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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

FINAL STATE INTERACTION IN $B^0 \to J/\psi\rho^0$

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In this research, the hadronic decay of $B^0 \to J/\psi\rho^0$ is analyzed by using the QCD factorization (QCDF) and final state interaction (FSI). The $D^{+(*)}D^{-(*)}$ for $d\bar{d}$ and $D^{0(*)}\bar{D}^{0(*)}$ for $u\bar{u}$ contributions via the exchange of charmed mesons are chosen for the intermediate states. These amplitudes are calculated by using QCDF and used in the absorptive part of the diagrams. The experimental branching ratio of $B^0 \to J/\psi\rho^0$ decay is $(2.7 \pm 0.4) \cdot 10^{-5}$ and our results according to the QCDF method and FSI are $0.87 \cdot 10^{-5}$ and $(0.057{-}4.18) \cdot 10^{-5}$, respectively.

В представленной работе адронный распад $B^0 \to J/\psi\rho^0$ анализируется с учетом КХД-факторизации (ФКХД) и взаимодействия в конечном состоянии (ВКС). Вклады $D^{+(*)}D^{-(*)}$ для $d\bar{d}$ и $D^{0(*)}\bar{D}^{0(*)}$ для $u\bar{u}$ рассматриваются в качестве промежуточных состояний, возникающих при обмене очарованными мезонами. Эти амплитуды вычисляются с помощью учета ФКХД и используются в частях диаграмм, отвечающих за поглощение. Экспериментальное значение отношения расщепления в распаде $B^0 \to J/\psi\rho^0$ берется равным $(2{,}7 \pm 0{,}4) \cdot 10^{-5}$. Наши результаты, полученные в рамках учета ФКХД и ВКС, — это $0{,}87 \cdot 10^{-5}$ и $(0{,}057{-}4{,}18) \cdot 10^{-5}$ соответственно.

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INTRODUCTION

B-meson nonleptonic decays are significant for testing theoretical frameworks and searching for new physics beyond the Standard Model. The next-to-leading order low-energy effective Hamiltonian is used for the weak interaction matrix elements and the FSI. The importance of the FSI in hadronic processes has been identified for a long time. Recently, its applications in D and B decays have attracted extensive interests and attention of theorists.

Since the hadronic matrix elements are fully controlled by nonperturbative QCD, the most important problem is how to evaluate them properly. Factorization method enables one to separate the nonperturbative QCD effects from the perturbative parts and to calculate the latter in terms of the field theory order by order. Several factorization approaches have been proposed to analyze B-meson decays, such as the naive factorization approach, the QCD factorization approach, the perturbative QCD approach and Soft Collinear Effective Theory (SCET); none provided an estimate of the FSI at the hadronic level. These approaches, successfully explain many phenomena; however, there are still some problems which are not easy to describe within this framework.

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These may be some hints for the need of FSI in B decays. FSI effects are nonperturbative in nature [1]. FSI is one of the ways to solve the nonperturbative QCD for the long-distance case. In many decay modes, the FSI may play a crucial role [2]. In this way, the CKM matrix elements and color factor are suppressed and the CKM's most favored two-body intermediate states are the only ones that have been taken into consideration [3].

The FSI can be considered as a soft rescattering style for certain intermediate two-body hadronic channel $B^0 \to D^{+(*)}D^{-(*)}$ and $B^0 \to D^{0(*)}\bar{D}^{0(*)}$ decays [4]. Therefore, they can be treated as the one-particle exchange processes at the hadron loop level (HLL). We calculated the $B^0 \to J/\psi\rho^0$ decay according to QCDF method and selected the next-to-leading order Wilson coefficients at the scale m_b [5] and obtained the $\mathrm{BR}(B^0 \to J/\psi\rho^0) = 0.87 \cdot 10^{-5}$ that is small in factorization approach. The FSI can give sizable corrections and we can include it. Rescattering amplitude can be derived by calculating the absorptive part of triangle diagrams. Since $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$, intermediate states for $d\bar{d}$ are $D^{+(*)}D^{-(*)}$ and for $u\bar{u}$ are $D^{0(*)}\bar{D}^{0(*)}$. We calculated the $B^0 \to D^+D^-$ and $B^0 \to D^0\bar{D}^0$ decays amplitudes as the intermediate states by using the QCDF method. The experimental result of this decay is $\mathrm{BR}(B^0 \to J/\psi\rho^0) = (2.7 \pm 0.4) \cdot 10^{-5}$ [6] and we calculated that according to HLL method. In this case, the branching ratio of $B^0 \to J/\psi\rho^0$ for $\eta = 1 \sim 3.5$ is $(0.057–4.18) \cdot 10^{-5}$.

This paper is organized as follows. We present the calculation of QCDF for $B^0 \to J/\psi\rho^0$ decay in Sec. 1. In Sec. 2, we calculate the amplitudes of the intermediate states of $B^0 \to D^{+(*)}D^{-(*)}, D^{0(*)}\bar{D}^{0(*)}$ decays. Then, we present the calculation of HLL for $B^0 \to J/\psi\rho^0$ decay in Sec. 3. In Sec. 4, we give the numerical results, and in the last section, we have a short conclusion.

1. QCD FACTORIZATION OF $B^0 \to J/\psi\rho^0$ DECAY

To compare QCDF with FSI, we explore QCDF analysis. In this case we only have color-suppressed tree and color-allowed penguin topology. These contributions are small, but it is interesting and necessary to discuss them. In the factorization method, Feynman diagrams for the $B^0 \to J/\psi\rho^0$ decay are shown in Fig. 1 and the amplitude when both mesons are vector is given by

$$M(B^0 \to J/\psi(p_1, \epsilon_1)\rho^0(p_2, \epsilon_2)) = -i\frac{G_F}{2} f_{J/\psi} m_{J/\psi} \Big\{ (\epsilon_1^* \cdot \epsilon_2^*)(m_B + m_\rho) A_1^{B\rho}(m_{J/\psi}^2) - \\ - (\epsilon_1^* \cdot p_B)(\epsilon_2^* \cdot p_B) \frac{2A_2^{B\rho}(m_{J/\psi}^2)}{m_B + m_\rho} - i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu \epsilon_2^\nu p_2^\alpha p_B^\beta \times \\ \times \frac{2V^{B\rho}(m_{J/\psi}^2)}{m_B + m_\rho} \Big\} \{a_2 V_{cb} V_{cd}^* + (a_3 + a_5 + a_7 + a_9)\lambda_p\}, \quad (1)$$

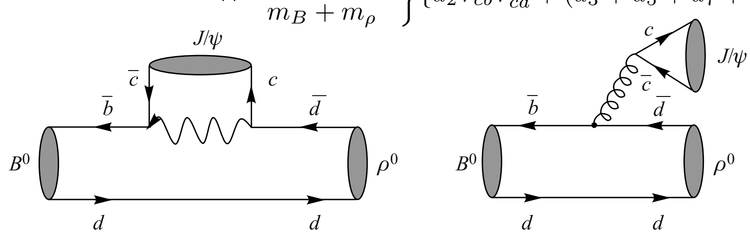

Fig. 1. Diagrams describing the decay of $B^0 \to J/\psi\rho^0$

here $f_{J/\psi}$ is the decay constant and ϵ^* is the polarization vector of vector meson. $A_1^{B\rho}(m^2_{J/\psi})$, $A_2^{B\rho}(m^2_{J/\psi})$ and $V^{B\rho}(m^2_{J/\psi})$ are the decay form factors. We select $\epsilon_{0123} = +1$ and

$$\begin{aligned} a_i &= c_i + \frac{1}{N_c} c_{i+1} \quad (i = \text{odd}), \\ a_i &= c_i + \frac{1}{N_c} c_{i-1} \quad (i = \text{even}), \\ \lambda_p &= \sum_{p=u,c} V_{pb} V^*_{pd}, \end{aligned} \tag{2}$$

where c_i are the Wilson coefficients, with i running from $i = 1, \ldots, 10$, and N_c is the number of color in QCD. In the rest frame of the decaying B meson only longitudinally polarized ρ and J/ψ are produced. $\epsilon^*_\rho \cdot p_B$ and $\epsilon^*_{J/\psi} \cdot p_B$ are then given by

$$\epsilon^*_V \cdot p_B = \frac{m_B}{m_V} |\mathbf{p}| \quad (V = \rho \text{ or } J/\psi), \tag{3}$$

where $|\mathbf{p}|$ is the absolute value of the 3-momentum of the ρ (or the J/ψ) in the B rest frame.

2. AMPLITUDES OF INTERMEDIATE STATES

In this section, before analyzing FSI in $B^0 \to J/\psi \rho^0$ decay we introduce the factorization approach in detail. The effective weak Hamiltonian for B decays consists of a sum of local operators Q_i multiplied by QCDF coefficients c_i and products of elements of the quark mixing matrix [7]. The factorization approach of the heavy meson decays can be expressed in terms of different topologies of various decays mechanism such as tree, penguin and annihilation.

Since $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$, intermediate states for $d\bar{d}$ are $D^{+(*)}D^{-(*)}$ and for $u\bar{u}$ are $D^{0(*)}\bar{D}^{0(*)}$. The Feynman diagrams of the $B^0 \to D^{+(*)}D^{-(*)}$ decays are shown in Fig. 2 and the amplitudes read

$$\begin{aligned} M(B^0 \to D^+ D^-) = {} & i\frac{G_F}{\sqrt{2}} f_D F_0^{BD}(m_B^2 - m_D^2) \Big\{ (a_1 + a_2) V_{cb} V^*_{cd} + \\ & + [a_4 + a_{10} + r_\chi^D (a_6 + a_8)] \lambda_p \Big\} + \\ & + i\frac{G_F}{\sqrt{2}} f_B f_D^2 \left\{ b_1 V_{cb} V^*_{cd} + \left[b_3 + 2b_4 - \frac{1}{2} b_{3,\text{ew}} + \frac{1}{2} b_{4,\text{ew}} \right] \lambda_p \right\}, \end{aligned} \tag{4}$$

$$\begin{aligned} M(B^0 \to D^{+*} D^{-*}) = {} & -i\frac{G_F}{\sqrt{2}} f_{D^*} m_{D^*} V_{cb} V^*_{cd} \Big\{ (\epsilon_1^* \cdot \epsilon_2^*)(m_B + m_{D^*}) A_1^{BD^*}(m^2_{D^*}) - \\ & - (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B) \frac{2A_2^{BD^*}(m^2_{D^*})}{m_B + m_{D^*}} \Big\} \Big\{ (a_1 + a_2) V_{cb} V^*_{cd} + [a_4 + a_{10} + r_\chi^{D^*}(a_6 + a_8)] \lambda_p \Big\} + \\ & + i\frac{G_F}{\sqrt{2}} f_B f^2_{D^*} \left\{ b_1 V_{cb} V^*_{cd} + \left[b_3 + 2b_4 - \frac{1}{2} b_{3,\text{ew}} + \frac{1}{2} b_{4,\text{ew}} \right] \lambda_p \right\}, \end{aligned} \tag{5}$$

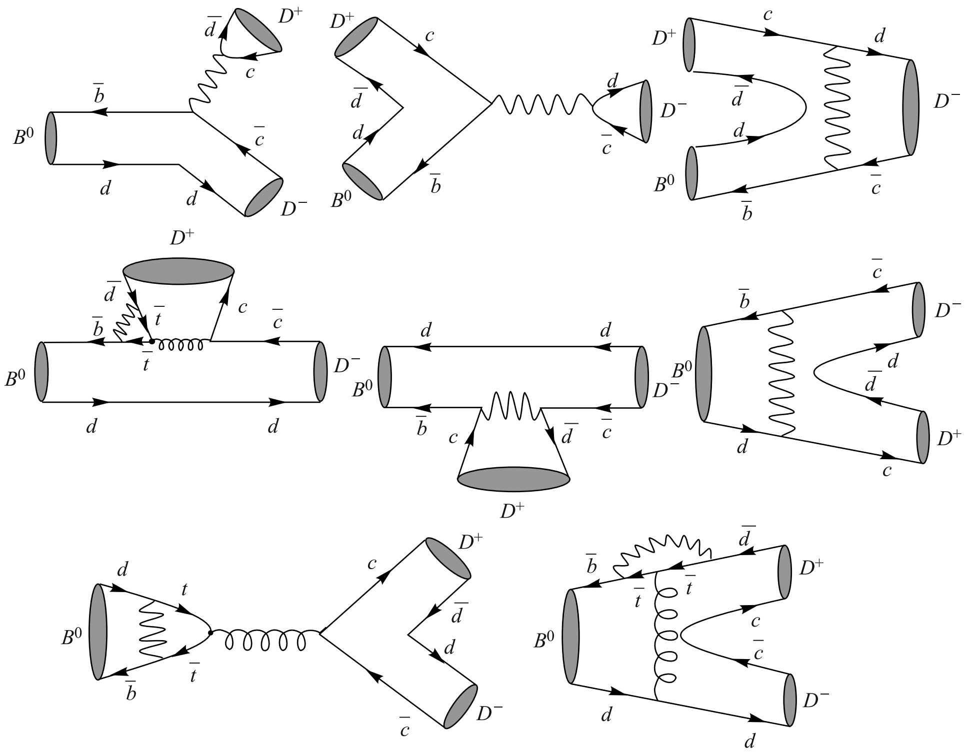

Fig. 2. $B^0 \to D^{+(*)}D^{-(*)}$ decays diagrams

$$M(B^0 \to D^{+*}D^-) = \sqrt{2}G_F m_{D^*} V_{cb}V_{cd}^*(\epsilon_{D^*}\cdot p_D)\left[f_{D^*}F_1^{BD}(m_{D^*}^2) + f_D A_0^{BD^*}(m_D^2)\right]\times$$

$$\times\left\{(a_1+a_2)V_{cb}V_{cd}^* + [a_4 + a_{10} + r_\chi^{D^*}(a_6+a_8)]\lambda_p\right\}+$$

$$+\, i\frac{G_F}{\sqrt{2}}f_B f_{D^*}^2\left\{b_1 V_{cb}V_{cd}^* + \left[b_3 + 2b_4 - \frac{1}{2}b_{3,\mathrm{ew}} + \frac{1}{2}b_{4,\mathrm{ew}}\right]\lambda_p\right\}, \quad (6)$$

where $A_0^{BD^*}$, $A_1^{BD^*}$ and $A_2^{BD^*}$ are form factors for $B \to D^*$; F_0^{BD} and F_1^{BD} are form factors for $B \to D$ transitions [8] and

$$b_1 = \frac{C_F}{N_c^2}c_1 A_1^i, \quad b_3 = \frac{C_F}{N_c^2}[c_3 A_1^i + c_5(A_3^i + A_3^f) + N_c C_6 A_3^f],$$

$$b_{3,\mathrm{ew}} = \frac{C_F}{N_c^2}[c_9 A_1^i + c_7(A_3^i + A_3^f) + N_c C_8 A_3^f],$$

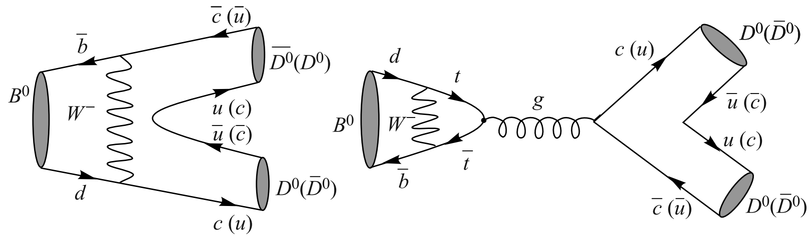

Fig. 3. $B^0 \to D^{0(*)}\bar{D}^{0(*)}$ decays diagrams

$$\begin{aligned} b_4 &= \frac{C_F}{N_c^2}[c_4 A_1^i + c_6 A_2^i], \quad b_{4,\mathrm{ew}} = \frac{C_F}{N_c^2}[c_{10} A_1^i + c_8 A_2^i], \\ A_1^i &\approx A_2^i = 2\pi\alpha_s\left[9\left(X_A - 4 + \frac{\pi^2}{3}\right) + r_\chi^{D^+} r_\chi^{D^-} X_A^2\right], \\ A_3^i &= 0, \quad A_3^f \approx 6\pi\alpha_s(r_\chi^{D^+} + r_\chi^{D^-})(2X_A^2 - X_A), \\ r_\chi^{D^{\pm(0)}} &= \frac{2m^2_{D^{\pm(0)}}}{(m_b - m_c)(m_{d(u)} + m_c)}. \end{aligned} \tag{7}$$

In the QCDF method the process of $B^0 \to D^{0(*)}\bar{D}^{0(*)}$ only occurs via annihilation between b and $\bar{d}$, so the FSI must be seriously considered to solve the $B^0 \to D^{0(*)}\bar{D}^{0(*)}$ decays and we follow [9], for these decays, the Feynman diagrams are shown in Fig. 3 and the amplitudes read

$$M(B^0 \to D^{0(*)}\bar{D}^{0(*)}) = i\frac{G_F}{\sqrt{2}} f_{B^0} f_{D^{0(*)}} f_{\bar{D}^{0(*)}} (b_1 + 2b_4 + 2b_{4,\mathrm{ew}})\lambda_p. \tag{8}$$

3. FINAL STATE INTERACTION OF $B^0 \to J/\psi\rho^0$ DECAY

When the FSI method for decay is calculated, two-body intermediate states such as D^+D^-, $D^{+*}D^{-*}$, $D^{+*}D^-$, D^+D^{-*}, $D^0\bar{D}^0$, $D^{0*}\bar{D}^{0*}$, $D^{0*}\bar{D}^0$ and $D^0\bar{D}^{0*}$ are produced. The quark model for $B^0 \to D^{+(*)}D^{-(*)} \to J/\psi\rho^0$ and $B^0 \to D^{0(*)}\bar{D}^{0(*)} \to J/\psi\rho^0$ diagrams is shown in Figs. 4 and 5. The hadronic level diagrams are shown in Figs. 6 and 7. In this framework we choose the t-channel one-particle exchange processes. For calculation, the relevant Lagrangian density are defined as follows [2, 10, 11]:

$$\begin{aligned} \pounds_{DD\rho} &= ig_{DD\rho}(D\rho^\mu\partial_\mu\bar{D} - \partial_\mu D\rho^\mu\bar{D}), \\ \pounds_{D^*D\rho} &= -g_{D^*D\rho}\epsilon^{\mu\nu\alpha\beta}(D\partial_\mu\rho_\nu\partial_\alpha\bar{D}^*_\beta + \partial_\mu D^*_\nu\partial_\alpha\rho_\beta\bar{D}), \\ \pounds_{D^*D^*\rho} &= ig_{D^*D^*\rho}\{\partial_\mu D^*_\nu\rho^\mu\bar{D}^{*\nu} - D^*_\nu\rho_\mu\partial^\mu\bar{D}^{*\nu} + (D^{*\nu}\partial_\mu\rho_\nu - \partial_\mu D^*_\nu\rho^\nu)\bar{D}^{*\mu} + \\ &\qquad + D^{*\mu}(\rho^\nu\partial_\mu\bar{D}^*_\nu - \partial_\mu\rho_\nu\bar{D}^{*\nu})\}, \\ \pounds_{\psi DD} &= ig_{\psi DD}\psi_\mu(\partial^\mu D\bar{D} - D\partial^\mu\bar{D}), \\ \pounds_{\psi D^*D} &= -g_{\psi D^*D}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\psi_\nu(\partial_\alpha D^*_\beta\bar{D} + D\partial_\alpha\bar{D}^*_\beta), \\ \pounds_{\psi D^*D^*} &= -ig_{\psi D^*D^*}\{\psi^\mu(\partial_\mu D^{*\nu}\bar{D}^*_\nu - D^{*\nu}\partial_\mu\bar{D}^*_\nu) + (\partial_\mu\psi_\nu D^{*\nu} - \psi_\nu\partial_\mu D^{*\nu})\bar{D}^{*\mu} + \\ &\qquad + D^{*\mu}(\psi^\nu\partial_\mu\bar{D}^*_\nu - \partial_\mu\psi_\nu\bar{D}^{*\nu})\}. \end{aligned} \tag{9}$$

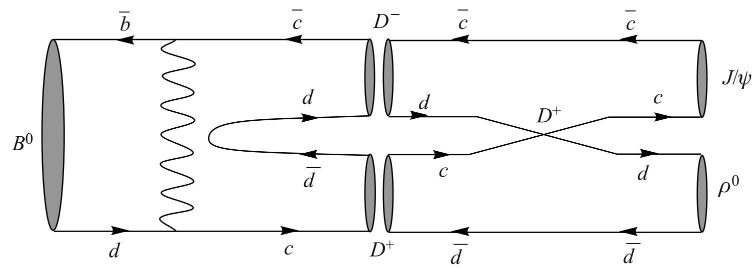

Fig. 4. Quark level diagram for $B^0 \to D^{+(*)}D^{-(*)} \to J/\psi\rho^0$

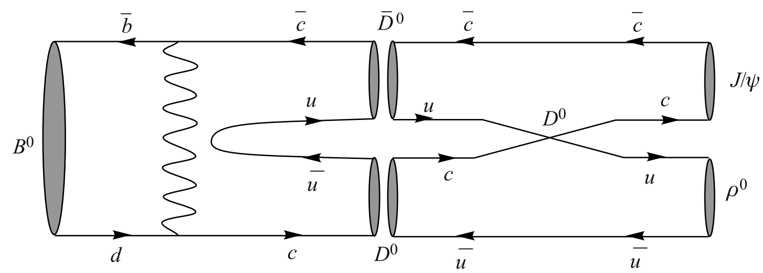

Fig. 5. Quark level diagram for $B^0 \to D^{0(*)}\bar{D}^{0(*)} \to J/\psi\rho^0$

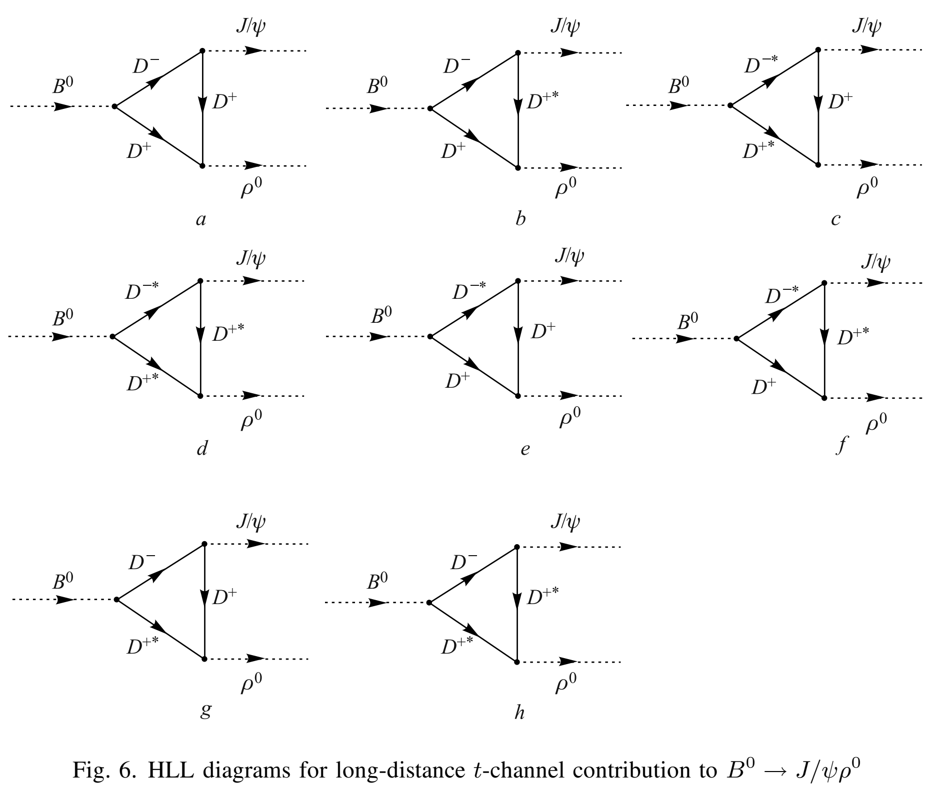

Fig. 6. HLL diagrams for long-distance t-channel contribution to $B^0 \to J/\psi\rho^0$

Here $\epsilon_{0123} = +1$ and we define the charm meson iso-doublets as

$$\begin{aligned} \bar{D}^T &= (\bar{D}^0, D^-), \qquad D = (D^0, D^+), \\ \bar{D}^{*T} &= (\bar{D}^{*0}, D^{*-}), \quad D^* = (D^{*0}, D^{*+}). \end{aligned} \tag{10}$$

With the above preparation we can write out the decay amplitude involving HLL contributions with the following formula:

$$\begin{aligned} M(B(p_B) \to M(p_1)M(p_2) \to M(p_3)M(p_4)) = \frac{1}{2}\int \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}\times \\ \times (2\pi)^4\delta^4(p_B - p_1 - p_2)M(B \to M_1M_2)G(M_1M_2 \to M_3M_4), \end{aligned} \tag{11}$$

for which both intermediate mesons (M_1, M_2) are pseudoscalar. And

$$\begin{aligned} M(B(p_B) \to M(p_1)M(p_2) \to M(p_3)M(p_4)) = -i\frac{G_F}{2\sqrt{2}}\int \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}\times \\ \times (2\pi)^4\delta^4(p_B - p_1 - p_2)f_{D^*}m_{D^*}V_{cb}V_{cd}^*\Big[(\epsilon_1^* \cdot \epsilon_2^*)(m_B + m_1)A_1^{BM_1}(m_2^2) - \\ - (\epsilon_1^* \cdot p_B)(\epsilon_2^* \cdot p_B)\frac{2A_2^{BM_1}(m_2^2)}{m_B + m_1}\Big]G(M_1M_2 \to M_3M_4), \end{aligned} \tag{12}$$

in which both mesons are vectors. Also $G(M_1M_2 \to M_3M_4)$ involves hadronic vertices factor, which is defined as

$$\begin{aligned} \langle D(p_3)\rho(\varepsilon_2, p_2)|i\pounds|D(p_1)\rangle &= -ig_{DD\rho}\epsilon_2(p_1 + p_3), \\ \langle D^*(\epsilon_3, p_3)\rho(\epsilon_2, p_2)|i\pounds|D(p_1)\rangle &= -i\sqrt{2}g_{D^*D\rho}\epsilon_{\mu\nu\alpha\beta}\varepsilon_2^\mu\epsilon_3^{*\nu}p_1^\alpha p_2^\beta, \\ \langle D^*(\epsilon_3, p_3)\rho(\epsilon_2, p_2)|i\pounds|D^*(\epsilon_1, p_1)\rangle &= -i\epsilon_1^\mu\epsilon_2^\nu\epsilon_3^\alpha[2p_{2\mu}g_{\nu\alpha} - (p_1 + p_2)_\alpha g_{\mu\nu} + 2p_{1\nu}g_{\mu\alpha}], \\ \langle D(p_3)\psi(\epsilon_2, p_2)|i\pounds|D(p_1)\rangle &= -ig_{\psi DD}\epsilon_2(p_1 + p_3), \\ \langle D^*(\epsilon_3, p_3)\psi(\epsilon_2, p_2)|i\pounds|D(p_1)\rangle &= -ig_{\psi D^*D}\epsilon_{\mu\nu\alpha\beta}\epsilon_2^\mu\epsilon_3^{*\nu}p_1^\alpha p_2^\beta, \\ \langle D^*(\epsilon_3, p_3)\psi(\epsilon_2, p_2)|i\pounds|D^*(\epsilon_1, p_1)\rangle &= -i\epsilon_1^\beta\epsilon_2^\eta\epsilon_3^\lambda[2p_{2\beta}g_{\eta\lambda} - (p_1 + p_2)_\lambda g_{\beta\eta} + 2p_{1\eta}g_{\beta\lambda}]. \end{aligned} \tag{13}$$

The amplitudes of the mode $B^0 \to D^-(p_1)D^+(p_2) \to J/\psi(\epsilon_3, p_3)\rho^0(\epsilon_4, p_4)$, where D^+ and D^{+*} mesons are exchanged at t-channel respectively, are given by

$$\begin{aligned} Abs(6a) = \int_{-1}^{1} \frac{|\mathbf{p}_1|d(\cos\theta)}{4\pi m_B}M(B^0 \to D^+D^-)(-i)g_{\psi DD}(\epsilon_3 \cdot p_1)\times \\ \times (-i)g_{DD\rho}(\epsilon_4 \cdot p_2)\frac{F^2(q^2, m_D^2)}{q^2 - m_D^2}, \end{aligned} \tag{14}$$

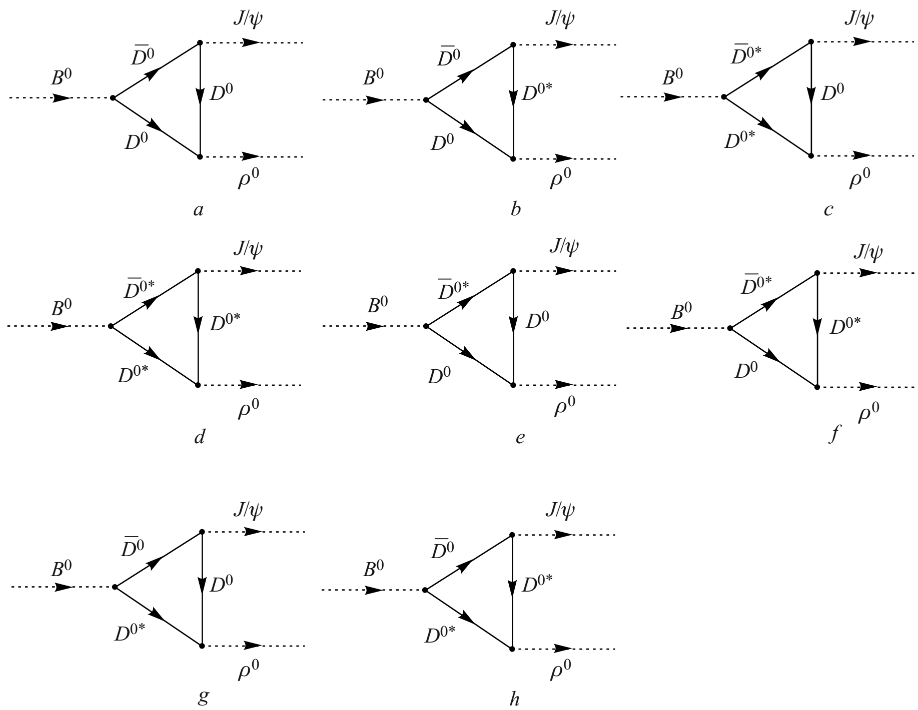

Fig. 7. HLL diagrams for long-distance t-channel contribution to $B^0 \to J/\psi\rho^0$

$$Abs(6b) = \int_{-1}^{1} \frac{|\mathbf{p}_1| d(\cos\,\theta)}{16\pi m_B} M(B^0 \to D^+ D^-)(-i) g_{\psi D^* D} \epsilon_{\rho\sigma\lambda\eta} \epsilon_3^\rho p_1^\lambda p_3^\eta \times$$
$$\times (-i)\sqrt{2} g_{D^* D\rho} \epsilon_{\mu\nu\alpha\beta} \epsilon_4^\mu p_2^\alpha p_4^\beta \left(g_{\sigma\nu} - \frac{q_\sigma q_\nu}{m_{D^*}^2} \right) \frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2}. \quad (15)$$

The amplitudes of the $B^0 \to D^{-*}(\epsilon_1, p_1) D^{+*}(\epsilon_2, p_2) \to J/\psi(\epsilon_3, p_3)\rho^0(\epsilon_4, p_4)$, where D^+ and D^{+*} are exchanged respectively, read as

$$Abs(6c) = \int_{-1}^{1} \frac{|\mathbf{p}_1| d(\cos\,\theta)}{16\pi m_B} \left(-i\frac{G_F}{\sqrt{2}} \right) f_{D^*} m_{D^*} V_{cb} V_{cd}^* \times$$
$$\times (-i) g_{\psi D^* D} \epsilon_{\mu\nu\alpha\beta} \epsilon_3^\mu p_3^\nu p_1^\beta (-i)\sqrt{2} g_{D^* D\rho} \epsilon_{\rho\sigma\lambda\eta} \epsilon_4^\rho p_4^\sigma p_2^\eta \times$$
$$\times \left[(m_B^2 + m_{D^*}^2) A_1^{BD^*}(m_{D^*}^2) \left(-g_\chi^\alpha + \frac{p_1^\alpha p_{1\chi}}{m_{D^*}^2} \right) \left(-g^{\lambda\chi} + \frac{p_2^\lambda p_2^\chi}{m_{D^*}^2} \right) - \right.$$
$$- \frac{2A_2^{BD^*}(m_{D^*}^2)}{(m_B^2 + m_{D^*}^2)} \left(-p_B^\alpha + \frac{(p_1 \cdot p_B) p_1^\alpha}{m_{D^*}^2} \right) \times$$
$$\left. \times \left(-p_B^\lambda + \frac{(p_2 \cdot p_B) p_2^\lambda}{m_{D^*}^2} \right) \right] \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2}, \quad (16)$$

and

$$Abs(6d) = \int_{-1}^{1} \frac{|p_1| d(\cos\theta)}{16\pi m_B} \Big(- i\frac{G_F}{\sqrt{2}}\Big) f_{D^*} m_{D^*} V_{cb} V_{cd}^* \times$$
$$\times (-i) g_{\psi D^* D^*} \epsilon_3^{\rho} [2p_{1\rho} g_{\mu\beta} - (p_1 + p_3)_\beta g_{\mu\rho} + 2p_{3\mu} g_{\beta\rho}] \times$$
$$\times (-i) g_{D^* D^* \rho} \epsilon_4^{\lambda} [2p_{2\lambda} g_{\alpha\nu} - (p_2 + p_4)_\alpha g_{\nu\lambda} + 2p_{4\nu} g_{\alpha\lambda}] \times$$
$$\times \Big(- g^{\alpha\beta} + \frac{q^\alpha q^\beta}{m_{D^*}^2}\Big) \Big[(m_B^2 + m_{D^*}^2) A_1^{BD^*}(m_{D^*}^2) \Big(- g^{\mu\chi} + \frac{p_1^\mu p_1^\chi}{m_{D^*}^2}\Big) \Big(- g_\chi^\nu + \frac{p_2^\nu p_{2\chi}}{m_{D^*}^2}\Big) -$$
$$- \frac{2A_2^{BD^*}(m_{D^*}^2)}{(m_B^2 + m_{D^*}^2)} \Big(- p_B^\mu + \frac{(p_1 \cdot p_B) p_1^\mu}{m_{D^*}^2}\Big) \Big(- p_B^\nu + \frac{(p_2 \cdot p_B) p_2^\nu}{m_{D^*}^2}\Big) \Big] \frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2}. \quad (17)$$

The amplitudes of the mode $B^0 \to D^{-*}(\epsilon_1, p_1) D^+(p_2) \to J/\psi(\epsilon_3, p_3) \rho^0(\epsilon_4, p_4)$, where D^+ and D^{+*} are exchanged respectively, become

$$Abs(6e) = \int_{-1}^{1} \frac{|\mathbf{p}_1| d(\cos\theta)}{16\pi m_B} \sqrt{2} G_F V_{cb} V_{cd}^* \times$$
$$\times (-2i) g_{DD\rho} (\epsilon_4 \cdot p_2) (-i) g_{\psi D^* D} \epsilon_{\mu\nu\alpha\beta} \epsilon_3^\mu p_1^\alpha p_3^\beta \times$$
$$\times [f_{D^*} F_1^{BD}(m_{D^*}^2) + f_D A_0^{BD^*}(m_D^2)] \Big(- p_2^\nu + \frac{(p_1 \cdot p_2) p_1^\nu}{m_{D^*}^2}\Big) \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2}, \quad (18)$$

and

$$Abs(6f) = \int_{-1}^{1} \frac{|\mathbf{p}_1| d(\cos\theta)}{16\pi m_B} \sqrt{2} G_F V_{cb} V_{cd}^* \times$$
$$\times (-i) g_{\psi D^* D^*} \epsilon_3^{\nu} [2p_{1\nu} g_{\mu\beta} - (p_1 + p_3)_\beta g_{\mu\nu} + 2p_{3\mu} g_{\beta\nu}] \times$$
$$\times (i) \sqrt{2} g_{D^* D\rho} \epsilon_{\rho\sigma\lambda\eta} \epsilon_4^\rho p_2^\lambda p_4^\eta [f_{D^*} F_1^{BD}(m_{D^*}^2) + f_D A_0^{BD^*}(m_D^2)] \times$$
$$\times \Big(- g^{\alpha\sigma} + \frac{q^\alpha q^\sigma}{m_{D^*}^2}\Big) \Big(- p_2^\mu + \frac{(p_1 \cdot p_2) p_1^\mu}{m_{D^*}^2}\Big) \frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2}. \quad (19)$$

The amplitudes of the mode $B^0 \to D^-(p_1) D^{+*}(\epsilon_2, p_2) \to J/\psi(\epsilon_3, p_3) \rho^0(\epsilon_4, p_4)$, where D^+ and D^{+*} are exchanged respectively, read as

$$Abs(6g) = \int_{-1}^{1} \frac{|\mathbf{p}_1| d(\cos\theta)}{16\pi m_B} \sqrt{2} G_F V_{cb} V_{cd}^* \times$$
$$\times (-2i) g_{\psi DD} (\epsilon_3 \cdot p_1) (i) \sqrt{2} g_{D^* D\rho} \epsilon_{\mu\nu\alpha\beta} \epsilon_4^\mu p_2^\alpha p_4^\beta \times$$
$$\times [f_{D^*} F_1^{BD}(m_{D^*}^2) + f_D A_0^{BD^*}(m_D^2)] \Big(- p_1^\nu + \frac{(p_1\, p_2) p_2^\nu}{m_{D^*}^2}\Big) \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2}, \quad (20)$$

and

$$Abs(6h) = \int_{-1}^{1} \frac{|\mathbf{p}_1| d(\cos\theta)}{16\pi m_B} \sqrt{2} G_F V_{cb} V_{cd}^* \times$$
$$\times (-i) g_{D^* D^* \rho} \epsilon_4^\mu [2p_{2\mu} - (p_2+p_4)_\alpha g_{\mu\nu} + 2p_{4\nu} g_{\alpha\mu}] \times$$
$$\times (-i) g_{\psi D^* D} \epsilon_{\rho\sigma\lambda\eta} \epsilon_3^\rho p_1^\lambda p_3^\eta [f_{D^*} F_1^{BD}(m_{D^*}^2) + f_D A_0^{BD^*}(m_D^2)] \times$$
$$\times \left(- g^{\beta\sigma} + \frac{q^\beta q^\sigma}{m_{D^*}^2}\right)\left(- p_1^\nu + \frac{(p_1 \cdot p_2) p_2^\nu}{m_{D^*}^2}\right) \frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2}. \quad (21)$$

In writing the above amplitudes, we have used $\mathbf{p} \cdot \boldsymbol{\epsilon} = 0$. The effective vertices of strong interaction for the rescattering process, such as $g_{DD\rho}$, $g_{DD\psi}$, etc., are gained from data provided the flavor $SU(3)$ symmetry holds. However, since the t-channel exchanged particles P and V are off their mass shell, a phenomenological form factor $F(q^2, m_i^2)$ is introduced to compensate the off-shell effect at the vertices. Because the effective coupling constants, for example, $g_{DD\rho}$, are obtained from the data where the three particles are all on-shell, while in our case the exchanged $D^{+(*)}$ and $D^{0(*)}$ mesons are off-shell, a compensation form factor is needed and we take it as suggested in [1, 12]

$$F(q^2, m_i^2) = \left(\frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2}\right)^n. \quad (22)$$

The form factor (i.e., $n = 1$) normalized to unity at $q^2 = m_i^2 \cdot m_i$ and q are the physical parameters of the exchange particle and Λ is a phenomenological parameter. It is obvious that for $q^2 \to 0$ $F(q^2, m_i^2)$ becomes a number. If $\Lambda \gg m_i$ then $F(q^2, m_i^2)$ turns to be unity, whereas, as $q^2 \to \infty$ the form factor approaches zero and the distance becomes small and the hadron interaction is no longer valid. It is noted that the parameter Λ would smear the errors caused by assuming the dominating of one-particle exchange. Since Λ should not be far from the m_i and q, we choose

$$\Lambda = m_i + \eta \Lambda_{\mathrm{QCD}}, \quad (23)$$

where η is the phenomenological parameter and the coefficient for the strong interaction scale. Its value in the form factor shows importance of the strong interaction and is expected to be of the order of unity and can be determined from the measured rates, and

$$q^2 = p_1^2 + p_3^2 - 2E_1 E_3 + 2|\mathbf{p}_1||\mathbf{p}_3| \cos\theta, \quad (24)$$

where θ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_3$. Now we want to calculate the amplitudes of Figs. 7, *a–h*. By using the equations of $Abs(6a)$ and $Abs(6b)$, the amplitudes of Figs. 7, *a* and *b*, are given by

$$Abs(7a) = \int_{-1}^{1} \frac{|\mathbf{p}_1| d(\cos\theta)}{4\pi m_B} M(B^0 \to D^0 \bar{D}^0)(-i) g_{\psi DD}(\epsilon_3 \cdot p_1) \times$$
$$\times (-i) g_{DD\rho}(\epsilon_4 \cdot p_2) \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2}, \quad (25)$$

and

$$Abs(7b) = \int_{-1}^{1} \frac{|\mathbf{p}_1| d(\cos\theta)}{16\pi m_B} M(B^0 \to D^0 \bar{D}^0)(-i) g_{\psi D^* D} \epsilon_{\rho\sigma\lambda\eta} \epsilon_3^{\rho} p_1^{\lambda} p_3^{\eta} \times$$
$$\times (-i)\sqrt{2} g_{D^* D\rho} \epsilon_{\mu\nu\alpha\beta} \epsilon_4^{\mu} p_2^{\alpha} p_4^{\beta} \left(g_{\sigma\nu} - \frac{q_\sigma q_\nu}{m_{D^*}^2} \right) \frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2}, \quad (26)$$

and the amplitudes of Figs. 7, *c–h* are similar to equations $Abs(6c) - Abs(6h)$. The decay amplitude of $B^0 \to J/\psi\rho^0$ due to all contributions related to $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ via the HLL diagrams is

$$A(B^0 \to J/\psi\rho^0) = \{Abs(7a) + Abs(7b) - Abs(6a) - Abs(6b)\}/\sqrt{2}. \quad (27)$$

4. NUMERICAL RESULTS

The Wilson coefficients c_i have been calculated in different schemes. In this paper we will use consistently the naive dimensional regularization (NDR) scheme. The values of c_i at $\mu = m_b$ with the next-to-leading order (NLO) QCD corrections are given by [5, 8]

$$\begin{aligned} c_1 &= 1.117, & c_2 &= -0.257, \\ c_3 &= 0.017, & c_4 &= -0.044, \\ c_5 &= 0.011, & c_6 &= -0.056, \\ c_7 &= -1 \cdot 10^{-5}, & c_8 &= 5 \cdot 10^{-4}, \\ c_9 &= -0.010, & c_{10} &= 0.002. \end{aligned} \quad (28)$$

The relevant input parameters which are used: $m_{B^0} = 5.279$ GeV, $m_{J/\psi} = 3.1$ GeV, $m_\rho = 0.78$ GeV, $m_{D^\pm} = m_{D^0} = 1.87$ GeV, $m_{D^{\pm *}} = m_{D^{0*}} = 2.01$ GeV, $f_{B^0} = 0.176$ GeV, $f_{D^\pm} = f_{D^0} = 0.222$ GeV, $f_{D^{\pm *}} = f_{D^{0*}} = 0.23$ GeV, $V_{ub} = 0.0043, V_{ud} = 0.974$, $V_{cb} = 0.042$, $V_{cd} = 0.230$ [6]; $f_{J/\psi} = 0.405$ GeV [13]; $A_1^{B\rho}(0) = A_2^{B\rho}(0) = 0.28$, $V^{B\rho}(0) = 0.33$ [5]; $A_1^{BD^*}(m_{D^*}^2) = 1.1$, $A_2^{BD^*}(m_{D^*}^2) = 1.82$, $F_0^{BD}(m_D^2) = 0.86$, $F_1^{BD}(m_{D^*}^2) = 0.95$, $A_0^{BD^*}(m_D^2) = 2.73$ [8]; $g_{DD\rho} = g_{D^*D^*\rho} = 2.52$, $g_{\psi DD} = g_{\psi D^* D^*} = 7.71$, $g_{D^* D\rho} = 2.82$, $g_{\psi D^* D} = 8.64$ [2].

Using the parameters relevant for the $B^0 \to J/\psi\rho^0$ decay, we get flavor averaged branching ratio for the QCDF method as

$$\mathrm{BR}(B^0 \to J/\psi\rho^0) = 0.87 \cdot 10^{-5}. \quad (29)$$

After considering intermediate states we obtain

1) The amplitude of $B^0 \to D^+ D^-$ decay

$$M(B^0 \to D^+ D^-) = 2.10 \cdot 10^{-7} \quad (30)$$

and the branching ratio is given by [8]

$$\mathrm{BR}(B^0 \to D^+ D^-) = \tau_{B^0} \sqrt{\lambda(m_{B^0}^2, m_{D^+}^2, m_{D^-}^2)} \, \frac{|M(B^0 \to D^+ D^-)|^2}{16\pi m_B^3}, \quad (31)$$

The branching ratio of $B^0 \to J/\psi\rho^0$ decay with $\eta = 1{-}3.5$ and experimental data (in units of 10^{-5})

Branching ratio	η						
	1	1.5	2	2.5	3	3.5	EXP [6]
BR($B^0 \to J/\psi\rho^0$)	0.057	0.25	0.68	1.43	2.47	4.18	2.7 ± 0.4

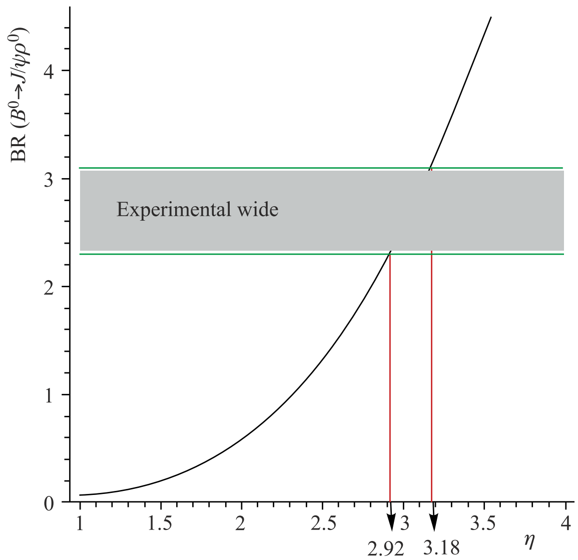

Fig. 8. The variation of the branching ratio of $B^0 \to J/\psi\rho^0$ with $\eta = 1{-}4$

where $\lambda(m^2_{B^0}, m^2_{D^+}, m^2_{D^-}) = m^4_{B^0} - 2m^4_{D^\pm} - 2m^2_{B^0}m^2_{D^\pm}$, so

$$\mathrm{BR}(B^0 \to D^+D^-) = 3.28 \cdot 10^{-4} \quad (\mathrm{EXP} = 2.11 \cdot 10^{-4}). \tag{32}$$

2) The amplitude and branching ratio of $B^0 \to D^0\bar{D}^0$ decay [9]

$$M(B^0 \to D^0\bar{D}^0) = 7.95 \cdot 10^{-8},$$
$$\mathrm{BR}(B^0 \to D^0\bar{D}^0) = 5.68 \cdot 10^{-5} \quad (\mathrm{EXP} < 6 \cdot 10^{-5}).$$

Now, according to FSI, we are able to obtain the branching ratios of $B^0 \to J/\psi\rho^0$ decay with different values of η shown in the table and Fig. 8.

CONCLUSION

In this work, we have calculated the contribution of the t-channel FSI, i.e., inelastic rescattering processes to the branching ratio of $B^0 \to J/\psi\rho^0$ decay. For evaluating the FSI effects, we have only considered the absorptive part of the HLL because both hadrons produced via the weak interaction are on their mass shells.

We have calculated the branching ratio of $B^0 \to J/\psi\rho^0$ decay by using QCDF method and FSI. The experimental result of this decay is $\mathrm{BR}(B^0 \to J/\psi\rho^0) = (2.7 \pm 0.4) \cdot 10^{-5}$ [6] and according to QCDF and FSI our results are $\mathrm{BR}(B^0 \to J/\psi\rho^0) = 0.87 \cdot 10^{-5}$ and

$(0.057{-}4.18) \cdot 10^{-5}$, respectively. We have considered that the value of FSI has a good agreement with the experimental result.

There exist some phenomenological parameters in our calculations on FSI such as η in (23) and many other sources of uncertainties, for example, the coupling constant $g_{D^*D\rho}$ etc., the neglected subdominant contributions in the FSI, the estmate of pure QCDF contribution, etc. We have introduced the phenomenological parameter η; its value in the form factor is expected to be of order unity and can be determined from the measured rates. For a given exchanged particle, we have used $\eta = 1{-}3.5$, and the branching ratios are $0.057{-}4.18$. We have considered that the factor of η is important to obtain branching ratio. According to Fig. 8, we have seen that, if $\eta = 2.92{-}3.18$ is selected, the branching ratio of the $B^0 \to J/\psi\rho^0$ decay approaches the experimental value.

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