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PRODUCTION OF $\pi^0\rho^0$ PAIR IN ELECTRON–POSITRON ANNIHILATION IN THE NAMBU–JONA-LASINIO MODEL

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The process $e^+e^- \to \pi^0\rho^0$ is described in the framework of the expanded NJL model in the energy region from 0.9 to 1.5 GeV. The contribution of intermediate state with vector mesons $\omega(782)$, $\phi(1020)$, and $\omega'(1420)$, where ω' is the first radial excitation of ω meson, was taken into account. Results obtained are in satisfactory agreement with experimental data.

Процесс $e^+e^- \to \pi^0\rho^0$ описан в рамках расширенной модели Намбу–Йона-Лазинио в области энергий от 0,9 до 1,5 ГэВ. Были учтены вклады промежуточных состояний с векторными мезонами $\omega(782)$, $\phi(1020)$, $\omega'(1420)$, где ω' является первым радиальным возбуждением ω-мезона. Полученные результаты находятся в удовлетворительном согласии с экспериментальными данными.

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INTRODUCTION

This work is devoted to the description of the process $e^+e^- \to \pi^0\rho^0$ recently measured in experiments of the years 2006 and 2011 [1,2]. It is also concerned with a theoretical study of the process $e^+e^- \to \pi^0\rho^0$ within the ω-, ω'-, and ϕ-mesons energy range. This process was recently measured at the CMD-2 detector at the VEPP-2M e^+e^- collider [1–3].

The cross section of hadron production in the e^+e^- annihilation in the energy region $\sqrt{s} < 1.03$ GeV can be described in the vector meson dominance model (VDM) framework and is determined by the transitions of light vector mesons (ω, ω', ϕ) to the final states.

It is one of the series of works [4,5], where the process $e^+e^- \to \pi^0\omega$, $\pi^0\gamma$ was described in the framework of the expanded NJL model [6,7]. The results obtained were found to be in satisfactory agreement with the known experimental data [1,2]. The main formalism including the $SU(2) \times SU(2)$ chiral NJL model coincides with one of the papers [4,5]. The standard NJL Lagrangian which describes interactions of photons, pions, and vector ρ and ω mesons with quarks is presented in [8,9].

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1. AMPLITUDE OF THE PROCESS $e^+e^- \to \omega,\ \omega',\ \phi \to \pi^0\rho^0$

The amplitude can be written down in the form:

$$T = \bar{e}\,\gamma_\mu\,e\,\varepsilon_{\mu\lambda\alpha\beta}\frac{p_\pi^\alpha p_\rho^\beta}{m\cdot s}\{B_{\gamma+\omega}+B_\phi+B_{\omega'}\}\,\varepsilon_\lambda(\rho), \tag{1}$$

where $s = (p_1(e^+)+p_2(e^-))^2$.

The quantity $B_{\gamma+\omega}$ to the contribution of the amplitude from the process with intermediate photons and ω mesons is as follows:

$$B_{\gamma+\omega} = \frac{M_\omega^2 + iM_\omega\Gamma_\omega}{M_\omega^2 - s + iM_\omega\Gamma_\omega}\frac{1}{g_{\rho_1}}V_{\rho\pi^0\gamma}(s). \tag{2}$$

The quantity B_ϕ corresponds to the contribution with ϕ meson in the intermediate state [7]:

$$B_\phi = \frac{s\sqrt{2}\sin\theta_{\omega\phi}}{s - M_\phi^2 + iM_\phi\Gamma_\phi}\frac{1}{g_{\rho_1}}V_{\rho\pi^0\gamma}(s), \tag{3}$$

where $\sin\theta_{\omega\phi} = -0.0523$.

The quantity $B_{\omega'}$ to the contribution from the intermediate radial excitation of the ω-meson state, $\omega' \to \pi^0\rho$, is taken from paper [10]

$$B_{\omega'} = \frac{s}{s - M_{\omega'}^2\Gamma_{\omega'}}\left(-\frac{\cos(\beta+\beta_0)}{\sin(2\beta_0)} - \Gamma\frac{\cos(\beta-\beta_0)}{\sin(2\beta_0)}\right)\frac{1}{g_{\rho_1}}V_{\rho'\pi^0\gamma}(s), \tag{4}$$

where $\Gamma \approx 1/2$ will be specified below (see (8)) and

$$\begin{aligned}
V_{\rho\pi^0\gamma}(s) &= g_{\pi_1}\left(\frac{\sin(\beta+\beta_0)g_{\rho_1}I_0^{(3)}}{\sin(2\beta_0)} + \frac{\sin(\beta-\beta_0)g_{\rho_2}I_1^{(3)}}{\sin(2\beta_0)}\right)\frac{1}{g_{\rho_1}}V_{\rho'\pi^0\gamma},\\
V_{\rho'\pi^0\gamma}(s) &= -g_{\pi_1}\left(\frac{\cos(\beta+\beta_0)g_{\rho_1}I_0^{(3)}}{\sin(2\beta_0)} + \frac{\cos(\beta-\beta_0)g_{\rho_2}I_1^{(3)}}{\sin(2\beta_0)}\right),
\end{aligned} \tag{5}$$

$$I_n^{(3)} = -\int\frac{d^4k\cdot m^2 f^n(k^{\perp 2})\Theta(\Lambda^2-|k^{\perp 2}|)}{i\pi^2(k^2-m^2+i0)}\times \\ \times\frac{1}{((k+p_\rho)^2-m^2+i0)((k+p_\pi)^2-m^2+i0)}. \tag{6}$$

The radially-excited states were introduced in the NJL model with the help of the form factor in the quark–meson interaction:

$$\begin{aligned}
f(k^{\perp 2}) &= (1-d|k^{\perp 2}|)\Theta(\Lambda^2-|k^{\perp 2}|),\\
k^\perp &= k - \frac{(kp)p}{p^2},\quad d = 1.78\ \text{GeV}^{-2},
\end{aligned} \tag{7}$$

where k and p are the quark and meson momenta, respectively. The cut-off parameter $\Lambda = 1.03$ GeV is taken from [11]. The coupling constants $g_{\pi_1} = g_\pi$ and $g_{\rho_1} = g_\rho$ are the same as in the standard NJL version. The constants $g_{\pi_2} = 3.20$, $g_{\rho_2} = 9.87$, being the mixing angles $\beta_0 = 61.53°$ and $\beta = 76.78°$, were defined in [10]. The standard value of the ϕ–ω mixing angle $\theta_{\omega\phi} \approx -3°$ is used [9]. So, for the numerical calculations we use the values from the Particle Data Group [12]: $\Gamma_\omega = 8.49$ MeV, $\Gamma_{\omega'} = 215$ MeV, $M_\omega = 782$ MeV, $M_\rho = 775$ MeV, $M_{\omega'} = 1420$ MeV, $M_\phi = 1020$ MeV, $\Gamma_\phi = 4.26$ MeV, $M_\pi = 139$ MeV. The γ–ω transition differs from the above just by a factor $1/3$ compared with γ–ρ. In the amplitudes with excited mesons we have to take into account the γ–ρ_2 and γ–ω_2 transitions (γ–$\omega_1(\rho_1)$ transitions are the same as in the standard γ–$\omega(\rho)$ cases) that can be expressed via the γ–$\omega(\rho)$ transition with the additional factor [7, 10]

$$\Gamma = \frac{I_2^f}{\sqrt{I_2 I_2^{f^2}}} \approx 0.47. \tag{8}$$

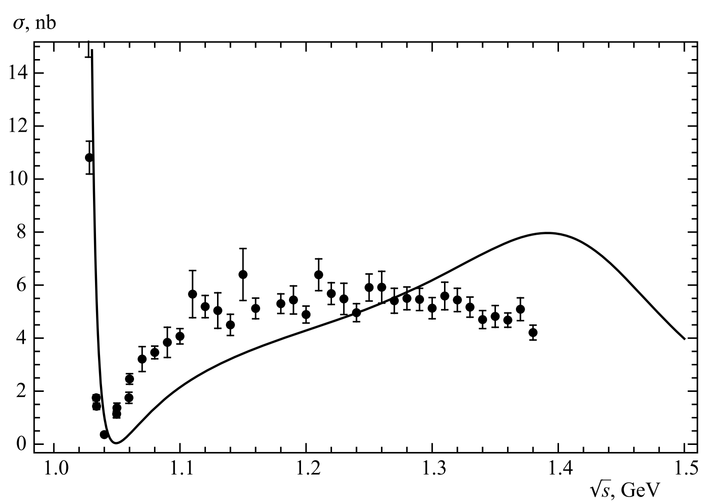
2. TOTAL CROSS SECTION

In (7), m is the constituent quark mass ($m_u = m_d = 280$ MeV). For calculation of the total cross section of the process we use:

$$\sigma(s) = \frac{3\alpha^2 g_\rho^2}{32\pi^3 s^3 f_\pi^2} \lambda^{3/2}(s, M_\rho^2, M_\pi^2) |B_{\gamma+\omega} + B_\phi + B_{\omega'}|^2, \tag{9}$$

where $f_\pi = 93$ MeV is the pion decay constant, and $\lambda(s, M_\rho^2, M_\pi^2) = (s - M_\pi^2 - M_\pi^2)^2 - 4M_\rho^2 M_\pi^2$, g_ρ is the vector meson coupling constant $g_\rho \approx 6.14$ corresponding to the standard relation $g_\rho^2 \approx 3$. The total cross section in the region $0.9 < \sqrt{s} < 2$ GeV is presented in the Figure.

In the Table the behavior of the cross section in the region $m_\pi + m_\rho = \sqrt{s_{\rm th}} < \sqrt{s} = 1.1$ GeV is presented. In this region the cross section has a resonance character. In conclusion, we would like to note the distinction between the $\pi^0\rho^0$ and $\pi^0\omega$ processes, where the



Total cross section as a function $\sqrt{s}$ of the $e^+e^- \to \pi^0\rho^0$ process in the NJL model. Points are experimental data [3]

The magnitude of the total cross section in the resonance region $0.915 < \sqrt{s} < 1.1$ **GeV**

$\sqrt{s}$, GeV	σ, nb	$\sqrt{s}$, GeV	σ, nb	$\sqrt{s}$, GeV	σ, nb	$\sqrt{s}$, GeV	σ, nb
0.915	0	0.95	3.4	1.02	796	1.054	0.13
0.916	0.022	0.956	4.27	1.026	58.2	1.056	0.2
0.918	0.11	0.962	5.2	1.03	15.8	1.06	0.38
0.922	0.38	0.972	7.1	1.04	1.03	1.07	0.88
0.926	0.71	0.98	9.11	1.048	0.05	1.08	1.36
0.932	1.3	1	22	1.05	0.04	1.09	1.78
0.944	2.2	1.01	58.6	1.052	0.07	1.1	2.14

ϕ resonance is not seen in the $\pi^0\omega$ process. A similar situation takes place in the process $e^+e^- \to \pi^0\gamma$ which was supported by experimental data.

As a by-product of our analysis we obtain the partial decay of the process $\phi \to \rho^0\pi^0$, $\Gamma_{\phi\to\rho^0\pi^0} \approx 0.5$ MeV which is in good agreement with PDG data [12].

CONCLUSIONS

The cross section of the process $e^+e^- \to \pi^0\rho^0$ was measured in the Spherical Neutral Detector (SND) experiment at the VEPP-2M collider in the energy region $\sqrt{s} = 980$–1380 MeV [1–3].

Our calculations for the process $e^+e^- \to \pi^0\rho^0$ showed the presence of two regions of enhancement of the cross section in the energy range below 1.020 and 1.4 GeV. The first one appears in the region of the ϕ-meson mass and looks like a very high narrow peak. The second one is a smooth peak, it lies in the region of ω'-meson mass.

Notwithstanding, the process $e^+e^- \to \pi^0\rho^0$ is similar to the process $e^+e^- \to \omega\pi^0$, but in our result in the ϕ-meson mass region we have a very narrow peak which will be in agreement with experiment.

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