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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

ON THE POLARIZATION OF THE FINAL NUCLEON IN NC ELASTIC $\nu_\mu(\bar{\nu}_\mu)-N$ SCATTERING

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New short baseline neutrino experiments open new possibilities of high-precision study of different neutrino processes. We present here results of the calculation of the polarization of final nucleon in NC elastic $\nu_\mu(\bar{\nu}_\mu)$–nucleon scattering. In a numerical analysis the sensitivity to the different choices of the axial and axial strange form factors is examined. Measurements of the polarization of the final proton in elastic $e-p$ scattering drastically changed our knowledge about the electromagnetic form factors of the proton. From measurement of the nucleon polarization in the NC elastic scattering, new additional information about the axial $G_A(Q^2)$ and the strange axial $G_A^s(Q^2)$ form factors of the nucleon could be inferred.

Новые эксперименты по осцилляциям нейтрино открывают новые возможности для очень точных исследований нейтринных процессов. Здесь мы приводим результаты вычислений поляризации конечного адрона в упругом $\nu_\mu(\bar{\nu}_\mu)$-нуклонном рассеянии. В численном анализе исследована чувствительность к выбору параметризации аксиального и странного аксиального формфакторов. Измерения поляризации конечного протона в упругом $e-p$-рассеянии существенно изменили наши представления об электромагнитных формфакторах протона. Измерение поляризаций нуклона в эластичном ν-нуклонном рассеянии дает возможность получить дополнительную информацию об аксиальном $G_A(Q^2)$ и странном аксиальном $G_A^s(Q^2)$ формфакторах.

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INTRODUCTION

The study of the Neutral Current (NC) elastic scattering of neutrino and antineutrino on the nucleon,

$$\nu_\mu + N \to \nu_\mu + N, \quad (1)$$

$$\bar{\nu}_\mu + N \to \bar{\nu}_\mu + N, \quad (2)$$

is an important source of information on weak form factors of the nucleon. The effective Standard Model (SM) Lagrangian of these processes has the form

$$\mathcal{L}_I = -\frac{G_F}{\sqrt{2}}\bar{\nu}_\mu\gamma^\alpha(1-\gamma_5)\nu_\mu\, j_\alpha^{\mathrm{NC}}. \quad (3)$$

Here

$$j_\alpha^{\rm NC} = 2j_\alpha^3 - 2\sin^2\theta_W j_\alpha^{\rm em} \tag{4}$$

is the NC of quarks. In Eq. (4), θ_W is the weak angle, $j_\alpha^{\rm em}$ is the electromagnetic current of quarks and

$$j_\alpha^3 = \sum_{a=1}^{3} \bar\psi_{aL}\gamma_\alpha \frac{1}{2}\tau_3\psi_{aL} \tag{5}$$

is the third component of the isovector current ($\psi_{aL}(a = 1,2,3)$ is the left-handed quark doublet).

For the matrix element of the process (1) we have

$$\langle f|(S-1)|i\rangle = -i\frac{G_F}{\sqrt{2}}N_{k'}N_k\bar u(k')\,\gamma^\alpha(1-\gamma_5)\,u(k)_p\langle p'|J_\alpha^{\rm NC}|p\rangle_p(2\pi)^4\,\delta(p'-p-q). \tag{6}$$

Here $k(p)$ and $k'(p')$ are momenta of the initial and final neutrinos (nucleons); $q = k-k' = p'-p$ is the momentum transfer; $J_\alpha^{\rm NC}$ is the hadronic neutral current in the Heisenberg representation and $N_k = \left(\frac{1}{(2\pi)^3 2k^0}\right)^{1/2}$ is the standard normalization factor.

We will take into account the light u, d, s quarks. In this case, for the SM neutral current we have

$$J_\alpha^{\rm NC} = (V_\alpha^3 - A_\alpha^3) - \frac{1}{2}(V_\alpha^s - A_\alpha^s) - 2\sin^2\theta_W J_\alpha^{\rm em}. \tag{7}$$

Here V_α^3 and A_α^3 are third components of isovector currents V_α^i and A_α^i, V_α^s and A_α^s are strange vector and axial currents. The electromagnetic current is connected with V_α^3 by the relation

$$J_\alpha^{\rm em} = V_\alpha^3 + V_\alpha^0, \tag{8}$$

where V_α^0 is the isoscalar current.

Using isospin symmetry from (8) for the one-nucleon matrix elements, we find

$${}_{p,n}\langle p'|V_\alpha^3|p\rangle_{p,n} = \pm\frac{1}{2}\left[{}_p\langle p'|J_\alpha^{\rm em}|p\rangle_p - {}_n\langle p'|J_\alpha^{\rm em}|p\rangle_n\right]. \tag{9}$$

The isovector A_α^i satisfies the relation

$$[A_\alpha^i, T_k] = ie_{ikl}A_\alpha^k, \tag{10}$$

where T_k is the operator of the total isotopic spin. From this relation we find

$${}_{p,n}\langle p'|A_\alpha^3|p\rangle_{p,n} = \pm\frac{1}{2}{}_p\langle p'|A_\alpha^{1+i2}|p\rangle_n. \tag{11}$$

The one-nucleon matrix elements of the electromagnetic current are given by the relations

$${}_{p,n}\langle p'|J_\alpha^{\rm em}|p\rangle_{p,n} = N_{p'}N_p\bar u(p')\left[\gamma^\alpha F_1^{p,n}(Q^2) + \frac{i}{2M}\sigma_{\alpha\beta}q^\beta F_2^{p,n}(Q^2)\right]u(p), \tag{12}$$

where $F_1^{p,n}(Q^2)$ and $F_2^{p,n}(Q^2)$ are the Dirac and Pauli form factors of the proton (neutron) and $Q^2 = -q^2$.

For the matrix element of the CC axial current A_α^{1+i2} we have

$$
{}_p\langle p'|A_\alpha^{1+i2}|p\rangle_n = N_{p'}N_p\bar{u}(p')\left[\gamma^\alpha\gamma_5 G_A(Q^2) + \frac{1}{2M}\gamma_5 q^\alpha G_P(Q^2)\right]u(p), \tag{13}
$$

where $G_A(Q^2)$ and $G_P(Q^2)$ are the axial and pseudoscalar form factors of the nucleon.

From time reversal invariance of strong interactions it follows that the matrix elements of the strange vector and axial currents have the same form as the matrix elements of J_α^{em} and A_α^{1+i2}, respectively (see [1]).

Finally, for the one-nucleon matrix element of the neutral current we find [1]

$$
{}_{p,n}\langle p'|J_\alpha^{\mathrm{NC}}|p\rangle_{p,n} = N_{p'}N_p\bar{u}(p')J_\alpha^{\mathrm{NC}(p,n)}u(p). \tag{14}
$$

Here

$$
J_\alpha^{\mathrm{NC}(p,n)} = V_\alpha^{\mathrm{NC}(p,n)} - A_\alpha^{\mathrm{NC}(p,n)}, \tag{15}
$$

where

$$
\begin{aligned}
V_\alpha^{\mathrm{NC}(p,n)} &= \gamma_\alpha F_1^{\mathrm{NC}(p,n)}(Q^2) + \frac{i}{2M}\sigma_{\alpha\beta}q^\beta F_2^{\mathrm{NC}(p,n)}(Q^2),\\
A_\alpha^{\mathrm{NC}(p,n)} &= \gamma_\alpha\gamma_5 G_A^{\mathrm{NC}(p,n)}(Q^2).
\end{aligned} \tag{16}
$$

In this relation we have

$$
F_{1,2}^{\mathrm{NC}(p,n)}(Q^2) = \pm\frac{1}{2}(F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2)) - \frac{1}{2}\ F_{1,2}^s(Q^2) - 2\sin^2\theta_W F_{1,2}^{p,n}(Q^2) \tag{17}
$$

and

$$
G_A^{\mathrm{NC}(p,n)}(Q^2) = \pm\frac{1}{2}G_A(Q^2) - \frac{1}{2}\ G_A^s(Q^2), \tag{18}
$$

where $F_{1,2}^s(Q^2)$ and $G_A^s(Q^2)$ are vector and axial strange form factors of the nucleon.

Thus, the matrix elements of the processes (1) and (2) are determined by the known electromagnetic form factors of the proton and the neutron, the axial form factor of the nucleon $G_A(Q^2)$ and the strange form factors of the nucleon.

Information on the axial form factor $G_A(Q^2)$ is inferred from study of the CC quasi-elastic (CCQE) processes:

$$
\nu_\mu + n \to \mu^- + p, \quad \bar{\nu}_\mu + p \to \mu^+ + n. \tag{19}
$$

The axial form factor is usually parameterized by the dipole formula

$$
G_A(Q^2) = \frac{g_A}{\left(1 + \dfrac{Q^2}{M_A^2}\right)^2}. \tag{20}
$$

Here $g_A \simeq 1.27$ is the axial constant, and M_A is a parameter (the «axial mass»).

[1] It is obvious that pseudoscalar form factors do not make contribution to the matrix element of the processes (1) and (2).

The values of the parameter M_A determined from the data of different experiments under the assumption that impulse approximation is valid (neutrino interacts with a quasi-free nucleon in a nucleus and other nucleons are spectators) are quite different.

From analysis of the old bubble chamber data on measurements of the cross section of the process $\nu_\mu + n \to \mu^- + p$ on deuterium target and of the process $\bar{\nu}_\mu + p \to \mu^+ + n$ on proton target it was found [2] that

$$M_A = (1.016 \pm 0.026)\ \text{GeV}. \tag{21}$$

The value of the parameter M_A obtained from the measurement of the CCQE cross section in the NOMAD experiment (carbon target) [3]

$$M_A = (1.05 \pm 0.02 \pm 0.06)\ \text{GeV} \tag{22}$$

is in agreement with (21).

However, from fit of the data of more recent experiments larger average values of the parameter M_A were obtained. From the data of the MINOS experiment (iron target) it was found [4] that

$$M_A = 1.26^{+0.12+0.08}_{-0.10-0.12}\ \text{GeV}. \tag{23}$$

The K2K experiment (H_2O target) obtained [5]

$$M_A = (1.20 \pm 0.12)\ \text{GeV}. \tag{24}$$

The high-statistics MiniBooNE experiment (carbon target) inferred [6]

$$M_A = (1.35 \pm 0.17)\ \text{GeV}. \tag{25}$$

There could be many different reasons for such a disagreement. It could be a problem of systematics and normalization. Target nuclei in the different experiments are different. The difference of the values of M_A obtained from the data of different experiments could be due to various nuclei effects (see [7, 8]).

The axial form factor $G_A(Q^2)$ is a fundamental characteristic of the nucleon. It is of a great theoretical interest. CCQE processes (19) are the dominant neutrino processes in the GeV energy range. The modern high-precision neutrino oscillation experiments require a percentage-level knowledge of the axial form factor and cross sections of CCQE processes (19). In several dedicated neutrino experiments (T2K [9], MINERVA [10], ArgoNeuT [11]), new measurements of CCQE cross section will be performed.

In [12] we proposed a measurement of the polarization of the recoil nucleon in CCQE processes (19) for a determination of the axial form factor.

A measurement of the polarization of the recoil protons in the elastic $e{-}p$ scattering drastically changed our understanding of the electromagnetic form factors of the proton (see [13, 14]). Before these measurements were done the results of the analysis of the data of the numerous experiments on measurements of the cross section of elastic scattering of unpolarized electrons on unpolarized protons indicated that the ratio $R(Q^2)$ of the electric and magnetic form factors of the proton does not depend on Q^2 and is close to one. A measurement of the ratio of the transverse and longitudinal polarizations of the proton allows one to determine the ratio of the electric and magnetic form factors in a direct model independent way. After these measurements were done, it was established that the ratio $R(Q^2)$ decreased linearly with Q^2 ($R \simeq 1$ at $Q^2 \simeq 1$ GeV^2 and $R = 0.28 \pm 0.09$ at $Q^2 = 5.6$ GeV^2). The measurement of the polarization significantly changed the theoretical models for the structure of the nucleon.

In this paper, we present the results of calculation of the polarization of the recoil nucleon in the elastic NC processes (1) and (2). It is natural to expect that measurements of the polarization of the nucleon in CCQE scattering and NC elastic scattering could provide important information about the axial form factor of the nucleon. From our point of view, it is worthwhile to consider a possibility of such measurements in modern high-statistics short baseline neutrino experiments in which hundreds of thousands of neutrino events are observed.

Here we present the results of the calculation of the the polarization of the final nucleon in the case of the definite neutrino energy and free nucleon target. In order to obtain the polarization in a realistic neutrino experiment with a spectrum of initial neutrinos (antineutrinos), one needs to average the expressions presented below over the spectrum. Let us notice that the numerator and the denominator in the expressions (28) and (32) must be averaged separately. In modern neutrino experiments, nuclear targets such as carbon, oxygen, iron or argon are used. It was shown in many papers (see, for example, [7, 8]) that nuclear effects are important and must be taken into account. We do not consider nuclear effects here.

Investigation of the NC processes (1) and (2) allows one to obtain an information about strange form factors of the proton (see, for example, [1]). Strange vector form factors can be inferred from experiments on the study of the P-odd asymmetry in the elastic scattering of longitudinally polarized electrons on unpolarized proton and other targets. From many experiments performed at different values of Q^2, it follows that strange vector form factors are small, compatible with zero. For example, from analysis of the data of the recent HAPPEX experiment at JLab [15], it was found that at $Q^2 \simeq 0.62$ GeV^2 the charge and magnetic strange form factors are

$$G_E^s = 0.047 \pm 0.034, \quad G_M^s = 0.070 \pm 0.067. \tag{26}$$

Information about G_A^s was obtained from the data of the BNL experiment on the measurement of the cross sections of the NC processes (1) and (2) [16]. From analysis of the data it was found that

$$G_A^s(0) = -0.21 \pm 0.10. \tag{27}$$

There exists, however, a strong correlation between the values of the parameters $G_A^s(0)$ and M_A. Taking into account this correlation, we can conclude that $-0.25 < G_A^s(0) < 0$ (see [1]).

1. POLARIZATION OF THE FINAL NUCLEON IN NC ELASTIC SCATTERING

We will present here the result of the calculation of the polarization of the final nucleon in NC processes (1) and (2). In the covariant density matrix formalism the 4-vector of the polarization of the final nucleon produced in (1) and (2) is given by the expression

$$\xi^\tau = \frac{\mathrm{Tr}\,[\gamma^\tau \gamma_5\ \rho_f]}{\mathrm{Tr}\,[\rho_f]}. \tag{28}$$

Here the final density matrix ρ_f is determined by the expression

$$\rho_f = L^{\alpha\beta}\Lambda(p')J_\alpha^{\mathrm{NC}(p,n)}\Lambda(p)J_\beta^{\mathrm{NC(p,n)}}\Lambda(p'), \tag{29}$$

where

$$L^{\alpha\beta} = \mathrm{Tr}\left[\gamma^\alpha(1-\gamma_5)\not{k}\gamma^\beta(1-\gamma_5)\not{k}'\right], \tag{30}$$

$J_\beta^{\mathrm{NC}(p,n)}$ is given by (15) and $\Lambda(p) = \not{p} + M$. Taking into account the relation

$$\Lambda(p')\gamma^\tau\gamma_5\Lambda(p') = 2M\left(g^{\tau\sigma} - \frac{p'^\tau p'^\sigma}{M^2}\right)\Lambda(p')\gamma_\sigma\gamma_5, \tag{31}$$

we can rewrite the expression for the 4-vector of the polarization in the form

$$\xi^\tau = \left(g^{\tau\sigma} - \frac{(p')^\tau(p')^\sigma}{M^2}\right)\frac{L^{\alpha\beta}\,\mathrm{Tr}\left[\gamma_\sigma\gamma_5\Lambda(p')J_\alpha^{\mathrm{NC}(p,n)}\Lambda(p)J_\beta^{\mathrm{NC}(p,n)}\right]}{L^{\alpha\beta}\,\mathrm{Tr}\left[\Lambda(p')J_\alpha^{\mathrm{NC}(p,n)}\Lambda(p)J_\beta^{\mathrm{NC}(p,n)}\right]}. \tag{32}$$

After lengthy calculations for the vector of polarization in the rest frame of the initial nucleon we will find the following expression:

$$\boldsymbol{\xi} = \frac{1}{J_0\,E}\left\{(\mathbf{k}+\mathbf{k}')P_+ + \mathbf{q}\;\left[-\frac{E+E'}{M}\;P_+ + \left(1+\frac{E-E'}{M}\right)\;(P_- - P_p)\right]\right\}. \tag{33}$$

Here

$$P_+ = [y\,G_M^{\mathrm{NC}} + (2-y)G_A^{\mathrm{NC}}]\,G_E^{\mathrm{NC}}, \tag{34}$$

$$P_- - P_p = -[(2-y)\,G_A^{\mathrm{NC}} + yG_M^{\mathrm{NC}}]\left[G_A^{\mathrm{NC}} + \frac{\tau}{y}(2-y)F_2^{\mathrm{NC}}\right] \tag{35}$$

and

$$J_0^{\nu,\bar\nu} = 2(1-y)\left[(G_A^{\mathrm{NC}})^2 + \frac{\tau(G_M^{\mathrm{NC}})^2 + (G_E^{\mathrm{NC}})^2}{1+\tau}\right] + \\ + \frac{My}{E}\;\left[(G_A^{\mathrm{NC}})^2 - \frac{\tau(G_M^{\mathrm{NC}})^2 + (G_E^{\mathrm{NC}})^2}{1+\tau}\right] + y^2\,(G_M^{\mathrm{NC}} \mp G_A^{\mathrm{NC}})^2 \pm 4y\,G_M^{\mathrm{NC}}\,G_A^{\mathrm{NC}}. \tag{36}$$

The quantities $J_0^{\nu,\bar\nu}$ are connected to the cross sections of the processes (1) and (2) by the relations

$$J_0^{\nu,\bar\nu} = \frac{4\pi}{G_F^2}\;\frac{d\sigma^{\nu,\bar\nu}}{dQ^2}. \tag{37}$$

In Eqs. (33)–(35) E and E' are the energies of the initial and final neutrinos in the lab. frame

$$y = \frac{pq}{pk}, \quad \tau = \frac{Q^2}{4M^2}, \quad G_M^{\mathrm{NC}} = F_1^{\mathrm{NC}} + F_2^{\mathrm{NC}}, \quad G_E^{\mathrm{NC}} = F_1^{\mathrm{NC}} - \tau F_2^{\mathrm{NC}}. \tag{38}$$

From (33) it follows that the polarization vector lays in the scattering plane. We expand this vector along the two orthogonal unit vectors $\mathbf{e}_L$ and $\mathbf{e}_T$ determined as follows:

$$\mathbf{e}_L = \frac{\mathbf{p}'}{|\mathbf{p}'|} = \frac{\mathbf{q}}{|\mathbf{q}|}, \qquad \mathbf{e}_T = \mathbf{e}_L \times \mathbf{n}, \qquad \mathbf{n} = \frac{\mathbf{q}\times\mathbf{k}}{|\mathbf{q}\times\mathbf{k}|}. \tag{39}$$

We have

$$\boldsymbol{\xi} = \xi_T \mathbf{e}_T + \xi_L \mathbf{e}_L, \tag{40}$$

where ξ_T and ξ_L are the transverse and longitudinal polarizations. From (33)–(35) for the transverse polarization we obtain the expression

$$\xi_T^{\nu,\bar{\nu}} = -\frac{2\sin\theta_N}{J_0^{\nu,\bar{\nu}}} \left[\pm\, y\, G_M^{\mathrm{NC}} + (2-y) G_A^{\mathrm{NC}} \right] G_E^{\mathrm{NC}}, \tag{41}$$

where θ_N is the angle between momenta of the initial neutrino and the final nucleon.

It is obvious that $\xi_T = s_T$, where s_T is the transverse polarization in the rest frame of the recoil nucleon. For the longitudinal polarization in the rest frame of the recoil nucleon we find

$$s_L^{\nu,\bar{\nu}} = \frac{q_0}{|\mathbf{q}| J_0^{\nu,\bar{\nu}}} \left[\pm y\, G_M^{\mathrm{NC}} + (2-y)\, G_A^{\mathrm{NC}} \right] \left[(2-y)\, G_M^{\mathrm{NC}} \pm y \left(\frac{1+\tau}{\tau} \right) G_A^{\mathrm{NC}} \right]. \tag{42}$$

In the case of the NC processes (1) and (2) only the energy E'_N of the final nucleon and the scattering angle θ_N can be measured. In terms of these quantities we have

$$Q^2 = 2M(E'_N - M), \quad y = \frac{(E'_N - M)}{E}, \quad q_0 = E'_N - M, \quad |\mathbf{q}| = \sqrt{E'^{\,2}_N - M^2}. \tag{43}$$

The neutrino energy E is determined by the relation

$$E = \frac{M(E'_N - M)}{M - E'_N + p'_N \cos\theta_N}, \qquad p'_N = \sqrt{E'^{\,2}_N - M^2}. \tag{44}$$

2. COMMENTS

Our comments are based on the following two characteristic features of the NC neutrino processes.

i) From (18) it follows that the axial form factor appears only in the combinations $G_A - G_A^s$ — if measurements are on protons, and $G_A + G_A^s$ — if measurements are on neutrons.

ii) From (17) and (38) we find the following expressions for the the NC magnetic and electric form factors of the proton and the neutron:

$$G_{M,E}^{\mathrm{NC}(p)} = \frac{1}{2}(1 - 4\sin^2\theta_W)\; G_{M,E}^p - \frac{1}{2}\; G_{M,E}^n - \frac{1}{2}\; G_{M,E}^s \tag{45}$$

and

$$G_{M,E}^{\mathrm{NC}(n)} = \frac{1}{2}(1 - 4\sin^2\theta_W)\; G_{M,E}^n - \frac{1}{2}\; G_{M,E}^p - \frac{1}{2}\; G_{M,E}^s, \tag{46}$$

in which $\sin^2\theta_W$ enters in the combination $(1 - 4\sin^2\theta_W)$. From analysis of the existing experimental data it follows that [17]

$$\sin^2\theta_W = 0.23116 \pm 0.00012. \tag{47}$$

This implies that $(1 - 4\sin^2\theta_W) \simeq 0.075$ and, consequently, the NC charge form factor of the proton is very small:

$$G_{E\,p}^{\mathrm{NC}} \simeq \frac{1}{2}\,[0.075\,G_E^p - G_E^n - G_E^s] \simeq 0. \tag{48}$$

• The transverse polarizations of the final protons and nucleons are determined in (41) and are directly proportional to the NC charge form factors G_E^{NC}.

Equation (48) implies that the transverse polarization of the proton is strongly suppressed, which makes its measurement a very difficult task.

Of interest could be the transverse polarization of the final neutron. It exhibits a simple linear dependence on $G_A + G_A^s$:

$$G_A + G_A^s = \frac{2}{2-y}\left[\frac{1}{2\,\sin\theta_N}\,\frac{(J_0\,s_T)_n^{\nu,\,\bar\nu}}{G_{E,n}^{\mathrm{NC}}} \pm y\,G_{M,n}^{\mathrm{NC}}\right], \tag{49}$$

$$\simeq \frac{-1}{2-y}\left[-\frac{1}{\sin\theta_N}\,\frac{(J_0\,s_T)_n^{\nu,\,\bar\nu}}{G_E^p + G_E^s} \pm y\,G_{M,n}^{\mathrm{NC}}\right]. \tag{50}$$

In the last line we have used

$$G_{E\,n}^{\mathrm{NC}} = \frac{1}{2}\,[0.075 G_E^n - G_E^p - G_E^s] \simeq -\frac{1}{2}\,(G_E^p + G_E^s)\,. \tag{51}$$

• The longitudinal polarization is determined in (42). Note that the electric form factors do not enter this expression and the longitudinal polarization is expressed only in terms of G_M^{NC} and the axial form factors. From (45) and (46) it follows that for both protons and neutrons it is expressed entirely in terms of the best measured magnetic form factors $G_M^{p,n}$, the small strange vector form factor G_M^s and the axial form factors $(G_A \pm G_A^s)$.

In order to measure the longitudinal polarization, the neutrino detector must be placed in a magnetic field.

These expressions considerably simplify forming the sum of the longitudinal polarizations of ν and $\bar\nu$. Then measurements on protons and neutrons provide two linear equations for $(G_A \pm G_A^s)$:

$$G_A - G_A^s = \frac{\sqrt{\tau(1+\tau)}}{[y^2(1+\tau) + \tau(2-y)^2]}\,\frac{(J_0\,s_L)_p^{\nu+\bar\nu}}{G_{M,p}^{\mathrm{NC}}}, \tag{52}$$

$$G_A + G_A^s = -\frac{\sqrt{\tau(1+\tau)}}{[y^2(1+t) + \tau(2-y)^2]}\,\frac{(J_0\,s_L)_n^{\nu+\bar\nu}}{G_{M,n}^{\mathrm{NC}}}. \tag{53}$$

• If both the transverse and longitudinal polarizations can be measured, then we can determine their ratio (like in the case of elastic $e{-}p$ scattering):

$$\left(\frac{s_L}{s_T}\right)^{\nu,\bar\nu} = \frac{-\,M}{|\mathbf{q}|\sin\theta_N}\,\frac{\left[\tau(2-y)\,G_M^{\mathrm{NC}} \pm y\,(1+\tau)\,G_A^{\mathrm{NC}}\right]}{G_E^{\mathrm{NC}}}. \tag{54}$$

As the transverse polarization of the proton is strongly suppresses, we consider the polarization of the final neutron only. From (54), for $G_A + G_A^s$ we obtain

$$G_A + G_A^s = \frac{\pm\,2}{y}\sqrt{\frac{\tau}{1+\tau}}\left[\frac{2\,\sin\theta_N}{G_{E,n}^{\mathrm{NC}}}\,\left(\frac{s_L}{s_T}\right)^{\nu,\bar\nu} + \sqrt{\frac{\tau}{1+\tau}}\,(2-y)G_{M,n}^{\mathrm{NC}}\right]. \tag{55}$$

An advantage of the ratio s_L/s_T is that many of the systematic uncertainties cancel.

3. NUMERICAL ANALYSIS

Here we present the results of the study of the sensitivity of the transverse and longitudinal polarizations to the different choices of the axial form factors G_A and G_A^s. For comparison, we present also the cross sections for the same values of G_A and G_A^s.

We use the following commonly used parameterizations for the form factors, summarized in [13]:

$$G_D = \frac{1}{\left(1+\dfrac{Q^2}{M_V^2}\right)^2}, \quad M_V^2 = 0.71, \quad G_{M,p} = \mu_p\, G_D, \quad G_{M,n} = \mu_n\, G_D,$$
$$G_{E,p} = (1.06 - 0.14\, Q^2)\, G_D, \quad G_{E,n} = -a\,\frac{\mu_n \tau}{1+b\tau}\, G_D, \quad a = 1.25, \quad b = 18.3, \tag{56}$$
$$G_A^s = \frac{g_a^s}{\left(1+\dfrac{Q^2}{M_A^2}\right)^2}, \quad G_M^s = G_E^s = 0,$$

where $\mu_p = 2.79$ and $\mu_n = -1.91$ are the magnetic moments of the proton and neutron. Our free parameters are M_A and g_a^s. We have calculated the effect of the different axial form factors on the polarizations, considering the following choices of M_A and g_a^s:

1) $M_A = 1.016$, $g_a^s = 0$ — full line;
2) $M_A = 1.016$, $g_a^s = -0.21$ — dashed line;

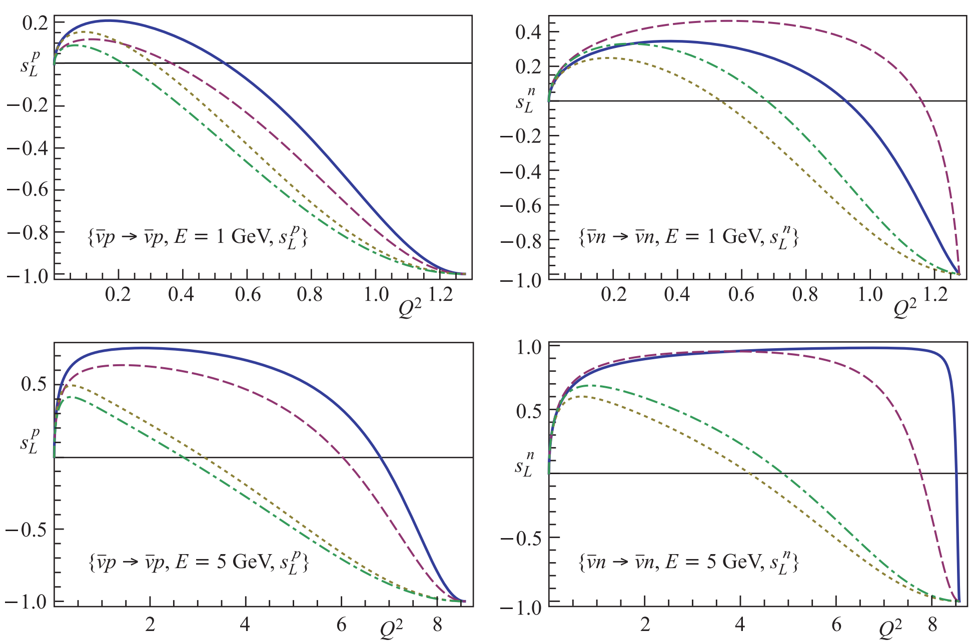

Fig. 1. The dependence of the longitudinal polarization of protons s_L^p (left) and neutrons s_L^n (right) in $\bar{\nu}-N$ scattering on the different choices of M_A and g_A^s, given in Eq. (56). The polarizations are shown for two neutrino energies: $E = 1$ GeV (top) and $E = 5$ GeV (bottom)

3) $M_A = 1.35$, $g_a^s = 0$ — dotted line;
4) $M_A = 1.35$, $g_a^s = -0.21$ — dash-dotted line.

Note that we assume the same Q^2-dipole form for the axial and strange axial form factors.

We present the polarizations for two values of the neutrino energy: $E = 1$ GeV and $E = 5$ GeV. The plots are given as functions of Q^2 in the interval $Q^2_{\rm min} \leqslant Q^2 \leqslant Q^2_{\rm max}$, where $Q^2_{\rm min}$ is determined from $E'_N \geqslant M$, and $Q^2_{\rm max}$ — by the condition $\cos\theta_N \leqslant 1$. Once we fix E and Q^2, the scattering angle θ_N is determined via (44). We have

$$\cos\theta_N = \sqrt{\frac{\tau}{1+\tau}}\left(1+\frac{M}{E}\right) \leqslant 1, \tag{57}$$

which implies $Q^2_{\rm max} = 4ME^2/(2E+M)$ and

$$\sin\theta_N = \sqrt{\frac{Q^2}{Q^2+4M^2}\left(\frac{4M^2}{Q^2}-\frac{2M}{E}-\frac{M^2}{E^2}\right)}. \tag{58}$$

We examine separately $\nu - N$ and $\bar\nu - N$ elastic scattering.

• First we show the polarizations in $\bar\nu - N$ elastic scattering.

The plots in Fig. 1 show the longitudinal polarization. It is clearly seen that for both the protons and the neutrons it exhibits a strong sensitivity to the choice of M_A. This sensitivity becomes very clearly pronounced for higher energies, at $Q^2 \geqslant 1$ GeV^2 ($E = 5$ GeV). For example, at $Q^2 = 3$ GeV^2 the proton polarization changes from $s_L^p \simeq 0$ at $M_A = 1.0$ to

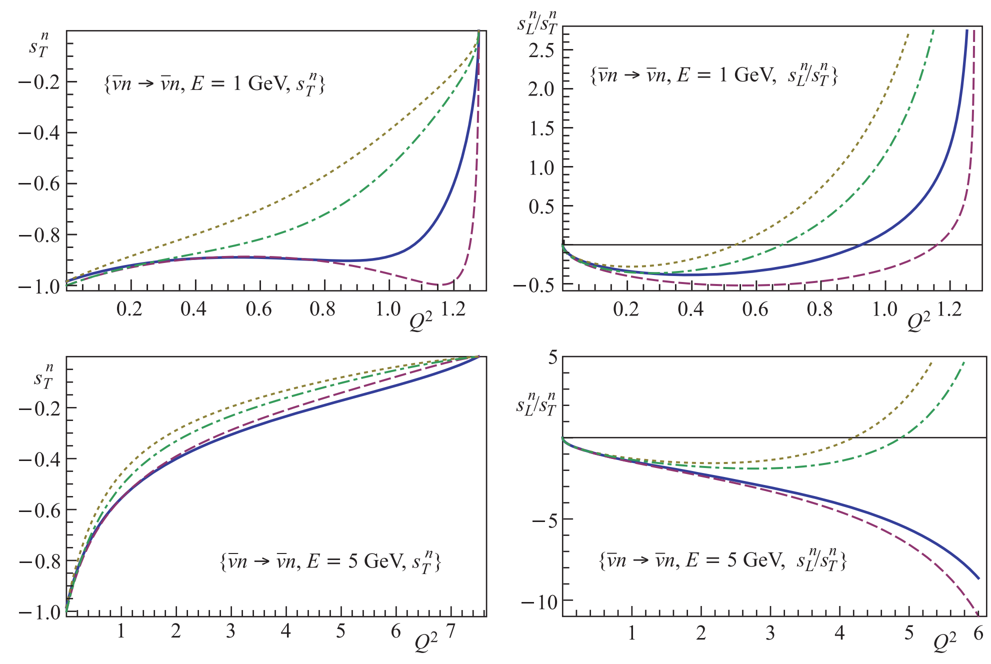

Fig. 2. The dependence of the transverse polarization of the neutron s_T^n (left) and the ratio s_L^n/s_T^n (right) in $\bar\nu - n$ scattering on the different choices of M_A and g_A^s, given in Eq. (56). The polarizations are shown for two neutrino energies: $E = 1$ GeV (top) and $E = 5$ GeV (bottom)

$s_L^p \simeq 0.7$ at $M_A = 1.35$ with almost no sensitivity to g_A^s, the neutron polarization exhibits similar behaviour at high energies (the lower plots in Fig. 1). A sensitivity to both the axial and strange axial form factors we find at lower energies in the neutron polarization. For example, at $Q^2 \simeq 0.8$ GeV^2 ($E = 1$ GeV), s_L^n varies from -0.4 to $+0.4$ depending on the choices 1)–4) (the upper plots in Fig. 1).

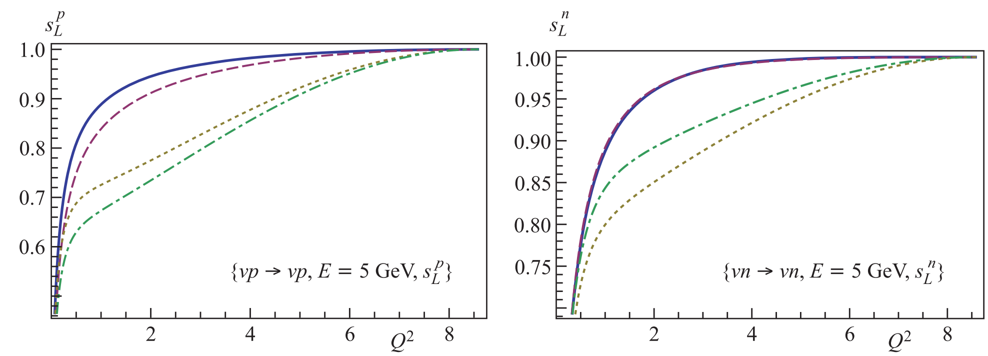

Fig. 3. The longitudinal polarization in $\nu-N$ scattering at $E = 5$ GeV of protons (left) and neutrons (right) for the different choices of M_A and g_A^s as shown in Eq. (56)

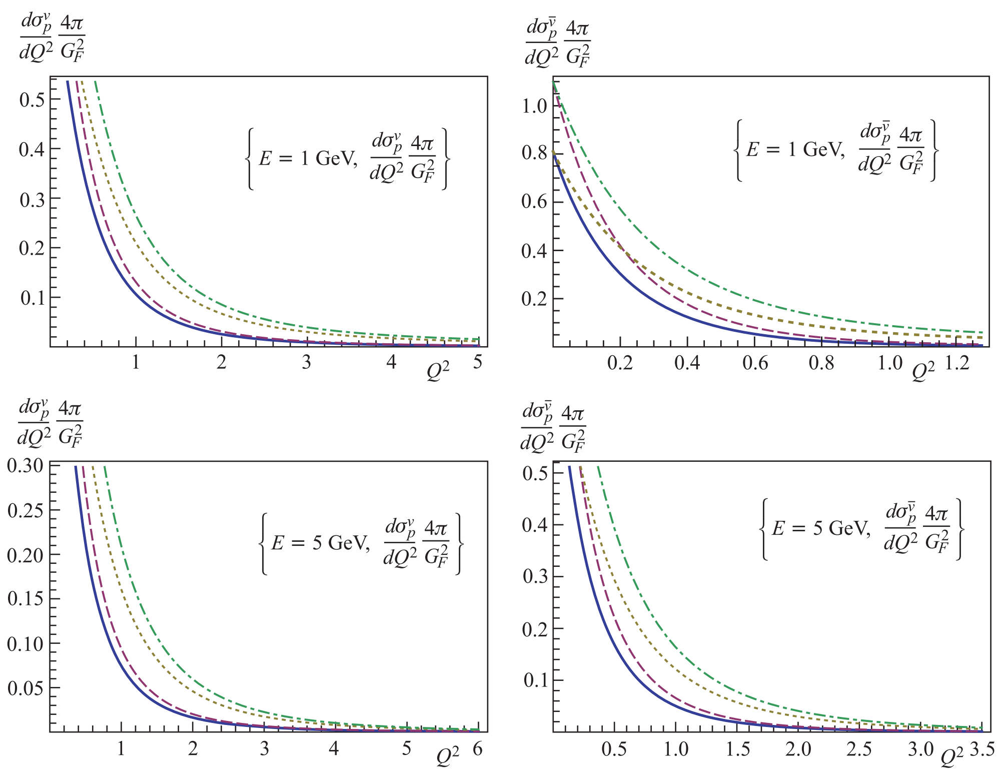

Fig. 4. The dependence of $d\sigma_p^\nu/dQ^2$ (left) and $d\sigma_p^{\bar{\nu}}/dQ^2$ (right) (multiplied by $4\pi/G_F^2$) on the different choices of M_A and g_A^s, given in Eq. (56), at $E = 5$ GeV for protons (top) and neutrons (bottom)

In Fig. 2 we show the transverse polarization s_T^n of the neutron and the ratio s_L^n/s_T^n, Eq. (54). At lower energies, s_T^n exhibits a sensitivity to both the axial and strange axial form factors, and almost no sensitivity at $E = 5$ GeV (the left plots in Fig. 2). At lower energies the Q^2-dependence of s_L^n/s_T^n distinguishes all four choices (56) and becomes sensitive only to M_A at higher energies $Q^2 \geqslant 3$ GeV^2.

As our estimates showed, the transverse polarization of the proton is very small, $s_T^p \simeq 0.02{-}0.06$.

• The polarization in $\nu{-}N$ scattering is big, but shows much weaker sensitivity to the axial form factors. For illustration, in Fig. 3 we show the longitudinal polarizations at $E = 5$ GeV, where the sensitivity is the biggest one.

• In Fig. 4, we show the differential cross sections (multiplied by $4\pi/G_F^2$) for $\nu{-}p$ and $\bar{\nu}{-}p$ elastic scatterings at $E = 5$ GeV^2 for the same M_A and g_A^s, Eq. (56). As compared to the polarization, the sensitivity to the axial form factors is much weaker.

CONCLUSIONS

In a recent paper [12] we suggested that measurements of the polarization of the final nucleon in quasi-elastic $\nu{-}N$ scattering could provide additional information about the axial form factor G_A. Here we present the results of the calculations of the polarization of the final nucleons in elastic $\nu_\mu(\bar{\nu}_\mu){-}N$ scattering.

The NC form factors, which determine the process, are expressed in terms of the electromagnetic, axial and strange vector and axial form factors of the nucleon. We have examined numerically the sensitivity of the final nucleon polarization to the axial and strange axial form factors.

Most sensitive to the axial form factors are the longitudinal polarizations of the proton and neutron in antineutrino nucleon scattering. This sensitivity increases for higher energies. Also the value of the polarizations is large. In order to measure the longitudinal polarization, like in the case of $e{-}p$ scattering, a magnetic field must be applied.

The transverse polarization of the proton is extremely small and thus, very difficult to be measured. This is a consequence of the smallness of the NC electric form factor of the proton. On the other hand, the transverse polarization of the neutron is unsuppressed and can be large. Big and most sensitive to the axial and strange axial form factors is the transverse polarization in $\bar{\nu}{-}n$ scattering at small $Q^2 \leqslant 1$ GeV^2. Its determination, however, requires difficult measurement of left-right asymmetry in the scattering of the final neutron.

The measurement of the polarization of the final nucleons in NC elastic neutrino–nucleon scattering is a challenge. However, such measurements could be an important source of information about axial and strange form factors of nucleon in the same way as measurement of the polarization of the final proton in elastic $e{-}p$ scattering were very important for the electromagnetic form factors of the proton. From our point of view, it is worthwhile to consider a possibility for such measurements short-baseline neutrino experiments.

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