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ON 2D $\mathcal{N} = (4,4)$ SUPERSPACE SUPERGRAVITY

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We review some recent results obtained in studying superspace formulations of 2D $\mathcal{N} = (4,4)$ matter-coupled supergravity. For a superspace geometry described by the minimal supergravity multiplet, we first describe how to reduce to components the chiral integral by using «ectoplasm» superform techniques as in arXiv:0907.5264 and then we review the bi-projective superspace formalism introduced in arXiv:0911.2546. After that, we elaborate on the curved bi-projective formalism providing a new result: the solution of the covariant type-I twisted multiplet constraints in terms of a weight-$(-1,-1)$ bi-projective superfield.

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INTRODUCTION

In the literature, two superspace frameworks have been developed to study supersymmetric field theories with eight real supercharges. They go under the names of harmonic superspace (HS) [1, 2] and projective superspace (PS) [3, 4][2]. Although in some respects similar, the two formalisms differ in the structure of the off-shell supermultiplets and the supersymmetric action principle. For these reasons the two approaches often prove to be complementary one to each other[3]. This proves to be confirmed when one considers curved extensions of the HS and PS approaches.

An HS description of 4D $\mathcal{N} = 2$ conformal supergravity was given twenty years ago [8]. This is based on a prepotential formulation but its relationship to standard, curved superspace geometrical methods has not been elaborated in detail yet. On the other hand, first for five-dimensional [9, 10] and then for four-dimensional [11, 12] supergravity we recently proposed a PS approach to study supergravity-matter systems in a covariant geometric way[4]. In many respects the PS formalism resembles the covariant Wess–Zumino superspace approach to 4D $\mathcal{N} = 1$ supergravity [20] even if the PS supergravity prepotential structure is still not completely understood.

Together with the formulation of general supergravity-matter systems in superspace, one has to face the problem of reduction to components which is important for many applications.

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[2]See [5] for a review on flat 4D $\mathcal{N} = 2$ projective superspace.

[3]For global supersymmetry, the relationship between the harmonic and projective superspaces has been described in [6]. See also [7] for a recent discussion.

[4]See [13–16] for recent developments and applications. Note that the curved PS is built on the superconformal projective multiplets of [17, 18], for a curved geometry projective superfields were first used in studying field theory in 5D $\mathcal{N} = 1$ anti-de Sitter superspace [19].

Even if in principle trivial, in supergravity theories, the components reduction of supersymmetric actions has always represented a challenging technical task. At the present time, the state-of-the-art methods are represented by superspace normal coordinates [21–24] and the so-called «ectoplasm» [25–27] techniques[1]. As described in [23,24], a crucial property of the normal coordinates approach is its universality. On the other hand, the ectoplasm, which is based on the use of superforms, is a very general method to construct locally supersymmetric invariants [25, 26]. Moreover, in conjunction with additional ideas, the ectoplasm technique has proven to be flexible enough to provide the most efficient approach to component reduction in supergravity [27].

As part of a program aimed to develop efficient off-shell superspace formulations for matter-coupled supergravity theories with eight real supercharges in various dimensions, this year we studied some topics in the case of 2D $\mathcal{N} = (4,4)$ supergravity [28, 29]. A better understanding of locally conformal matter systems coupled to 2D $\mathcal{N} = (4,4)$ supergravity is interesting in studying WZNW/Liouville-type systems, nonlinear sigma models and $\mathcal{N} = (4,4)$ noncritical strings. Moreover, being some aspects of 2D superspace supergravity simpler compared to D > 2, a better understanding of the 2D case could shed light on unclear aspects of the higher dimensional cases.

The main scope of this note is to review some results we recently obtained in [28, 29]. In particular, in [29], by using ectoplasm techniques, we derived the chiral action principle in components for the case of the minimal supergravity geometry of Gates et al. [30].

In [28] the main result is represented by the formulation of a curved bi-projective superspace for 2D $\mathcal{N} = (4,4)$ conformal supergravity extending the flat case studied in [31–34][2]. This includes the definition of a large class of matter multiplets coupled to 2D $\mathcal{N} = (4,4)$ conformal supergravity and a manifestly locally supersymmetric and super-Weyl invariant action principle in bi-projective superspace[3].

At the end of the paper, we include a new result. Elaborating on the curved bi-projective formalism of [28], we provide the solution of the covariant type-I twisted multiplet (TM-I) constraints [31, 37, 38] in terms of a weight-$(-1,-1)$ bi-projective superfield. This is a new interesting development of [28] considering, for example, that the TM-I is the constrained prepotential of the type-II twisted multiplet [38,39] which describes the supergravity conformal compensator.

The paper is organized as follows. In Sec. 1 we review the superspace geometry of the minimal multiplet of [30]. According to [29], in Sec. 2 we describe how to derive the 2D $\mathcal{N} = (4,4)$ superspace integration measure in components by using the ectoplasm technique. Section 3 is devoted to a review of the bi-projective superspace formalism of [28]. We then conclude with Sec. 4 which contains the bi-projective prepotential for the covariant TM-I.

[1] We refer the reader to [24] and [27] for a more detailed list of references on normal coordinates and ectoplasm techniques.

[2] It is worth to note that for 2D $\mathcal{N} = (4,4)$ supersymmetry, harmonic superspace has been introduced in [35]. A prepotential formulation for 2D $\mathcal{N} = (4,4)$ conformal supergravity has been given in the so-called bi-harmonic superspace [36].

[3] Note that in this note we will focus on the geometry given by the minimal supergravity multiplet of [30] even if the bi-projective superfields were first defined in [28] on a new extended superspace geometry having tangent space group described by the $SO(1,1) \times SU(2)_L \times SU(2)_R$ group.

1. 2D $\mathcal{N}=(4,4)$ MINIMAL SUPERGRAVITY IN SUPERSPACE

In this section we review some aspects of the off-shell 2D $\mathcal{N}=(4,4)$ minimal supergravity multiplet first introduced in [30]. We focus on the curved superspace geometry underlining the minimal supergravity. For our 2D notations and conventions the reader should see [28].

Consider a curved 2D $\mathcal{N}=(4,4)$ superspace, which we will denote by $\mathcal{M}^{2|4,4}$. This is locally parametrized by coordinates $z^M=(x^m,\theta^{\mu\imath},\bar{\theta}^\mu_\imath)$, where $m=0,1$, $\mu=+,-$ and $\imath=\underline{1},\underline{2}$. In the light-cone coordinates the superspace is locally parametrized by $z^M=(x^{++},x^{=},\theta^{+\imath},\bar{\theta}^+_\imath,\theta^{-\imath},\bar{\theta}^-_\imath)$. The Grassmann variables are related one to each other by the complex conjugation rule $(\theta^{\mu\imath})^*=\bar{\theta}^\mu_\imath$.

In [30] the tangent space group was chosen to be $SO(1,1)\times SU(2)_\mathcal{V}$ where $\mathcal{M}$ and $\mathcal{V}_{ij}$ denote the corresponding Lorentz and $SU(2)_\mathcal{V}$ generators. The covariant derivatives $\nabla_A=(\nabla_a,\nabla_{\alpha i},\bar{\nabla}^i_\alpha)$ (or $\nabla_A=(\nabla_{++},\nabla_{=},\nabla_{+i},\bar{\nabla}^i_+,\nabla_{-i},\bar{\nabla}^i_-)$) of the minimal geometry are

$$\nabla_A=E_A+\Omega_A\mathcal{M}+(\Phi_\mathcal{V})_A{}^{kl}\,\mathcal{V}_{kl}. \tag{1.1}$$

Here $E_A=E_A{}^M(z)\partial_M$ is the supervielbein, with $\partial_M=\partial/\partial z^M$, $\emptyset_A(z)$ is the Lorentz connection and $(\Phi_\mathcal{V})_A{}^{kl}(z)$ is the $SU(2)_\mathcal{V}$ connections. The action of the Lorentz generator on the covariant derivatives is

$$[\mathcal{M},\nabla_{\alpha i}]=\frac{1}{2}(\gamma^3)_\alpha{}^\beta\nabla_{\beta i},\quad [\mathcal{M},\bar{\nabla}^i_\alpha]=\frac{1}{2}(\gamma^3)_\alpha{}^\beta\bar{\nabla}^i_\beta,\quad [\mathcal{M},\nabla_a]=\varepsilon_{ab}\nabla^b, \tag{1.2a}$$

$$[\mathcal{M},\nabla_{\pm i}]=\pm\frac{1}{2}\nabla_{\pm i},\quad [\mathcal{M},\bar{\nabla}^i_\pm]=\pm\frac{1}{2}\bar{\nabla}^i_\pm,\quad [\mathcal{M},\nabla_{\underset{=}{++}}]=\pm\nabla_{\underset{=}{++}}. \tag{1.2b}$$

The generator $\mathcal{V}_{ij}$ acts on the covariant derivatives according to the rules

$$[\mathcal{V}_{kl},\nabla_{\alpha i}]=\frac{1}{2}C_{i(k}\nabla_{\alpha l)},\quad [\mathcal{V}_{kl},\bar{\nabla}^i_\alpha]=-\frac{1}{2}\delta^i_{(k}\bar{\nabla}_{\alpha l)},\quad [\mathcal{V}_{kl},\nabla_a]=0. \tag{1.3}$$

It is worth to note that the operator $\mathcal{V}_{kl}$ generates a diagonal $SU(2)_\mathcal{V}$ subgroup inside a $SU(2)_L\times SU(2)_R$ whose generators $\mathbf{L}_{ij}$ and $\mathbf{R}_{ij}$ satisfy

$$[\mathbf{L}_{kl},\nabla_{+i}]=\frac{1}{2}C_{i(k}\nabla_{+l)},\quad [\mathbf{L}_{kl},\bar{\nabla}^i_+]=-\frac{1}{2}\delta^i_{(k}\bar{\nabla}_{+l)},\quad [\mathbf{L}_{kl},\nabla_{-i}]=[\mathbf{L}_{kl},\bar{\nabla}^i_-]=0, \tag{1.4a}$$

$$[\mathbf{R}_{kl},\nabla_{-i}]=\frac{1}{2}C_{i(k}\nabla_{-l)},\quad [\mathbf{R}_{kl},\bar{\nabla}^i_-]=-\frac{1}{2}\delta^i_{(k}\bar{\nabla}_{-l)},\quad [\mathbf{R}_{kl},\nabla_{+i}]=[\mathbf{R}_{kl},\bar{\nabla}^i_+]=0. \tag{1.4b}$$

Moreover, it holds $[\mathbf{L}_{kl},\nabla_a]=[\mathbf{R}_{kl},\nabla_a]=0$. In terms of $\mathbf{L}_{ij}$ and $\mathbf{R}_{ij}$ the generator $\mathcal{V}_{ij}$ is

$$\mathcal{V}_{kl}=\mathbf{L}_{kl}+\mathbf{R}_{kl}. \tag{1.5}$$

The generators $\mathbf{L}_{ij}$ and $\mathbf{R}_{ij}$ will be largely used in Sec. 3.

Note also that in [28] an extended supergravity multiplet has been formulated whose superspace geometry is based on the $SO(1,1)\times SU(2)_L\times SU(2)_R$ tangent space group. The minimal multiplet arises from the extended one after partially gauge fixing the super-Weyl transformations and gauge fixing the local chiral $SU(2)_\mathcal{C}$ transformations generated by

$$\mathcal{C}_{kl}=\mathbf{L}_{kl}-\mathbf{R}_{kl},\quad [\mathcal{C}_{kl},\nabla_{\alpha i}]=\frac{1}{2}C_{i(k}(\gamma^3)_\alpha{}^\beta\nabla_{\beta l)}. \tag{1.6}$$

The minimal supergravity gauge group is given by local general coordinate and tangent space transformations of the form

$$\delta_{\mathcal{K}} \nabla_A = [\mathcal{K}, \nabla_A], \quad \mathcal{K} = K^C \nabla_C + K\mathcal{M} + (K_{\mathcal{V}})^{kl} \mathcal{V}_{kl}, \tag{1.7}$$

with the gauge parameters obeying natural reality conditions, but otherwise arbitrary superfields. Given a tensor superfield $\mathcal{U}(z)$, with its indices suppressed, it transforms as

$$\delta_{\mathcal{K}} \mathcal{U} = \mathcal{K}\mathcal{U}. \tag{1.8}$$

The minimal covariant derivatives algebra has the form

$$[\nabla_A, \nabla_B\} = T_{AB}{}^C \nabla_C + R_{AB}\mathcal{M} + (R_{\mathcal{V}})_{AB}^{kl} \mathcal{V}_{kl}, \tag{1.9}$$

where T_{AB}^C is the torsion, R_{AB} is the Lorentz curvature and $(R_{\mathcal{V}})_{AB}{}^{kl}$ is the $SU(2)_{\mathcal{V}}$ curvature.

In [30] it was proved that the off-shell 2D $\mathcal{N} = (4,4)$ minimal supergravity multiplet is described by the constraints[1]

$$\{\nabla_{\alpha i}, \nabla_{\beta j}\} = -4iC_{ij}C_{\alpha\beta}N\mathcal{M} + 4i(\gamma^3)_{\alpha\beta}N\mathcal{V}_{ij}, \tag{1.10a}$$

$$\{\nabla_{\alpha i}, \bar{\nabla}^j_\beta\} = 2i\delta_i^j(\gamma^a)_{\alpha\beta}\nabla_a - 4\delta_i^j\left(iC_{\alpha\beta}\mathcal{T} + (\gamma^3)_{\alpha\beta}\mathcal{S}\right)\mathcal{M} + 4\left(i(\gamma^3)_{\alpha\beta}\mathcal{T} + C_{\alpha\beta}\mathcal{S}\right)\mathcal{V}_i{}^j, \tag{1.10b}$$

$$\begin{aligned}[\nabla_a, \nabla_{\beta j}] = {} & \left(i(\gamma_a)_\beta^\gamma\mathcal{S} + \varepsilon_{ab}(\gamma^b)_\beta^\gamma\mathcal{T}\right)\nabla_{\gamma j} - \varepsilon_{ab}(\gamma^b)_\beta^\gamma N\bar{\nabla}_{\gamma j} + \\ & + (\gamma_a)_\beta^\gamma(\bar{\nabla}_{\gamma j}N)\mathcal{M} - \varepsilon_{ab}(\gamma^b)_\beta^\gamma(\bar{\nabla}_\gamma^k N)\mathcal{V}_{jk},\end{aligned} \tag{1.10c}$$

$$\begin{aligned}[\nabla_a, \nabla_b] = {} & -\frac{1}{2}\varepsilon_{ab}\left(i(\nabla^{\gamma k}\bar{N})\nabla_{\gamma k} + i(\bar{\nabla}_k^\gamma N)\bar{\nabla}_\gamma^k + \right. \\ & + \left(\frac{i}{16}[\nabla^{\alpha(k}, \bar{\nabla}^{l)}_\alpha]\bar{N} - \frac{i}{16}[\bar{\nabla}^{\alpha(k}, \bar{\nabla}^{l)}_\alpha]N\right)\mathcal{V}_{kl} + \\ & + \left(\frac{i}{4}(\gamma^3)^{\alpha\beta}[\bar{\nabla}_{\alpha k}, \bar{\nabla}^k_\beta]N - \frac{i}{4}(\gamma^3)^{\alpha\beta}[\nabla_{\alpha k}, \nabla^k_\beta]\bar{N}\right)\mathcal{M} + \\ & \left. + \left(8\mathcal{T}^2 + 8\mathcal{S}^2 + 8\bar{N}N\right)\mathcal{M}\right).\end{aligned} \tag{1.10d}$$

Here the dimension-1 components of the torsion obey the reality conditions

$$(N)^* = \bar{N}, \quad (\mathcal{T})^* = \mathcal{T}, \quad (\mathcal{S})^* = \mathcal{S}. \tag{1.11}$$

The N, $\mathcal{S}$ and $\mathcal{T}$ superfields are Lorentz scalars and are invariant under $SU(2)_{\mathcal{V}}$ transformations.

The components of the dimension-1 torsion obey differential constraints imposed by the Bianchi identities. At dimension-3/2 the Bianchi identities give

$$\nabla_{\alpha i}N = 0, \quad \nabla^i_\alpha\mathcal{S} = \frac{i}{2}(\gamma^3)_\alpha^\beta\bar{\nabla}^i_\beta N, \quad \nabla^i_\alpha\mathcal{T} = -\frac{1}{2}\bar{\nabla}^i_\alpha N. \tag{1.12}$$

[1]The algebra of covariant derivatives here is written according to the notation of [28] and is equivalent to the one given in [30] up to trivial redefinitions of the torsion superfields.

We conclude this section by noting that, besides the $SO(1,1) \times SU(2)_{\mathcal{V}}$ tangent space group transformations, the minimal supergravity multiplet provides a representation of the superconformal group through local super-Weyl transformations. This is completely analogue to the analysis of Howe and Tucker [40]: super-Weyl transformations are «scale» variations of the covariant derivatives such that the torsion constraints remain invariant. In the case of the 2D $\mathcal{N} = (4,4)$ minimal supergravity multiplet, the super-Weyl transformations are generated by two real superfields $\mathbf{S}, \mathbf{S}_{ij} = \mathbf{S}_{ji}$, $(\mathbf{S})^* = \mathbf{S}$, $(\mathbf{S}_{ij})^* = \mathbf{S}^{ij}$, through the following infinitesimal variation of the spinor covariant derivative [28, 30]:

$$\tilde{\delta}\nabla_{\alpha i} = \frac{1}{2}\mathbf{S}\nabla_{\alpha i} + (\gamma^3)_\alpha^\beta \mathbf{S}_i^j \nabla_{\beta j} + (\gamma^3)_\alpha^\gamma (\nabla_{\gamma i}\mathbf{S})\mathcal{M} + (\nabla_\alpha^k \mathbf{S})\mathcal{V}_{ik}. \tag{1.13}$$

The first term in the previous equation is a local superscale transformation, while the second term is related to a compensating chiral $SU(2)_{\mathcal{C}}$ transformation of the covariant derivatives [28]. The $\mathbf{S}$ and $\mathbf{S}_{ij}$ superfields have to satisfy the differential constraint

$$(\nabla_{\alpha i}\mathbf{S}_{kl}) = -\frac{1}{2}(\gamma^3)_\alpha^\beta C_{i(k\nabla_{\beta k)}}\mathbf{S}. \tag{1.14}$$

This is the dimension-1/2 differential constraint of a twisted-II multiplet [38, 39].

To ensure the invariance of the supergravity constraints, the dimension-1 torsion components of the minimal multiplet have to transform according to the following rules [28]:

$$\tilde{\delta}N = \mathbf{S}N + \frac{i}{8}(\gamma^3)^{\gamma\delta}(\nabla_{\gamma k}\nabla_\delta^k \mathbf{S}), \tag{1.15a}$$

$$\tilde{\delta}\mathcal{T} = \mathbf{S}\mathcal{T} + \frac{i}{16}(\gamma^3)^{\gamma\delta}([\nabla_{\gamma k}, \bar{\nabla}_\delta^k]\mathbf{S}), \tag{1.15b}$$

$$\tilde{\delta}\mathcal{S} = \mathbf{S}\mathcal{S} + \frac{1}{16}([\nabla_{\gamma k}, \bar{\nabla}^{\gamma k}]\mathbf{S}). \tag{1.15c}$$

The transformations of the $\bar{\nabla}_\alpha^i$ covariant derivative can be trivially obtained by complex conjugation of (1.13), while for the vector covariant derivative it holds

$$\tilde{\delta}\nabla_a = \mathbf{S}\nabla_a + \frac{i}{2}(\gamma_a)^{\gamma\delta}(\nabla_{\gamma k}\mathbf{S})\bar{\nabla}_\delta^k + \frac{i}{2}(\gamma_a)^{\gamma\delta}(\bar{\nabla}_\gamma^k \mathbf{S})\nabla_{\delta k} + \varepsilon_{ab}(\nabla^b \mathbf{S})\mathcal{M} - \varepsilon_{ab}(\nabla^b \mathbf{S}^{kl})\mathcal{V}_{kl}. \tag{1.16}$$

2. ECTOPLASM AND 2D $\mathcal{N} = (4,4)$ SUPERSPACE INTEGRATION

The aim of this section is to review the results of [29] about the component reduction of the chiral integral in 2D $\mathcal{N} = (4,4)$ minimal supergravity. According to the ectoplasm paradigm for component reduction of superspace actions in supergravity, the search of supersymmetric invariants is related to the study of closed superforms [25][1]. Before the description of the results in [29], let us give a brief review of the ectoplasmic construction of supersymmetric actions.

[1]Note that a mathematical construction giving the formal, but physically uncomplete, bases for the ectoplasm methods can be found in the theory of integration over surfaces in supermanifolds developed in [41–43].

Consider a curved superspace $\mathcal{M}^{d|\delta}$ with d space-time and δ fermionic dimensions, and let $\mathcal{M}^{d|\delta}$ be parametrized by local coordinates $z^M = (x^{\hat{m}}, \theta^{\hat{\mu}})$, where $\hat{m} = 1, \ldots, d$ and $\hat{\mu} = 1, \ldots, \delta$. The corresponding superspace geometry is described by covariant derivatives

$$\nabla_A = (\nabla_{\hat{a}}, \nabla_{\hat{\alpha}}) = E_A + \Phi_A, \quad E_A := E_A^M \partial_M, \quad \Phi_A := \Phi_A \cdot \mathbb{J} = E_A^M \Phi_M. \tag{2.1}$$

Here $\mathbb{J}$ denotes the generators of the structure group (with all indices of $\mathbb{J}$s suppressed), E_A is the inverse vielbein, and $\Phi = dz^M \Phi_M = E^A \Phi_A$ the connection. The vielbein $E^A := dz^M E_M^A$ and its inverse E_A are such that $E_A^M E_M^B = \delta_A^B$ and $E_M^A E_A^N = \delta_M^N$. The covariant derivatives obey the algebra

$$[\nabla_A, \nabla_B\} = T_{AB}^C \nabla_C + R_{AB} \cdot \mathbb{J}, \tag{2.2}$$

with T_{AB}^C the torsion, and R_{AB} the curvature.

Next, consider a super d-form

$$J = \frac{1}{d!} dz^{M_d} \wedge \ldots \wedge dz^{M_1} J_{M_1 \ldots M_d} = \frac{1}{d!} E^{A_d} \wedge \ldots \wedge E^{A_1} J_{A_1 \ldots A_d} \tag{2.3}$$

constrained to be closed

$$dJ = 0 \Longleftrightarrow \nabla_{[B} J_{A_1 \ldots A_d\}} - \frac{d}{2} T_{[BA_1|}^C J_{C|A_2 \ldots A_d\}} = 0. \tag{2.4}$$

Then, consider the following integral over the bosonic space-time coordinates:

$$S = \frac{1}{d!} \int d^d x \varepsilon^{\hat{m}_1 \ldots \hat{m}_d} J_{\hat{m}_1 \ldots \hat{m}_d} = \frac{1}{d!} \int d^d x \varepsilon^{\hat{m}_1 \ldots \hat{m}_d} E_{\hat{m}_d}{}^{A_d} \cdots E_{\hat{m}_1}{}^{A_1} J_{A_1 \ldots A_d}. \tag{2.5}$$

Due to the closure of the super d-form J, the functional S turns out to be such that: (i) S is independent of the Grassmann variables θ's; and (ii) S is invariant under general coordinate transformations on $\mathcal{M}^{d|\delta}$ and structure group transformations. Now, define the component vielbein as $e_{\hat{m}}{}^{\hat{a}} = E_{\hat{m}}{}^{\hat{a}}|_{\theta=0}$ where its inverse $e_{\hat{a}}{}^{\hat{m}}$ is such that $e_{\hat{m}}{}^{\hat{b}} e_{\hat{b}}{}^{\hat{n}} = \delta_{\hat{m}}^{\hat{n}}$, $e_{\hat{a}}{}^{\hat{n}} e_{\hat{n}}{}^{\hat{b}} = \delta_{\hat{a}}^{\hat{b}}$. If one defines the gravitini fields according to $\Psi_{\hat{a}}{}^{\hat{\alpha}} := -e_{\hat{a}}{}^{\hat{m}} E_{\hat{m}}{}^{\hat{\alpha}}|_{\theta=0}$, the functional (2.5) can be rewritten as

$$S = \frac{1}{d!} \int d^d x \varepsilon^{\hat{m}_1 \cdots \hat{m}_d} E_{\hat{m}_d}{}^{A_d} \cdots E_{\hat{m}_1}{}^{A_1} J_{A_1 \ldots A_d} \Big|_{\theta=0}, \tag{2.6a}$$

$$= \frac{1}{d!} \int d^d x \mathrm{e}^{-1} \varepsilon^{\hat{a}_1 \ldots \hat{a}_d} \Big(J_{\hat{a}_1 \ldots \hat{a}_d} - d \Psi_{\hat{a}_1}{}^{\hat{\alpha}} J_{\hat{\alpha} \hat{a}_2 \ldots \hat{a}_d} + \frac{d(d-1)}{2} \Psi_{\hat{a}_2}{}^{\hat{\alpha}_2} \Psi_{\hat{a}_1}{}^{\hat{\alpha}_1} J_{\hat{\alpha}_1 \hat{\alpha}_2 \hat{a}_3 \ldots \hat{a}_d} + \\ + \ldots + (-)^d \Psi_{\hat{a}_d}{}^{\hat{\alpha}_d} \cdots \Psi_{\hat{a}_1}{}^{\hat{\alpha}_1} J_{\hat{\alpha}_1 \ldots \hat{\alpha}_d} \Big) \Big|_{\theta=0}, \tag{2.6b}$$

where $\mathrm{e}^{-1} = [\det e_{\hat{a}}{}^{\hat{m}}]^{-1}$. Besides the closure condition (2.4), depending on the case under consideration, the superform J obeys some additional covariant constraints imposed on its components $J_{A_1 \ldots A_d}$. In cases related to component reductions of superspace actions, the components $J_{A_1 \ldots A_d}$ are all function of a single superfield $\mathcal{L}$, spinor covariant derivatives of it and torsion components. The maximum number of derivatives of $\mathcal{L}$ in a given component

$J_{A_1 \ldots A_d}$ depends on its mass dimension. The cohomology equation (2.4) iteratively defines the $J_{A_1 \ldots A_d}$ components with higher dimension in terms of derivatives and torsion multiplying the lower dimensional components.

Let us now consider the case of 2D $\mathcal{N} = (4,4)$ minimal supergravity. On general grounds we can easily construct a locally supersymmetric invariant as

$$S = \int d^2x d^4\theta d^4\bar{\theta} E^{-1} \mathcal{L}, \quad E^{-1} := [\mathrm{Ber}\, E_A{}^M]^{-1}, \tag{2.7}$$

where $\mathcal{L}$ is a scalar and $SU(2)$-invariant but otherwise unconstrained superfield.

For practical application, one is interested to have the previous action principle ready for components reduction. In particular, we want to find two fourth-order differential operators $\Delta^{(4)}$ and $\overline{\mathcal{D}}^{(4)}$ such that

$$S = \frac{1}{2} \int d^2x \mathrm{e}^{-1} \Delta^{(4)} \overline{\mathcal{D}}^{(4)} \mathcal{L} \Big|_{\theta=0}. \tag{2.8}$$

Here with $\Phi|_{\theta=0}$ we indicate the limit where all the Grassmann variables in a superfield Φ are set to zero. In (2.8) the operator $\overline{\mathcal{D}}^{(4)}$ defined by

$$\overline{\mathcal{D}}^{(4)} = \left(\bar{\nabla}^{(2)\alpha\beta} + 4i\bar{N}(\gamma^3)^{\alpha\beta} \right) \bar{\nabla}^{(2)}_{\alpha\beta} \tag{2.9}$$

is the chiral projection operator satisfying

$$\bar{\nabla}^i_\gamma \overline{\mathcal{D}}^{(4)} \Psi = \bar{\nabla}^i_\gamma \left(\bar{\nabla}^{(2)\alpha\beta} + 4i\bar{N}(\gamma^3)^{\alpha\beta} \right) \bar{\nabla}^{(2)}_{\alpha\beta} \Psi = 0 \tag{2.10}$$

for any general scalar and $SU(2)$-invariant superfield Ψ. Here the operator $\bar{\nabla}^{(2)}_{\alpha\beta}$ is

$$\bar{\nabla}^{(2)}_{\alpha\beta} = \frac{1}{2} C_{ij} \left(\bar{\nabla}^i_\alpha \bar{\nabla}^j_\beta + \bar{\nabla}^i_\beta \bar{\nabla}^j_\alpha \right). \tag{2.11}$$

The chiral projector (2.9) for the minimal supergravity was recently computed in [44] by Gates and Morrison.

The operator $\Delta^{(4)}$ is called the «chiral» density projector operator. We computed it in [29] by using ectoplasm techniques. The fact that there exists the factorization $\Delta^{(4)}\overline{\mathcal{D}}^{(4)}$ in (2.8) is due to the existence of covariantly chiral superfield and integration over the chiral subspace for the 2D $\mathcal{N} = (4,4)$ minimal supergravity[1]. In the ectoplasm framework the factorization results are trivial.

The construction of the density projector operator using ectoplasm lies in the existence of a «chiral» closed two-form which is function of an unconstrained covariantly chiral superfield U such that $\bar{\nabla}^i_\alpha U = 0$. The chiral superfield U plays the role of the chiral Lagrangian and can be thought as $\overline{\mathcal{D}}^{(4)} \mathcal{L}$ by using the chiral projector.

[1] One expects similar factorizations every time invariant subspaces of a given curved superspace exist.

The components $J_{AB}=\left(J_{\alpha i\beta j}, J_{\alpha i}{}_{\beta}^{j}, J_{\alpha\beta}^{i\,j}, J_{\alpha i b}, J_{\alpha b}^{i}, J_{ab}\right)$ of the closed super two-form we are interested in turn out to be

$$J_{\alpha\beta}^{i\,j}=\left(2(\gamma^3)_{\alpha\beta}\nabla^{(2)ij}-C_{\alpha\beta}C^{ij}(\gamma^3)^{\gamma\delta}\nabla^{(2)}_{\gamma\delta}\right)U, \tag{2.12a}$$

$$J_{\alpha b}^{i}=-\frac{i}{3}\varepsilon_{bc}(\gamma^c)_{\alpha}{}^{\gamma}\nabla_{\gamma k}\nabla^{(2)ik}U, \tag{2.12b}$$

$$J_{ab}=-\frac{1}{8}\varepsilon_{ab}\left(\nabla^{(4)}+4iN(\gamma^3)^{\alpha\beta}\nabla^{(2)}_{\alpha\beta}\right)U, \tag{2.12c}$$

$$J_{\alpha i\beta j}=J_{\alpha i}{}_{\beta}^{j}=J_{\alpha i b}=0. \tag{2.12d}$$

Here we have introduced second- and fourth-order spinorial derivative operators via the equations

$$\nabla^{(2)}_{\alpha\beta}=\frac{1}{2}\left(\nabla_{\alpha k}\nabla_{\beta}^{k}+\nabla_{\beta k}\nabla_{\alpha}^{k}\right), \quad \nabla^{(2)}_{ij}=\frac{1}{2}\left(\nabla_{\gamma i}\nabla_{j}^{\gamma}+\nabla_{\gamma j}\nabla_{i}^{\gamma}\right), \quad \nabla^{(4)}=\frac{1}{3}\nabla^{(2)kl}\nabla^{(2)}_{kl}. \tag{2.13}$$

The complex closed super two-form (2.12a)–(2.12d) satisfies Eq. (2.4) where the supergravity geometry is the 2D $\mathcal{N}=(4,4)$ minimal one of Sec. 1. A way to derive (2.12a)–(2.12d) is to first take the following ansatz for the lower dimensional components[1]: $J_{\alpha i\beta j}=J_{\alpha i}{}_{\beta}^{j}=0$, $J_{\alpha\beta}^{i\,j}=\left(a(\gamma^3)_{\alpha\beta}\nabla^{(2)}_{ij}+bC_{\alpha\beta}C_{ij}(\gamma^3)^{\gamma\delta}\nabla^{(2)}_{\gamma\delta}+C_{\alpha\beta}C_{ij}F\right)U$, where $F=\left(b_1N+b_2\bar{N}+b_3\mathcal{S}+b_4\mathcal{T}\right)$ and a,b,b_1,b_2,b_3,b_4 are constants. Imposing the closure equation (2.4) on the components J_{AB}, one fixes the constants and iteratively expresses the higher dimensional components $J_{\alpha ib}$, $J_{\alpha b}^{i}$ and J_{ab} in terms of derivatives of the lower dimensional one. This procedure gives the result (2.12a)–(2.12d).

It would be interesting to rederive the previous closed super two-form by using the powerful arguments recently developed in [27] and, in particular, find a 2D $\mathcal{N}=(4,4)$ «chiral» closed super 1-form such that from its square wedge product one can derive the closed 2-form just introduced.

To conclude, let us give the component form of the action (2.8) by using the ectoplasm functional (2.6b). In the 2D $\mathcal{N}=(4,4)$ case, Eq. (2.6b) becomes

$$S=\frac{1}{2}\int d^2x\mathrm{e}^{-1}\varepsilon^{ab}\Big(J_{ab}-2\left(\bar{\psi}_{a}{}_{i}^{\alpha}J_{\alpha b}^{i}+\psi_{a}{}^{\alpha i}J_{\alpha ib}\right)-\bar{\psi}_{a}{}_{i}^{\alpha}\bar{\psi}_{b}{}_{j}^{\beta}J_{\alpha\beta}^{i\,j}- \\ -2\psi_{a}{}^{\alpha i}\bar{\psi}_{b}{}_{j}^{\beta}J_{\alpha i}{}_{\beta}^{j}-\psi_{a}{}^{\alpha i}\psi_{b}{}^{\beta j}J_{\alpha i\beta j}\Big)\Big|_{\theta=0}. \tag{2.14}$$

By using the previous expression, Eqs. (2.12a)–(2.12d) and the chiral superfield $U=\overline{\mathcal{D}}^{(4)}\mathcal{L}$,

[1] In [29] we derived a real closed super two-form which is function of U and its antichiral complex conjugate $\bar{U}$. It is easy to observe that the chiral and antichiral sectors are algebraically independent under Eq. (2.4). Then, relaxing the reality condition, one can find the closed super two-form (2.12a)–(2.12d) with computations equal to the one given in [29]; the result is in fact identical but with the antichiral sector formally turned off.

one finds the component action (2.8) to be

$$S = \int d^2x \mathrm{e}^{-1} \left(\frac{1}{8}\nabla^{(4)} + \frac{i}{2}N(\gamma^3)^{\alpha\beta}\nabla^{(2)}_{\alpha\beta} + \frac{i}{3}\bar{\psi}_{a}{}_{i}^{\gamma}(\gamma^a)_{\gamma}{}^{\delta}\nabla_{\delta j}\nabla^{(2)ij} - \right.$$
$$\left. -\varepsilon^{ab}\bar{\psi}_{a}{}_{i}^{\alpha}\bar{\psi}_{b}{}_{j}^{\beta}(\gamma^3)_{\alpha\beta}\nabla^{(2)ij} - \frac{1}{2}\varepsilon^{ab}\bar{\psi}_{a}{}_{i}^{\alpha}\bar{\psi}_{b}{}^{i}_{\alpha}(\gamma^3)^{\gamma\delta}\nabla^{(2)}_{\gamma\delta} \right) \overline{\mathcal{D}}^{(4)}\mathcal{L}\Big|_{\theta=0}. \quad (2.15)$$

The terms in the brackets then define the «chiral» density projector operator $\Delta^{(4)}$.

3. CURVED BI-PROJECTIVE SUPERSPACE

In Sec. 1 we have reviewed the geometric description of 2D $\mathcal{N} = (4, 4)$ minimal supergravity in superspace [30]. Let us now turn to discuss a large family of off-shell supermultiplets coupled to supergravity, which can be used to describe supersymmetric matter. We introduced them in [28] under the name of covariant bi-projective supermultiplets. These supermultiplets are a curved-superspace extension of the 2D multiplets introduced in the flat case in [31–34]. The formalism possesses clear similarities with the bi-harmonic superspace approach of [35, 36]. Moreover, curved bi-projective superspace is a 2D extension of the curved projective approach recently developed in the cases of 5D $\mathcal{N} = 1$ supergravity [9, 10] and 4D $\mathcal{N} = 2$ supergravity [11, 12].

It is useful to introduce auxiliary isotwistors coordinates $u_i^{\oplus} \in \mathbb{C}^2\backslash\{0\}$ and $v_i^{\boxplus} \in \mathbb{C}^2\backslash\{0\}$ in addition to the superspace coordinates $z^M = (x^m, \theta^{\mu\imath}, \bar{\theta}^{\mu}_{\imath})$. All the coordinates $u_i^{\oplus}$, $v_i^{\boxplus}$ and z^M are defined to be inert under the action of the structure group.

The next step is to introduce superfields which are functions of z^M and also of the extra $u^{\oplus}$ and $v^{\boxplus}$ variables and have well-defined supergravity gauge transformations. We define a weight-(m, n) bi-isotwistor superfield $U^{(m,n)}(z, u^{\oplus}, v^{\boxplus})$ to be holomorphic on an open domain of $\{\mathbb{C}^2\backslash\{0\}\} \times \{\mathbb{C}^2\backslash\{0\}\}$ with respect to the homogeneous coordinates $(u_i^{\oplus}, v_j^{\boxplus})$ for $\mathbb{C}P^1 \times \mathbb{C}P^1$, and be characterized by the conditions:

(i) it is a homogeneous function of $(u^{\oplus}, v^{\boxplus})$ of degree (m, n), that is,

$$U^{(m,n)}(z, c_L\, u^{\oplus}, v^{\boxplus}) = (c_L)^m U^{(m,n)}(z, u^{\oplus}, v^{\boxplus}), \quad c_L \in \mathbb{C}\backslash\{0\}, \quad (3.1a)$$
$$U^{(m,n)}(z, u^{\oplus}, c_R\, v^{\boxplus}) = (c_R)^n U^{(m,n)}(z, u^{\oplus}, v^{\boxplus})\ , \quad c_R \in \mathbb{C}\backslash\{0\}; \quad (3.1b)$$

(ii) the minimal supergravity gauge transformations act on $U^{(m,n)}$ as follows (remember that $\mathcal{V}_{ij} = (\mathbf{L}_{ij} + \mathbf{R}_{ij})$):

$$\delta_{\mathcal{K}} U^{(m,n)} = \left(K^C\nabla_C + K\mathcal{M} + (K_{\mathcal{V}})^{kl}\mathcal{V}_{kl}\right) U^{(m,n)}, \quad (3.2a)$$
$$\mathbf{L}_{kl} U^{(m,n)} = -\frac{1}{2(u^{\oplus}u^{\ominus})}\left(u^{\oplus}_{(k}u^{\oplus}_{l)}D^{\ominus\ominus} - m u^{\oplus}_{(k}u^{\ominus}_{l)}\right) U^{(m,n)}, \quad (3.2b)$$
$$\mathbf{R}_{kl} U^{(m,n)} = -\frac{1}{2(v^{\boxplus}v^{\boxminus})}\left(v^{\boxplus}_{(k}v^{\boxplus}_{l)}D^{\boxminus\boxminus} - n\, v^{\boxplus}_{(k}v^{\boxminus}_{l)}\right) U^{(m,n)}, \quad (3.2c)$$
$$\mathcal{M} U^{(m,n)} = \frac{m-n}{2} U^{(m,n)}, \quad (3.2d)$$

where we have introduced

$$D^{\ominus\ominus} = u^{\ominus i}\frac{\partial}{\partial u^{\oplus i}}, \qquad D^{\boxminus\boxminus} = v^{\boxminus i}\frac{\partial}{\partial v^{\boxplus i}}, \tag{3.3a}$$

$$(u^{\oplus}u^{\ominus}) := u^{\oplus i}u_i^{\ominus} \neq 0, \qquad (v^{\boxplus}v^{\boxminus}) := v^{\boxplus i}v_i^{\boxminus} \neq 0. \tag{3.3b}$$

The previous equations involve two new isotwistors $u^{\ominus}$ and $v^{\boxminus}$ which are subject to the only conditions (3.3b) and are otherwise completely arbitrary. One can prove that, due to (3.1a), the superfield $(\mathbf{L}_{kl}U^{(m,n)})$ is independent of $u^{\ominus}$ even if the transformations in (3.2b) explicitly depend on it; similarly $(\mathbf{R}_{kl}U^{(m,n)})$ is independent of $v^{\boxminus}$. Then $\mathcal{V}_{kl}U^{(m,n)}$ and, in particular, $\delta_{\mathcal{K}}U^{(m,n)}$ are independent of $u^{\ominus}$ and $v^{\boxminus}$. One can prove that the homogeneity condition is closely related to (3.2b), (3.2c) and the independence of $u^{\ominus}$ and $v^{\boxminus}$. The reader should see [11] for a more detailed discussion on the $SU(2)$ transformations of isotwistor-like superfields. Note that, even if the supergravity gauge group of the minimal multiplet possesses only $SU(2)_{\mathcal{V}}$ transformations in (3.2a), it is useful to keep manifest the $SU(2)_L$ and $SU(2)_R$ parts [28].

Using the u, v isotwistors, one can define the covariant derivatives

$$\nabla_+^{\oplus} := u_i^{\oplus}\nabla_+^i, \quad \bar{\nabla}_+^{\oplus} := u_i^{\oplus}\bar{\nabla}_+^i, \quad \nabla_-^{\boxplus} := v_i^{\boxplus}\nabla_-^i, \quad \bar{\nabla}_-^{\boxplus} := v_i^{\boxplus}\bar{\nabla}_-^i. \tag{3.4}$$

A crucial property of 2D bi-isotwistor superfields is that the anticommutator of any of the covariant derivatives $\nabla_+^{\oplus}$, $\bar{\nabla}_+^{\oplus}$, $\nabla_-^{\boxplus}$, $\bar{\nabla}_-^{\boxplus}$ is zero when acting on $U^{(m,n)}$. It holds

$$0 = \{\nabla_+^{\oplus}, \nabla_+^{\oplus}\}U^{(m,n)} = \{\nabla_+^{\oplus}, \bar{\nabla}_+^{\oplus}\}U^{(m,n)} = \{\nabla_+^{\oplus}, \nabla_-^{\boxplus}\}U^{(m,n)} = \dots \tag{3.5}$$

The proof of this important relation is given in [28]. With the definitions (i) and (ii) assumed, the set of bi-isotwistor superfields results to be closed under the product of superfields and the action of the $\nabla_+^{\oplus}$, $\bar{\nabla}_+^{\oplus}$, $\nabla_-^{\boxplus}$, $\bar{\nabla}_-^{\boxplus}$ derivatives. In fact, given a weight-(m,n) $U^{(m,n)}$ and a weight-(p,q) $U^{(p,q)}$ bi-isotwistor superfields the superfield $(U^{(m,n)}U^{(p,q)})$ is a weight-$(m+p, n+q)$ bi-isotwistor superfield. Moreover, the superfields $(\nabla_+^{\oplus}U^{(m,n)})$, $(\bar{\nabla}_+^{\oplus}U^{(m,n)})$ and $(\nabla_-^{\boxplus}U^{(m,n)})$, $(\bar{\nabla}_-^{\boxplus}U^{(m,n)})$ are respectively weight-$(m+1, n)$ and weight-$(m, n+1)$ bi-isotwistor superfields.

Due to (3.5), one can consistently define analyticity constraints. Let us then introduce 2D $\mathcal{N} = (4,4)$ covariant bi-projective superfields. We define a weight-(m,n) covariant bi-projective supermultiplet $Q^{(m,n)}(z, u^{\oplus}, v^{\boxplus})$ to be a bi-isotwistor superfield satisfying (i), (ii), (3.1a)–(3.2d) and to be constrained by the analyticity conditions

$$\nabla_+^{\oplus}Q^{(m,n)} = \bar{\nabla}_+^{\oplus}Q^{(m,n)} = 0, \quad \nabla_-^{\boxplus}Q^{(m,n)} = \bar{\nabla}_-^{\boxplus}Q^{(m,n)} = 0. \tag{3.6}$$

The consistency of the previous constraints is indeed guaranteed by Eq. (3.5).

For the coupling to conformal supergravity, it is important to derive consistent super-Weyl transformations of the matter multiplets. One can prove that the transformation (remember that $\mathcal{C}_{ij} = (\mathbf{L}_{ij} - \mathbf{R}_{ij})$)

$$\tilde{\delta}Q^{(m,n)} = \left(\frac{m+n}{2}\mathbf{S} - \mathbf{S}^{kl}\mathcal{C}_{kl}\right)Q^{(m,n)} \tag{3.7}$$

preserves the analyticity conditions (3.6). Note the presence of the $SU(2)_{\mathcal{C}}$ term in (3.7) which is due to the compensating $SU(2)_{\mathcal{C}}$ transformations that appear in the super-Weyl transformation of the minimal supergravity covariant derivatives (1.13).

Let us also remind that, if $Q^{(m,n)}(z,u^{\oplus},v^{\boxplus})$ is a bi-projective multiplet, its complex conjugate is not covariantly analytic. However, one can introduce a generalized, analyticity-preserving conjugation, $Q^{(m,n)} \to \widetilde{Q}^{(m,n)}$, defined as

$$\widetilde{Q}^{(m,n)}(u^{\oplus},v^{\boxplus}) \equiv \bar{Q}^{(m,n)}\left(\overline{u^{\oplus}} \to \widetilde{u}^{\oplus}, \overline{v^{\boxplus}} \to \widetilde{v}^{\boxplus}\right), \tag{3.8a}$$

$$\widetilde{u}^{\oplus} = i\sigma_2 u^{\oplus}, \quad \widetilde{v}^{\boxplus} = i\sigma_2\, v^{\boxplus}, \tag{3.8b}$$

with $\bar{Q}^{(m,n)}(\overline{u^{\oplus}},\overline{v^{\boxplus}})$ the complex conjugate of $Q^{(m,n)}$ and $\overline{u^{\oplus}},\overline{v^{\boxplus}}$ the complex conjugates of $u^{\oplus},v^{\boxplus}$. Then $\widetilde{Q}^{(m,n)}(z,u^{\oplus},v^{\boxplus})$ is a weight-(m,n) bi-projective multiplet. One can see that $\widetilde{\widetilde{Q}}^{(m,n)} = (-1)^{m+n}Q^{(m,n)}$, and therefore real supermultiplets can be consistently defined when $(m+n)$ is even.

The simplest example of bi-projective superfield is given by the covariant twisted-II multiplet (TM-II) [28]. Consider a superfield T_{ij} satisfying a set of analyticity-like differential constraints [39]

$$\nabla_{+(k}T_{i)j} = \bar{\nabla}_{+(k}T_{i)j} = 0, \quad \nabla_{-(k}T_{|i|j)} = \bar{\nabla}_{-(k}T_{|i|j)} = 0. \tag{3.9}$$

The superfield T_{ij} is a Lorentz scalar and possesses the $SU(2)$ transformations

$$\mathbf{L}_{kl}T_{ij} = \frac{1}{2}C_{i(k}T_{l)j}, \quad \mathbf{R}_{kl}T_{ij} = \frac{1}{2}C_{j(k}T_{|i|l)}. \tag{3.10}$$

Note that T_{ij} has no symmetry conditions imposed in the i and j indices but satisfies the reality condition $(T_{ij})^* = T^{ij}$.

We have already seen an example of TM-II described by the super-Weyl transformation parameters $(\mathbf{S},\mathbf{S}_{ij})$ constrained by (1.14). In fact, if one decomposes T_{ij} in its symmetric and antisymmetric parts $T_{ij} = W_{ij} + (1/2)C_{ij}F$, where $W_{ij} = W_{ji}$ and both W_{ij} and F are real $(W_{ij})^* = W^{ij}$, $(F)^* = F$, then the constraints (3.9) are equivalent to (1.14) with (F, W_{ij}) taking the place of $(\mathbf{S},\mathbf{S}_{ij})$.

By contracting the $u^{\oplus}$, $v^{\boxplus}$ isotwistors with T_{ij}, the superfield $T^{\oplus\boxplus}(z,u,v)$ is defined as

$$T^{\oplus\boxplus}(u,v) := u_i^{\oplus}v_j^{\boxplus}T^{ij}. \tag{3.11}$$

Then, the analyticity conditions (3.9) are equivalent to (3.6). Moreover, the $SU(2)$ transformations (3.10) can be written exactly as Eqs. (3.2b), (3.2c) with $T^{\oplus\boxplus}$ considered as a weight-(1,1) isotwistor superfield. Therefore, $T^{\oplus\boxplus}$ satisfies all the conditions of a weight-(1,1) bi-projective superfield. By definition $T^{\oplus\boxplus}$ describes a regular holomorphic tensor field on the whole product of two complex projective spaces $\mathbb{C}P^1 \times \mathbb{C}P^1$. More general multiplets can have poles and more complicate analytic properties on $\mathbb{C}P^1 \times \mathbb{C}P^1$. For instance, one can easily define 2D bi-projective superfields with infinite number of superfields in a way completely analogue to the more studied curved 4D–5D cases [9–12]. The twisted-II multiplet plays a special role also because it represents the conformal compensator for the minimal supergravity.

The constraints of the covariant TM-II can be solved in terms of a prepotential described by the so-called covariant twisted-I multiplet (TM-I) [28]. The TM-I can be described by the superfields W, P and Q that are defined to be invariant under the action of the Lorentz $\mathcal{M}$ and $SU(2)$s $\mathbf{L}_{ij}$, $\mathbf{R}_{ij}$ generators. Moreover, the TM-I superfields are chosen to be invariant under super-Weyl transformations $\tilde{\delta}W = \tilde{\delta}P = \tilde{\delta}Q = 0$ and enjoy the following constraints[1] [31, 37, 38]:

$$\bar{\nabla}^i_\alpha W = 0, \quad \nabla_{\gamma k}Q = \frac{1}{2}(\gamma^3)_\gamma{}^\delta \bar{\nabla}_{\delta k}\bar{W}, \quad \nabla_{\alpha i}P = -\frac{i}{2}\bar{\nabla}_{\alpha i}\bar{W}, \tag{3.12a}$$

$$(W)^* = \bar{W}, \quad (P)^* = P, \quad (Q)^* = Q. \tag{3.12b}$$

In (3.12a) we have omitted some constraints that can be obtained by complex conjugation. The superfield $T^{\oplus\boxplus}$ of the TM-II can then be described in terms of a TM-I by the aid of the following equations [28]:

$$T^{\oplus\boxplus} = u_i^\oplus v_j^\boxplus T^{ij} = \frac{i}{4}u_i^\oplus v_j^\boxplus[\nabla^i_+, \nabla^j_-]W = \frac{i}{4}u_i^\oplus v_j^\boxplus[\bar{\nabla}^i_+, \bar{\nabla}^j_-]\bar{W} = u_i^\oplus v_j^\boxplus (T_{ij})^* = \widetilde{(T^{\oplus\boxplus})}. \tag{3.13}$$

We can now provide a bi-projective superfield action principle. This is invariant under the supergravity gauge group and super-Weyl transformations. Let the Lagrangian $\mathcal{L}^{(0,0)}$ be a real bi-projective superfield of weight-$(0,0)$. Consider a TM-II described by $T^{\oplus\boxplus}$ with $W, (\bar{W})$ the chiral superfield of the TM-I prepotential. Associated with $\mathcal{L}^{(0,0)}$, we introduce the action principle

$$S = \frac{1}{4\pi^2}\oint (u^\oplus du^\oplus)\oint (v^\boxplus dv^\boxplus)\int d^2x d^8\theta E \frac{W\bar{W}}{(T^{\oplus\boxplus})^2}\mathcal{L}^{(0,0)}, \quad E^{-1} = \mathrm{Ber}\,(E_A{}^M). \tag{3.14}$$

By construction, the functional is invariant under the rescaling $u_i^\oplus(t) \to c_L(t)\, u_i^\oplus(t)$, for an arbitrary function $c_L(t) \in \mathbb{C}\backslash\{0\}$, where t denotes the evolution parameter along the first closed integration contour. Similarly, (3.14) is invariant under rescalings $v_I^\boxplus(s) \to c_R(s)\, v_I^\boxplus(s)$, for an arbitrary function $c_R(s) \in \mathbb{C}\backslash\{0\}$, where s denotes the evolution parameter along the second closed integration contour. Note that (3.14) has clear similarities with the action principles in four- and five-dimensional curved projective superspace [9–12].

The action (3.14) can be proved to be invariant under arbitrary local supergravity gauge transformations (1.7). The invariance under general coordinates and Lorentz transformations is trivial. One can prove the invariance under the two $SU(2)_L$ and $SU(2)_R$, and then $SU(2)_\mathcal{V}$ in (1.7), transformations. By using that under super-Weyl transformations E varies like $\tilde{\delta}E = 2\mathbf{S}E$ and the transformations $\tilde{\delta}\mathcal{L}^{(0,0)} = -\mathbf{S}^{kl}\mathcal{C}_{kl}\mathcal{L}^{(0,0)}$, $\tilde{\delta}T^{\oplus\boxplus} = (\mathbf{S} - \mathbf{S}^{kl}\mathcal{C}_{kl})T^{\oplus\boxplus}$ and $\tilde{\delta}W = W$, one sees that S is super-Weyl invariant. Moreover, it is important to note that one can prove [28] that if $\mathcal{L}^{(0,0)}$ is a function of some supermultiplets to which the TM-II

[1]Note that the covariant TM-I constraints given here are equivalent to the differential constraints (1.12) of the torsion components $N, \mathcal{S}$ and $\mathcal{T}$ of the minimal supergravity multiplet. However, the two multiplets possess a crucial difference: the superfields (W, P, Q) are invariant under super-Weyl transformations, while $(N, \mathcal{S}, \mathcal{T})$ are not and transform inhomogeneously according to (1.15a)–(1.15c). This difference emphasizes that, even if they consistently satisfy the same differential constraints, $(W,\ P,\ Q)$ are matter superfields, while $(N, \mathcal{S}, \mathcal{S})$ are supregravity torsion components.

compensator does not belong, then the action S is independent of the superfields $T^{\oplus\boxplus}$, W and $\bar{W}$ chosen.

It would be clearly of interest to reduce the bi-projective action principle (3.14) to components and find the bi-projective density operator analogously to the chiral action of Sec. 2. One could derive the action (3.14) in components by using the «projective-invariance» techniques similarly to the 5D $\mathcal{N}=1$ [9] and 4D $\mathcal{N}=2$ [16] cases. Alternatively, and more interestingly, one could use ectoplasm [25–27] or normal coordinates techniques [23, 24].

4. A BI-PROJECTIVE PREPOTENTIAL FOR THE COVARIANT TM-I

This section is devoted to some new results on the bi-projective superspace formalism of [28]. In particular, here we give the solution of the covariant twisted-I multiplet constraints (3.12a) in terms of a weight-$(-1,-1)$ real but otherwise unconstrained bi-projective superfield $V^{(-1,-1)}$. Although in this paper for simplicity we are focusing on the minimal supergravity described in Sec. 1, it is important to point out that all the results in this section remain true without any modifications if one considers the extended $SU(2)_L \times SU(2)_R$ superspace supergravity geometry of [28].

Let us start by giving the result. Consider the superfields

$$W = \frac{1}{4\pi^2}\oint\frac{(u^\oplus du^\oplus)}{(u^\oplus u^\ominus)}\oint\frac{(v^\boxplus dv^\boxplus)}{(v^\boxplus v^\boxminus)}\bar{\nabla}_+^\ominus\bar{\nabla}_-^\boxminus V^{(-1,-1)}, \tag{4.15a}$$

$$X = -\frac{1}{4\pi^2}\oint\frac{(u^\oplus du^\oplus)}{(u^\oplus u^\ominus)}\oint\frac{(v^\boxplus dv^\boxplus)}{(v^\boxplus v^\boxminus)}\bar{\nabla}_+^\ominus\nabla_-^\boxminus V^{(-1,-1)}; \tag{4.15b}$$

these turn out to describe a covariant twisted-I multiplet where the superfields P and Q have been reabsorbed into the complex superfield X defined as

$$X = Q + iP, \quad \bar{X} = (X)^*. \tag{4.16}$$

According to (4.15a), (4.15b), and provided that $V^{(-1,-1)}$ is a weight-$(-1,-1)$ bi-projective superfield, the W and X superfields are invariant under Lorentz, $SU(2)_L$, $SU(2)_R$ and super-Weyl transformations. Moreover, they satisfy the following differential constraints:

$$\bar{\nabla}_+^i W = 0, \qquad \bar{\nabla}_+^i X = 0, \qquad \bar{\nabla}_+^i \bar{X} = -\nabla_+^i W, \tag{4.17a}$$

$$\bar{\nabla}_-^i W = 0, \qquad \nabla_{-i} X = 0, \qquad \bar{\nabla}_-^i X = \nabla_-^i W. \tag{4.17b}$$

The previous equations, once used (4.16), are indeed equivalent to (3.12a).

Let us provide some details of the proof that, as stated above, W and X defined in terms of $V^{(-1,-1)}$ satisfy all the properties of the covariant TM-I.

First, let us note that it holds

$$\oint\frac{(u^\oplus du^\oplus)}{(u^\oplus u^\ominus)}\oint\frac{(v^\boxplus dv^\boxplus)}{(v^\boxplus v^\boxminus)}\{\bar{\nabla}_+^\ominus, \bar{\nabla}_-^\boxminus\}V^{(-1,-1)} = 0. \tag{4.18}$$

Analogously, the integral $\oint\frac{(u^\oplus du^\oplus)}{(u^\oplus u^\ominus)}\oint\frac{(v^\boxplus dv^\boxplus)}{(v^\boxplus v^\boxminus)}\{\bar{\nabla}_+^\ominus, \nabla_-^\boxminus\}V^{(-1,-1)}$ is also zero. Then, one can freely anticommute the derivatives and consider only the commutator part in Eqs. (4.15a), (4.15b). Equation (4.18) can be proved by using the minimal covariant derivatives algebra, the following relation:

$$\mathcal{V}^{\ominus\boxminus}V^{(-1,-1)} = -D^{\ominus\ominus}(u^{\oplus}v^{\boxminus})V^{(-1,-1)} - D^{\boxminus\boxminus}(v^{\boxplus}u^{\ominus})V^{(-1,-1)}, \tag{4.19}$$

which easily follows from (3.2b), (3.2c), and by using the fact that it holds

$$\oint \frac{(u^{\oplus}du^{\oplus})}{(u^{\oplus}u^{\ominus})}D^{\ominus\ominus}f_L^{(0)}(u^{\oplus}) = 0, \quad \oint \frac{(v^{\boxplus}dv^{\boxplus})}{(v^{\boxplus}v^{\boxminus})}D^{\boxminus\boxminus}f_R^{(0)}(v^{\boxplus}) = 0 \tag{4.20}$$

for any function $f_L^{(0)}(u^{\oplus})$ homogeneous of degree zero in $u^{\oplus}$ and any function $f_R^{(0)}(v^{\boxplus})$ homogeneous of degree zero in $v^{\boxplus}$.

It is important to note that W and X do not depend on the isotwistors $u^{\ominus}$ and $v^{\boxminus}$, even if they explicitly appear on the right-hand side of (4.15a), (4.15b). In particular, (4.15a), (4.15b) are invariant under arbitrary «projective» transformations of the form

$$(u_i^{\ominus}, u_i^{\oplus}) \to (u_i^{\ominus}, u_i^{\oplus})P_L, \qquad P_L = \begin{pmatrix} a_L & 0 \\ b_L & c_L \end{pmatrix} \in GL(2,\mathbb{C}), \tag{4.21}$$

$$(v_i^{\boxminus}, v_i^{\boxplus}) \to (v_i^{\boxminus}, v_i^{\boxplus})P_R, \qquad P_R = \begin{pmatrix} a_R & 0 \\ b_R & c_R \end{pmatrix} \in GL(2,\mathbb{C}). \tag{4.22}$$

These transformations express the homogeneity with respect to $u^{\oplus}$, $v^{\boxplus}$ and the independence of $u^{\ominus}$, $v^{\boxminus}$. The invariance of (4.15a), (4.15b) under the a and c part of the transformations is trivial. Let us see that it is true also for b-transformations. For example, consider $\delta_{b_R}v^{\ominus} = b_R v^{\oplus}$ in (4.15a)

$$\delta_{b_R}W = \frac{1}{4\pi^2}\oint \frac{(u^{\oplus}du^{\oplus})}{(u^{\oplus}u^{\ominus})}\oint \frac{(v^{\boxplus}dv^{\boxplus})}{(v^{\boxplus}v^{\boxminus})} b_R \bar{\nabla}_+^{\ominus}\bar{\nabla}_-^{\boxplus}V^{(-1,-1)} = 0, \tag{4.23}$$

which is zero being $V^{(-1,-1)}$ a bi-projective superfield. By using (4.18) and then considering the b_L-transformation, it similarly holds that $\delta_{b_L}W = 0$. Analogously, the invariance under (4.21), (4.22) of the right-hand side of (4.15b) follows.

The Lorentz invariance of (4.15a), (4.15b) is trivial. Let us prove the $SU(2)$ invariance. By using (3.2c), (1.4a) and then (4.20), one can prove

$$\mathbf{R}_{kl}W = -\frac{1}{4\pi^2}\oint \frac{(u^{\oplus}du^{\oplus})}{(u^{\oplus}u^{\ominus})}\oint \frac{(v^{\boxplus}dv^{\boxplus})}{(v^{\boxplus}v^{\boxminus})}D^{\boxminus\boxminus}\frac{v^{\boxplus}_{(k}v^{\boxplus}_{l)}}{2(v^{\boxplus}v^{\boxminus})}\bar{\nabla}_+^{\ominus}\bar{\nabla}_-^{\boxminus}V^{(-1,-1)} = 0. \tag{4.24}$$

In a very similar manner one obtains W and X under the action of $\mathbf{L}_{kl}$ and $\mathbf{R}_{kl}$.

By using Eqs. (1.13), (1.4a), (1.4b), (3.2b)–(3.2d), (3.6) and (4.20), it is not difficult to prove that W and X in (4.15a), (4.15b) are invariant under super-Weyl transformations. We leave this computation to the reader.

To prove that W and X describe a covariant TM-I, it is left to prove that (4.17a), (4.17b) are satisfied. Let us prove that W in (4.15a) is chiral. By using

$$(u^{\oplus}u^{\ominus})\delta_j^i = (u^{\oplus i}u_j^{\ominus} - u_j^{\oplus}u^{\ominus i}) \tag{4.25}$$

and the analyticity of $V^{(-1,-1)}$, we find

$$\bar{\nabla}^i_+ W = \frac{1}{4\pi^2}\oint \frac{(u^\oplus du^\oplus)}{(u^\oplus u^\ominus)^2}\oint \frac{(v^\boxplus dv^\boxplus)}{(v^\boxplus v^\boxminus)}\Big(\frac{1}{2}u^{\oplus i}\{\bar{\nabla}^\ominus_+, \bar{\nabla}^\ominus_+\}\bar{\nabla}^\boxminus_- - u^{\ominus i}\{\bar{\nabla}^\oplus_+, \bar{\nabla}^\ominus_+\}\bar{\nabla}^\boxminus_- + \\ + u^{\ominus i}\bar{\nabla}^\ominus_+\{\bar{\nabla}^\oplus_+, \bar{\nabla}^\boxminus_-\}\Big)V^{(-1,-1)}. \quad (4.26)$$

By considering that it holds

$$\{\bar{\nabla}^i_+, \bar{\nabla}^j_+\} = 0, \quad \{\bar{\nabla}^i_+, \bar{\nabla}^j_-\} = -4C^{ij}\bar{T}\mathcal{M} - 4\bar{T}\mathcal{V}^{ij} \quad (4.27)$$

and

$$\mathcal{V}^{\oplus\boxminus}V^{(-1,-1)} = D^{\boxminus\boxminus}(u^\oplus v^\boxplus)V^{(-1,-1)}, \quad (4.28)$$

which follows from (3.2b), (3.2c), one easily obtains

$$\bar{\nabla}^i_+ W = 0. \quad (4.29)$$

Similarly, one finds that $\bar{\nabla}^i_- W = 0$ and then $\bar{\nabla}^i_\alpha W = 0$. Analogously, it can be derived that $\bar{\nabla}^i_+ X = 0$ and $\nabla_{-i}X = 0$.

Let us now turn our attention to the equation $\bar{\nabla}_{+i}\bar{X} = -\nabla_{+i}W$ in (4.17a). We obtain

$$\nabla_{+i}W = \frac{1}{4\pi^2}\oint \frac{(u^\oplus du^\oplus)}{(u^\oplus u^\ominus)^2}\oint \frac{(v^\boxplus dv^\boxplus)}{(v^\boxplus v^\boxminus)}\Big(u^\oplus_i \nabla^\ominus_+\bar{\nabla}^\ominus_+\bar{\nabla}^\boxminus_- - u^\ominus_i\{\nabla^\oplus_+, \bar{\nabla}^\ominus_+\}\bar{\nabla}^\boxminus_- + \\ + u^\ominus_i\bar{\nabla}^\ominus_+\{\nabla^\oplus_+, \bar{\nabla}^\boxminus_-\}\Big)V^{(-1,-1)}. \quad (4.30)$$

By using the minimal supergravity anticommutators

$$\{\nabla^i_+, \bar{\nabla}^j_+\} = 2iC^{ij}\nabla_{++}, \quad \{\nabla_{+i}, \bar{\nabla}_{-j}\} = 4iC_{ij}\mathcal{S}\mathcal{M} - 4C_{ij}\mathcal{T}\mathcal{M} + 4\mathcal{T}\mathcal{V}_{ij} - 4i\mathcal{S}\mathcal{V}_{ij}, \quad (4.31)$$

it follows that

$$\nabla_{+i}W = \frac{1}{4\pi^2}\oint \frac{(u^\oplus du^\oplus)}{(u^\oplus u^\ominus)}\oint \frac{(v^\boxplus dv^\boxplus)}{(v^\boxplus v^\boxminus)}\left(\frac{u^\oplus_i}{(u^\oplus u^\ominus)}\nabla^\ominus_+\bar{\nabla}^\ominus_+\bar{\nabla}^\boxminus_- + 2iu^\ominus_i\nabla_{++}\bar{\nabla}^\boxminus_-\right)V^{(-1,-1)}. \quad (4.32)$$

Next, we compute $\bar{\nabla}_{+i}\bar{X}$

$$\bar{\nabla}_{+i}\bar{X} = \frac{1}{4\pi^2}\oint \frac{(u^\oplus du^\oplus)}{(u^\oplus u^\ominus)^2}\oint \frac{(v^\boxplus dv^\boxplus)}{(v^\boxplus v^\boxminus)}\Big(-u^\oplus_i\nabla^\ominus_+\bar{\nabla}^\ominus_+\bar{\nabla}^\boxminus_- + u^\oplus_i\{\nabla^\ominus_+, \bar{\nabla}^\ominus_+\}\bar{\nabla}^\boxminus_- - \\ - u^\ominus_i\{\nabla^\ominus_+, \bar{\nabla}^\oplus_+\}\bar{\nabla}^\boxminus_- + u^\ominus_i\nabla^\ominus_+\{\bar{\nabla}^\oplus_+, \bar{\nabla}^\boxminus_-\}\Big)V^{(-1,-1)}. \quad (4.33)$$

One can simplify the previous equation and prove that $\bar{\nabla}^i_+\bar{X} = -\nabla^i_+ W$. Analogous computations can be used to derive $\bar{\nabla}^i_- X = \nabla^i_- W$. Then, W and X in Eqs. (4.15a), (4.15b) satisfy all the defining properties of the covariant type-I twisted multiplet.

Let us conclude by giving some comments on the results of this section. First of all, the prepotential solution of the TM-I has clear analogies with the weight-zero real projective prepotential of the chiral field strength of an Abelian vector multiplet in 4D $\mathcal{N}=2$ superspace supergravity [11,12]. We remind that in the 4D case, the projective prepotential $V^{(0)}$ possesses the gauge freedom $\delta_g V^{(0)} = \Lambda^{(0)} + \widetilde{\Lambda}^{(0)}$ where $\Lambda^{(0)}$ is a weight-zero covariantly arctic superfield and $\widetilde{\Lambda}^{(0)}$ is its analyticity preserved conjugate. The 4D arctic superfield $\Lambda^{(0)}$ is such that in the north chart of $\mathbb{C}P^1$ it does not possess any poles. In our 2D bi-projective case the solution (4.15a), (4.15b) turns out to possess a gauge freedom

$$\delta_g V^{(-1,-1)} = \Lambda_L^{(-1,-1)} + \widetilde{\Lambda}_L^{(-1,-1)} + \Lambda_R^{(-1,-1)} + \widetilde{\Lambda}_R^{(-1,-1)}, \tag{4.34}$$

generated by the superfields $\Lambda_L^{(-1-1)}(u^{\oplus}, v^{\boxplus})$ and $\Lambda_R^{(-1-1)}(u^{\oplus}, v^{\boxplus})$ together with their conjugates. Here the superfields $\Lambda_L^{(-1-1)}$ and $\Lambda_R^{(-1-1)}$ are such that $\Lambda_L^{(-1-1)}$ does not possess poles on the north chart of the left $\mathbb{C}P^1$ having homogeneous coordinates $u^{\oplus}$ and $\Lambda_L^{(-1-1)}$ does not possess poles on the north chart of the right $\mathbb{C}P^1$ having homogeneous coordinates $v^{\boxplus}$. In this way either the $u^{\oplus}$ or the $v^{\boxplus}$ contour integral in the definition of the field strengths (4.15a), (4.15b) is zero.

Considering that the covariant TM-I describes a prepotential for the TM-II, it is clear that one can solve the type-II twisted multiplet constraints in terms of $V^{(-1,-1)}$ by using Eqs. (4.15a) and (3.13)[1]. Now, given a TM-II described by the superfield $\mathbf{T}^{\oplus\boxplus}$ and its projective prepotential $\mathbf{V}^{(-1,-1)}$, one can construct an action by considering the bi-projective Lagrangian[2]

$$\boldsymbol{\mathcal{L}}^{(0,0)} = \mathbf{V}^{(-1,-1)}\mathbf{T}^{\oplus\boxplus}. \tag{4.35}$$

The action (3.14), with the previous Lagrangian, is then invariant under (4.34).

To conclude we stress again that, if one considers the $SO(1,1)\times SU(2)_L \times SU(2)_R$ extended supergravity geometry of [28], all the main results in this section, in particular (4.15a), (4.15b), remain unchanged even if the computations described here become a bit more complicated.

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[1]It is worth noting that in the flat case a similar prepotential solution of the TM-II constraints has been described by Siegel in [45] by using a form of bi-projective superspace.

[2]Here the TM-II and its prepotential do not have to be the supergravity conformal compensator; this is why we have used the bold characters to distinguish it by the one in (3.14).

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