\,\mathrm{interf}} \simeq \sqrt[4]{\Omega_g r_g H_0}$ does not depend on the radius $R_\perp$. Thus, the strong graviton enforces particles to rotate around a certain center in the plane orthogonal to the gravitational wave vector with equal velocities.

One can see that in the mean field approximation, $\mathcal{N} = 1$, $\overline{D} = 0$, $\mathcal{N}_{(b)} = 0$, the diffeo-invariant sector of the strong gravitation plane waves coincides with a free field theory:

$$\mathsf{H}_g = \sum_{\mathbf{k},\mathbf{k}^2 \neq 0} \omega_{\mathbf{k}} [g^+_{\mathbf{k}}(0) g^-_{-\mathbf{k}}(0)]. \tag{5.11}$$

This fact is well known as the BRT solutions [16]. The equivalence of nonlinear theory of gravitons in the diffeo-invariant measurable space-time with the free theory recalls us the equivalence of the nonlinear unnormalizable theory $(\partial_\mu \sinh f)^2$ with a free scalar field theory $(\partial_\mu \Phi)^2$, that is proved by the change of variables $\Phi = \sinh f$. The latter can be treated as a choice of the Cartesian field coordinates along geodesic in the superspace of field variables [33].

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[1] There is one more argument in favor of the geometry of similarity and relative units, namely, the large deficit of the mass of luminous matter M_L: $M/M_L > 10$, in all superclusters with a mass of $M \geqslant 10^{15} M_\odot$ and a size of $R \geqslant 5$ Mpc [26]. In this case, the Newtonian velocity is less than the cosmic one by an order of magnitude. In terms of relative units of the Conformal Cosmology, this deficit can be caused by galaxy deceleration in the course of the cosmological evolution of galaxy masses.

6. CREATION OF GRAVITONS IN THE DILATON VACUUM BACKGROUND

Straightforward calculations define a set of evolution equations for the Lagrangian L_g (2.14) and the Hamiltonian H_g (4.5)

$$\partial_{\langle D\rangle}\mathsf{H}_g = 2\mathsf{L}_g, \tag{6.1}$$

$$\partial_{\langle D\rangle}\mathsf{T}_g = 2\,\mathrm{e}^{-2\langle D\rangle}\mathsf{L}_g, \tag{6.2}$$

$$\partial_{\langle D\rangle}\mathsf{L}_g = 2\mathsf{H}_g - 2\,\mathrm{e}^{-2\langle D\rangle}\mathsf{T}_g, \tag{6.3}$$

where $\mathsf{T}_g = \sqrt{\mathsf{H}_g^2 - \mathsf{L}_g^2}$.

Note, the GR equations in terms of the spin-connection coefficients (6.1)–(6.3) coincide with the evolution equations for the parameters of squeezing r_b and rotation θ_b [34–36]

$$\partial_{\langle D\rangle} r_b = \cos 2\theta_b, \tag{6.4}$$

$$\omega_{\rm so} - \partial_{\langle D\rangle}\theta_b = \coth 2r_b \sin 2\theta_b \tag{6.5}$$

of the Bogoliubov transformations $A^+ = B^+\cosh r\,\mathrm{e}^{i\theta} + B^-\sinh r\,\mathrm{e}^{i\theta}$ for a squeezed oscillator (SO) $\partial_{\langle D\rangle}A^\pm = \pm i\omega_{\rm so}A^\pm + A^\mp$. Indeed, Eqs. (6.4), (6.5) establish similar relations for the expectation values of various combinations of the operators $A^\pm$ with respect to the Bogoliubov vacuum $B^-|\rangle = 0$ (see details in [26])

$$N_b \equiv \langle|A^+A^-|\rangle = \frac{\cosh 2r_b - 1}{2} \equiv \omega_{\rm so}^{-1} :\mathsf{H}_b:, \tag{6.6}$$

$$\frac{i}{4}\langle A^-A^- - A^+A^+\rangle = \frac{\sinh 2r_b \sin 2\theta_b}{2} \equiv \omega_{\rm so}^{-1}\mathsf{T}_b, \tag{6.7}$$

$$\frac{1}{4}\langle A^+A^+ + A^-A^-\rangle = \frac{\sinh 2r_b \cos 2\theta_b}{2} \equiv \omega_{\rm so}^{-1}\mathsf{L}_b. \tag{6.8}$$

On the other hand, Eqs. (2.14), (4.9), (4.12), and (4.11), show up that the graviton action (2.11) has a bilinear oscillator-like form

$$\begin{aligned}
\mathsf{H}_g &= \sum_{\mathbf{k}} \underline{\mathcal{H}}_{\mathbf{k}}, & \underline{\mathcal{H}}_{\mathbf{k}} &= \frac{\omega_{\mathbf{k}}}{2}[g^+_{\mathbf{k}} g^-_{-\mathbf{k}} + g^-_{\mathbf{k}} g^+_{-\mathbf{k}}],\\
\mathsf{L}_g &= \sum_{\mathbf{k}} \underline{\mathcal{L}}_{\mathbf{k}}, & \underline{\mathcal{L}}_{\mathbf{k}} &= \frac{\omega_{\mathbf{k}}}{2}[g^+_{\mathbf{k}} g^+_{-\mathbf{k}} + g^-_{\mathbf{k}} g^-_{-\mathbf{k}}],\\
\mathsf{T}_g &= \sum_{\mathbf{k}} \underline{\mathcal{T}}_{\mathbf{k}}, & \underline{\mathcal{T}}_{\mathbf{k}} &= \frac{i\omega_{\mathbf{k}}}{2}[g^+_{\mathbf{k}} g^+_{-\mathbf{k}} - g^-_{\mathbf{k}} g^-_{-\mathbf{k}}],
\end{aligned} \tag{6.9}$$

where

$$g^\pm_{\mathbf{k}} = \frac{\left[\overline{g}_{\mathbf{k}}\sqrt{\omega_{\mathbf{k}}} \mp i p_{\mathbf{k}}/\sqrt{\omega_{\mathbf{k}}}\right]}{\sqrt{2}} \tag{6.10}$$

are the classical variables in the holomorphic representation [35]. The form (6.10) suggests itself to replace the variables $g^\pm_{\mathbf{k}}$ by creation and annihilation graviton operators. Evidently, in this case we have to postulate the existence of a stable vacuum $|0\rangle$. As a consequence, it is reasonable to suppose that the classical graviton Hamiltonian (see Eqs. (6.9)) is the quantum

Hamiltonian averaged over coherent states [37]. One may speculate that such a procedure reflects a transformation of a genuine quantum Hamiltonian (describing the initial dynamics of the Universe) to the classical Hamiltonian, associated with present-day dynamics. Having the correspondence between two sets of equations (6.1)–(6.3) for the GR and (6.6)–(6.8) for the SO, we are led to the ansatz that the SO is the quantum version of our graviton Hamiltonian (see also [31]).

In the limit $a = \mathrm{e}^{\langle D\rangle} \to 0$ the classical graviton equations (6.1)–(6.3) take very simple form

$$\partial^2_{\langle D\rangle}\mathsf{H}_g = 4\mathsf{H}_g. \tag{6.11}$$

Its quantum version, in the same limit, differs by the vacuum energy

$$\partial^2_{\langle D\rangle}\langle|:\mathsf{H}_b:|\rangle = 4\langle|\left[:\mathsf{H}_b: + \frac{\omega_{\rm so}}{2}\right]|\rangle. \tag{6.12}$$

Both equations are supplemented by the zero initial data $\mathsf{H}_g = 0$, $\partial_{\langle D\rangle}\mathsf{H}_g = 0$. One can see that classical equation has zero vacuum solution $\mathsf{H}_g \equiv 0$, whereas the quantum one takes the nonzero solution $\langle|:\mathsf{H}_b:|\rangle = \omega_{\rm so}\dfrac{\cosh 2\langle D\rangle - 1}{2}$. This solution means that dilaton plays the role of the squeezing parameter. The cause of the vacuum creation is the vacuum Casimir energy. This energy is a result of the normal ordering of the graviton Hamiltonian [26]

$$\mathsf{H}_g = \mathsf{H}_b =: \mathsf{H}_b : + \frac{\omega_{\rm so}}{2}, \quad \mathsf{L}_g = \mathsf{L}_b, \quad \mathsf{T}_g = \mathsf{T}_b, \tag{6.13}$$

where

$$\omega_{\rm so} = \mathrm{e}^{-2\langle D\rangle}\omega_c. \tag{6.14}$$

This is a central point of our construction.

The normal ordering of classical Hamiltonian after its quantization gives the Casimir-type vacuum energy $\omega_c = 0.09235/(2r_g)$ [38], where r_g is the radius of the sphere. One can obtain the primordial Casimir energy density of gravitons Ω_g normalized on the critical energy $\rho_{\rm cr} = M^2_{\rm Pl}3/(8\pi)$

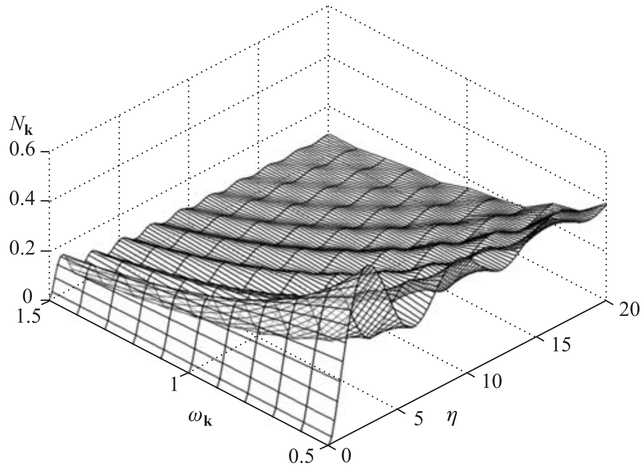
$$\Omega_g = 0.09235 \cdot \frac{1}{M^2_{\rm Pl}H_0^2 r_g^4}. \tag{6.15}$$

The exact solution of Eqs. (6.1)–(6.3) is shown in the Figure for $\omega_{\rm so} = 1/2$ and $H_g = 1$. This solution shows us that during some time of relaxation $\eta_g \sim 2{H_g}^{-1}$ we obtain almost the constant density, the value of which depends on the instance of the creation $1 + z_g$.

In accordance with this solution, at the tremendous redshift $(1 + z_g) = \mathrm{e}^{\langle D_g\rangle} = a_g^{-1}$, $z_g \to \infty$ is reduced to the zeroth mode dilaton integral of motion $\Omega_{\langle D\rangle}$, which corresponds to the z-dependence of the Hubble parameter[1] $H(z) = H_0(1 + z)^2$.

At this moment, the Universe was empty, and all particle densities had the zero initial data. The next step is the creation of gravitons induced by the direct dilaton interaction. A hypothetic observer being at the first instance of the Universe at the horizon value $r_g =$

[1]Note that the same dilaton vacuum regime $H(z) = H_0(1 + z)^2$ is compatible with the SNeIa data [25] in the geometry of similarity (1.10) [9].



The creation of the Universe distribution $[N_{\mathbf{k}} = N_b]$ (6.6) versus dimensionless time η and energies $0.5 \leqslant \omega_{\mathbf{k}} = \omega_{\mathrm{so}}$ at the initial data $N_{\mathbf{k}}(\eta = 0) = 0$ and the Hubble parameter $H(\eta) = 1/(1 + 2\eta)$

$1/(2H_g)$ in the primordial volume $V_g = 4\pi r_g^3/(3 \cdot 8) = H_0^3(1 + z_g)^6/(3 \cdot 8)$ observes the vacuum creation of gravitons (and any other massless particles) with the primordial density

$$\Omega_g = 0.09235 \cdot \frac{16H_0^2}{M_{\mathrm{Pl}}^2}(1 + z_g)^8 \tag{6.16}$$

defined by the Casimir energy. The question which remains to answer is how to define the instance of the creation z_g?

7. STANDARD MODEL OF ELEMENTARY PARTICLES

In order to estimate the instance of creation of gravitons $(1 + z_g)$, one can add the Hamiltonian of the Standard Model (SM): $\mathsf{H}_g \to \mathsf{H} = \mathsf{H}_g + \mathsf{H}_{\mathrm{SM}}$, when in the limit $(1 + z_h) \to \infty$ and $a_h \to 0$ all particles become nearly massless $\sqrt{\mathbf{k}^2 + a_h^2 M_0^2} \to \omega_{\mathbf{k}}$. In this case, the same mechanism of intensive particle creation works also for any scalar fields including four Higgs bosons [36]

$$\Omega_h(z_h) = 4\Omega_g(z_h). \tag{7.1}$$

The decays of the Higgs sector including longitudinal vector W and Z bosons approximately preserve this partial energy density for the decay products. These products are Cosmic Microwave Background (CMB) photons and n_ν neutrino. Therefore, one obtains

$$(1 + n_\nu)\Omega_{\mathrm{CMB}}(z_h) \approx 4\Omega_g(z_h). \tag{7.2}$$

In the conformal cosmological model [9], in the dilaton regime $H(z) = a'/a = (1+z)^2 H_0$, there is the coincidence of two epochs:

• the creation of SM bosons in the Universe in electroweak epoch

$$1 + z_W = \left[\frac{M_W}{H_0}\right]^{1/3} = 0.37 \cdot 10^{15}, \tag{7.3}$$

when the horizon $H(z_W) = (1 + z_W)^2 H_0 = (1 + z_W)^2 = 1.5 \cdot 10^{-42}$ GeV contains only a single W boson;

• and the CMB origin time

$$1 + z_{\mathrm{CMB}} = [\lambda_{\mathrm{CMB}} H_0]^{-1/2} = [10^{-29} \cdot 2.35/1.5]^{1/2} = 0.39 \cdot 10^{15}, \tag{7.4}$$

when the horizon contains only a single CMB photon with mean wave length λ_{CMB} that is approximately equal to the inverse temperature $\lambda_{\mathrm{CMB}}^{-1} = T_{\mathrm{CMB}} = 2.35 \cdot 10^{-13}$ GeV.

One can see that these values of the z-factor $(1+z_h) \simeq (1+z_W) = (1+z_{\mathrm{CMB}})$ are less in three times than the Planck value (3.15). This difference can be explained by the retardation factor defined by the cubic root of the Weinberg coupling constant in SM $\alpha_W^{1/3} \simeq 1/3$. It arises due to the lifetime of the SM bosons. In particular, remember that the W-boson lifetime is proportional to α_W^{-1}, and the proper time of massive particles is proportional to the cube of z-factor.

In the same epoch $z_h \approx z_W \approx z_{\mathrm{CMB}}$, the primordial graviton density (6.16) coincides with the CMB density normalized to a single degree of freedom. Therefore, there are observational evidences that the instance of the creation of graviton $z_g \simeq 0.37 \cdot 10^{15}$ is very close to the instance of the creation of Higgs particles $z_h \simeq z_W = 0.37 \cdot 10^{15}$. In particular, for $n_\nu = 3$ we got $z_g = 0.37 \cdot 10^{15} = z_W = z_h$ (as it was supposed in [31]).

The coincidence of the epoch z_g with the first two ones $z_h \simeq z_W = z_{\mathrm{CMB}}$ gives us a hope to solve cosmological problems with the aid of the quantum dilaton squeezing of the Higgs particles (1.10) in the Dirac geometry of similarity, without the classical inflation (see also [9]).

While adding the SM sector to the theory in order to preserve the conformal symmetry, we should exclude the unique dimensional parameter from the SM Lagrangian, i.e., the Higgs term with a negative squared mass. However, following Kirzhnits [39], we can include the vacuum expectation of the Higgs field (its zeroth harmonic) $\langle\phi\rangle$. The latter appears as certain external initial data or a condensate. In our construction we can choose it in the most simple form: $\langle\phi\rangle = \mathrm{const} = \langle\phi\rangle_I = 246$ GeV, which could be considered as the initial condition at the beginning of the Universe. The fact that the Higgs vacuum expectation is equal to its present-day value allows us to preserve the status of the SM as the proper quantum field theory during the whole Universe evolution. The standard vacuum stability conditions

$$\langle 0|0\rangle|_{\phi=\langle\phi\rangle} = 1, \quad \langle 0|0\rangle'|_{\phi=\langle\phi\rangle} = 0 \tag{7.5}$$

yield the following constraints on the Coleman–Weinberg effective potential of the Higgs field:

$$V_{\mathrm{eff}}(\langle\phi\rangle) = 0, \quad V'_{\mathrm{eff}}(\langle\phi\rangle) = 0. \tag{7.6}$$

It results in a zero contribution of the Higgs field vacuum expectation into the Universe energy density. In other words, the SM mechanism of a mass generation can be completely repeated.

However, the origin of the observed conformal symmetry breaking is not a dimensional parameter of the theory but a certain nontrivial (and very simple at the same moment) set of the initial data. In particular, one obtains that the Higgs boson mass is determined from the equation $V''_{\rm eff}(\langle\phi\rangle) = M_H^2$. Note that in our construction the Universe evolution is provided by the dilaton, without making use of any special potential and/or any inflation field. In this case, we have no reason to spoil the renormalizability of the SM by introducing the nonminimal interaction between the Higgs boson and the gravity [40].

The SM interactions include the amplitudes $M_{B\gamma\gamma}$ of the gamma processes $h \to \gamma\gamma$, $W^+W^- \to \gamma\gamma$, $ZZ \to \gamma\gamma$ and the process of the transition of double W to neutral kaon $M_{WW\to K_0}$ (described by the triangle anomaly [11, 36]) with creation from the Dirac sea of quarks and electron that form the baryon asymmetry of the Universe. The interaction Hamiltonian in Eq. (4.16) can be written in the form

$$H_{\rm int} = \sum_{f_j=W,Z,h} \frac{1}{2}(b_j^+ b_j^+ + b_j^- b_j^-) M_{B_j\gamma\gamma} + \\ + \frac{i}{2}\left(b^+_{W^+} b^+_{W^-} M_{WW\to K_0} - b^-_{W^+} b^-_{W^-} M_{WW\to \overline{K}_0}\right). \quad (7.7)$$

The Bogoliubov transformation

$$a^+a^- + \frac{1}{2} = \quad (7.8)$$

$$= \frac{1}{2}\left[\left(b^+b^- + \frac{1}{2}\right)\cosh 2r + \left(b^+b^+ + b^-b^-\right)\sinh 2r\cos 2\theta + i\left(b^+b^+ - b^-b^-\right)\sinh 2r \sin 2\theta\right] \quad (7.9)$$

shows us that the intensive creation of particle goes from the vacuum $1/2$ in the first term, while the next terms give additional contributions due to interactions (7.7). Here one can mark that the evolution equation (4.16) gives the double contribution of the Higgs particle decays in comparison with the vector boson ones.

The additional photons arising due to interactions in (7.7) in the quantum evolution equation (4.16) describe the Cosmic Microwave Background power spectrum anisotropy [41]. The numbers of these additional processes (i.e., emitters) determine the multipole momenta ℓ_B at the instance of the processes marked by z-factor. Therefore, the z-dependence of the multipole momenta ℓ_B can be obtained as the ratio of the length of the horizon and the size of emitters given by their Compton lengths, so that multipole momenta proportional to the numbers of emitters ℓ_B have the z-factor like the cubed conformal mass $\ell_B \sim M_B^3$.

The observational fact is that the cubic root of the ratio of the third and the second peak momenta $(800/546)^{1/3} = 1.136 \simeq M_Z/M_W$ in the Cosmic Microwave Background power spectrum coincides with the ratio of the Z and W masses in good agreement with the experimentally defined value $M_Z/M_W = 1.134$. This agreement allows us to interpret the first peak (that is two times greater than the last two in accordance with the evolution equation (4.16) and the Bogoliubov transformation (7.8)) as contribution of the two-photon decay of the Higgs particle with the mass $m_h = 2(220/546)^{1/3} M_W = 118$ GeV lying just in the region preferred by the results of the Standard Model parameters fitted from the experimental data of LEP [11]. Note that the vacuum postulate in the form of the Dirac constraint of zero momentum of the local volume element (A.20) (with negative contribution into the energy constraint) forbids the Sakharov-type oscillations associated with the CMB power spectrum in the ΛCDM model [5].

CONCLUSION

In the present paper, we considered the Hilbert action expressed in terms of the Maurer–Cartan forms of the joint nonlinear realization of the affine and conformal symmetries following the ideas of the affine $A(4)$ and conformal symmetry C $[1, 2, 8, 18]$. The dilaton D was introduced as a representation of the Poincaré group in the tangent Minkowskian space-time. The dilaton zeroth mode $\langle D \rangle$ coincides by definition with the redshift-factor logarithm of the Hubble evolution. Thus, we studied the redshift-factor evolution of the GR and SM in terms of the Maurer–Cartan forms. Diffeo-invariance of the Maurer–Cartan forms leaves a single graviton degree of freedom instead of two.

The Maurer–Cartan forms are objects of the Lorentz (relativistic) transformations in the tangent Minkowskian space-time. It was shown that relativistic properties of the Maurer–Cartan forms and homogeneity hypothesis fix the vacuum state that corresponds to the dominance of the scalar field zeroth mode energy density. This density is in agreement with SNeIa data in the cosmological model based on the Dirac geometry of similarity. The SNeIa cosmological evolution of the metrics [25] is induced by the homogeneous scalar field zeroth mode, without the inflation hypothesis and the Λ-term.

The vacuum is defined as the state with the minimal energy of the Universe. This energy is identified with the zeroth mode momentum $P_{\langle D \rangle}$. It solves the energy-time problems of the Einstein theory at the level of the Hilbert action. This solution includes the construction of the z-ordering operator of evolution $\hat{U} = T_{\langle D \rangle} \exp\left(-i \int d\langle D \rangle P_{\langle D \rangle}\right)$. Vacuum stability can be achieved by the ordinary quantization of the zeroth dilaton harmonics. The corresponding uncertainty principle gives us the dilaton initial data at the Planck epoch, while the conformal symmetry unambiguously leads to the Dirac geometry of similarity compatible with SNeIa data in the vacuum regime.

We have provided a few arguments in favor of that the exact evolution of the GR as a theory of spontaneous conformal symmetries breaking is related to the equations for the quantum squeezed oscillator. In the suggested cosmological model, the Planck epoch coincides with the electroweak one. It was shown that there is intensive vacuum creation of gravitons and SM bosons due to their Casimir energies at the Planck epoch in the vacuum background of the Empty Universe. The rough estimation [11] shows us that the considered approach to the GR and SM dilaton evolution can yield the vacuum creation of matter in the Universe in agreement with its present-day energy budget. The Early Universe behaves like a factory of electroweak bosons and Higgs scalars. It gives us a possibility to identify three peaks in the Cosmic Microwave Background power spectrum with the contributions of photonic decays and annihilation processes of primordial Higgs, W, and Z bosons in agreement with the QED coupling constant, Weinberg's angle, and Higgs' particle mass of about 118 GeV.

The problems of calculations of the S-matrix elements and their transformational properties at the level of quantum theory and its renormalizability will be discussed elsewhere.

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Appendix A

DIRAC HAMILTONIAN APPROACH TO GR IN TERMS OF THE MAURER–CARTAN FORMS

In this appendix we adapt the standard Dirac approach to the conventional scalar curvature action in terms of the Maurer–Cartan forms including the electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the scalar one Q

$$W[g, A, Q] = -\int d^4x \sqrt{-g} \left(\frac{1}{6} R^{(4)}(g) - \frac{1}{4} F_{\mu\alpha} F_{\nu\beta} g^{\mu\nu} g^{\alpha\beta} + \partial_\mu Q \partial_\nu Q g^{\mu\nu} \right) \tag{A.1}$$

in units

$$\hbar = c = M_{\rm Pl}\sqrt{3/(8\pi)} = 1. \tag{A.2}$$

Using the definition of the metric components given by Eqs. (2.1)–(2.3), we obtain the action

$$\widetilde{W} = \int d^4x \, N \left[\mathcal{L}_D + \mathcal{L}_g + \mathcal{L}_A + \mathcal{L}_Q \right], \tag{A.3}$$

where

$$\begin{aligned}
\mathcal{L}_D &= -v_D^2 - \frac{4}{3} \mathrm{e}^{D/2} \triangle \mathrm{e}^{-D/2}, \\
\mathcal{L}_g &= \frac{1}{6} \left[v_{(ab)}^2 - \mathrm{e}^{-4D} R^{(3)}(\mathbf{e}) \right], \\
\mathcal{L}_A &= \frac{1}{2} \left[\mathrm{e}^{2D} v_{(b)(A)}^2 - \mathrm{e}^{-2D} F_{ij} F^{ij} \right], \\
\mathcal{L}_Q &= \mathrm{e}^{2D} (v_Q + v_D \widetilde{Q})^2 - \mathrm{e}^{-2D} \left(\partial_{(b)} \widetilde{Q} + \partial_{(b)} D \widetilde{Q} \right)^2
\end{aligned} \tag{A.4}$$

is the set of Lagrangian densities,

$$\begin{aligned}
v_Q &= \frac{1}{N} \left[(\partial_0 - N^l \partial_l) \widetilde{Q} + \partial_l N^l / 3 \right], \\
v_D &= \frac{1}{N} \left[(\partial_0 - N^l \partial_l) D + \partial_l N^l / 3 \right], \\
v_{(ab)} &= \frac{1}{N} \left[\omega_{(ab)}^R (\partial_0 - N^l \partial_l) + \partial_{(a)} N_{(b)}^\perp + \partial_{(b)} N_{(a)}^\perp \right], \\
v_{(b)(A)} &= \frac{1}{N} \mathbf{e}_{(a)}^i \left[\partial_0 A_i - \partial_i A_0 + F_{ij} N^j \right]
\end{aligned}$$

are velocities of the metric components and fields, $\triangle = \partial_i [\mathbf{e}_{(a)}^i \mathbf{e}_{(a)}^j \partial_j]$ is the Laplace operator, and $R^{(3)}(\mathbf{e})$ is a three-dimensional spatial curvature expressed in terms of triads $\mathbf{e}_{(a)i}$ and the coefficients of the spin-connection

$$R^{(3)} = R^{(3)}(\mathbf{e}) - \frac{4}{3}\mathrm{e}^{7D/2}\triangle \mathrm{e}^{-D/2}, \tag{A.5}$$

$$R^{(3)}(\mathbf{e}) = -2\partial_i\,[\mathbf{e}^i_{(b)}\sigma_{(c)|(bc)}] - \sigma_{(c)|(bc)}\sigma_{(a)|(ba)} + \sigma_{(c)|(df)}\sigma_{(f)|(d)(c)}, \tag{A.6}$$

$$\sigma_{(c)|(ab)} = [\omega^L_{(ab)}(\partial_{(c)}) + \omega^R_{(ac)}(\partial_{(b)}) - \omega^R_{(bc)}(\partial_{(a)})],$$

$$\omega^R_{(ab)}(\partial_{(c)}) = \frac{1}{2}\left[\mathbf{e}^j_{(a)}\partial_{(c)}\mathbf{e}^j_{(b)} + \mathbf{e}^i_{(b)}\partial_{(c)}\mathbf{e}^i_{(a)}\right], \tag{A.7}$$

$$\omega^L_{(ab)}(\partial_{(c)}) = \frac{1}{2}\left[\mathbf{e}^j_{(a)}\partial_{(c)}\mathbf{e}^j_{(b)} - \mathbf{e}^i_{(b)}\partial_{(c)}\mathbf{e}^i_{(a)}\right]. \tag{A.8}$$

Using the Legandre transformations in (A.3) $v^2/B = pv - Bp^2/4$, we got momenta

$$P_{(ba)} = \frac{v_{(ab)}}{3}, \quad P_D = 2v_D, \quad P_Q = 2v_Q, \quad P_{A(b)} = v_{A(b)}. \tag{A.9}$$

So that one can write total action (A.3) in the Dirac Hamiltonian form [12]

$$\widetilde{W} = \int d^4x \left[\sum_F P_F\partial_0 F - \mathcal{C}\right], \tag{A.10}$$

$$\sum_F P_F\partial_0 F = P_Q\left(\partial_0\widetilde{Q} + \partial_0 D\widetilde{Q}\right) + P_{(ab)}\omega^R(\partial_0) + P_{A(b)}\partial_0 A_{(b)} - P_D\partial_0 D, \tag{A.11}$$

$$\mathcal{C} = N\mathcal{H} + N_{(b)}T_{(b)} + A_{(0)}\partial_{(b)}P_{A(b)} + \lambda_{(0)}P_D + \lambda_{(b)}\partial_k\mathbf{e}^k_{(b)}, \tag{A.12}$$

where

$$\mathcal{H} = -\frac{\delta\widetilde{W}}{\delta N} = \mathcal{H}_D + \mathcal{H}_g + \mathcal{H}_A + \mathcal{H}_Q, \tag{A.13}$$

$$\mathcal{H}_D = -\frac{P_D^2}{4} - \frac{4}{3}\mathrm{e}^{7D/2}\triangle\mathrm{e}^{-D/2}, \tag{A.14}$$

$$\mathcal{H}_g = \left[6P^2_{(ab)} + \frac{\mathrm{e}^{-4D}}{6}R^{(3)}(\mathbf{e})\right], \tag{A.15}$$

$$\mathcal{H}_A = \frac{\mathrm{e}^{-2D}}{2}\left[P_{i(A)}P^i_{(A)} + F_{ij}F^{ij}\right], \tag{A.16}$$

$$\mathcal{H}_Q = \mathrm{e}^{-2D}\left[\frac{P_Q^2}{4} + \left(\partial_{(b)}Q + \partial_{(b)}DQ\right)^2\right], \tag{A.17}$$

$$T^0_{(c)} = -P_{(ab)}\omega^R_{(ab)}(\partial_{(c)}) + \sum_{F=A,Q} P_F\partial_{(c)}F \tag{A.18}$$

are the energy-momentum tensor components [10]. Dirac [12] added the secondary class gauge constraints

$$\partial_k\mathbf{e}^k_{(b)} = 0, \tag{A.19}$$

$$P_D = 2\frac{v_D}{N} \equiv \frac{2\mathrm{e}^{3D}}{3N}\left[\partial_0\mathrm{e}^{-3D} - \partial_l\left(N^l\mathrm{e}^{-3D}\right)\right] = 0. \tag{A.20}$$

So that the first three of them fix the spatial coordinates by the accepted way [12], and the constraint $P_{\overline{D}} = 0$ removes negative contribution of nonzero Fourier harmonics of dilaton momentum.

Appendix B
DILATON COSMOLOGICAL PERTURBATION THEORY

The comparison of the cosmological perturbation theory in the ΛCDM model with the Hamiltonian approach to the same cosmological perturbation theory [10] reveals essential differences of these approaches and their physical consequences.

In order to keep the vacuum postulate, Dirac [12] imposed the minimal surface constraint $P_{\overline{D}} = 0$ and excluded the negative contribution $-P^2_{\overline{D}}$ of the spatial local volume element momentum to the energy constraint. In the dilaton version, the local dilaton $\overline{D}$ is associated with the acoustic waves used for the explanation of the CMB power spectrum [5]. Thus, the vacuum postulate $P_{\overline{D}} = 0$ excludes the dynamics of acoustic waves $P_{\overline{D}} \neq 0$ with negative contribution into the energy density.

In order to demonstrate these consequences, we consider the case when the simplex components $\mathbf{e}_{(b)i}dx^i = \omega^{(3)}_{(b)} = dx_{(b)}$ are total differentials. The latter means that the coefficients of the spin-connection are equal to zero together with the three-dimensional curvature $R^{(3)} = 0$. In this case, the transverse components of the shift vector can be defined by

$$T_{(0)(a)} = -\mathbf{e}^i_{(b)}\frac{\delta S_{\rm U}}{\delta N_i} = -\partial_{(b)}p_{(b)(a)} + \sum_{f=\overline{\phi},\overline{Q},\widetilde{F}} p_f\partial_{(a)}f = 0, \tag{B.1}$$

$$p_{(b)(a)} = \frac{1}{6\mathcal{N}}\left(\frac{2}{3}\delta_{(a)(b)}\partial_{(c)}\mathcal{N}_{(c)} - \partial_{(a)}\mathcal{N}_{(b)} - \partial_{(b)}\mathcal{N}_{(a)}\right). \tag{B.2}$$

While the shift-vector longitudinal component is given by the Dirac constraint $\partial_\eta\, \mathrm{e}^{-3\overline{D}} = \partial_{(b)}\left(\mathrm{e}^{-3\overline{D}}\mathcal{N}_{(b)}\right)$. The lapse function and dilaton in the first order in the Newton coupling constant take the forms [10]

$$\mathrm{e}^{-\overline{D}/2} = 1 + \frac{1}{2}\int d^3y\left[G_{(+)}(x,y)\overline{T}^{(\mu)}_{(+)}(y) + G_{(-)}(x,y)\overline{T}^{(\mu)}_{(-)}(y)\right], \tag{B.3}$$

$$\mathcal{N}\,\mathrm{e}^{-7\overline{D}/2} = 1 - \frac{1}{2}\int d^3y\left[G_{(+)}(x,y)\overline{T}^{(\nu)}_{(+)}(y) + G_{(-)}(x,y)\overline{T}^{(\nu)}_{(-)}(y)\right], \tag{B.4}$$

where $G_{(\pm)}(x,y)$ are the Green functions satisfying the equations

$$[\pm m^2_{(\pm)} - \triangle]G_{(\pm)}(x,y) = \delta^3(x-y), \tag{B.5}$$

$$m^2_{(\pm)} = H_0^2\frac{3(1+z)^2}{4}\left[14(\beta\pm 1)\Omega_{(0)}(z)\mp\Omega_{(1)}(z)\right], \tag{B.6}$$

$$\beta = \sqrt{1 + [\Omega_{(2)}(z) - 14\Omega_{(1)}(z)]/[98\Omega_{(0)}(z)]}, \tag{B.7}$$

$$\overline{T}^{(\mu)}_{(\pm)} = \overline{T}_{(0)} \mp 7\beta[7\overline{T}_{(0)} - \overline{T}_{(1)}], \tag{B.8}$$

$$\overline{T}^{(\nu)}_{(\pm)} = [7\overline{T}_{(0)} - \overline{T}_{(1)}] \pm (14\beta)^{-1}\overline{T}_{(0)} \tag{B.9}$$

in which the local currents and $\Omega_{(n)}(z)$ are given by the equation

$$\Omega_{(n)}(z) = \sum_{J=0,2,3,4,6} (2J)^n (1+z)^{2-J} \Omega_J, \quad \Omega_J = \langle \mathcal{T}_J \rangle / H_0^2, \tag{B.10}$$

and $\Omega_{J=0,2,3,4,6}$ are partial density of states: rigid, radiation, matter, curvature, Λ-term, respectively; $\Omega_{(0)}(0) = 1$, and H_0 is the Hubble parameter.

In the case of point mass distribution in a finite volume V_0 with the zeroth pressure and the density

$$\overline{\mathcal{T}}_{(0)}(x) = \frac{\overline{\mathcal{T}}_{(1)}(x)}{6} \equiv \frac{3}{4a^2} M \left[\delta^3(x-y) - \frac{1}{V_0} \right], \tag{B.11}$$

solutions (B.3), (B.4) take the Schwarzschild-type form

$$\mathrm{e}^{-\overline{D}/2} = 1 + \frac{r_g}{4r} \left[\frac{1+7\beta}{2}\, \mathrm{e}^{-m_{(+)}(z)r} + \frac{1-7\beta}{2} \cos m_{(-)}(z) r \right]_{H_0=0} = 1 + \frac{r_g}{4r},$$

$$\mathcal{N}\, \mathrm{e}^{-7\overline{D}/2} = 1 - \frac{r_g}{4r} \left[\frac{14\beta+1}{28\beta}\, \mathrm{e}^{-m_{(+)}(z)r} + \frac{14\beta-1}{28\beta} \cos m_{(-)}(z) r \right]_{H_0=0} = 1 - \frac{r_g}{4r},$$

where $\beta = 5/7$, $m_{(+)} = 3m_{(-)}$, $m_{(-)} = H_0\sqrt{3(1+z)\Omega_M/2}$. These solutions have spatial oscillations and the nonzero shift of the coordinate origin.

One can see that in the infinite volume limit $H_0 = 0$, $a = 1$ these solutions coincide with the isotropic version of the Schwarzschild solutions: $\mathrm{e}^{-\overline{D}/2} = 1 + r_g/4r$, $\mathcal{N}\,\mathrm{e}^{-7\overline{D}/2} = 1 - r_g/4r$, $N^k = 0$.

In contrast to standard cosmological perturbation theory [5] the diffeo-invariant version of the perturbation theory does not contain time derivatives that are responsible for the CMB «primordial power spectrum» in the inflationary model.

The next differences are a nonzero shift vector and spatial oscillations of the scalar potentials determined by $\hat{m}^2_{(-)}$. The dominance of rigid state $\Omega_{\mathrm{Stiff}} \sim 1$ determines the parameter of spatial oscillations $\hat{m}^2_{(-)} = \dfrac{6}{7} H_0^2 [\Omega_R (z+1)^2 + \dfrac{9}{2} \Omega_{\mathrm{Mass}}(z+1)]$. The redshifts in the recombination epoch $z_r \sim 1100$ and the clustering parameter $r_{\mathrm{clust}} = \dfrac{\pi}{\hat{m}_{(-)}} \sim \dfrac{\pi}{H_0 \Omega_R^{1/2}(1+z_r)} \sim 130\,\mathrm{Mpc}$ recently discovered in the researches of a large-scale periodicity in redshift distribution [42] lead to a reasonable value of the radiation-type density $10^{-4} < \Omega_R \sim 3 \cdot 10^{-3} < 5 \cdot 10^{-2}$ at the time of this epoch.

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