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ON THE ROLE OF DENSITY-DEPENDENT SYMMETRY ENERGY AND MOMENTUM-DEPENDENT INTERACTIONS IN MULTIFRAGMENTATION

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The multifragmentation for the different parameterizations of the density-dependent symmetry energy is studied using an isospin-dependent quantum molecular dynamics (IQMD) model. We analyze the sensitivity of fragment production towards various forms of the density-dependent symmetry energy. The inclusion of momentum-dependent interactions (MDI) results in a larger variation of fragment production. We here highlighted the collective response of the MDI and symmetry energy towards the fragmentation of colliding nuclei at intermediate energies.

Исследуется процесс мультифрагментации для различных параметризаций зависимости энергии симметрии от плотности с помощью модели изоспин-зависимой квантовой молекулярной динамики. Анализируется чувствительность образования фрагментов к различным формам энергии симметрии, зависящей от плотности. Добавление зависящих от импульса взаимодействий приводит к большему разнообразию вылетающих фрагментов. Подчеркивается, что зависящие от импульса взаимодействия и энергия симметрии оказывают совместное влияние на фрагментацию сталкивающихся ядер при промежуточных энергиях.

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INTRODUCTION

Nuclear physics, in general, and heavy-ion collision, in particular, are of central interest in investigating the nature of equation of state. The study of multifragmentation gives us an insight into the reaction dynamics and into the hot and dense nuclear matter formed during a reaction. In general, fragments produced during the collision depend upon the incident energy, mass of the colliding nuclei as well as on the impact parameter of the reaction. At low incident energies, reaction dynamics is dominated by the attractive mean-field potential [1, 2]. With the increase in the incident energy, repulsive nucleon–nucleon scattering becomes dominant. This results in the breaking of colliding nuclei into free nucleons (FNs) ($A = 1$), light charged particles (LCPs) ($2 \leqslant A \leqslant 4$), and intermediate mass fragments ($5 \leqslant A \leqslant A_{\rm tot}/6$). The size and multiplicity of these fragments depend on the above-model ingredient or reaction inputs.

All the previous investigations revealed that the energy transferred in the IQMD events is governed by the elementary nucleon–nucleon cross section, that was assumed to be independent of the nuclear environment [3, 4]. In a contribution [4], the demand of a reduced isospin-dependent cross section has been suggested for the better explanation of the experimental data. It was also observed that the reduced isospin-dependent cross section is valid for

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the soft as well as for the soft momentum-dependent equation of state [4]. In addition to the above-said factors, one of the major parameters in this direction is the symmetry energy. The symmetry energy is an important term which affects the collision of the nuclei at intermediate energies. The form and strength of the symmetry energy is one of the hot topics these days. The knowledge of the symmetry energy is not only relevant for the nuclear physics, it is also vital for the other branches, such as astrophysics. The symmetry energy tends to vary with the density as the reaction proceeds. The term symmetry energy, $E_{\mathrm{sym}}(\rho)$, implies an estimate of the energy cost to convert all the protons to neutrons in nuclear matter at a fixed density ρ [5]

$$E_{\mathrm{sym}} = E_{\mathrm{sym}}(\rho, 1) - E_{\mathrm{sym}}(\rho, 0). \tag{1}$$

Till now our understanding for the nucleon–nucleon interaction has come from the study of nuclear matter at normal density ($\rho = 0.16$ fm^{-3}) [5]. As in the present scenario, symmetry energy is taken as 32 MeV corresponding to normal nuclear matter density. It has been observed that the nuclear matter density is below the normal density when fragmentation takes place. So, it is an interesting and important goal of heavy-ion physics to extract information about the symmetry energy of the nuclear matter at densities higher and lower than the normal nuclear matter density [6, 7]. A large number of studies have been done on the density dependence of the symmetry energy in the recent past [5–8]. The equation below gives us the theoretical conjecture of how the symmetry energy varies against ρ [5–8]

$$E(\rho) = E(\rho_0)(\rho/\rho_0)^{\gamma}, \tag{2}$$

γ tells us about the stiffness of the symmetry energy [5–8]. Constraining the above-said equation for the different values of gamma and its effects on various observables which define nuclear matter, equation of state (EOS) is a much talked about thing in the present day nuclear physics research. For the present study, different values of γ have been taken into account.

At the same time, we also know that apart from the density dependence of symmetry energy, relative velocities of nucleons also affect the nuclear interaction. The momentum dependence of the nuclear equation of state has been reported to affect the multifragmentation and nuclear dynamics drastically [9–11]. So, it could be of interest to see the collective effect of symmetry energy as well as momentum-dependent interactions (MDI) on the fragmentation. No such study is reported in the literature so far. Our present aim is at least twofold: 1) to study the effect of density-dependent symmetry energy on fragmentation and 2) to understand the collective effect of momentum-dependent interactions and density-dependent symmetry energy on fragment production.

The present work has been carried out within the framework of isospin-dependent quantum molecular dynamics (IQMD) model [13]. This paper is organized as follows. We discuss the model briefly in Sec. 1. Our results are presented in Sec. 2, and we summarize the results in Sec. 3.

1. ISOSPIN-DEPENDENT QUANTUM MOLECULAR DYNAMICS (IQMD) MODEL

Though statistical models can be used for fragmentation, their utility is limited due to the fact that dynamics is not considered in these models. For the present study therefore N-body models are better suited. Among various models, the QMD model [12] and its isospin-dependent version IQMD [13] are based on the molecular dynamics picture, where nucleons interact via two- and three-body mutual interactions. Intensive calculations by Aichelin and coworkers demonstrate that these models carry essential dynamics and can explain the phenomena of collective flow [14], multifragmentation [15], particle production [16] and isospin dynamics [17], etc. As noted in [18], relativistic effects do not play a role at incident energies addressed in the present work [18].

In IQMD model, the nucleons of target and projectile interact via two- and three-body Skyrme forces, Yukawa potential, and Coulomb interactions. In addition to the use of explicit charge states of all baryons and mesons, a symmetry potential between protons and neutrons corresponding to the Bethe–Weizsäcker mass formula has been included.

The hadrons propagate using classical Hamilton equations of motion:

$$\frac{d\mathbf{r}_i}{dt} = \frac{d\langle H\rangle}{dp_i}, \quad \frac{d\mathbf{p}_i}{dt} = \frac{d\langle H\rangle}{dr_i}, \tag{3}$$

with

$$\langle H\rangle = \langle T\rangle + \langle V\rangle = \sum_i \frac{p_i^2}{2m_i} + \sum_i \sum_{j>i} \int f_i(\mathbf{r},\mathbf{p},t) V^{ij}(\mathbf{r}',\mathbf{r}) f_j(\mathbf{r}',\mathbf{p}',t)\, d\mathbf{r}\, d\mathbf{r}'\, d\mathbf{p}\, d\mathbf{p}'. \tag{4}$$

The baryon–baryon potential V^{ij}, in the above relation, reads as

$$\begin{aligned} V^{ij}(\mathbf{r}'-\mathbf{r}) &= V^{ij}_{\mathrm{Skyrme}} + V^{ij}_{\mathrm{Yukawa}} + V^{ij}_{\mathrm{Coul}} + V^{ij}_{\mathrm{sym}} + V_{\mathrm{MDI}} = \\ &= \left(t_1\delta(\mathbf{r}'-\mathbf{r}) + t_2\delta(\mathbf{r}'-\mathbf{r})\rho^{\gamma-1}\left(\frac{\mathbf{r}'+\mathbf{r}}{2}\right)\right) + t_3\frac{\exp\left(|\mathbf{r}'-\mathbf{r}|/\mu\right)}{\left(|\mathbf{r}'-\mathbf{r}|/\mu\right)} + \frac{Z_iZ_je^2}{|\mathbf{r}'-\mathbf{r}|} + \\ &\quad + t_6\frac{1}{\varrho_0}T_3^iT_3^j\delta(\mathbf{r}_i{}'-\mathbf{r}_j) + t_7\ \ln^2[t_8(\mathbf{p}_i{}'-\mathbf{p})^2+1]\delta(\mathbf{r}_i'-\mathbf{r}). \end{aligned} \tag{5}$$

Here Z_i and Z_j denote the charges of ith and jth baryons, and T_3^i, T_3^j are their respective T_3 components (i.e., $1/2$ for protons and $-1/2$ for neutrons). The parameters μ and $t_1, \ldots, t_6$ are adjusted to the real part of the nucleonic optical potential. For the density dependence of nucleon optical potential, the standard Skyrme-type parameterization is employed. The potential part resulting from the convolution of the distribution function with the Skyrme interactions V_{Skyrme} reads as

$$V_{\mathrm{Skyrme}} = \alpha\left(\frac{\rho_{\mathrm{int}}}{\rho_0}\right) + \beta\left(\frac{\rho_{\mathrm{int}}}{\rho_0}\right)^{\gamma}. \tag{6}$$

The two of the three parameters of equation of state are determined by demanding that at normal nuclear matter density, the binding energy should be equal to 16 MeV:

$$\kappa = 9\rho^2\frac{\partial^2}{\partial\rho^2}\left(\frac{E}{A}\right), \tag{7}$$

κ is the compressibility factor. The different values of compressibility give rise to soft (S) and hard (H) equations of state (EOS). It is worth mentioning that as shown by Puri and coworkers, Skyrme forces are very successful in the analysis of low-energy phenomena such as fusion, fission, and cluster-radioactivity, where nuclear potential plays an important role [19]. The momentum-dependent interactions can be incorporated by parameterizing the momentum dependence of real part of optical potential [20].

For the present analysis, a soft (S) equation of state has been employed along with reduced isospin-dependent nucleon–nucleon cross section (0.9 of σ_{NN}).

2. RESULTS AND DISCUSSION

We have simulated the reactions of $^{197}\mathrm{Au}_{79} + {}^{197}\mathrm{Au}_{79}$ at different incident energies. The phase space generated by the IQMD model has been analyzed using the minimum spanning tree (MST) method [12]. Two nucleons are bound together in a fragment if their distance is less than 4 fm. Phase space is analyzed at 200 fm/c. In recent years, several improvements have also been suggested [15, 21].

In Fig. 1, we display the symmetry energy as a function of (ρ/ρ_0) to study the variation of the symmetry energy with density. As in the case of the stiff dependence, the symmetry energy tends to increase at supradensity $(\rho \geqslant \rho_0)$, and in the case of the soft dependence, the symmetry energy decreases above the normal nuclear matter density. We here use different forms of symmetry energies to study the multifragmentation. Also, the role of MDI in fragment production has been analyzed subjected to the different forms of symmetry energies.

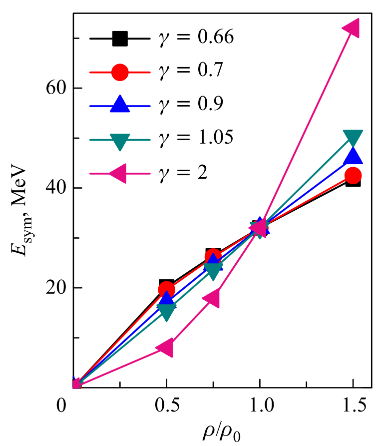

Fig. 1. The various forms of the density dependence of the symmetry energy

In view of the findings of Chen et al. [22], it is believed that the best estimate of the density dependence of the symmetry energy can be extracted from the heavy-ion reaction studies using Eq. (2), by varying the value of γ between 0.6 and 1.05. As for now, $\gamma = 0.69$ is among the best candidates to study the density-dependent symmetry energy in recent theoretical calculations [6]. This form of the density dependence of the symmetry energy is consistent with the parameterization adopted by the H. Heiselberg and Hjorth-Jensen in their study on neutron stars [23].

In Fig. 2, we display the time evolution of free nucleons (FNs) and light charged particles (LCPs) for the symmetry energy of 32, 0 MeV and for 32 $(\rho/\rho_0)^\gamma$ MeV for $\gamma = 0.69$ to study the effect of symmetry energy on fragmentation, without MDI (plots *a* and *c*) and with MDI (plots *b* and *d*). The trends observed through our simulations show the more sensitivity of LCPs towards the symmetry energy. In contrary to the free nucleons, light charged particles seem to be affected by the different forms of the symmetry energy. The larger sensitivity of LCPs towards symmetry energy is due to the pairing nature of LCPs, because the symmetry energy term $\propto (N - Z)^2$ contributes considerably. With zero symmetry energy we see that fewer LCPs are produced whereas the maximum production scale with the symmetry energy. The reduced value of γ leads to mild effect compared to full symmetry energy strength. The inclusion of MDI results in larger variation of fragment production for $\gamma = 0.69$.

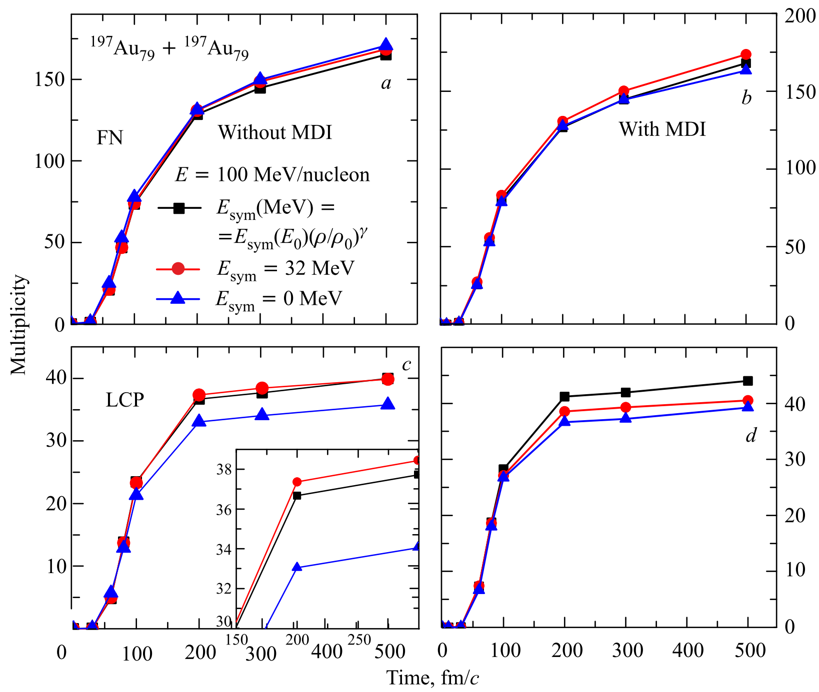

Fig. 2. The time evolution for the free nucleons ($A = 1$) (a, b) and light charged particles ($2 \leqslant A \leqslant 4$) ($c, d$) at $E = 100$ MeV/nucleon for the system $^{197}Au_{79} + {}^{197}Au_{79}$

It is worth to carry out the investigation for the density dependence of the symmetry energy for the different values of γ, i.e., the stiffness of the symmetry energy. Therefore, we perform a comparative study by parameterizing the percentage change in the FN and LCP multiplicity for the different values of γ.

In Fig. 3, we display the percentage change in the production of FNs and LCPs at different colliding geometries for the reaction $^{197}Au_{79} + {}^{197}Au_{79}$ for the different parameterizations of the density dependence of the symmetry energy, i.e., $\gamma = 0.66, 0.7, 0.9, 1.33$, and 2, without MDI (plots a, c) and with MDI (plots b, d). The FN and LCP multiplicity seems to be sensitive towards the different forms of the symmetry energy. Without MDI, the effect of the symmetry energy seems to be more at the semicentral collisions, i.e., $\hat{b} = 0.5$ for all the parameterizations of the density dependence of the symmetry energy.

The percentage change in multiplicity is larger in the case of the LCPs as compared to the FNs for all the parameterizations of the density dependence of the symmetry energy. We see that without MDI, the symmetry energy affects the fragment production within 2.7% for FN and 4.6% for LCP on average. LCPs seem to be more sensitive towards the symmetry energy as concluded above in Fig. 2. However, in Fig. 3, a, c, the FN multiplicity as a function of scaled impact parameter shows the larger change in the multiplicity for lesser values of gamma. After achieving a saturation value at $\gamma = 0.7$, the peak value tends to decrease for the value of γ above and below it. The findings of D. V. Shetty et al. [6] and H. Heiselberg et al. [23] indicate that for $\gamma = 0.69$ the symmetry energy provides a better description of the

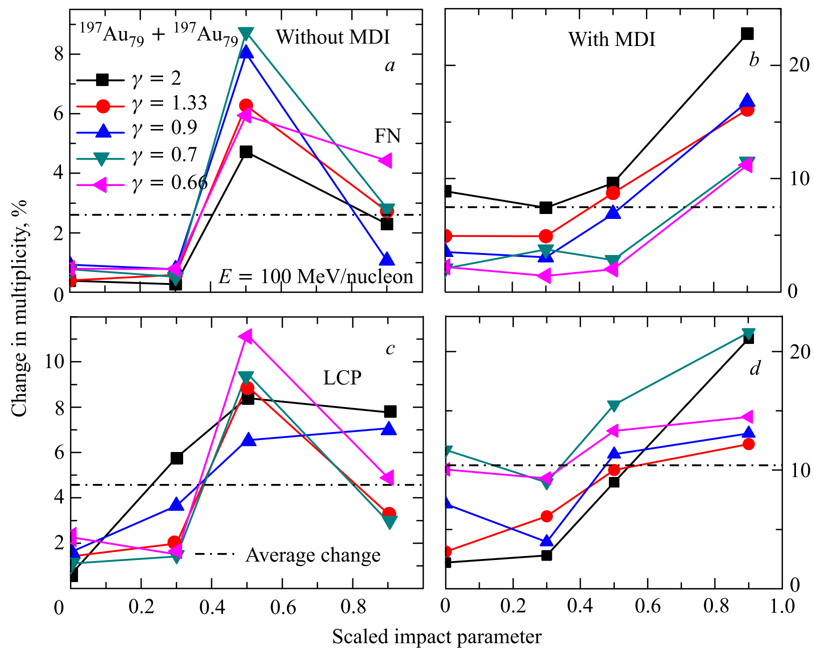

Fig. 3. The same as in Fig. 2, but for the percentage change in multiplicity as a function of scaled impact parameter for different parameterizations of the density dependence of the symmetry energy

nuclear matter equation of state at subnuclear densities. The maximum change is said to be observed for the $\gamma = 0.7$ and $\gamma = 0.66$ for the free nucleons and the LCP, respectively. In central collisions, nearly no effect is seen. This happens due to the violent phase of collision in central collision.

In all the cases, the effect of drastic variation of the form and strength of the symmetry energy is less than 10%. However, our findings are based upon the incident energy of 100 MeV/nucleon. This energy is not large enough to see the huge variation in the production of fragments because the density achieved in collision is not so large. This is in agreement with the findings of G. Lehaut et al. [24], where it was concluded that isospin content of the system has no major influence at low incident energies. In a major contribution, I. Dutt et al. [25] provides an evidence that at low incident energies, which belong to smaller baryonic densities, the isospin dependence of mean-field potential was shown to yield the same result obtained with potentials that have no isospin. The sensitivity of the fragment production can be easily observed for the different values of γ.

However, we want to probe the phenomenon of density dependence of the symmetry energy through fragmentation. But the detected fragments correspond to the subsaturation density, therefore it is not possible to see a large variation in the fragment production with density-dependent symmetry energy.

We observe that the inclusion of MDI results in the larger variation of the fragment production for the various forms of symmetry energies. The MDI and symmetry energy affect the fragment production within 7.5% for FN and 10.3% for LCP on average for $\gamma = 0.66$ to 2. Therefore, our findings give a clear indication of the role of MDI in multifragmentation and in the momentum dependence of the symmetry energy. In [26], it has been reported that the momentum dependence of the symmetry potential has a significant role in nuclear reactions.

In Fig. 4, we show the incident energy dependence of the mean IMF multiplicity and comparison with the ALADIN data [28] for the central collisions. The momentum-dependent forces and the symmetry energy seem to be less sensitive at low incident energies. The IMF multiplicity seems to be the least sensitive at the incident energy of 100 MeV/nucleon. This is due to the less density achieved during the collision which is not sufficient to see the effect of the density-dependent symmetry energy.

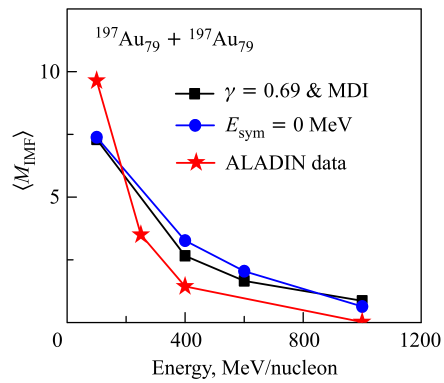

Fig. 4. The same as in Fig. 2, but for the incident energy dependence of mean IMF multiplicity

As the energy increases, the MDI and the symmetry energy seem to affect the IMF production. At higher incident energies, the mean field does not play any role. Though, MDI destabilizes the nuclei; a careful analysis is made by Puri et al. [9, 27] and found that up to 200 fm/c, emission of the nucleons with MDI is quite small. The repulsion produced due to MDI and symmetry energy tends to reduce the IMF production.

The maximum effect can be observed around the incident energy of 400 MeV/nucleon. Therefore the larger role of the symmetry energy can be observed at higher incident energies due to the larger variation in the density achieved during the collision. The effect of the symmetry energy and MDI seems to decrease at the incident energy above 400 MeV/nucleon because of the lesser overlapping time of the target and projectile due to the violent collision. The results obtained through the inclusion of MDI and $\gamma = 0.69$ favour the data.

The correlation between the mean intermediate mass fragment (IMF) multiplicity with scaled impact parameter is plotted for the system $^{197}Au_{79} + {}^{197}Au_{79}$ in Fig. 5.

The theoretical trends are in accordance with the data of ALADIN setup [28]. The momentum cut of the order of 150 MeV/c has been applied to analyze the phase space. Indeed, the maximum of IMF multiplicity is reached at semiperipheral collisions. In the case of central collisions, the violent collisions reduce the fragment production. While in the case of peripheral collisions, the IMF production again decreases due to the lesser overlapping of the target and projectile. The inclusion of MDI and $\gamma = 0.69$ concludes a significant shift in the peak IMF multiplicity.

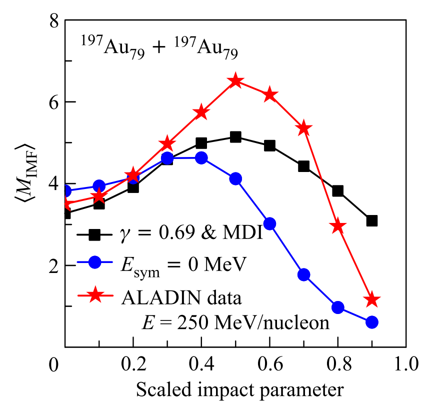

Fig. 5. Impact parameter dependence of the mean IMF multiplicity at E = 250 MeV/nucleon for the system $^{197}Au_{79} + {}^{197}Au_{79}$

3. SUMMARY

We have investigated the effect of the density-dependent symmetry energy and momentum-dependent interactions on the production of fragments. Our calculations show larger sensitivity of LCPs towards symmetry energy. The fragment production seems to be affected by the symmetry energy at semiperipheral and peripheral collisions. Overall, we see that the symmetry energy affects the fragment production within 2.7% for FN and 4.6% for LCP on average. This mild sensitivity can be attributed to the fact that the density at which the fragmentation takes place is not high enough to see the role of symmetry energies. Also, the inclusion of MDI concludes larger variation, i.e., 7.5% for FN and 10.3% for LCP on average. It is concluded that the momentum dependence of the symmetry energy plays a significant role in multifragmentation.

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