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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

INCOHERENT PHOTOPRODUCTION OF PSEUDOSCALAR MESONS OFF NUCLEI AT FORWARD ANGLES

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Recent advances in the photon tagging facilities together with the novel, high-resolution fast calorimetry make it possible to perform photoproduction cross section measurements of pseudoscalar mesons on nuclei with a percent level accuracy. The extraction of the radiative decay widths, needed for testing the symmetry breaking effects in QCD, from these measurements at small angles is done by the Primakoff method. This method requires theoretical treatment of all processes participating in these reactions at the same percent level. The most updated description of general processes, including the nuclear coherent amplitude, is done in our previous paper. In this work, in the framework of the Glauber multiple scattering theory, we obtain analytical expressions for the incoherent cross section of the photoproduction of pseudoscalar mesons off nuclei accounting for the mesons absorption in nuclei and the Pauli suppression at forward production angles. As illustrations of the obtained formulas, we calculate the incoherent cross section for photoproduction from a closed shell nucleus, ^{16}O, and from an unclosed shell nucleus, ^{12}C. These calculations allow one to compare different approaches and estimate their impact on the incoherent cross section of the processes under consideration.

Последние достижения в получении пучков меченых фотонов совместно с современной быстрой калориметрией высокого разрешения позволяют измерять сечения фоторождения псевдоскалярных мезонов на ядрах с процентной точностью. Извлечение радиационных ширин распадов псевдоскалярных мезонов, необходимых для проверки эффектов нарушения симметрии КХД, проводится измерением при малых углах методом Примакова. Этот метод требует теоретической обработки всех процессов, участвующих в этих реакциях, на том же процентном уровне. Наиболее полное описание всех возможных механизмов, включая когерентную ядерную амплитуду, было проведено нами ранее. В настоящей работе в рамках глауберовской теории многократного рассеяния получены аналитические выражения для некогерентного сечения процессов фоторождения псевдоскалярных мезонов на ядрах, учитывающих поглощение мезонов в ядре и подавление за счет принципа Паули при малых углах. Для иллюстрации полученных формул вычислено некогерентное сечение фоторождения на ядре с заполненной оболочкой ^{16}O и незаполненной оболочкой ^{12}C. Эти вычисления позволяют сравнить различные приближения и оценить их влияние на некогерентные сечения рассматриваемых процессов.

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INTRODUCTION

Recent years have seen a renewed interest in the study of photoproduction of pseudoscalar mesons off complex nuclei at few GeV energies due to the advent of high duty-cycle electron accelerators together with new photon tagging facilities and novel experimental technologies. It has now become possible to perform photoproduction cross section measurements of pseudoscalar mesons off nuclei with a percent level accuracy.

Photoproduction of pseudoscalar mesons off nuclei at small angles,

$$\gamma + A \to \mathrm{Ps} + A\,(\mathrm{Ps} = \pi^0, \eta, \eta'), \tag{1}$$

is provided mainly by the possibility of extraction of their radiative decay widths from experimental data. These reactions at small angles proceed via production of a heavy nucleus in the Coulomb field, the Primakoff effect [1]. On the other hand, photoproduction in the nuclear strong field complicates the extraction of decay widths from experimental data and requires detailed theoretical examination.

The precision measurement of π^0 radiative decay width via the Primakoff effect in the photoproduction of pseudoscalar mesons off nuclear targets has recently been completed at Jefferson Lab [2, 3]. The energy upgrade of the CEBAF accelerator to 12 GeV will provide a great opportunity to extend the current program to the η and η' mesons.

In a recent article [4] we discussed that a precise knowledge of the radiative decay widths of light pseudoscalar mesons provides an accurate test of chiral anomalies and mixing effects that are due to isospin breaking by the difference of the masses of light quarks. This is of great importance to understand fundamental properties of QCD at low energies [5–7]. The precision extraction of the decay widths requires state-of-the-art theoretical descriptions of all processes participating in the reaction. In our recent work [4] we considered explicitly the photoproduction of pseudoscalar mesons in the Coulomb field of nuclei and their coherent photoproduction off the nuclear strong field. In the present work we will consider the incoherent photoproduction of pseudoscalar mesons off nuclei, i.e.,

$$\gamma + A \to \mathrm{Ps} + A'. \tag{2}$$

Here A' is the target final state including all possible nuclear excitations and its breakup. As in our previous work [4], here we will discuss the incoherent photoproduction of π^0 meson, bearing in mind that the obtained expressions, after obvious modifications, can be applied to photoproduction of the other pseudoscalar mesons.

The necessity for such investigations is two-fold. As mentioned above, the advent of precise experimental data demands the accuracy of theoretical predictions to be at relevant levels. Thus, the widely used expressions for the incoherent photoproduction of pseudoscalar mesons [8, 9] cannot be considered as satisfactory. The reason for this is that the nucleon spin-nonflip amplitude [1] is zero at zero production angle due to conservation of angular momentum.

[1] The photoproduction of pseudoscalar mesons off the nucleon is described by four independent amplitudes [10]. In the coherent photoproduction off spinless nuclei only the spin-nonflip amplitude survives. As for the incoherent photoproduction, this amplitude is the dominant one [11], though all four amplitudes are involved in the process.

As was first shown by Fäldt [12], such behavior of the nucleon amplitude requires particular consideration for the coherent photoproduction process off nuclei. On the other hand, in the incoherent production, as we will show later, similar corrections to the cross section take place and their neglect leads to meaningless results for the incoherent photoproduction at forward angles.

Despite the fact that the incoherent production off nuclei in nondiffractive processes was considered in the works of Fäldt [13, 14], we are compelled to study this process again since the expressions obtained in [13, 14] are not suitable for the production at small angles [1]. The Pauli suppression is at work for production at small angles and its correct accounting is crucial for the process under consideration.

1. THE INCOHERENT PHOTOPRODUCTION OFF NUCLEI

In the Glauber multiple scattering theory [15] the amplitude of the process as a result of which the nuclear wave function in the initial state $\Phi_i(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$ transforms to the wave function of the final state $\Phi_f(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$ is

$$F_{fi}(\mathbf{q}, \Delta) = \frac{ik}{2\pi}\int d^2b\, \mathrm{e}^{i\mathbf{q}\cdot\mathbf{b}} d^3r_1, \ldots, d^3r_A \Phi_f^*(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)\times \\ \times \sum_{j=1}^{A} \Gamma_j(\mathbf{b}, \mathbf{r}_1, \ldots, \mathbf{r}_A)\Phi_i(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A), \tag{3}$$

$$\Gamma_j(\mathbf{b}, \mathbf{r}_1, \ldots, \mathbf{r}_A) = \Gamma_p(\mathbf{b} - \mathbf{s}_j)\, \mathrm{e}^{i\Delta z_j} \prod_{i\neq j}^{A} \big[1 - \Gamma_s(\mathbf{b} - \mathbf{s}_i)\theta(z_i - z_j)\big].$$

Here the two-dimensional vectors $\mathbf{b}$ and $\mathbf{s}_i$ are the impact parameter and the nucleon's transverse coordinate, respectively; z_i is the nucleon's longitudinal coordinate in the nucleus; $\mathbf{q}$ and Δ are the transverse and longitudinal components of the transferred momenta: $q^2 = 4kp\sin^2(\theta/2)$, $\Delta = M^2/(2k)$. Here k and p are the photon and meson momenta and M is the meson mass. The profile functions $\Gamma_{p,s}(\mathbf{b} - \mathbf{s})$, by definition, are the two-dimensional Fourier transformations of the elementary amplitudes for the photoproduction of pseudoscalar meson off the nucleon, $f_p = f(\gamma + N \to \mathrm{Ps} + N)$, and elastic meson–nucleon scattering, $f_s = f(\mathrm{Ps} + N \to \mathrm{Ps} + N)$:

$$\Gamma_{p,s}(\mathbf{b} - \mathbf{s}) = \frac{1}{2\pi ik}\int \mathrm{e}^{i\mathbf{q}\cdot(\mathbf{b}-\mathbf{s})} f_{p,s}(q)\, d^2q. \tag{4}$$

The summed cross section, including both the coherent and the incoherent production processes, can be obtained if one uses the closure approximation, i.e., assumes that in the limits of the produced particle energies (usually 50–100 MeV) the final states of the nucleus form a complete set:

$$\sum_n \Phi_f\Phi_f^* = 1. \tag{5}$$

[1] For instance, the incoherent cross section would be strongly suppressed as a result of the Pauli exclusion principle, which is not the case in [14] (see, for instance, expression (33b)).

Under this assumption the summed cross section depends only on the ground-state wave function [13]:

$$\frac{d\sigma}{d\Omega} = \sum_f |F_{if}|^2 = \frac{k^2}{(2\pi)^2} \int \mathrm{e}^{i\mathbf{q}\cdot(\mathbf{b}-\mathbf{b}')} d^2b\, d^2b'\, d^3r_1, \ldots, d^3r_A\, |\Phi_i(\mathbf{r}_1, \ldots, \mathbf{r}_A)|^2 \times \\ \times \sum_{j,j'} \Gamma_j(\mathbf{b}, \mathbf{r}_1, \ldots, \mathbf{r}_A, \Delta)\Gamma^*_{j'}(\mathbf{b}', \mathbf{r}_1, \ldots, \mathbf{r}_A, \Delta). \quad (6)$$

Let us assume that the ground state of the nucleus can be described by means of the independent particle model. Introducing the single-particle density $\rho(r)$, we have

$$| \Phi_i(\mathbf{r}_1, \ldots, \mathbf{r}_A) |^2 = \prod_{k=1}^{A} \frac{\rho(r_k)}{A}; \quad \int \rho(\mathbf{r})\, d^3r = A. \quad (7)$$

Separating the contributions of diagonal and nondiagonal terms in the sum in Eq. (6), one gets [13, 16]

$$\begin{aligned} \frac{d\sigma_d}{d\Omega} &= \frac{k^2}{(2\pi)^2} \int \mathrm{e}^{i\mathbf{q}\cdot(\mathbf{b}-\mathbf{b}')} d^2b\, d^2b'\, d^2s\, dz \Gamma_p(\mathbf{b}-\mathbf{s})\Gamma^*_p(\mathbf{b}'-\mathbf{s})\rho(\mathbf{s}, z)\, \mathrm{e}^{-E(\mathbf{b},\mathbf{b}',z)}, \\ E(\mathbf{b}, \mathbf{b}', z) &= \int_z^\infty \big[\Gamma_s(\mathbf{b}-\mathbf{s}) + \Gamma^*_s(\mathbf{b}'-\mathbf{s}) - \Gamma_s(\mathbf{b}-\mathbf{s})\Gamma^*_s(\mathbf{b}'-\mathbf{s})\big]\rho(\mathbf{s}, z')\, d^2s\, dz', \\ \frac{d\sigma_{\rm nd}}{d\Omega} &= \frac{(A-1)}{A} \frac{k^2}{(2\pi)^2} 2\Re \int \mathrm{e}^{i\mathbf{q}\cdot(\mathbf{b}-\mathbf{b}') + i\Delta(z_2 - z_1)} \theta(z_2 - z_1) d^2b d^2b' d^2s_1 d^2s_2 dz_1 dz_2 \times \\ &\times \Gamma_p(\mathbf{b}-\mathbf{s}_1)\Gamma^*_p(\mathbf{b}'-\mathbf{s}_2)\big[1 - \Gamma_s(\mathbf{b}'-\mathbf{s}_2)\big]\rho(\mathbf{s}_1, z_1)\rho(\mathbf{s}_2, z_2)\mathrm{e}^{-E(\mathbf{b},\mathbf{b}',z_1,z_2)}, \\ E(\mathbf{b}, \mathbf{b}', z_1, z_2) &= \int_{z_1}^{z_2} \Gamma_s(\mathbf{b}-\mathbf{s})\rho(\mathbf{s}, \mathbf{z}) d^2s\, dz + \\ &+ \int_{z_2}^\infty \big[\Gamma_s(\mathbf{b}-\mathbf{s}) + \Gamma^*_s(\mathbf{b}'-\mathbf{s}) - \Gamma_s(\mathbf{b}-\mathbf{s})\Gamma^*_s(\mathbf{b}'-\mathbf{s})\big]\rho(\mathbf{s}, \mathbf{z})\, d^2s\, dz. \end{aligned} \quad (8)$$

These expressions are general and can be applied to diffractive production off nuclei (for instance, in photoproduction of vector mesons $V(\rho, \omega, \phi, \psi)$) and to nondiffractive processes like production of pseudoscalar mesons.

The incoherent processes at large momentum transfer were considered in detail in [13] [1]. Our main goal here is to investigate the incoherent photoproduction of pseudoscalar mesons at small momentum transfers. In this kinematic region one can neglect the mesons multiple scattering to nonforward angles, the effect important at high momentum transfers [17]. In

[1] In [12, 13] the final formulas have been cited for the production processes with the same absorption in nuclei for the initial and final hadrons, which significantly facilitates the calculations, but it is not appropriate for photoproduction.

this approximation the differential cross section Eq. (8) takes the form

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_d}{d\Omega} + \frac{d\sigma_{\rm nd}}{d\Omega} = \int \rho(\mathbf{s},z)|h(\mathbf{s},z)|^2 d^2s\,dz + \frac{(A-1)}{A}\left|\int \rho(\mathbf{s},z)h(\mathbf{s},z)d^2s\,dz\right|^2,$$
$$h(\mathbf{s},z) = \frac{ik}{2\pi}\int \mathrm{e}^{i\mathbf{q}\cdot\mathbf{b}+i\Delta z}d^2b\Gamma_p(\mathbf{b}-\mathbf{s})\exp\Big(-\int_z^\infty \Gamma_s(\mathbf{b}-\mathbf{s}')\rho(\mathbf{s}',z')d^2s'dz'\Big). \tag{9}$$

The second term in this expression includes the coherent cross section. To obtain the incoherent cross section, one has to subtract it from the summed cross section (Eq. (9)):

$$\frac{d\sigma_{\rm inc}}{d\Omega} = \int \rho(\mathbf{s},z)|h(\mathbf{s},z)|^2 d^2s\,dz - \frac{1}{A}\left|\int \rho(\mathbf{s},z)h(\mathbf{s},z)\,d^2s\,dz)\right|^2. \tag{10}$$

The well-known approximation that is valid for intermediate and heavy nuclei for convolution of the profile function $\Gamma_s(\mathbf{b}-\mathbf{s})$ with nuclear density [17] is

$$\int \Gamma_s(\mathbf{b}-\mathbf{s})\rho(s,z)\,d^2s\,dz = \frac{\sigma'}{2}\int \rho(\mathbf{b},z)dz, \tag{11}$$

where $\sigma' = (4\pi/ik)f_s(0)$. On general grounds it is known that the spin-nonflip nucleon amplitude in photoproduction of pseudoscalar mesons would vanish at zero angles. Thus, we choose for the elementary photoproduction amplitude on the nucleon the parametrization

$$f_p(q) = (\mathbf{n}\cdot\mathbf{q})\varphi(q). \tag{12}$$

Here $\varphi(0) \neq 0$; $\mathbf{n} = \dfrac{\mathbf{k}\times\boldsymbol{\epsilon}}{k}$, with ϵ the polarization vector of the photon. The relevant cross section for photoproduction off the nucleon is $d\sigma_0/d\Omega = (1/2)q^2|\varphi(q)|^2$, and the relevant profile function becomes

$$\Gamma_p(\mathbf{b}-\mathbf{s}) = -i\mathbf{n}\cdot\frac{\partial\Gamma(\mathbf{b}-\mathbf{s})}{\partial\mathbf{b}}, \quad \Gamma(\mathbf{b}-\mathbf{s}) = \frac{1}{2\pi ik}\int \mathrm{e}^{i\mathbf{q}\cdot(\mathbf{b}-\mathbf{s})}\varphi(q)\,d^2q. \tag{13}$$

Accounting for the fact that the profile function $\Gamma_p(\mathbf{b}-\mathbf{s})$ changes much slower than the nuclear density $\rho(\mathbf{s},z)$, it is straightforward to obtain from Eqs. (9) and (13)

$$h(\mathbf{s},z) = \left((\mathbf{n}\cdot\mathbf{q}) - i\frac{\sigma'}{2}\mathbf{n}\cdot\frac{\partial T(\mathbf{s},z)}{\partial\mathbf{s}}\right)\varphi(q)\,\mathrm{e}^{i\mathbf{q}\cdot\mathbf{s}+i\Delta z}\exp\Big(-\frac{\sigma'}{2}T(s,z)\Big),$$
$$T(s,z) = \int_z^\infty \rho(\mathbf{s},z')\,dz'. \tag{14}$$

Substituting this expression into Eq. (10), we get for the incoherent cross section of the process under consideration:

$$\frac{d\sigma_{\rm inc}}{d\Omega} = \frac{d\sigma_0}{d\Omega}(q)\left(N(0,\sigma) - \frac{|F(q,\Delta)|^2}{A}\right) + |\varphi(q)|^2\frac{\sigma^2}{8}\times$$
$$\times\int\rho(\mathbf{s},z)\left|\frac{\partial T(\mathbf{s},z)}{\partial s}\right|^2\exp\Big(-\sigma T(\mathbf{s},z)\Big)\,d^2s\,dz,$$
$$F(q,\Delta) = \int {\rm e}^{i\mathbf{q}\cdot\mathbf{s}+i\Delta z}\rho(\mathbf{s},z)\,d^2s\,dz\exp\Big(-\frac{\sigma'}{2}T(\mathbf{s},z)\Big) - \frac{\pi\sigma'}{q}\times$$
$$\times\int J_1(qs)\rho(\mathbf{s},z_2)\frac{\partial\rho(\mathbf{s},z_1)}{\partial s}s\,ds\,dz_1\,dz_2\,{\rm e}^{i\Delta z_1}\exp\Big(-\frac{\sigma'}{2}T(s,z_1)\Big),$$
$$N(0,\sigma) = \int\frac{1-\exp\left(-\sigma\int\rho(s,z)\,dz\right)}{\sigma}d^2s. \tag{15}$$

Let us mention the main difference between this expression and the one widely used for the incoherent cross section [8, 9]:

$$\frac{d\sigma_{\rm inc}}{d\Omega} = \frac{d\sigma_0}{d\Omega}(q)N(0,\sigma)\,[1-G(t)]\,. \tag{16}$$

At small momentum transfer this expression goes to zero ($G(0)=1$). On the other hand, this is not the case for Eq. (15), which differs from zero at $q=0$. The complete suppression takes place only if one neglects the final state interaction of the produced meson ($\sigma=0$). This fact has been well known for many years for incoherent cross section of elastic scattering [17] and for diffractive photoproduction [18, 19].

2. PAULI CORRELATIONS

Up to now we have assumed the factorization for the nuclear ground-state wave function (see Eq. (7)). As has been shown above, even in such a simplified model the orthogonality and completeness of final states nuclear wave functions (closure approximation) lead to suppression of the pseudoscalar mesons' yield at forward angles different from the common one due to the mesons' final-state interactions in nuclei.

Let us estimate the impact of the nucleon–nucleon correlations on the incoherent cross section. Here we are constrained by the correlations due to the Pauli exclusion principle, as their contribution is the largest one and they determine the behavior of the cross section at small momentum transfers.

The ground-state wave function of the nucleus can be cast in the form [19]

$$\rho^{(A)}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A) = |\Phi_i(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)|^2 =$$
$$= \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\cdots\rho(\mathbf{r}_A) + \sum_{\rm contraction}\Big[\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\cdots\rho(\mathbf{r}_A)\Big] + \ldots, \tag{17}$$

where the pair contraction is defined by $\Delta(\mathbf{r}_1,\mathbf{r}_2)=\rho^{(2)}(\mathbf{r}_1,\mathbf{r}_2)-\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)$, and the single-particle and two-particle densities

$$\begin{aligned}\rho(\mathbf{r}_1)&=\int\rho^{(A)}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)\,d^3r_2\,d^3r_3\cdots d^3r_A,\\ \rho^{(2)}(\mathbf{r}_1,\mathbf{r}_2)&=\int\rho^{(A)}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)\,d^3r_3\,d^3r_4\cdots d^3r_A.\end{aligned}\tag{18}$$

Let us introduce the so-called mixed density [8]

$$d(\mathbf{r}_1,\mathbf{r}_2)=\sum_{j=1}^{A}\phi_j(\mathbf{r}_1)\phi_j^*(\mathbf{r}_2),\tag{19}$$

where $\phi_j(\mathbf{r})$ is the wave function of the single nucleon, so the nuclear density is given by $\rho(\mathbf{r})=d(\mathbf{r},\mathbf{r})$. The two-body density can be expressed through the mixed density

$$\rho^{(2)}(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{A(A-1)}\big[d(\mathbf{r}_1,\mathbf{r}_1)d(\mathbf{r}_2,\mathbf{r}_2)-d(\mathbf{r}_1,\mathbf{r}_2)\,d(\mathbf{r}_2,\mathbf{r}_1)\big].\tag{20}$$

Substituting this expression into the general formula, Eq. (6), for the summed cross section and using the relation

$$\int d(\mathbf{r}_1,\mathbf{r}_2)\,d(\mathbf{r}_2,\mathbf{r}_1)\,d^3r_2=d(\mathbf{r}_1,\mathbf{r}_1),\tag{21}$$

we obtain

$$\begin{aligned}\frac{d\sigma_{\rm inc}}{d\Omega}&=\int d(\mathbf{s},z,\mathbf{s},z)|h(\mathbf{s},z)|^2\,d^2s\,dz-\\ &\quad-\int d(\mathbf{s}_1,z_1,\mathbf{s}_2,z_2)\,d^*(\mathbf{s}_2,z_2,\mathbf{s}_1,z_1)h(\mathbf{s}_1,z_1)h^*(\mathbf{s}_2,z_2)\,d^2s_1\,dz_1\,d^2s_2\,dz_2.\end{aligned}\tag{22}$$

It is straightforward to check that in the Born approximation ($\sigma=0$) this expression takes the well-known form [8]

$$\begin{aligned}\frac{d\sigma_{\rm inc}}{d\Omega}&=A\frac{d\sigma_0}{d\Omega}(1-G(q,\Delta));\\ G(q,\Delta)&=\int \mathrm{e}^{i\mathbf{q}(\mathbf{s}_1-\mathbf{s}_2)+i\Delta(z_1-z_2)}\,d(\mathbf{s}_1,z_1,\mathbf{s}_2,z_2)\,d^*(\mathbf{s}_2,z_2,\mathbf{s}_1,z_1)\,d^2s_1\,d^2s_2\,dz_1\,dz_2.\end{aligned}\tag{23}$$

To proceed with the calculations for the incoherent cross section, one has to choose the model for the mixed density. We will consider the photoproduction from light nuclei, for which the shell model with harmonic-oscillator wave functions works well.

Let us first consider a closed shell nucleus, for instance, ^{16}O. The mixed density in the independent particle model for this nucleus with harmonic-oscillator wave functions [8] is

$$d(\mathbf{r}_1,\mathbf{r}_2)=\frac{4}{a_0^3\pi^{3/2}}\left(1+2\frac{\mathbf{r}_1\mathbf{r}_2}{a_0^2}\right)\exp\left(-\frac{r_1^2+r_2^2}{2a_0^2}\right).\tag{24}$$

Substituting this equation into Eq. (22), we calculate the incoherent cross section.

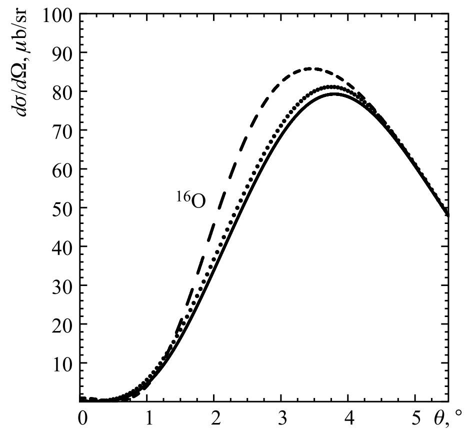

Fig. 1. Incoherent differential cross section for π^0 photoproduction off ^{16}O nucleus as a function of production angle in the lab system. The long-dashed line is calculated by Eq. (15) with single-nucleon density relevant to harmonic-oscillator wave functions; the dotted line is the incoherent cross section taking into account Pauli correlations, Eqs. (22) and (24); the solid line is the standard parametrization as in Eq. (16)

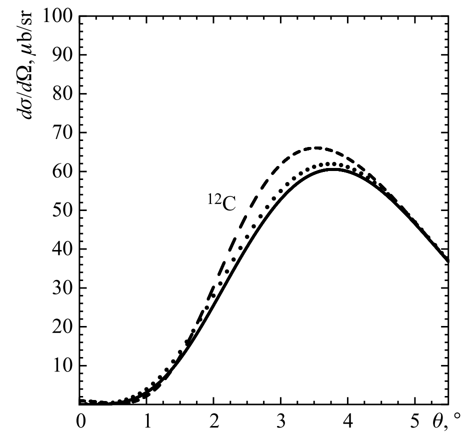

Fig. 2. Incoherent differential cross section for π^0 photoproduction off ^{12}C nucleus as a function of the production angle in the lab system. The curves have the same meaning as in Fig. 1

In Fig. 1 we plot the dependence of the incoherent cross section for photoproduction of π^0 mesons off ^{16}O nucleus as a function of the production angle. The oscillator parameter for ^{16}O is $a_0 = 1.81$ fm [21], and the total cross section of $\pi^0 N$ interaction is $\sigma = 27$ mb. The solid line is calculated by Eq. (16). The long-dashed and dotted curves are the results of calculations according to Eqs. (15) and (22), respectively.

For the ^{12}C nucleus, we choose the mixed density in the matrix form [22]:

$$d(\mathbf{r}_1, \mathbf{r}_2) = \frac{2}{a_0^3 \pi^{3/2}} \left[1 + \frac{2}{3a_0^2} \left(2\mathbf{r}_1 \cdot \mathbf{r}_2 + i\boldsymbol{\sigma} \cdot (\mathbf{r}_1 \times \mathbf{r}_2)\right)\right] \exp\left(-\frac{r_1^2 + r_2^2}{2a_0^2}\right). \tag{25}$$

In this case the single-particle nucleon density is expressed through mixed density by the relation

$$\rho(\mathbf{r}) = \mathrm{Tr}(d(\mathbf{r}, \mathbf{r})). \tag{26}$$

The results of our calculations for ^{12}C using mixed density, Eq. (25), with oscillator parameter $a_0 = 1.6$ fm [17] are shown in Fig. 2. The solid line is calculated using Eq. (16). The long-dashed and the dotted curves have the same meaning as in Fig. 1.

As seen from the figures, the correct accounting of pion absorption changes the behavior of the incoherent cross sections at forward angles which can be crucial for the extraction of radiative decay widths of pseudoscalar mesons.

3. SUMMARY AND CONCLUSIONS

In recent years it has become possible to perform high-precision measurements of photoproduction cross sections for light pseudoscalar mesons on nuclei. The extraction of meson radiative decay widths with high accuracy from the Primakoff type of experiments requires state-of-the-art theoretical descriptions of all participating processes. We have calculated the incoherent photoproduction cross sections for π^0 using the Glauber multiple scattering theory. On that way, we have revised the existing approaches for incoherent nondiffractive photoproduction at forward angles and obtained new expressions (see Eq. (15)) which correctly account for meson absorption in the final state. This expression correctly takes into account the fact that the differential cross section on a nucleon is zero at zero production angle. Moreover, the zero in the nucleon cross section leads to additional terms in the incoherent cross section without which the incoherent cross section would be negative at very small angles.

The Pauli correlations between nucleons on which photoproduction takes place are properly taken into account as their effects are dominant at forward angles. To account for the Pauli suppression, the ground-state wave functions of nuclei have to be antisymmetrized as the nucleons are fermions. This can be done if one works using the mixed densities, as in [8].

Taking into account the above considerations, the general expressions of the incoherent photoproduction cross sections for pseudoscalar mesons off nuclei were obtained for the first time (see Eq. (22)). Using these new expressions and taking the mixed and single-nucleon densities in correspondence with the harmonic-oscillator model, we have performed calculations for a closed shell light nucleus (oxygen) and an unclosed shell nucleus (carbon).

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