Письма в ЭЧАЯ. 2011. Т. 8, № 6(169). С. 853–860

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

# ON UNIFIED FIELD THEORIES, DYNAMICAL TORSION AND GEOMETRICAL MODELS: II

*D. J. Cirilo-Lombardo*

Joint Institute for Nuclear Research, Dubna

We analyze in this letter the same space-time structure as that presented in our previous reference (Part. Nucl., Lett. 2010. V. 7, No. 5. P. 299–307), but relaxing now the condition a priori of the existence of a potential for the torsion. We show through exact cosmological solutions from this model, where the geometry is Euclidean $R \otimes O_3 \sim R \otimes SU(2)$, the relation between the space-time geometry and the structure of the gauge group. Precisely this relation is directly connected with the relation of the spin and torsion fields. The solution of this model is explicitly compared with our previous ones and we find that: i) the torsion is not identified directly with the Yang–Mills type strength field, ii) there exists a compatibility condition connected with the identification of the gauge group with the geometric structure of the space-time: this fact leads to the identification between derivatives of the scale factor $a$ with the components of the torsion in order to allow the Hosoya–Ogura ansatz (namely, the alignment of the isospin with the frame geometry of the space-time), and iii) of two possible structures of the torsion the «tratorial» form (the only one studied here) forbids wormhole configurations, leading only to cosmological instanton space-time in eternal expansion.

В данной статье мы изучаем ту же пространственно-временную структуру, что и ранее (Письма в ЭЧАЯ. 2010. Т. 7, № 5(161). С. 491), но не предполагая изначально существования потенциала для кручения. Используя точные космологические решения для данной модели с евклидовой геометрией $R \otimes O_3 \sim R \otimes SU(2)$, мы получаем соотношение между пространственно-временной геометрией и структурой калибровочной группы. Именно это соотношение приводит к связи спина и кручения. При сравнении решений в данной модели с найденными ранее оказывается, что: 1) кручение не связано напрямую с напряженностью поля Янга–Миллса, 2) существует условие совместности, связанное с отождествлением калибровочной группы и геометрической структуры пространства-времени, что ведет к отождествлению производных скалярного фактора $a$ и компонент кручения и получению анзатца Хосойя–Огура (выстраивания изоспина с базовой геометрией пространства-времени), и 3) из двух возможных структур «траторная» структура кручения, которая только и рассмотрена в данной работе, запрещает «кротовые» конфигурации, позволяя только космологическое инстантонное вечно расширяющееся пространство-время.

PACS: 04.50.-h; 12.10.-g

## 1. MOTIVATION AND SUMMARY OF THE RESULTS

In a previous work we presented a new model of a nondualistic Unified Theory. The model in [1], absolutely consistent from the mathematical and geometrical points of view, was based on a manifold equipped with an underlying hypercomplex structure and zero nonmetricity, that lead to the important fact that the Torsion of the space-time structure turns to be totally antisymmetric. As is well known, in the particular case of totally antisymmetric torsion tensor this type of affine geometrical frameworks have the geodesics and the minimal length equations equivalents. But recently it was demostrated that more strong conditions need to be imposed from a point of view of the conservation laws into the space-time (in the Noether sense) to confirm or not that the equivalence principle is not violated. As a result, in [2] it was shown that the geodesic equation, the minimal length equation and the equation of motion for particles in the space-time under consideration are not coincident in the general case, and the only one reliable result is the explicit computation of the conserved currents by the Papapetrou method that is based on the Bianchi identities that are a characteristic ingredient of all types of space-time. Strongly motivated for the reason described above, we analyze here the same space-time structure as that presented in our previous reference, but relaxing now the condition a priori of the existence of a potential for the torsion: then it has not to be totally antisymmetric only. As in [1], we show through exact cosmological solutions from this model, where the geometry is Euclidean $R \otimes O_3 \sim R \otimes SU(2)$, the relation between the space-time geometry and the structure of the gauge group. Precisely this relation is directly connected with the relation of the spin and torsion fields. The solution of this model is explicitly compared with our previous ones of Refs. [1, 3] and we find that: i) the torsion is not identified directly with the Yang–Mills type strength field, ii) there exists a compatibility condition connected with the identification of the gauge group with the geometric structure of the space-time: this fact leads to the identification between derivatives of the scale factor $a$ with the components of the torsion in order to allow the Hosoya–Ogura ansatz (namely, the alignment of the isospin with the frame geometry of the space-time), and iii) of two possible structures of the torsion the «tratorial» form (the only one studied here) forbids wormhole configurations, leading only to cosmological instanton space-time in eternal expansion.

The organization of this letter is as follows: in Sec. 2 we review and generalize the geometrical framework where the theory is based. Section 3 is devoted to the analysis and reduction of the dynamical equations of the model. The exact solution, in the context of [1, 3], is obtained and analyzed from the geometrical and topological points of view in Sec. 4. Finally, the concluding remarks about the obtained results are discussed. Our conventions and mathematical notation are the same as in [1].

## 2. THE SPACE-TIME MANIFOLD AND THE GEOMETRICAL ACTION

The starting point is a hypercomplex construction of the (metric compatible) space-time manifold [1]

$$M, g_{\mu\nu} \equiv e_\mu \cdot e_\nu, \tag{1}$$

where for each point $\in M$ $\exists$ a local space affine $A$. The connection over $A$ $\widetilde{\Gamma}$ defines a generalized affine connection $\Gamma$ on $M$ specified by $(\nabla, K)$, where $K$ is an invertible $(1,1)$ tensor over $M$. We will demand that the connection be compatible and rectilinear:

$$\nabla K = KT, \quad \nabla g = 0, \tag{2}$$

where $T$ is the torsion, and $g$ (the space-time metric, used to raise and to lower indices and determine the geodesics) is preserved under parallel transport. This generalized compatibility condition ensures that the affine generalized connection $\Gamma$ maps autoparallels of $\Gamma$ on $M$ in straight lines over the affine space $A$ (locally). The first equation is equal to the condition determining the connection in terms of the fundamental field in the UFT nonsymmetric. For instance, $K$ can be identified with the fundamental tensor in the nonsymmetric fundamental theory. This fact gives us the possibility to restrict the connection to an (anti-)Hermitian theory.

The covariant derivative of a vector with respect to the generalized affine connection is given by

$$\begin{aligned} A^{\mu}{}_{;\nu} &\equiv A^{\mu}{}_{,\nu} + \Gamma^{\mu}{}_{\alpha\nu} A^{\alpha}, \\ A_{\mu;\nu} &\equiv A_{\mu}{}_{,\nu} + \Gamma^{\alpha}{}_{\mu\nu} A_{\alpha}. \end{aligned} \tag{3}$$

The generalized compatibility condition (2) determines the 64 components of the connection by the 64 equations as follows:

$$K_{\mu\nu;\alpha} = K_{\mu\rho} T^{\rho}{}_{\nu\alpha}, \quad \text{where} \quad T^{\rho}{}_{\nu\alpha} \equiv 2\Gamma^{\rho}{}_{[\alpha\nu]}.$$

The metric is uniquely determined by the metricity condition that puts 40 restrictions on the partial derivatives of the metric

$$g_{\mu\nu,\rho} = 2\Gamma_{(\mu\nu)\rho}.$$

The space-time curvature tensor, that is defined in the usual way, has two possible contractions: the Ricci tensor $R^{\lambda}_{\mu\lambda\nu} = R_{\mu\nu}$ and the second contraction $R^{\lambda}_{\lambda\mu\nu} = 2\Gamma^{\lambda}{}_{\lambda[\nu\mu]}$ is identically zero due to the metricity condition (2). In order to find a symmetry of the torsion tensor if we denote the inverse of $K$ by $\widehat{K}$, $\widehat{K}$ is uniquely specified by $\widehat{K}^{\alpha\rho}\ K_{\alpha\sigma} = K^{\alpha\rho}\ \widehat{K}_{\alpha\sigma} = \delta^{\rho}_{\sigma}$. As was pointed out in [1], by inserting explicitly the torsion tensor as the antisymmetric part of the connection in (4) and multiplying by $\widehat{K}^{\alpha\nu}/2$, this results after straighforward computations in

$$(\ln\sqrt{-K})_{,\mu} - \Gamma^{\nu}_{(\mu\nu)} = 0, \tag{6}$$

where $K = \det\,(K_{\mu\rho})$. Notice that from expression (6) we arrive at the following relation between the determinants $K$ and $g$: $K/g = \text{const}$. Now we can write

$$\Gamma^{\nu}{}_{\alpha\nu,\beta} - \Gamma^{\nu}{}_{\beta\nu,\alpha} = \Gamma^{\nu}{}_{\nu\beta,\alpha} - \Gamma^{\nu}{}_{\nu\alpha,\beta} \tag{7}$$

because the first term of (6) is the derivative of a scalar. Then, the torsion tensor has the symmetry

$$T^{\nu}{}_{\nu[\beta,\alpha]} = T^{\nu}{}_{\nu[\alpha,\beta]} = 0. \tag{8}$$

The second important point is to consider, as in the previous work of the author [1], the extended curvature

$$\mathcal{R}^{ab}_{\mu\nu} = R^{ab}_{\mu\nu} + \Sigma^{ab}_{\mu\nu} \tag{9}$$

with

$$\begin{aligned} R^{ab}_{\mu\nu} &= \partial_{\mu}\omega^{ab}_{\nu} - \partial_{\nu}\omega^{ab}_{\mu} + \omega^{ac}_{\mu}\omega_{\nu c}{}^{b} - \omega^{ac}_{\nu}\omega_{\mu c}{}^{b}, \\ \Sigma^{ab}_{\mu\nu} &= -\left(e^{a}_{\mu}e^{b}_{\nu} - e^{a}_{\nu}e^{b}_{\mu}\right). \end{aligned} \tag{10}$$

We assume here $\omega^{ab}_{\nu}$ is a $SO\,(d-1,1)$ connection and $e^{a}_{\mu}$ is a vierbein field. Equation (3) can be obtained, for example, using the formulation that was pioneering introduced in seminal works by E. Cartan a long time ago [1]. It is well known that in such a formalism the gravitational field is represented as a connection one form associated with some group which contains the Lorentz group as subgroup. The typical example is provided by the $SO\left(d,1\right)$ de Sitter gauge theory of gravity. In this specific case, the $SO\left(d,1\right)$ gravitational gauge field $\omega_{\mu}^{AB}=-\omega_{\mu}^{BA}$ is broken into the $SO\left(d-1,1\right)$ connection $\omega_{\mu}^{ab}$ and the $\omega_{\mu}^{da}=e_{\mu}^{a}$ vierbein field, with the dimension $d$ fixed. Then, the de Sitter (anti-de Sitter) curvature

$$\mathcal{R}_{\mu\nu}^{ab}=\partial_{\mu}\omega_{\nu}^{AB}-\partial_{\nu}\omega_{\mu}^{AB}+\omega_{\mu}^{AC}\omega_{\nu C}^{\ \ B}-\omega_{\nu}^{AC}\omega_{\mu C}^{\ \ B} \tag{11}$$

splits in the curvature (3).

Now we define the following geometrical object:

$$\mathcal{R}^{a}{}_{\mu}=\lambda\left(e^{a}{}_{\mu}+f^{a}{}_{\mu}\right)+R^{a}{}_{\mu}\qquad\left(M^{a}{}_{\mu}\equiv e^{a\nu}M_{\nu\mu}\right). \tag{12}$$

The action will contain, as usual, $\mathcal{R}=\det\left(\mathcal{R}^{a}{}_{\mu}\right)$ as the geometrical object that defines the dynamics of the theory. The particularly convenient definition of $\mathcal{R}^{a}{}_{\mu}$ makes it easy to establish the equivalent expression in the spirit of the Unified theories developed time ago by Eddington, Einstein, and Born and Infeld, for example:

$$\sqrt{\det\mathcal{R}^{a}{}_{\mu}\mathcal{R}_{a\nu}}=\sqrt{\det\left[\lambda^{2}\left(g_{\mu\nu}+f^{a}{}_{\mu}f_{a\nu}\right)+2\lambda R_{(\mu\nu)}+2\lambda f^{a}{}_{\mu}R_{[a\nu]}+R^{a}{}_{\mu}R_{a\nu}\right]}, \tag{13}$$

where $R_{\mu\nu}=R_{(\mu\nu)}+R_{[\mu\nu]}$.

The important point to consider in this simple Cartan-inspired model is that, although a cosmological constant $\lambda$ is required, the expansion of the action in four dimensions lead automatically to the Hilbert–Einstein part when $f_{\mu}^{a}=0$. Explicitly $(R=g^{\alpha\beta}R_{\alpha\beta})$

$$\begin{aligned}S=\int d^{4}x\,(e+f)\Big\{&\lambda^{4}+\lambda^{3}\left(R+f_{\mu}^{a}R_{a}^{\mu}\right)+\\&+\frac{\lambda^{2}}{2!}\left[R^{2}-R^{\mu\nu}R_{\mu\nu}+\left(f_{\mu}^{a}R_{a}^{\mu}\right)^{2}-f^{\mu\nu}f^{\rho\sigma}R_{\mu\rho}R_{\nu\sigma}\right]+\\&+\frac{\lambda}{3!}\Big[R^{3}-3RR^{\mu\nu}R_{\mu\nu}+2R^{\mu\alpha}R_{\alpha\beta}R_{\mu}^{\beta}+\left(f_{\mu}^{a}R_{a}^{\mu}\right)^{3}-3\left(f_{\mu}^{a}R_{a}^{\mu}\right)f^{\mu\nu}f^{\rho\sigma}R_{\mu\rho}R_{\nu\sigma}+\\&+2f^{\mu\nu}R_{\mu}^{\alpha}R_{\alpha\beta}R_{\nu}^{\beta}\Big]+\det\left(R_{\mu\nu}\right)\Big\}.\end{aligned} \tag{14}$$

## 3. THE DYNAMICAL EQUATIONS

In this case, the variation with respect to the metric remains the same as before in Eq. (9) [1]: (e.g., $\delta_{g}\sqrt{G}=\sqrt{G}/2\left(G^{-1}\right)^{\mu\nu}\delta_{g}G=0$). The variation with respect to the connection gives immediately

$$\begin{aligned}\frac{\delta\sqrt{G}}{\delta\Gamma_{\mu\nu}^{\omega}}=\Big\{&-\nabla_{\sigma}\left[\sqrt{G}\left(G^{-1}\right)^{\alpha\nu}\mathcal{R}_{\alpha}^{\sigma}\right]\delta_{\omega}^{\mu}+\\&+\nabla_{\omega}\left[\sqrt{G}\left(G^{-1}\right)^{\alpha\nu}\mathcal{R}_{\alpha}^{\mu}\right]+\sqrt{G}\left(G^{-1}\right)^{\alpha\nu}\mathcal{R}_{\alpha}^{\sigma}\Gamma_{[\sigma\omega]}^{\mu}\Big\},\end{aligned} \tag{15}$$

where the general form of Palatini's identity has been used. By defining $\Sigma^{\nu\sigma} \equiv \sqrt{G}\left(G^{-1}\right)^{\alpha\nu}\mathcal{R}^{\sigma}_{\alpha}$, the above equation can be written in a more suggestive form, but due to Eq. (9) of [1] it is identically zero (due to the lack of energy momentum tensor) and the only information, up till now, at our disposal is through the antisymmetric part of the variation with respect to the metric Eq. (12) of [1]

$$R_{\mu\nu} = -\lambda\left(g_{\mu\nu} + f_{\mu\nu}\right) \Rightarrow R_{[\mu\nu]} = \left(\nabla_\alpha + 2T_\alpha\right)\left(T^{\alpha}_{\mu\nu} + T_\nu\delta^{\alpha}_{\mu} - T_\mu\delta^{\alpha}_{\nu}\right) = -2\lambda f_{\mu\nu}. \tag{16}$$

Now we have to explore the role played by $f_{\mu\nu}$:

i) if $f_{\mu\nu}$ plays the role of the electromagnetic field, then we have a one-form vector potential from which $f_{\mu\nu}$ is derived. This fact leads to the usual Euler–Lagrange equations, where the variation is made with respect to the electromagnetic potential $a_\tau$:

$$\frac{\delta\sqrt{G}}{\delta a_\tau} = \nabla_\rho\left(\frac{\partial\sqrt{G}}{\partial f_{\rho\tau}}\right) \equiv \nabla_\rho\mathbb{F}^{\rho\tau} = 0. \tag{17}$$

Explicitly

$$\nabla_\rho\left[\frac{\lambda^2 N^{\mu\nu}\left(\delta^{\sigma}_{\mu}\ f^{\rho}_{\nu} + \delta^{\sigma}_{\nu}\ f^{\rho}_{\mu}\right)}{2\mathbb{R}}\right] = 0, \tag{18}$$

where $N^{\mu\nu}$ is given by expression (32) of [1]. The set of equations to solve for this particular case is

$$R_{(\mu\nu)} = \overset{\circ}{R}_{\mu\nu} - T^{\alpha}_{\mu\rho}T^{\rho}_{\alpha\nu} = -\lambda g_{\mu\nu}, \tag{19a}$$

$$R_{[\mu\nu]} = \left(\nabla_\alpha + 2T_\alpha\right)\left(T^{\alpha}_{\mu\nu} + T_\nu\delta^{\alpha}_{\mu} - T_\mu\delta^{\alpha}_{\nu}\right) = -\lambda f_{\mu\nu}, \tag{19b}$$

$$\nabla_\rho\left[\frac{\lambda^2 N^{\mu\nu}\left(\delta^{\sigma}_{\mu}\ f^{\rho}_{\nu} + \delta^{\sigma}_{\nu}\ f^{\rho}_{\mu}\right)}{2\mathbb{R}}\right] = 0; \tag{19c}$$

from this set, the link between $T$ and $f$ will be determined.

ii) $f_{\mu\nu}$ has only the role of the antisymmetric part of a fundamental (nonsymmetric) tensor $K$. Then, the variation of the geometrical Lagrangian $\delta_f\sqrt{G}$ gives the same information as $\delta_g\sqrt{G}$, which means that the remaining equations are

$$R_{(\mu\nu)} = \overset{\circ}{R}_{\mu\nu} - T^{\alpha}_{\mu\rho}T^{\rho}_{\alpha\nu} = -\lambda g_{\mu\nu}, \tag{20a}$$

$$R_{[\mu\nu]} = \left(\nabla_\alpha + 2T_\alpha\right)\left(T^{\alpha}_{\mu\nu} + T_\nu\delta^{\alpha}_{\mu} - T_\mu\delta^{\alpha}_{\nu}\right) = -\lambda f_{\mu\nu}, \tag{20b}$$

**Analysis and Reduction of the Dynamical Equations.** The important equation that appears in the two sets recently described (independently of the specific role of the antisymmetric tensor $f_{\mu\nu}$) brings us a lot of information about the link between $T$ and $f$, i.e., (19b) and (20b). Precisely this equation $R_{[\mu\nu]} = -\lambda f_{\mu\nu}$ plus the condition $\nabla_\alpha T^{\alpha}_{\mu\nu} = 0$ lead immediately to

$$\nabla_\mu T_\nu - \nabla_\nu T_\mu = -\left(\lambda f_{\mu\nu} + 2T_\alpha T^{\alpha}_{\mu\nu}\right); \tag{21}$$

then the quantity in the RHS is the definition of the *minimal coupling* electromagnetic tensor $\mathcal{F}_{\mu\nu}$ in a space-time with torsion. Two cases naturally arise:

i) if we assume the existence of the potential vector, we have

$$\nabla_\mu T_\nu - \nabla_\nu T_\mu \equiv \mathcal{F}_{\mu\nu} = -\lambda\left(\overbrace{\partial_\mu a_\nu - \partial_\nu a_\mu}^{f_{\mu\nu}}\right) - 2T_\alpha T^\alpha_{\mu\nu}; \tag{22}$$

a link between $a_\nu$ and $T_\nu$ clearly appears: $T_\nu = -\lambda a_\nu$.

ii) If $f_{\mu\nu}$ has only the role of the antisymmetric part of a fundamental (nonsymmetric) tensor $K$, it acquires a potential automatically, being in this manner *an exact form* where $T_\nu$ takes the role of potential vector. Clearly, now $f$ cannot be potential for the torsion from this point of view.

From above statements over the «trace» of the torsion, it is clearly seen that two ansatz appear as candidates for the torsion tensor structure: the «tratorial» structure $T^\alpha_{\mu\nu} \sim \left(\delta^\alpha_\mu a_\nu - \delta^\alpha_\nu a_\mu\right)$ and the «product» structure $T^\alpha_{\mu\nu} = k^\alpha f_{\mu\nu}$, where the vector $k^\alpha$ is eigenvector of the antisymmetric tensor $f_{\mu\nu}$, in general.

The other possibility is to take $\nabla_\alpha T^\alpha_{\mu\nu} = -\lambda f_{\mu\nu}$, then $\nabla_\mu T_\nu - \nabla_\nu T_\mu = -2T_\alpha T^\alpha_{\mu\nu}$, but the interpretations are not so clean as before.

## 4. COSMOLOGICAL EUCLIDEAN SOLUTION IN UFT THEORY: WORMHOLE CONFIGURATIONS FORBIDDEN

To begin with, let us consider the problem involving the set of equations (19) with the usual definition for the $SU(2)$ electromagnetic field strength

$$f^\gamma = \frac{1}{2} f^\gamma_{\mu\nu} dx^\mu \wedge dx^\nu \tag{23}$$

and as before, we are going to seek for a classical solution of Eqs. (19) with the following spherically symmetric ansatz for the metric and gauge connection:

$$ds^2 = d\tau^2 + a^2(\tau)\, \sigma^i \otimes \sigma^i \equiv d\tau^2 + e^i \otimes e^i. \tag{24}$$

Here $\tau$ is the Euclidean time and the dreibein is defined by $e^i \equiv a(\tau)\, \sigma^i$. However, in the case of the set (19) we have assumed that the two-form $f^\gamma$ comes from a one-form potential $A$ which, as in the non-Abelian Born–Infeld model of [3], is defined as $A^a \equiv A^a_\mu dx^\mu = h\sigma^a$. But we need to wait here a minute. The reason is: we know that $\sigma^i$ one-form satisfies the $SU(2)$ Maurer–Cartan structure equation, as fundamental geometrical structure of the non-Abelian electromagnetic field

$$d_{su(2)}\sigma^a + \varepsilon^a_{bc}\sigma^b \wedge \sigma^c = 0, \tag{25}$$

but now due to the identification assumed in (24):

$$e^i \equiv a(\tau)\, \sigma^i \tag{26}$$

$$\Rightarrow de^a = T^a - e^a_b \wedge \sigma^b. \tag{27}$$

Here we make the difference between the exterior derivatives in the space-time with torsion and in the $SU(2)$ group manifold. It is clearly seen that a question of compatibility involving the identification of the gauge group with the geometrical structure of the space-time with torsion certainly exists. From (25)–(27) we see that

$$\partial_\tau a d\tau \wedge \sigma^a - a\varepsilon^a_{bc}\sigma^b \wedge \sigma^c = T^a - e^a_b \wedge \sigma^b. \tag{28}$$

If

$$e^a_b = -\varepsilon^a_{bc}\sigma^c \tag{29}$$

and

$$T^a = \delta^a_b \left(\partial_\tau a\right) d\tau \wedge \sigma^b, \tag{30}$$

the space-time and gauge group are fully compatible, then

$$d\sigma^a + \varepsilon^a_{bc}\sigma^b \wedge \sigma^c = 0 \tag{31}$$

is restored. Hence, the general form assumed for the torsion field, due to the symmetry conditions prescribed above, is

$$T^\alpha_{\beta\gamma} = \xi \left(\delta^\alpha_\beta u_\gamma - \delta^\alpha_\gamma u_\beta\right) + \varsigma k^\alpha \varepsilon_{\beta\gamma} \quad \left(\xi, \varsigma : \mathrm{const}\right). \tag{32}$$

Notice that the condition of compatibility that is imposed by such a type of «trator» form for the torsion tensor in order to restore the behavior of the volume form of the space-time with respect to the covariant derivative, here appears in a natural manner without introduction of any extra scalar field (dilaton) or passing to other frame (i.e., Jordan, Einstein, etc.). Moreover, if we continue without making the correspondences (29), (30), the equations of motion for the electromagnetic field itself bring these conditions as a result also, as we will see in the next paragraph. Notice that in the HO ansatz the frame and isospin indices are identified as for the case with the NBI Lagrangian of [3]. The electromagnetic field two-form

$$\begin{aligned} f^a = dA^a + \frac{1}{2}\varepsilon^a_{bc}A^b \wedge A^c = h\delta^a_b \left(\partial_\tau \ln a\right) d\tau \wedge \sigma^b + h\frac{T^a}{a} - \left(-h + \frac{1}{2}h^2\right)\varepsilon^a_{bc}\sigma^b \wedge \sigma^c = \\ = \left(-h + \frac{1}{2}h^2\right)\varepsilon^a_{bc}\sigma^b \wedge \sigma^c, \end{aligned} \tag{33}$$

where in the last equality conditions (29), (30) have been assumed. The dynamical equation is

$$\mathbb{F}^a_{bc} \equiv \frac{\partial L_G}{\partial F_a} \Rightarrow {}^*\mathbb{F}^a \equiv \frac{\lambda\sqrt{|g|}}{\sqrt{3}} h\mathbb{A}(-2h + h^2)d\tau \wedge \frac{e^a}{a^2} \equiv Mh(-2h + h^2)d\tau \wedge \frac{e^a}{a^2}. \tag{34}$$

Inserting it in the Yang–Mills type field equation (19c), we obtain

$$\begin{aligned} d^*\mathbb{F}^a + \frac{1}{2}\varepsilon^{abc}\left(A_b \wedge^* \mathbb{F}_c -^* \mathbb{F}_b \wedge A_c\right) = 0 \\ = Mh\ d\tau \wedge \sigma^b \wedge \sigma^c \left(-2h + h^2\right)(h-1), \end{aligned}$$

$$\mathbb{A} \equiv \lambda^4[(1+\alpha)^2 + \alpha/2]. \tag{35}$$

Then, there exists a nontrivial solution: $h = 1$ (with $s = 0$ in $\mathbb{A}$ as before in [1]). The electromagnetic field is immediately determined, and is as in the non-Abelian Born–Infeld model of our previous reference and in the result of Giddings and Strominger, namely

$$f^a_{\ bc} = -\frac{\varepsilon^a_{\ bc}}{a^2}, \quad f^a_{\ 0c} = 0, \tag{36}$$

i.e., we only have magnetic field.

Now by considering only a «trator» form for the torsion, Eq. (16b) is identically null due to the magnetic character of $f^a$ and the particular form of the symmetric coefficients of the connection. Inserting the torsion equation (30) into Eq. (19a), in a manner analogous to that of [1], we obtain

$$\left[\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{a^2}\right] = \frac{\lambda}{3}. \tag{37}$$

Integration of this last expression immediately leads to

$$a\,(\tau) = \left(\frac{\lambda}{3}\right)^{-1/2} \sinh\left[\left(\frac{\lambda}{3}\right)^{1/2}(\tau - \tau_0)\right]. \tag{38}$$

Then it is quite evident that this particular case does not lead to wormhole configurations: only eternal expansion exists with $a(\tau_0) = 0$ (the origin of the Euclidean time).

Now by considering only the product form for the torsion, Eq. (19c) does not change, but Eq. (19b) takes the form of a wave equation for the scale factor

$$\left[\Box a + (\partial_0 a)\left(\partial^0 a\right)\right] = \lambda$$

due to $T^\alpha_{\beta\gamma} = \varsigma k^\alpha \varepsilon_{\beta\gamma} \to \varepsilon_{ab}\left(\partial^0 a\right)$. It is not difficult to see that the $su(2)$ structure of the electromagnetic tensor is in some manner transferred to the structure of the torsion. But here we enter into conflict because the system of equations (38) turns out to be overdetermined: probably we need more freedom in the ansatz for $f^a_{bc}$ ($s \neq 0$, or $h = h(\tau)$). This fact will be studied in the near future.

**Acknowledgements.** I am very grateful to Professor Yu Xin who introduced me into the subject of the Unifed Theories based only on geometrical concepts and the Mach principle. Many thanks are due to Professors Yu. P. Stepanovsky, A. Dorokhov and E. Ivanov for their interest and discussions; and particularly to Professor S. Capozziello for his particular interest in the subject and sending us his extremely useful reference [4] on the subject.

## REFERENCES

1. *Cirilo-Lombardo D. J.* // Part. Nucl., Lett. 2010. V. 7, No. 5. P. 299–307 and refs. therein.
2. *Hammond R.* // Rep. Prog. Phys. 2002. V. 65. P. 599–649 and refs. therein.
3. *Cirilo-Lombardo D. J.* // Class. Quant. Grav. 2005. V. 22. P. 4987–5004 and refs. therein.
4. *Capozziello S. et al.* // Ann. Phys. (Leipzig). 2001. V. 10, No. 8. P. 713–727.

Received on February 1, 2011.