Письма в ЭЧАЯ. 2011. Т. 8, № 3(166). С. 452–461

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

LIGHT-CONE GAUGE FORMULATION FOR $AdS_4 \times \mathbb{CP}^3$ SUPERSTRING

D. V. Uvarov [1]

NSC Kharkov Institute of Physics and Technology, Kharkov, Ukraine

We review the Type IIA superstring on the $AdS_4 \times \mathbb{CP}^3$ background in the κ-symmetry light-cone gauge characterized by the choice of the lightlike directions from the $D = 3$ Minkowski boundary of AdS_4 both in the Lagrangian and Hamiltonian formulations.

PACS: 11.25.-w

INTRODUCTION

Testing of the AdS_4/CFT_3 duality proposal by Aharony, Bergman, Jafferis, and Maldacena (ABJM) [1] attracts significant attention. There exists the limit in the parameter space (N, k), where both of them go to infinity with their ratio. The 't Hooft coupling $\lambda = N/k$ in the $D = 3$ $\mathcal{N} = 6$ superconformal Chern–Simons-matter theory is kept fixed, and the dual bulk theory, that is, the M-theory on the $AdS_4 \times (S^7/\mathbf{Z}_k)$ background in this limit admits a description as free IIA superstring on $AdS_4 \times \mathbb{CP}^3$. Thus one arrives at some analog of the well-known AdS_5/CFT_4 duality [2] also involving the string theory in the bulk space although of Type IIB on $AdS_5 \times S^5$ background. This motivated broad application of the methods, used to study the AdS_5/CFT_4 duality, in particular, those of the integrable systems [2], and of the results obtained there to the ABJM duality case. However, the challenging feature of the AdS_4/CFT_3 correspondence of ABJM type is that there is less symmetry on both its sides compared with the maximally symmetric AdS_5/CFT_4 duality, and hence not all the quantities are determined by the symmetry properties.

This can be well illustrated already at the level of the classical string theory on the $AdS_4 \times \mathbb{CP}^3$ background that preserves 24 out of the 32 Type IIA supersymmetries and whose superisometry is described by the $OSp(4|6)$ supergroup. The initial proposal [4,5] to construct the string action on such a background using the supercoset approach [6,7] was based on the observation that the isometry groups $SO(2,3)$ and $SU(4)$ of the $AdS_4 = SO(2,3)/SO(1,3)$ and $\mathbb{CP}^3 = SU(4)/U(3)$ parts of the background match the bosonic subgroup $SO(2,3)\times SU(4)$ of the $OSp(4|6)$ supergroup. The resulting $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset action is the functional of 10 bosonic and 24 fermionic coordinates of the «reduced» superspace invariant under the $OSp(4|6)$ isometry supergroup of the $AdS_4 \times \mathbb{CP}^3$ background. It is by construction classically integrable and invariant under the 8-parameter κ-symmetry transformations

[1] E-mail: d_uvarov@hotmail.com, uvarov@kipt.kharkov.ua

[2] For the collection of recent reviews see [3].

allowing one to balance bosonic and fermionic physical degrees of freedom. In its turn it corresponds to the full Type IIA $AdS_4 \times \mathbb{CP}^3$ superstring action [8], in which the κ symmetry has been partially fixed by putting to zero those coordinates associated with 8 broken supersymmetries. Besides that, such gauge choice narrows down the possibilities of further gauge fixing and hence the action simplification within the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset model, it has the limited range of validity restricted to the classical string configurations propagating in $\mathbb{CP}^3$ or both parts of the background [4]. This motivates considering gauge conditions other than that used to obtain the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset action. One instance of such a gauge was proposed in [9]. In [10], we have considered another one, namely the fermionic light-cone gauge for which both lightlike directions are on the $D = 3$ Minkowski boundary of the AdS_4 space and the fermionic coordinates positively charged w.r.t. $SO(1,1)$ light-cone isometry group are put to zero . The resulting gauge-fixed Lagrangian includes fermions up to the 4th power and manifestly exhibits the $SU(3)$ symmetry subgroup of the $SU(4)$ global symmetry of $\mathbb{CP}^3$. To make contact with the $3D$-extended superconformal symmetry, that is, the symmetry group on the gauge side of ABJM duality, we have elaborated on the realization of both $osp(4|6)$ and $osp(4|8)$ superalgebras as $D = 3$ extended superconformal algebras [10, 11].

1. $AdS_4 \times \mathbb{CP}^3$ SUPERSTRING IN THE LIGHT-CONE GAUGE: LAGRANGIAN AND HAMILTONIAN APPROACHES

The complete $AdS_4 \times \mathbb{CP}^3$ superstring action [8] does not correspond to a $2D$-supercoset sigma-model but can be obtained by performing, by the double-dimensional reduction [12] of the $D = 11$ supermembrane, the action on the maximally supersymmetric $AdS_4 \times S^7$ background [13] given by

$$S = -\int\limits_V d^3\xi \sqrt{-g^{(3)}} + S_{\rm WZ}, \tag{1}$$

where $g^{(3)}$ is the determinant of the induced world-volume metric

$$g^{(3)}_{\underline{ij}} = \hat{E}^{\hat{m}}_{\underline{i}} \hat{E}_{\underline{j}\hat{m}}, \tag{2}$$

and the Wess–Zumino (WZ) term

$$S_{\rm WZ} = \pm \frac{1}{4} \int\limits_{\mathcal{M}_4} H_{(4)} \tag{3}$$

is presented as the integral of the closed 4-form

$$H_{(4)} = \frac{i}{2} \hat{E}^{\hat{\alpha}} \wedge \mathfrak{g}^{\hat{m}\hat{n}}{}_{\hat{\alpha}}{}^{\hat{\beta}} \hat{E}_{\hat{\beta}} \wedge \hat{E}_{\hat{m}} \wedge \hat{E}_{\hat{n}} + \varepsilon_{m'n'k'l'} \hat{E}^{m'} \wedge \hat{E}^{n'} \wedge \hat{E}^{k'} \wedge \hat{E}^{l'} \tag{4}$$

over the 4-dimensional auxiliary hypersurface $\mathcal{M}_4$, whose boundary coincides with the supermembrane world volume V. The $D = 11$ supervielbein bosonic components $\hat{E}^{\hat{m}} = (\hat{E}^{m'}, \hat{E}^{I'}) = (G^{0'm'}, \Omega^{8I'})$ consist of the AdS_4 and S^7 vielbeins $G^{0'm'}(d)$ and $\Omega^{8I'}(d)$, that are the Cartan forms corresponding to the $so(2,3)/so(1,3)$ and $so(8)/so(7)$ generators $M_{0'm'}$

and $V^{8'I'}$, respectively. Together with the fermionic 1-forms $\hat{E}^{\hat{\alpha}} \equiv \hat{E}^{\alpha A'}$ associated with the $osp(4|8)$ fermionic generators $O_{\hat{\alpha}} \equiv O_{\alpha A'}$, they satisfy the set of $osp(4|8)$ Maurer–Cartan (MC) equations [1]

$$\begin{aligned} dG^{0'm'} &= 2G^{m'}{}_{n'} \wedge G^{0'n'} + \frac{i}{4}\hat{E}^{\alpha A'} \wedge C_{A'B'}\Gamma^{m'}_{\alpha\beta}\hat{E}^{\beta B'}, \\ d\Omega^{8I'} &= 2\Omega^{I'J'} \wedge \Omega^{8J'} - \frac{i}{4}\hat{E}^{\alpha A'} \wedge \Gamma^5_{\alpha\beta}\gamma^{I'}_{A'B'}\hat{E}^{\beta B'}, \\ d\hat{E}^{\alpha A'} &= -\frac{1}{2}G^{m'n'} \wedge \hat{E}^{\beta A'}\Gamma_{m'n'\beta}{}^{\alpha} - G^{0'm'} \wedge \hat{E}^{\beta A'}\Gamma_{m'\beta}{}^{\gamma}\Gamma^5{}_{\gamma}{}^{\alpha} + \\ &\qquad + \frac{1}{2}\Omega^{I'J'} \wedge \hat{E}^{\alpha B'}\gamma^{I'J'}{}_{B'}{}^{A'} - \frac{1}{2}\Omega^{8I'} \wedge \hat{E}^{\alpha B'}\gamma^{I'}{}_{B'}{}^{A'}. \end{aligned} \tag{5}$$

The action (1) is by construction invariant under the κ-symmetry transformations with the local $D = 11$ Majorana spinor parameter $\kappa_{\hat{\beta}}(\xi)$

$$\hat{E}_{\hat{\alpha}}(\delta_\kappa) = \Pi_{\hat{\alpha}}{}^{\hat{\beta}}\kappa_{\hat{\beta}}(\xi), \tag{6}$$

where the matrix $\Pi_{\hat{\alpha}}{}^{\hat{\beta}}$ has the form

$$\Pi_{\hat{\alpha}}{}^{\hat{\beta}} = \delta^{\hat{\beta}}_{\hat{\alpha}} \mp \frac{1}{6\sqrt{-g^{(3)}}}\varepsilon^{\underline{ijk}}\hat{E}^{\hat{m}_1}_{\underline{i}}\hat{E}^{\hat{m}_2}_{\underline{j}}\hat{E}^{\hat{m}_3}_{\underline{k}}\mathfrak{g}_{\hat{m}_1\hat{m}_2\hat{m}_3\hat{\alpha}}{}^{\hat{\beta}}, \tag{7}$$

accompanied by $\hat{E}^{\hat{m}}(\delta_\kappa) = 0$. The matrix $\Pi_{\hat{\alpha}}{}^{\hat{\beta}}$ has the properties of the spinor projector that eliminates half components of the transformation parameter $\kappa_{\hat{\alpha}}(\xi)$.

The double-dimensional reduction of the $AdS_4 \times S^7$ supermembrane (1) to the $D = 10$ Type IIA superstring on $AdS_4 \times \mathbb{CP}^3$ proceeds following the general prescription elaborated in [12]. The form of the Kaluza–Klein ansatz for the $D = 11$ supervielbein bosonic components is

$$\hat{E}^{\hat{m}} = \{\mathrm{E}^{\hat{m}'},\ \mathrm{E}^{11} = \Phi(dy + A)\}, \tag{8}$$

where $\mathrm{E}^{\hat{m}'}$ are the $D = 10$ supervielbein bosonic components; A is the RR 1-form superfield, and $\Phi = \mathrm{e}^{2\phi/3}$ is related to the $D = 10$ IIA dilaton superfield ϕ. All of them do not depend neither on the target-space compact coordinate $y \in [0, 2\pi)$, to be identified with the world-volume compact direction, nor on its differential dy. For the fermionic components of the $D = 11$ supervielbein the reduction ansatz is

$$\hat{E}^{\hat{\alpha}} = \mathrm{E}^{\hat{\alpha}} + \mathrm{e}^{-2\phi/3}\chi^{\hat{\alpha}}\mathrm{E}^{11}, \tag{9}$$

where $\mathrm{E}^{\hat{\alpha}}$ are the $D = 10$ supervielbein fermionic components, and $\chi^{\hat{\alpha}}$ is the dilatino superfield. They are also chosen to be y- and dy-independent. The peculiarity in the present case is that the bosonic components of the supervielbein as corresponding to the $so(2,3)/so(1,3)$ and $su(4)/u(3)$ Cartan forms do receive contributions $\hat{E}^{\hat{m}'}_y$ proportional to

[1] The details of the γ-matrix algebra, spinor conventions and $osp(4|8)$ superalgebra are given in [10].

the differential dy of the target-space compact coordinate y so that the local Lorentz rotation in the tangent space has to be performed

$$\hat{E}^{\hat{m}} \to L^{\hat{m}}{}_{\hat{n}}\hat{E}^{\hat{n}}, \quad \hat{E}^{\hat{\alpha}} \to L^{\hat{\alpha}}{}_{\hat{\beta}}\hat{E}^{\hat{\beta}}, \quad L^{\hat{m}}{}_{\hat{n}} \in SO(1,10),\ L^{\hat{\alpha}}{}_{\hat{\beta}} \in \mathrm{Spin}(1,10) \qquad (10)$$

with the parameters determined by $\hat{E}_y^{\hat{m}'}$ to bring the supervielbein bosonic components to the Kaluza–Klein ansatz form (8).

The final necessary ingredient to perform the dimensional reduction of the supermembrane on the $AdS_4 \times S^7$ background is the 7-sphere realization as the Hopf fibration $\mathbb{CP}^3 \times U(1)$ [14, 15]. The corresponding changes of the basis for the generators of the $so(8)$ subalgebra of the $osp(4|8)$ and the Cartan forms are described in [10].

As the outcome of the dimensional reduction procedure, the kinetic term of the membrane action (1) reduces to the Nambu–Goto string action in the Kaluza–Klein frame

$$\int\limits_V d^3\xi\sqrt{-g^{(3)}} \to \int\limits_\Sigma d\tau d\sigma \Phi\sqrt{-g^{(2)}}, \qquad (11)$$

where $g^{(2)} = \det g_{ij}^{(2)}$ is the determinant of the induced world-sheet metric

$$g_{ij}^{(2)} = E_i^{\hat{m}'} E_{\hat{m}'j}. \qquad (12)$$

The membrane WZ term (3) reduces to the integral of the NS-NS 3-form over the auxiliary 3-dimensional hypersurface $\mathcal{M}_3$, whose boundary coincides with the superstring world-sheet Σ

$$\int\limits_{\mathcal{M}_4} H_{(4)} \to \int\limits_{\mathcal{M}_3} H_{(3)}. \qquad (13)$$

The explicit form of the $osp(4|8)$ Cartan forms and the $AdS_4 \times \mathbb{CP}^3$ superstring action is governed by the choice of the $OSp(4|8)/(SO(1,3) \times SO(7))$ supercoset element. The parameterization of the $OSp(4|8)/(SO(1,3) \times SO(7))$ representative, we consider, is

$$\mathscr{G} = \mathscr{G}_{OSp(4|6)/(SO(1,3)\times U(3))}\, \mathrm{e}^{yH}\, \mathrm{e}^{\theta_4^\mu Q_\mu^4 + \bar\theta^{\mu 4}\bar{Q}_{\mu 4}}\, \mathrm{e}^{\eta_{\mu 4}S^{\mu 4} + \bar\eta_\mu^4 \bar{S}_4^\mu}, \qquad (14)$$

where

$$\mathscr{G}_{OSp(4|6)/(SO(1,3)\times U(3))} = \mathrm{e}^{x^m P_m + \theta_a^\mu Q_\mu^a + \bar\theta^{\mu a}\bar{Q}_{\mu a}}\, \mathrm{e}^{\eta_{\mu a}S^{\mu a} + \bar\eta_\mu^a \bar{S}_a^\mu}\, \mathrm{e}^{z^a T_a + \bar{z}_a T^a}\, \mathrm{e}^{\varphi D} \qquad (15)$$

is the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset element [11]. It is parameterized by the $D = 3$ $\mathcal{N} = 6$ super-Poincare coordinates $x^m, \theta_a^\mu, \bar\theta^{\mu a}$ supplemented by the coordinates $\eta_{\mu a}, \bar\eta_\mu^a$ associated with the superconformal generators. Bosonic complex coordinates $z^a, \bar{z}_a$ parameterize $\mathbb{CP}^3$, and the coordinate φ is related to the radial direction of AdS_4. Other eight fermionic coordinates $\theta_4^\mu, \bar\theta^{\mu 4}$ and $\eta_{\mu 4}, \bar\eta_\mu^4$ are associated with the IIA supersymmetries broken by $AdS_4 \times \mathbb{CP}^3$ background. The expressions for the $osp(4|6)$ Cartan forms corresponding to the supercoset element (15) have been derived in [11].

In general, both the $osp(4|6)$ and $osp(4|8)$ Cartan forms are highly nonlinear functions of the superspace coordinates. Some simplification can be achieved by fixing the κ-symmetry gauge. In [10], we have considered the following light-cone gauge condition on the fermions

$$\mathfrak{g}^{+2}{}_{\hat\alpha}{}^{\hat\beta}\Theta_{\hat\beta} = (\mathfrak{g}^0 + \mathfrak{g}^2)_{\hat\alpha}{}^{\hat\beta}\Theta_{\hat\beta} = 0, \qquad (16)$$

where 32×32 $D=10$ γ-matrices $\mathfrak{g}^{\hat m'}{}_{\hat\alpha}{}^{\hat\beta}$ have been defined in Appendix A of [10] and $\Theta_{\hat\beta}=(\theta_{\nu B},\eta^{\nu}_{B},\bar\theta^{B}_{\nu},\bar\eta^{\nu B})$, that in components reads

$$\theta^2_A=\bar\theta^{2A}=\eta_{1A}=\bar\eta^A_1=0. \tag{17}$$

Corresponding light-cone directions lie on the $D=3$ Minkowski boundary of AdS_4, and the gauge condition is characterized by setting to zero odd coordinates associated with the generators of $osp(4|8)$ negatively charged w.r.t. the $so(1,1)$ generator $M^{+-}\equiv 2M^{02}$ from the AdS_4 boundary Lorentz group. Remaining superspace Grassmann coordinates

$$\theta^1_A=\theta^-_A=\theta_A,\quad \bar\theta^{1A}=\bar\theta^{-A}=\bar\theta^A,\quad \eta^1_A=\eta^-_A=\eta_A,\quad \bar\eta^{1A}=\bar\eta^{-A}=\bar\eta^A \tag{18}$$

become fermionic Goldstone fields of the $AdS_4\times\mathbb{CP}^3$ superstring in the gauge (17) [1].

Then, the gauge-fixed superstring action in the Polyakov representation can be brought to the form

$$\mathscr{S}=-\frac{1}{2}\int\limits_{\Sigma}d\tau d\sigma\gamma^{ij}g^{(2)}_{ij}\pm\int\limits_{\Sigma}d\tau d\sigma B_{(2)}. \tag{19}$$

The induced world-sheet metric is given by [2]

$$g^{(2)}_{ij}=\frac{1}{4}g^{AdS}_{ij}+g^{CP}_{ij}-\frac{\mathrm{e}^{-2\varphi}}{2}(\partial_i x^+\varpi_j+\partial_j x^+\varpi_i)+ \\ +(\partial_i x^+\partial_j z^M+\partial_j x^+\partial_i z^M)q_M+B\partial_i x^+\partial_j x^+, \tag{20}$$

where the AdS_4 metric is written in the Poincare coordinates

$$g^{AdS}_{ij}=\mathrm{e}^{-4\varphi}\left[\frac{1}{2}(\partial_i x^+\partial_j x^-+\partial_j x^+\partial_i x^-)+\partial_i x^1\partial_j x^1\right]+4\partial_i\varphi\partial_j\varphi, \tag{21}$$

and $\mathbb{CP}^3$ metric in the complex coordinates $z^M=(z^a,\bar z_a)$ equals

$$g^{CP}_{ij}=g_{MN}\partial_i z^M\partial_j z^N=g_{ab}\partial_i z^a\partial_j z^b+g^{ab}\partial_i\bar z_a\partial_j\bar z_b+g_a{}^b(\partial_i z^a\partial_j\bar z_b+\partial_j z^a\partial_i\bar z_b) \tag{22}$$

with the components

$$\begin{aligned} g_{ab}=&\frac{1}{4|z|^4}(|z|^2-\sin^2|z|+\sin^4|z|)\bar z_a\bar z_b,\\ g^{ab}=&\frac{1}{4|z|^4}(|z|^2-\sin^2|z|+\sin^4|z|)z^a z^b,\\ g_a{}^b=&\frac{\sin^2|z|}{2|z|^2}\delta^b_a+\frac{1}{4|z|^4}(|z|^2-\sin^2|z|-\sin^4|z|)\bar z_a z^b,\quad |z|^2=z^a\bar z_a. \end{aligned} \tag{23}$$

---

[1] Observe that among the physical fermions there are those associated with the unbroken, as well as with the broken space-time supersymmetries.

[2] The factor $1/4$ in front of the AdS metric is inherited from the $AdS_4\times S^7$ background, where the radius of AdS_4 space is twice less than that of S^7 determined by the maximal supersymmetry condition.

The world-sheet projection of the NS-NS 2-form potential in the light-cone gauge reduces to

$$B_{(2)} = \varepsilon^{ij} \left(\tilde{\omega}_i \partial_j x^+ + \tilde{c} \partial_i x^1 \partial_j x^+ + \partial_i x^+ \partial_j z^M \tilde{q}_M \right). \tag{24}$$

In (20) and (24), we have also introduced the following notation:

$$\begin{aligned}
\varpi_i &= i\,\mathrm{e}^{-2\varphi}(\partial_i\theta_a\bar{\theta}^a - \theta_a\partial_i\bar{\theta}^a) + i(\partial_i\theta_4\bar{\theta}^4 - \theta_4\partial_i\bar{\theta}^4) + i\,\mathrm{e}^{2\varphi}(\partial_i\eta_a\bar{\eta}^a - \eta_a\partial_i\bar{\eta}^a) + \\
&\qquad + i(\partial_i\eta_4\bar{\eta}^4 - \eta_4\partial_i\bar{\eta}^4),\\
\tilde{\omega}_i &= \mathrm{e}^{-2\varphi}(\hat{\eta}_a\hat{\partial}_i\bar{\theta}^a + \hat{\partial}_i\theta_a\hat{\bar{\eta}}^a) + \frac{\mathrm{e}^{-2\varphi}}{2}(\partial_i\theta_4\bar{\eta}^4 - \partial_i\eta_4\bar{\theta}^4 + \eta_4\partial_i\bar{\theta}^4 - \theta_4\partial_i\bar{\eta}^4), \qquad (25)\\
B &= 4\,\mathrm{e}^{-2\varphi}\eta_4\bar{\eta}^4(\hat{\eta}_a\hat{\bar{\eta}}^a - \mathrm{e}^{-2\varphi}\theta_4\bar{\theta}^4),\ \tilde{c} = \mathrm{e}^{-2\varphi}(2\hat{\eta}_a\hat{\bar{\eta}}^a + \mathrm{e}^{-2\varphi}\,\Theta),\ \Theta = \theta_4\bar{\theta}^4 + \eta_4\bar{\eta}^4,\\
q_M &= \mathrm{e}^{-\varphi}(\Omega_{aM}\hat{\bar{\eta}}^a\bar{\eta}^4 - \Omega^a_M\hat{\eta}_a\eta_4) + \mathrm{e}^{-2\varphi}\,\Theta\tilde{\Omega}_a{}^a{}_M,\\
\tilde{q}_M &= 2i\,\mathrm{e}^{-\varphi}\left[\Omega_{aM}\hat{\bar{\eta}}^a\bar{\theta}^4 + \Omega^a_M\hat{\eta}_a\theta_4 + \mathrm{e}^{-\varphi}(\theta_4\bar{\eta}^4 - \eta_4\bar{\theta}^4)\tilde{\Omega}_a{}^a{}_M\right].
\end{aligned}$$

The hats indicate that the Grassmann coordinates have been acted upon by the matrix $T_{\hat{a}}{}^{\hat{b}}$ [11]

$$T_{\hat{a}}{}^{\hat{b}} = \begin{pmatrix} \delta_a^b \cos|z| + \bar{z}_a z^b \dfrac{(1-\cos|z|)}{|z|^2} & i\varepsilon_{acb} z^c \dfrac{\sin|z|}{|z|} \\ -i\varepsilon^{acb}\bar{z}_c \dfrac{\sin|z|}{|z|} & \delta_b^a \cos|z| + z^a \bar{z}_b \dfrac{(1-\cos|z|)}{|z|^2} \end{pmatrix} \tag{26}$$

as

$$\hat{d}\theta_a(\hat{\eta}_a) = T_a{}^b d\theta_b(\eta_b) + T_{ab} d\bar{\theta}^b(\bar{\eta}^b), \quad \hat{d}\bar{\theta}^a(\hat{\bar{\eta}}^a) = T^a{}_b d\bar{\theta}^b(\bar{\eta}^b) + T^{ab} d\theta_b(\eta_b). \tag{27}$$

The $su(4)/u(3)$ Cartan forms $\Omega_a(d), \Omega^a(d)$ define the complex $\mathbb{CP}^3$ vielbein

$$\Omega_a(d) = \Omega_{a,b} dz^b + \Omega_a{}^{,b} d\bar{z}_b, \quad \Omega^a(d) = \Omega^a{}_{,b} dz^b + \Omega^{a,b} d\bar{z}_b \tag{28}$$

with the components

$$\begin{aligned}
\Omega_{a,b} &= \bar{z}_a\bar{z}_b \frac{\sin|z|(1-\cos|z|)}{2|z|^3} + \bar{z}_a\bar{z}_b\left(\frac{1}{|z|} - \frac{\sin|z|}{|z|^2}\right),\\
\Omega^{a,b} &= z^a z^b \frac{\sin|z|(1-\cos|z|)}{2|z|^3} + z^a z^b\left(\frac{1}{|z|} - \frac{\sin|z|}{|z|^2}\right), \qquad (29)\\
\Omega_a{}^{,b} = \Omega^b{}_{,a} &= \frac{\sin|z|}{|z|}\delta_a^b - \bar{z}_a z^b \frac{\sin|z|(1-\cos|z|)}{2|z|^3} + \bar{z}_a z^b\left(\frac{1}{|z|} - \frac{\sin|z|}{|z|^2}\right)
\end{aligned}$$

such that $ds^2_{CP^3} = \Omega_a(d)\Omega^a(d)$. The 1-form $\tilde{\Omega}_a{}^a(d)$

$$\tilde{\Omega}_a{}^a(d) = i\,\frac{\sin^2|z|}{|z|^2}(dz^a\bar{z}_a - z^a d\bar{z}_a) \tag{30}$$

corresponds to the bosonic part of the background RR 1-form potential superfield for the chosen gauge.

Further simplification of the fermionic light-cone gauge $AdS_4 \times \mathbb{CP}^3$ superstring action (19) can be achieved by fixing bosonic light-cone gauge. For the AdS background in the Poincare parameterization it appears more natural to impose bosonic gauge conditions on the phase-space variables [1] that implies working out the Hamiltonian formulation. In the Hamiltonian approach we introduce the momenta densities

$$\mathscr{P}_{\mathfrak{M}}(\tau,\sigma) = (p_\pm, p_1, p_\varphi, p_M) = \frac{\delta \mathscr{S}}{\delta \partial_\tau \mathscr{Q}^{\mathfrak{M}}}, \tag{31}$$

whose explicit form is

$$\begin{aligned} p_-(\tau,\sigma) &= -\frac{\mathrm{e}^{-4\varphi}}{8}\gamma^{\tau i}\partial_i x^+, \\ p_+(\tau,\sigma) &= -\gamma^{\tau i}\left(\frac{\mathrm{e}^{-4\varphi}}{8}\partial_i x^- + B\partial_i x^+ + q_M \partial_i z^M - \frac{\mathrm{e}^{-2\varphi}}{2}\varpi_i\right) - \\ &\quad - (\tilde{c}\partial_\sigma x^1 - \tilde{q}_M \partial_\sigma z^M + \tilde{\omega}_\sigma), \\ p_1(\tau,\sigma) &= -\frac{\mathrm{e}^{-4\varphi}}{4}\gamma^{\tau i}\partial_i x^1 + \tilde{c}\partial_\sigma x^+, \\ p_\varphi(\tau,\sigma) &= -\gamma^{\tau i}\partial_i \varphi, \\ p_M(\tau,\sigma) &= -\gamma^{\tau i}(g_{MN}\partial_i z^N + q_M \partial_i x^+) - \tilde{q}_M \partial_\sigma x^+. \end{aligned} \tag{32}$$

This allows one to write down the fermionic light-cone gauge superstring action (19) in terms of the phase-space variables

$$\mathscr{S} = \int\limits_{\Sigma} d\tau d\sigma \left(p_+\partial_\tau x^+ + p_-\partial_\tau x^- + p_1 \partial_\tau x^1 + p_\varphi \partial_\tau \varphi + p_M \partial_\tau z^M \right. \\ \left. - 4\,\mathrm{e}^{2\varphi} p_- \varpi_\tau + \tilde{\omega}_\tau \partial_\sigma x^+ + \frac{1}{\gamma^{\tau\tau}} T_1 + \frac{\gamma^{\tau\sigma}}{\gamma^{\tau\tau}} T_2 \right). \tag{33}$$

The last two summands introduce, via the Lagrange multipliers $1/\gamma^{\tau\tau}$ and $\gamma^{\tau\sigma}/\gamma^{\tau\tau}$, the Virasoro constraints expressed in terms of the phase-space variables

$$\begin{aligned} T_1 = 8\,\mathrm{e}^{4\varphi} p_+ p_- &+ \frac{\mathrm{e}^{-4\varphi}}{8}\partial_\sigma x^+ \partial_\sigma x^- + 2\,\mathrm{e}^{4\varphi} p_1^2 + \frac{\mathrm{e}^{-4\varphi}}{8}\partial_\sigma x^1 \partial_\sigma x^1 + \frac{1}{2}(p_\varphi^2 + \partial_\sigma\varphi\partial_\sigma\varphi) + \\ &+ \frac{1}{2}((p\cdot p) + \partial_\sigma z^M g_{MN} \partial_\sigma z^N) + 32\,\mathrm{e}^{8\varphi} p_-^2((q\cdot q) - B) + \\ &+ 8\,\mathrm{e}^{4\varphi} p_-(\tilde{\omega}_\sigma - (p\cdot q) - (q\cdot\tilde{q})\partial_\sigma x^+ + \tilde{c}\partial_\sigma x^1 - \tilde{q}_M \partial_\sigma z^M) - \\ &- (\frac{\mathrm{e}^{-2\varphi}}{2}\varpi_\sigma - q_M \partial_\sigma z^M - (p\cdot\tilde{q}) + 4\,\mathrm{e}^{4\varphi} p_1 \tilde{c})\partial_\sigma x^+ + \\ &+ \frac{1}{2}((\tilde{q}\cdot\tilde{q}) + B + 4\,\mathrm{e}^{4\varphi}\tilde{c}^2)\partial_\sigma x^+ \partial_\sigma x^+ \approx 0, \end{aligned} \tag{34}$$

$$T_2 = p_+\partial_\sigma x^+ + p_-\partial_\sigma x^- + p_1\partial_\sigma x^1 + p_\varphi \partial_\sigma \varphi + p_M \partial_\sigma z^M - 4\,\mathrm{e}^{2\varphi} p_- \varpi_\sigma + \tilde{\omega}_\sigma \partial_\sigma x^+ \approx 0, \tag{35}$$

[1]The discussion of the issues related to bosonic light-cone gauge fixing for strings on the AdS background can be found in [16].

where the following notation for the scalar product with the inverse $\mathbb{CP}^3$ metric $g^{MN}(z)$ has been introduced $q_M g^{MN} q_N = (q \cdot q)$, etc. The bosonic light-cone gauge conditions we impose are analogous to those of [16, 17]

$$x^+(\tau,\sigma) = \tau, \quad p_-(\tau,\sigma) = \frac{\tilde{P}_-}{4} = \text{ const.} \tag{36}$$

Using the equation of motion for p_+, it is possible to express $\gamma^{\tau\tau}$ through $\tilde{P}_-$

$$\gamma^{\tau\tau} = -2\,\mathrm{e}^{4\varphi}\tilde{P}_-. \tag{37}$$

Further, in order to bring the light-cone gauge Lagrangian to the more compact form one can perform the following rescalings of the superspace Grassmann coordinates:

$$\begin{gathered}\eta_a \to \mathrm{e}^{-2\varphi}\,\eta_a, \quad \bar{\eta}^a \to \mathrm{e}^{-2\varphi}\bar{\eta}^a; \quad \theta_4 \to \mathrm{e}^{-\varphi}\,\theta_4,\\ \bar{\theta}^4 \to \mathrm{e}^{-\varphi}\bar{\theta}^4; \quad \eta_4 \to \mathrm{e}^{-\varphi}\,\eta_4, \quad \bar{\eta}^4 \to \mathrm{e}^{-\varphi}\bar{\eta}^4.\end{gathered} \tag{38}$$

Then, the Lagrangian (33) takes the form

$$\begin{aligned}\mathscr{L}_{\mathrm{lc}} = p_\varphi\partial_\tau\varphi + p_1\partial_\tau x^1 + p_M\partial_\tau z^M - \\ - i\tilde{P}_-(\partial_\tau\theta_A\bar{\theta}^A - \theta_A\partial_\tau\bar{\theta}^A + +\partial_\tau\eta_A\bar{\eta}^A - \eta_A\partial_\tau\bar{\eta}^A) - \mathscr{H}_{\mathrm{lc}}.\end{aligned} \tag{39}$$

Corresponding light-cone gauge Hamiltonian is given by the utmost quartic in the Grassmann coordinates expression

$$\begin{aligned}\mathscr{H}_{\mathrm{lc}} = \mathrm{e}^{-4\varphi}\left[\hat{\eta}_a\hat{\partial}_\sigma\bar{\theta}^a + \hat{\partial}_\sigma\theta_a\hat{\bar{\eta}}^a + \frac{1}{2}(\partial_\sigma\theta_4\bar{\eta}^4 - \partial_\sigma\eta_4\bar{\theta}^4 + \eta_4\partial_\sigma\bar{\theta}^4 - \theta_4\partial_\sigma\bar{\eta}^4)\right] + \\ + \frac{\mathrm{e}^{-4\varphi}}{2\tilde{P}_-}\left[2\,\mathrm{e}^{4\varphi}p_1^2 + \frac{\mathrm{e}^{-4\varphi}}{8}\partial_\sigma x^1\partial_\sigma x^1 + \frac{1}{2}(p_\varphi^2 + \partial_\sigma\varphi\partial_\sigma\varphi) + \right.\\ + \frac{1}{2}((p\cdot p) + \partial_\sigma z^M g_{MN}\partial_\sigma z^N) + 2\,\mathrm{e}^{8\varphi}\tilde{P}_-^2((q\cdot q) - B) - 2\,\mathrm{e}^{4\varphi}\tilde{P}_-(p\cdot q) + \\ \left. + 2\,\mathrm{e}^{4\varphi}\tilde{P}_-\tilde{c}\partial_\sigma x^1 - 2\,\mathrm{e}^{4\varphi}\tilde{P}_-\tilde{q}_M\partial_\sigma z^M\right].\end{aligned} \tag{40}$$

As usual, in the light-cone gauge approach the $T_2 \approx 0$ constraint can be explicitly solved for $\partial_\sigma x^-$ that decouples from the Hamiltonian. Its only nontrivial content is the zero-mode part that defines the phase-space representation of the level matching condition

$$\frac{1}{\tilde{P}_-}\oint d\sigma(p_1\partial_\sigma x^1 + p_\varphi\partial_\sigma\varphi + p_M\partial_\sigma z^M - i(\partial_\sigma\theta_A\bar{\theta}^A - \theta_A\partial_\sigma\bar{\theta}^A + \partial_\sigma\eta_A\bar{\eta}^A - \eta_A\partial_\sigma\bar{\eta}^A)) = 0. \tag{41}$$

2. DISCUSSION

The highly nonlinear structure of the light-cone gauge Hamiltonian (40) precludes addressing directly to the problem of finding the $AdS_4 \times \mathbb{CP}^3$ fundamental superstring spectrum. It could be interesting to search for the reformulation of the superstring in terms of new variables, e.g., resembling twistors [18]. For the (super)particle models on the AdS-type backgrounds, the supertwistor formulations have been considered in [19, 20]. Alternatively, it is possible to consider the Hamiltonian (40) in various simplifying limits (see [21] for a review). One of them corresponding to the large light-cone momentum (string tension) is analogous to the stringy generalization [22] of the Penrose limit, in which the leading order Hamiltonian coincides with that of the superstring propagating on the background, that is, the Penrose limit of the original one. For the $AdS_5 \times S^5$ and $AdS_4 \times \mathbb{CP}^3$ backgrounds such light-cone gauge pp-wave Hamiltonians are quadratic and can be straightforwardly quantized [23, 24]. In our case [25], as well as for the $AdS_5 \times S^5$ superstring in the light-cone gauge of [16], this limit yields the Hamiltonian of the light-cone gauge superstring in flat superspace. The reason is the existence of two different kinds of Penrose spaces for $AdS \times S$ and also for $AdS_4 \times \mathbb{CP}^3$ backgrounds [22]. When the null geodesic, around which the limit is taken, lies only in the AdS part of the background, the resulting pp wave is necessarily flat, the nontrivial pp-wave background is obtained when the tangent vector to the null geodesic has nonzero components in the tangent space to S or $\mathbb{CP}^3$. This is the light-cone gauge considered in [26] and [27,28]. Studying the higher-order terms in the Lagrangian/Hamiltonian expansion around the pp-wave backgrounds of [23, 24] allows one to calculate on the gauge theory side the strong coupling corrections to the anomalous dimensions of the Berenstein–Maldacena–Nastace (BMN) operators [29]. It would be of interest to explore the significance of the corresponding series around the flat pp-wave background, in particular, its gauge theory interpretation.

Acknowledgements. It is a pleasure to thank A. A. Zheltukhin for valuable discussions and the Organizing Committee of the SQS'09 Workshop for warm hospitality at Bogoliubov Laboratory of Theoretical Physics and support.

REFERENCES

1. *Aharony O. et al.* $\mathcal{N} = 6$ Superconformal Chern–Simons-Matter Theories, M2-Branes and Their Gravity Duals // JHEP. 2008. V. 0810. P. 091; hep-th/0806.1218.
2. *Maldacena J.* The Large N Limit of Superconformal Field Theories and Supergravity // Adv. Theor. Math. Phys. 1998. V. 2. P. 231; hep-th/9711200;
Gubser S. S., Klebanov I. R., Polyakov A. M. Gauge Theory Correlators from Noncritical String Theory // Phys. Lett. 1998. B. V. 428. P. 105; hep-th/9802109;
Witten E. Anti-de Sitter Space and Holography // Adv. Theor. Math. Phys. 1998. V. 2. P. 253; hep-th/9802150.
3. J. Phys. A. 2009. V. 49, No. 25. Special Issue on Integrability and the AdS/CFT Correspondence.
4. *Arutyunov G., Frolov S.* Superstrings on $AdS_4 \times CP^3$ as a Coset Sigma-Model // JHEP. 2008. V. 0809. P. 129; hep-th/0806.4940.
5. *Stefanski B. J.* Green–Schwarz Action for Type IIA Strings on $AdS_4 \times CP^3$ // Nucl. Phys. B. 2009. V. 808. P. 80; hep-th/0806.4948.
6. *Metsaev R. R., Tseytlin A. A.* Type IIB Superstring Action in $AdS_5 \times S^5$ Background // Nucl. Phys. B. 1998. V. 533. P. 109; hep-th/9805028.
7. *Kallosh R., Rahmfeld J., Rajaraman A.* Near Horizon Superspace // JHEP. 1998. V. 9809. P. 002; hep-th/9805217.
8. *Gomis J., Sorokin D., Wulff L.* The Complete $AdS_4 \times CP^3$ Superspace for Type IIA Superstring and D Branes // JHEP. 2009. V. 0903. P. 015; hep-th/0811.1566.
9. *Grassi P. A., Sorokin D., Wulff L.* Simplifying Superstring and D-Brane Actions in $AdS_4 \times \mathbb{CP}^3$ Superbackground // JHEP. 2009. V. 0908. P. 060; hep-th/0903.5407.

10. *Uvarov D. V.* The $AdS_4 \times \mathbb{CP}^3$ Superstring in the Light-cone Gauge // Nucl. Phys. B. 2010. V. 826. P. 294; hep-th/0906.4699.
11. *Uvarov D. V.* The $AdS_4 \times \mathbb{CP}^3$ Superstring and $D = 3$ $\mathcal{N} = 6$ Superconformal Symmetry // Phys. Rev. D. 2009. V. 79. P. 106007; hep-th/0811.2813.
12. *Duff M. J. et al.* Superstrings in $D = 10$ from Supermembranes in $D = 11$ // Phys. Lett. B. 1987. V. 191. P. 70.
13. *de Wit B. et al.* The M-Theory Two-Brane in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ // Phys. Lett. B. 1998. V. 443. P. 143; hep-th/9808052.
14. *Nilsson B. E. W., Pope C.* Hopf Fibration of Eleven-Dimensional Supergravity // Class. Quant. Grav. 1984. V. 1. P. 499.
15. *Sorokin D. P., Tkach V. I., Volkov D. V.* Kaluza–Klein Theories and Spontaneous Compactification Mechanisms of Extra-Space Dimensions // Proc. «Quantum Gravity». M., 1984. P. 376–392;
Sorokin D. P., Tkach V. I., Volkov D. V. On the Relationship between Compactified Vacua of $D = 11$ and $D = 10$ Supergravities // Phys. Lett. B. 1985. V. 161. P. 301.
16. *Metsaev R. R., Tseytlin A. A.* Superstring Action in $AdS_5 \times S^5$: κ-Symmetry Light-Cone Gauge // Phys. Rev. 2001. D. V. 63. P. 046002; hep-th/0007036;
Metsaev R. R., Thorn C. B., Tseytlin A. A. Light-cone Superstring in the AdS Space-Time // Nucl. Phys. B. 2001. V. 596. P. 151; hep-th/0009171.
17. *Goddard P. et al.* Quantum Dynamics of a Massless Relativistic String // Nucl. Phys. B. 1973. V. 56. P. 109.
18. *Penrose R., Rindler W.* Spinors and Space-Time. Cambridge Univ. Press, 1986.
19. *Claus P., Rahmfeld J., Zunger Y.* A Simple Particle Action from a Twistor Parameterization of AdS_5 // Phys. Lett. B. 1999. V. 466. P. 181; hep-th/9906118.
20. *Bandos I. A. et al.* OSp Supergroup Manifolds, Superparticles, and Supertwistors // Phys. Rev. D. 2000. V. 61. P. 065009; hep-th/9907113.
21. *Arutyunov G., Frolov S.* Foundations of the $AdS_5 \times S^5$ Superstring. Part I // J. Phys. A. 2009. V. 42. P. 254003; hep-th/0901.4937.
22. *Blau M. et al.* Penrose Limits and Maximal Supersymmetry // Class. Quant. Grav. 2002. V. 19. P. 87; hep-th/0201081;
Blau M. et al. Penrose Limits, Supergravity, and Brane Dynamics // Class. Quant. Grav. 2002. V. 19. P. 4753; hep-th/0202111.
23. *Metsaev R. R.* Type IIB Green–Schwarz Superstring in Plane-Wave Ramond–Ramond Background // Nucl. Phys. B. 2002. V. 625. P. 70; hep-th/0112044;
Metsaev R. R., Tseytlin A. A. Exactly Solvable Model of Superstring in Plane-Wave Ramond–Ramond Background // Phys. Rev. D. 2002. V. 65. P. 126004; hep-th/0202109.
24. *Sugiyama K., Yoshida K.* Type IIA String and Matrix String on *pp* Wave // Nucl. Phys. B. 2002. V. 644. P. 128; hep-th/0208029;
Hyun S. J., Shin H. J. $N = (4, 4)$ Type IIA String Theory on *pp*-Wave Background // JHEP. 2002. V. 0210. P. 070; hep-th/0208074.
25. *Uvarov D. V.* Light-cone Gauge Hamiltonian for $AdS_4 \times \mathbb{CP}^3$ Superstring. hep-th/0912.1044.
26. *Arutyunov G., Frolov S.* Uniform Light-cone Gauge for Strings in $AdS_5 \times S^5$: Solving $su(1|1)$ Sector // JHEP. 2006. V. 0601. P. 055; hep-th/0510208;
Arutyunov G., Frolov S., Zamaklar M. Finite-Size Effects from Giant Magnons // Nucl. Phys. B. 2007. V. 778. P. 1; hep-th/0606126.
27. *Astolfi D. et al.* Finite-Size Corrections in the $SU(2) \times SU(2)$ Sector of Type IIA String Theory on $AdS_4 \times \mathbb{CP}^3$ // Nucl. Phys. B. 2009. V. 810. P. 150; hep-th/0807.1527.
28. *Sundin P.* The $AdS(4) \times \mathbb{CP}^3$ String and Its Bethe Equations in the Near Plane-Wave Limit // JHEP. 2002. V. 0902. P. 046; hep-th/0811.2775.
29. *Berenstein D. E., Maldacena J. M., Nastase H. S.* Strings in Flat Space and *pp* Waves from $N = 4$ Super Yang–Mills // JHEP. 2002. V. 0204. P. 013; hep-th/0202021.