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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

COMPARING SOME NUCLEON–NUCLEON POTENTIALS

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The aim is to compare a few Nucleon–Nucleon (NN) potentials especially Reid68, Reid68–Day, Reid93, UrbanaV14, ArgonneV18, Nijmegen 93, Nijmegen I and Nijmegen II. Although these potentials have some likenesses and are almost phenomenological, they include in general different structures and their own characteristics. The potentials are constructed in a manner that fit the NN scattering data or phase shifts and are compared in this way. A high-quality scale of a potential is that it fits the data with $\chi^2/N_{\rm data} \approx 1$, describes well the deuteron properties and gives satisfactory results in nuclear-structure calculations. However, these scales have some failures. Here, we first compare many potentials by confronting them with the data. Then, we try to compare the potential forms by considering the potential structures directly, and therefore regarding their substantial bases somehow. To do so, we note that since the potentials are written in different schemas, it is necessary to write them in a unique schema. On the other hand, because three major terms in the NN interaction are central, tensor and spin-orbit terms, so to perform a reduction plan and arrive at a common structure, we choose the Reid's potential form. Next, we compare the potentials for some states and address some other related issues as well.

Основной целью данного исследования является сравнение некоторых нуклон-нуклонных потенциалов, в частности Reid68, Reid68–Day, Reid93, UrbanaV14, ArgonneV18, Nijmegen 93, Nijmegen I и Nijmegen II. Хотя все эти потенциалы в некоторой степени похожи и имеют феноменологическую природу, их структура и характеристики различны. Потенциалы построены так, что описывают данные нуклон-нуклонного рассеяния или фазы рассеяния, и сравниваются по этим характеристикам. Высокий уровень потенциалов, описывающих экспериментальные данные с $\chi^2/N_{\rm data} \approx 1$, а также свойства дейтрона, позволяет получить удовлетворительные результаты в расчетах ядерной структуры, однако не всегда. В работе мы впервые сравниваем различные потенциалы, сопоставляя их с данными. Затем сравниваем формы потенциалов, рассматривая их структуры напрямую и, следовательно, их существенные базисы. Для этого потенциалы, записанные в различных схемах, должны быть переписаны в одной схеме. С другой стороны, так как три важнейших члена нуклон-нуклонного взаимодействия — центральное, тензорное и спин-орбитальное, то для того чтобы редуцировать и получить единую структуру, мы выбираем потенциал Рейда. Затем мы сравниваем потенциалы для одинаковых состояний и решаем некоторые другие смежные вопросы.

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INTRODUCTION

In the past decades, near a century, various models to describe the nucleon–nucleon interaction have been framed. In general, one can divide the main models into four categories — see [1] for a brief typical review on the subject including references therein.

For the models based on Quantum ChromoDynamics (QCD), the main examples are the «constituent quark models» (first in [2]), «Skyrme model» (first in [3]), Nambu–Jona-Lasinio (NJL) model [4], «lattice QCD» models (first in [5]), Moscow-type potentials [6], the Oxford potential [7], and many others. In these models/potentials, in general, the aim is to describe hadron–hadron processes in terms of quark and gluon degrees of freedom.

Effective Field Theory (EFT) is another outstanding approach to the NN problem; see [8,9] for recent studies and references therein. By breaking chiral symmetry of QCD Lagrangian in low energies, the main degrees of freedom are not quarks and gluons, but are pions and nucleons. Then, one employs a Chiral Perturbation Theory (CHPT) expansion in terms of the fields up to some orders. Some important χEFT potentials are presented by [1] Texas group (first in [10]), Brazil group (first in [11]), Munich group (first in [12]), Idaho group (first in [13]) and Bochum–Julich group (first in [14]). Chiral EFT potentials are nowadays more interesting to stand as the standard NN potentials.

The Boson Exchange (BE) models, as the name implies, use various meson exchanges in three NN interaction parts [2]. Most potentials in this category always employ the field-theoretical and dispersion-relations techniques. Among the first versions are Partovi–Lomon model (first in [16]), Stony Brook-group's potential (first in [17]), the super-soft-core potentials (first in [18]), Funabashi potentials (first in [19]), Paris-group (first in [20]), Bonn-group (first in [21]), Padua-group (first in [22]), Nijmegen-group (first in [23]) and Hamburg-group (first in [24]) potentials. The typical potentials such as the Virginia-group potential [25], the Bochum-group potential [26] and Tubingen group [27] are also mentionable.

The almost pure phenomenological NN potentials have many free parameters to be fitted with the experimental data. Even with less physical meanings, they are still important and applicable to nuclear-structure calculations. Some famous examples are Hamada–Johnston potential (first in [28]), Yale-group potential [29], Reid potentials (Reid68 [30], Reid68–Day [31] and Reid93 [32] by Nijmegen group), Urbana-group potentials (e.g., UrbanaV14 [33]), Argonne-group potentials (e.g., ArgonneV14 [34] and ArgonneV18 [35]) and some Nijmegen-group potentials [36].

In addition, there are some other typical and special NN potential models. Among them are the potentials based on «Mean Field Theory» (MFT) (first in [37]), which are interesting particularly in nuclear many-body calculations. The «Renormalization Group» (RG) approach (first in [38]) to the NN interaction is also creditable. In the RG potentials, by integrating out the high-momentum components of the various potentials, one could arrive at some model-independent low-momentum interactions with satisfactory results.

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[1] It is notable that we refer to the first presented version of the potentials by the various groups, while we quote some particular versions depended on the need.

[2] The NN interaction was divided into three parts first in [15]. The long-range (LR) part ($r \gtrsim 2$ fm) is always represented by a One-Pion-Exchange Potential (OPEP) tail attached to other parts. The intermediate/medium-range (MR) part ($1 \lesssim r \lesssim 2$ fm) is always owed to the scalar–meson exchanges (two pions and heavier mesons). The short-range (SR) part ($r \lesssim 1$ fm) is considered as the vector–bosons exchanges (heavier mesons and multipion exchanges as well as the QCD effects).

A definite fact about the models based on QCD (and even the old EFT models) is that they still need more quantitative improvements. These models describe the characteristic phenomena seen in the nucleon–nucleon, pion–nucleon and pion–pion scattering very well qualitatively but they almost fail quantitatively. Still, lattice QCD models give better quantitative results as the previous qualitative descriptions for the SR part mostly. In general, common features of the «QCD-inspired» models that decrease the demand for them are cumbersome mathematics, large number of parameters and limitations in applications essentially to very low energies. Nevertheless, nowadays the hybrid quark–meson models give satisfactory results; where for the LR and MR parts, they use the potentials from the other phenomenological and boson-exchange models, while just the SR part is discussed with the QCD techniques; as some examples, see [39–42] for a review of the QCD-inspired models.

Chiral EFT models have had a successful growth as they now show themselves as the standard two-nucleon and few-nucleon potentials. Their new high-quality potentials, such as the Idaho-group [13] and the Bochum–Julich-group [14] potentials, give the results very well as the famous high-quality phenomenological potentials next to having more theoretical and physical grounds. Indeed, they are becoming the best candidates to describe the NN interaction both qualitatively and quantitatively. Still, the proper renormalization of the chiral nuclear potentials and few-nucleon forces especially in the higher orders of the chiral expansion remain to be well addressed with these models; see [8, 9, 43].

The boson-exchange models have even a further old intimacy with the NN interaction facts. For example, in One-Boson-Exchange (OBE) potentials, for each set of the mesons, a role is given in one part of the interaction. In general, six nonstrange bosons, which are the pseudoscalar mesons π and η, the vector mesons ρ and ω, and two scalar bosons δ and σ, where the first meson in each group is isovector, while the second is isoscalar, with the masses below 1 GeV, are always considered. The π meson provides the tensor force, which is reduced at SR to the ρ meson. ω creates the spin-orbit force and the SR repulsion, and σ is responsible for the MR attraction and also provides a good parameterization of 2π system in S-state. Therefore, it is easy to understand why a model that includes theses four mesons can reproduce the major properties of the nuclear force [44]. In these models, besides the mentioned mesons, other different meson exchanges may be also included depending on the case. Then, the strength for any not considered meson exchanges (e.g., multimeson exchanges) is left as a free parameter to be determined by fitting the NN scattering data. Among the best BE potentials are the parameterized Paris [45] potential, CD Bonn [46] potential and Nijmegen 93, Nijmegen I, Nijmegen II [32] potentials.

On the other hand, the most important feature of the phenomenological NN potentials is their simplicity. General form of a potential allowed by the symmetries like rotation, translation, isospin and so on is always considered. There, the SR and MR parts are always determined in a phenomenological manner, while for the LR part, OPEP is often used. There are, however, some undesirable problems yet. Three-body forces and relativistic effects are included implicitly in these potentials. Further, the phenomenological models do not give much information about the NN dynamics and physics. For example, in a phenomenological potential that uses the Yukawa-type functions as $Y(r) = \dfrac{g}{4\pi r}\exp\left(\dfrac{-mc}{\hbar}r\right)$, the masses in the exponent and other free parameters are to be earned by fitting the NN scattering data; and similar for more complicated or other type functions. The same is true in most boson-exchange potentials, in which some parameters which have physical meaning, are free to be determined by fitting the data. That is a weakness because the nuclear force, in principle, should not depend on the external and by-hand controls so much. Nevertheless, some failures are indeed unavoidable though they could not decrease from the successes obtained by these potentials. Well reproducing the NN scattering data and neutron properties as well as giving satisfactory results in many nuclear-structure calculations are striking merits of the phenomenological potentials which make them still interesting.

Anyway, among many high-quality potentials, we try a basic comparison of some NN potentials as another step towards understanding the potential differences and similarities [1]. Here, we handle the coordinate-space potentials of Reid68 [30], Reid68–Day [31], UrbanaV14 [33], Reid93, Nijmegen 93, Nijmegen I, Nijmegen II [32] and ArgonneV18 [35]. In fact, we recast the exact potentials in one common form, and then compare the potential shapes in some channels to find likenesses and differences. This way may be applicable to many other potentials, but probably with other methods, to turn them into a nearly common form. This task seems to be important in that is fairly expected to have more acceptable NN models and potentials as the various comparisons then guide us to find the better ones.

The reaming parts of this note are organized as follows. In Sec. 1, we first study the common and important criteria to measure the potential's quality. Then, we compare many old and new potentials by confronting them with the experimental data, where giving a χ^2/N_{data} near the perfect value of 1 as the main criterion. There, we see that almost all famous potentials from the 1990s give satisfactory results with addressing some high-quality potentials. In Sec. 2, we sketch our reduction schema to compare some almost high-quality phenomenological NN potentials. We discuss on the main involved potential structures and details and on how to recast them into the wished form. In Sec. 3, based on the plots got for many different channels of the potentials, we make some comparisons among them with addressing their likenesses and differences after reduction of course. There, we also see more features and weakness of the potentials when they are plotted together. In Sec. 4, we make some closing remarks on nucleon–nucleon models, comparing the potentials, problems, challenges and future directions.

1. EVALUATING A FEW NN POTENTIALS

1.1. The Main Criteria. The quality measurement of the various NN potentials is possible through several methods. Giving satisfactory results in nuclear-structure calculations and deuteron properties (such as the ratio of D-wave to S-wave, quadratic magnetic-moment, electric quadruple-moment and binding energy) are two outstanding ways. It is of course necessary to mention that, in some potentials, these experimental parameters are used to fit the potential parameters and forms. Reproducing the phase shifts in different channels and comparing them with experimental values is another method for the Potential-Quality Measurement (PQM). Measuring the cross sections and polarizations in np scattering at high energies and analyzing power in low energies, next to many spin observations in pp scattering, are also the tasks to which a potential should respond. Especially, χ^2 associated with fitting

[1] Here, we have considered some early almost phenomenological NN potential forms commonly used in nuclear calculations. Further studies, by including even more new realistic potentials, are to be done later.

the experimental NN data by a potential is another desirable parameter for PQM as it is always considered in evaluating many potentials.

In the case of using χ^2 for PQM, however, there are some problems [47]. For instance, a notable point is that χ^2 is not a magic number as its relevance to the «quality» of a potential is indeed limited. For example, for a potential with many parameters but a weak theory with little physics, one may even gain the best fit of the data resulting in the optimized value of $\chi^2/\mathrm{datum} \approx 1$. On the other hand, there may be a model or potential based on a tight physical theory but include few parameters with each parameter having a physical meaning. Then, χ^2/datum of 2 or 3 may be reasonable. Therefore, χ^2 is just one aspect among many others that one can consider simultaneously to judge about the NN potential's quality. Other equally important criteria are the theoretical bases of a potential and its off-shell behavior to be tested with the off-shell NN data. The latter aspect is important when one uses the NN potential in nuclear calculations. In fact, χ^2/datum between 1 and 6 does not affect drastically the nuclear-structure results, while the off-shell differences are more important. Meanwhile, occasionally with the high-precision experimental data, χ^2 may reflect more or less the inerrancy of the data than the quality of the base theory. Another discussion is that if one can consider χ^2 for PQM, one should include both the pp and np data and not one of them. An important point here is that the pp potential must only be confronted with the pp data and the np potential with the np data. It is also mentionable that although the NN scattering data are improved in the recent decades for the energy regions of 350 MeV $\leqslant T_{\mathrm{lab}} \leqslant 2$ GeV, many potentials are valid up to the pion-production threshold energy of $T_{\mathrm{lab}} \approx 280$ MeV and do not necessarily include inelastic channels.

1.2. Confronting the Potentials with Data. *1.2.1. Some Old Potentials.* In the work performed in 1992 [48] by Nijmegen group, some potential forms, i.e., Hamada–Johnston potential [28], Reid soft-core (Reid68) potential [30], a super-soft-core potential [49], Funabashi potential [19, 50], Nijm78 potential [23], parameterized Paris (Paris80) potential [45], UrbanaV14 (Urb81) potential [33], ArgonneV14 (Arg84) potential [34], coordinate-space Full-Bonn (or Bonn87) potential [21] and Bonn89 potential [44, 51] were compared with some pp scattering data below $T_{\mathrm{lab}} = 350$ MeV. Later, they faced the potentials with the np (indeed all $pp + np$) data in [52].

Some potentials do not give good descriptions of the low-energy data mostly below 2 MeV. Although this is partly because of the inaccurate ${}^1S_0$ phase shift, there are other reasons such as fitting to the old data and mainly weak structures of the potentials. Still, one should note that because some potentials are originally fitted to each pp or np data, it is not so strange to give poor descriptions of the opposite data.

The Hamada–Johnston (HJ62) hard-core potential [28], such as most similar phenomenological potentials, uses OPEP in the LR part and some potential terms composed of the operators based on the symmetries with radial functions, which in turn have some free parameters to be fitted to the scattering data, in terms of internucleon distances for the MR and SR parts. The HJ62 potential is fitted to its time to both pp and np data; and when meet the Nijmegen 1993 pp database, it gives $\chi^2/N_{\mathrm{data}} = 6.1$ in the energy range of 2–350 MeV (we use for most the pp potentials here) and $\chi^2/N_{\mathrm{data}} = 3.7$ for the np data in the energy range of 5–350 MeV (we use for most the np potentials here).

The Ried68 soft-core potential [30] is fitted to both pp and np data of that time, and next to the LR OPEP part it uses the Yukawa-type functions by the pion masses for each partial wave up to the angular momentum of $J \leqslant 2$. The potentials are not regular at the origin because of a r^{-1} singularity. Later, B. D. Day extended the Reid68 potential for the upper partial waves. Day81 potential [31] describes the high-energy pp data good with $\chi^2/N_{\rm data} = 1.9$ and the np data with $\chi^2/N_{\rm data} = 10.7$ so bad!

The super-soft-core (TRS75) potential in [49] is fitted to both pp and np data and includes various meson exchanges and uses some step-like cut-off functions to regulate the potentials at the origin. It describes all pp data with $\chi^2/N_{\rm data} \approx 3.3$ and the high-energy np data with $\chi^2/N_{\rm data} \approx 3.6$. The Funabashi potential [19,50] is a similar meson-exchange field-theoretical potential as the former [49] with the bad overall behavior of $\chi^2/N_{\rm data} \approx 20$.

The charge-independent (CI) Nijm78 potential [23] includes various meson exchanges besides using Pomeron and other Regge-pole trajectories. It has both coordinate- and momentum-space versions and with 13 parameters gives a good description of the pp data with $\chi^2/N_{\rm data} \approx 2$. The parameterized Paris potential (Paris80) [45] is a meson-exchange potential that uses the dispersion theory to estimate the intermediate Two-Pion-Exchange Potential (TPEP). The ω-meson exchange in the SR part is included as a part of three-pion exchange with a special repulsive soft-core potential. The potential includes some static Yukawa functions with 13 originally needed parameters to fit the time $pp + np$ data. It gives a reasonable description of the low-energy and especially high-energy data in the energy range of 5–350 MeV with $\chi^2/N_{\rm data} = 2.2$ for the pp data and with $\chi^2/N_{\rm data} = 3.8$ for the np data.

The UrbanaV14 (Urb81) potential [33] is almost full phenomenological. It includes 14 different potential types which are central, spin–spin, tensor, spin–orbit, quadratic spin–orbit, centrifugal, centrifugal spin–spin, as well as other seven ones with dependence on the isospin. For the LR part, as usual, OPEP is used, while the MR part is parameterized with a TPEP with 14 parameters; and for the SR part, two Woods–Saxon potentials with 20 parameters are employed. The potentials are regulated with special cut-off functions. Describing of the pp data in the energy range of 5–350 MeV is bad with $\chi^2/N_{\rm data} = 5.9$; whereas for the np data in the same range, it is fair with $\chi^2/N_{\rm data} = 2.7$. That is because the potential was originally fitted to the np data and not to the pp data.

The ArgonneV14 (Arg84) potential [34] has similar structure as the Urb81 potential but with fewer parameters and it is fitted to the time np data in the energy range of 25–400 MeV. It provides as improvement compared with Urb81 in the energy range of 5–350 MeV just for the np data with $\chi^2/N_{\rm data} = 2.1$.

The Bonn-group comprehensive meson-exchange potentials use various field-theoretical techniques. The potentials are in terms of multiple OBEP and special TPEP (by an energy-independent σ-meson exchange) parts. The form factors truncate the potentials in the short distances and the SR repulsion comes from the ω-meson exchange. The first version named as Full-Bonn (Bonn87) potential [21] is in coordinate-space and uses various meson and two-pion exchanges, and is regularized at the origin by the dipole form factors. This potential does not describe all data good with giving $\chi^2/N_{\rm data} > 10$. Its updated version (Bonn89) [44, 51] gives good description of the pp and np data in the energy range of 5–350 MeV with $\chi^2/N_{\rm data} = 1.8$ and $\chi^2/N_{\rm data} = 3$, respectively; while the low-energy data descriptions are not so good. Other Bonn potentials (Bonn-A and Bonn-B) [44] with small differences from Bonn87 [21] are also not satisfactory in describing data.

So far, we see that from the older potentials, only Nijm78 and Bonn89 give satisfactory descriptions of the pp scattering data in the energy range of 0–350 MeV. By excluding the data of 0–2 MeV, Reid68 and Paris80 give a fair description as well. These potentials reproduce $\chi^2/N_{\rm data} \approx 2$ as they encounter the Nijmegen 1992 data [48]. When confronting with the

Nijmegen 1994 np scattering data [52], just Arg84 and Nijm93 (we describe later) give $\chi^2/N_{\rm data} \approx 2$, while Urb81 and Bonn89 give $\chi^2/N_{\rm data} \approx 3$ for the energies of 5–350 MeV. The other almost old potentials give a large or very large contribution to χ^2 especially in the low-energy region.

A reason not to reproduce so many good results by some potentials, when facing either the pp or np data, is that they are fitted only to the np or pp data, respectively, or to the $pp{+}np$ data. Second, some data, to which the original potentials are fitted, are old and incomplete nowadays; and third and maybe the most important one is that, some potentials have weak theoretical structures and bases. Fourth, about different results for very low energies, we first note that the pp $^1S_0$ phase shift in the energies of keV–2 MeV is very well-known; therefore, a small deviation for $^1S_0$ predicted by a potential gives rise to a large contribution to χ^2. Nevertheless, the last contribution should not be too large because most potentials are supposed to give good descriptions of the scattering-length and effective-range parameters. This means that the other phase shifts should often be improved to earn a better fit.

Indeed, one should note that, to give reasonable results, the potentials are necessary to fit both pp and np data, because a good fit to the pp (np) data does not automatically guarantee a good fit to the np (pp) data. An important conclusion is that only the potentials that are explicitly fitted to the pp (np) data give reasonable descriptions of the pp (np) data. Therefore, we assume that some potentials are not in fact NN potential but pp or np potential. For instance, Reid68, Nijm78, Paris80 and Bonn89 may be called the pp potentials, while Urb81 and Arg84 may be called the np potentials. Meanwhile, we should again mention that the potentials such as HJ62, Bonn87 and Bonn-A and Bonn-B do not describe well the pp and np scattering data.

1.2.2. New High-Quality Potentials. Nijmegen Partial-Wave Analysis (PWA) [53] improved more the NN phase shift analysis. The analysis was indeed a potential analysis, where the final phase shifts were the ones predicted by some «optimized» partial-wave potentials. In PWA, the SR and LR parts with a separation line in $r = 1.4$ fm are considered. The LR part in turn includes a detailed electromagnetic part and a detailed nuclear part — it is notable that in Nijmegen potentials, the mass differences between the charged and neutral pions and between proton and neutron are included; and because of their special SR parameterizations, the potentials are in contact with QCD. In the overall Nijmegen analysis in 1993 (PWA93) [53], for 1787 pp data and 2514 np data below $T_{\rm lab} = 350$ MeV, the «perfect» result of $\chi^2/N_{\rm data} = 0.99$ is obtained. Later, they performed another PWA up to the energy of 500 MeV that is above the pion-production threshold [54], where the more updated data, inelasticity and some other effects are included as well. For two newer PWA of the $pp + np$ data see also [55, 56].

Other generation of the Nijmegen-group potentials is Nijm93, NijmI, NijmII [32]. These potentials are based on the soft-core Nijm78 potential [23]. Nijm93, as a nonrelativistic meson-exchange potential, is an updated version of Nijm78, where the low-energy NN interaction is based on Regge-pole theory. This potential includes the charge-dependent (CD) terms, 13 parameters and exponential form factors. It gives a good description of both pp and np data from 0–350 MeV with $\chi^2/N_{\rm data} \approx 1.9$. The NijmI potential includes momentum-dependent terms that lead to nonlocal structure of the potential in the configuration space. In other words, the local representation of the OPE part is preserved, while the tracks of nonlocalities are included in the MR and SR parts by computing the second-order Feynman diagrams of the OBE parts. On the other hand, the NijmII nonrelativistic potential is fully local. In both the latter potentials, all 41 parameters are adjusted separately for each partial wave, and at very short distances the exponential form factors are used for regularization. The potentials fit all data well with $\chi^2/N_{\rm data} \approx 1.03$ and so have high quality. For a more recent generation of the high-quality Nijmegen (extended-soft-core) potentials, see [57].

On the other hand, the first disadvantage of the Reid68 [30] potential is the poor quality of the np data at the time of its construction. Another point is its r^{-1} singularity, and then its Fourier transform into the momentum space. To transform, the singularities are regularized by dipole form factors in Reid93 [32]. Here, OPEP is included besides the mass difference between the neutral and charged pions. In Reid93, the potentials are parameterized for each partial wave separately by combinations of the central, spin-orbit and tensor parts (with the local Yukawa functions) including the associated operators, while in Nijm93 the potential forms are the same for all partial waves. With 5 phenomenological parameters, it gives a good description of all data with $\chi^2/N_{\rm data} \approx 1.03$ and deuteron properties as the other high-quality Nijmegen potentials.

The ArgonneV18 (Arg94) potential [35] is a local potential that includes an electromagnetic (EM) part, a proper OPEP for the LR part that is regularized at short distances, and a phenomenological parameterizations for the MR and SR parts with the aid of the local Woods–Saxon potentials. The EM part is similar to that used in the Nijmegen PWA93 next to including the short-range terms and finite-size effects. The core functions are effective in $r = 0.5$ fm. The operators in Arg94 are more (eighteen) compared with a typical nonrelativistic OBE potential and also with the similar older phenomenological potentials such as Urb81 [33] and Arg84 [34]. With 40 adjustable parameters, it gives $\chi^2/N_{\rm data} \approx 1.03$ for 4301 pp and np data in the energy range of 0–350 MeV. In a later study [58], another extension of the Arg94 potential was made (called ArgonneV18pq potential), where various quadratic momentum dependences in the NN potentials were included to fit the data in the high partial waves with their effects in some nuclear applications.

And the last and the best version on the trail of the Bonn-group potentials is the CD-Bonn potential [46]. It is again based on relativistic meson-exchange theory. The charge-dependence (CD) and charge-symmetry breaking are included in all partial waves with $J \leqslant 4$. The charge-symmetry breaking is because of the OPE part of the potential and differences between the neutral- and charged-pion masses. This potential has a nonlocal structure arising from the covariant Feynman amplitudes. The potential may be called phenomenological because of fine-tuning of the partial waves to earn a wished χ^2 per datum. It fits 2932 pp data below 350 MeV available in 2000 with $\chi^2/N_{\rm data} = 1.01$ and 3058 np data with $\chi^2/N_{\rm data} = 1.02$, and so has high quality as well.

Now, among these newer potentials, which are fitted to the $pp + np$ data, the Nijm93 potential has indeed the lowest quality with $\chi^2/N_{\rm data} \approx 2$. Other potentials, which are NijmI, NijmII, Reid93, CD-Bonn and Arg94 potentials as well as the Nijmegen PWA93, all give $\chi^2/N_{\rm data} \approx 1$ that marks their high quality. Still, there are some other typical potentials that we address briefly below.

The Padua-group NN potential [22], which is based on meson-exchange theory by employing the phenomenological terms, describes the phase shifts and deuteron properties similar to the Paris80 [45], Arg84 [34] and Bonn-A [44] potentials. The Virginia-group potentials [25], as some special relativistic OBE models, have almost the same quality to fit the data as the Bonn87 [21] and Arg84 [34] potentials. The Bonn-B potential [44] was the starting point to build One-Solitary-Boson-Exchange potential (OSBEP) by Hamburg group (Ham95) [24]. It is shown [59] that the Ham95 potential describes the deuteron properties and the scattering data by R. A. Arndt et al. [60] similar to the Bonn-B potential. In fact, with 8 parameters, it describes 1292 pp data in the energy range of 1–300 MeV with $\chi^2/N_{\rm data} = 6.8$ and 2719 np data in the energy range of 0–300 MeV with $\chi^2/N_{\rm data} = 4.1$ near the Nijm93 results. Other potentials such as the Bochum-group potential [26] that uses meson exchange for the long distances and takes attention to the nuclear structure in the shorter distances, the Moscow-group potentials [6,61] that use a hybrid of the quark model of QCD and the meson-exchange picture, and the Oxford potential [7] as a QCD-inspired potential, claim to provide good descriptions of the NN data. So, in general, there are many high-quality models and potentials based on meson exchanges, QCD, and especially more recent chiral EFT potentials that we mention below.

1.2.3. A Summary. In summary, for the approach in which the criteria in Subsec. 1.1 and mainly χ^2 are considered for the Potential-Quality Measurement (PQM), we can make the following statements. Great progress in the NN data quality was achieved by Nijmegen group in the 1990s when more focus was started on the quantitative aspects of the NN potentials as well. Even the best NN potentials of the 1980s, such as Paris80 [45], Urb81 [33], Arg84 [34] and Bonn89 [44, 51], fit the NN data typically with at least $\chi^2/N_{\rm data} = 2$ that is above the perfect or wished value of $\chi^2/N_{\rm data} = 1$. A more completed and updated NN database by Nijmegen group [48,52,53] made more opportunities to build better potentials as discrepancies in the predictions could not be blamed on the bad fitting of the scattering data. Then, some new CD NN potentials were constructed in the early and mid 1990s. There are NijmI, NijmII and Reid93 potentials [32] by Nijmegen group, Arg94 potential [35] and CD-Bonn potential [46]. All these potentials have about 45 parameters and fit the NN data with nearly $\chi^2/N_{\rm data} \approx 1$, and so are the high-quality NN potentials.

On the other hand, there are satisfactory results from some chiral EFT potentials. Indeed, by using the same np data as in the CD-Bonn potential [46], the potentials in the next-to-leading (NL) and next-to-next-to-leading (NNL) orders of the chiral expansion, made by Bochum–Julich group, give a large $\chi^2/N_{\rm data}$ for the data below 350 MeV. However, in another development, they set up an NNNLO potential [14] whose parameters were fitted to the pp and np Nijmegen phase shifts [53] and the nn scattering length. This new potential gives a rather good description of the np data with $\chi^2/N_{\rm data} \geqslant 1.7$ and a rather bad description of the pp data with $\chi^2/N_{\rm data} \geqslant 2.9$ in the energy range of 0–290 MeV. It is notable that as the energy decreases, the data description by the potential becomes better and better. Still, the Idaho-group CHPT potentials appear to have higher quality. Indeed, the Idaho NNNLO potential (Idaho03) [62], reproduces the np and pp scattering data with almost $\chi^2/N_{\rm data} \approx 1.1$ and $\chi^2/N_{\rm data} \approx 1.5$ for the energy range of 0–290 MeV, respectively. So, the Idaho03 potential is another high-quality potential such as NijmI, NijmII, CD-Bonn, Reid93 and Arg94 at least in describing the NN scattering data and deuteron properties.

We should stress that most potentials use one-pion-exchange potentials (OPEPs) for the LR part, while correlated two-pion exchanges (TPEs) and other meson exchanges (always OBEs) are employed for the MR part. For the SR part, the heavy vector–boson exchanges and QCD effects or the phenomenological procedures are often used. Among the high-quality potentials, NijmII, Reid93, and Arg94 potentials are nonrelativistic with local functions that couple to the (nonrelativistic) operators composed of the various spin, isospin and angular momentum of the two-nucleon pairs. This approach is the simplest for calculations in the coordinate space. The NijmI potential also includes the p^2 terms attributable to the nonlocal contributions to the central force. The CD-Bonn potential is based on the relativistic meson-exchange theory and is nonlocal with including more momentum-dependent terms. In the Idaho03 potential, based on Chiral Perturbation Theory (CHPT), mesons and quarks degrees of freedom are included and its quality is high as in the NijmII, Reid93, Arg94 and CD-Bonn potentials — for some tests of the above high-quality potentials in nuclear-structure calculations see, for instance, [63, 64].

Now, according to the above discussions, we try to compare some potentials in a different and fairly substantial way, which is by considering their structures directly. Before doing so, we mention a plain comparison of some potentials in [65] slightly similar to the procedure we use here. Indeed, they have arrived at some effective low-momentum potentials by applying the renormalization-group (RG) methods to the potentials of Paris80, Bonn-A and CD-Bonn, NijmI and NijmII, Arg94, and Idaho03. Then, the resultant potentials have been compared with the model-independent RG potentials that reproduce the experimental phase shifts up to $T_{\rm lab} = 350$ MeV. The last comparison confirms the results outlined above from confronting the potentials with data, nearly.

2. REDUCING SOME POTENTIALS INTO THE REID POTENTIAL

2.1. The Basic Sketch. Among various NN potentials mentioned in the previous sections, here we consider the forms of Ried68 potential [30] and its extended version to higher orders by Day that is Day81 potential (or Full-Reid potential) [31], Reid93 potential [32], UrbanaV14 (Urb84) potential [33], ArgonneV18 (Arg94) potential [35] and Nijm93, NijmI, NijmII potentials [32]. These potentials are almost the phenomenological and boson-exchange ones, where the latter is the most important candidate for giving a true picture of the NN interaction nowadays with more confirmations from chiral EFT as well. In another moment, we try to extend the above list to include more potentials.

R. V. Reid in 1968 parameterized potentials in each partial wave up to $J \leqslant 2$ separately [30]. He used a central potential for the singlet- and triplet-uncoupled states, while for triplet-coupled states, a potential with central, tensor and the first-order spin-orbit forces was used as

$$V = V_C(r) + V_T(r)S_{12} + V_{SO}(r)\mathbf{L}\cdot\mathbf{S}, \qquad (2.1)$$

where $S_{12} = 3(\boldsymbol{\sigma}_1\cdot\hat{r})(\boldsymbol{\sigma}_2\cdot\hat{r}) - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$, $\mathbf{S} = (\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)/2$ and $\mathbf{L}\cdot\mathbf{S}$ are the usual tensor, spin and spin-orbit operators, respectively. In Reid68, for the long-range OPEP,

$$V_{\rm OPEP} = \left(\frac{g_{\rm pi}^2}{12}\right) m_{\rm pi}c^2 \left(\frac{m_{\rm pi}}{M}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left[(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + S_{12}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\right]\frac{{\rm e}^{-x}}{x}, \qquad (2.2)$$

where $m_{\rm pi} = 138.13$ MeV and $M = 938.903$ MeV are used for the pion and nucleon mass, respectively; $g_{\rm pi}^2 = 14$ with $g_{\rm pi}$ for the pion–nucleon coupling constant, and $x = \mu r$ with $\mu = m_{\rm pi}c/\hbar = r_0^{-1}$ ($\mu = 0.7$ fm^{-1} here) being the internucleon distance measured in the unit of the pion Compton's wavelength. There, to remove x^{-2} and x^{-3} behaviors at small x, a short-range interaction is also subtracted. The lack of the soft-core versions is that the potentials still have a x^{-1} singularity at the origin. The MR potentials are given by a sum of the Yukawa-type functions as ${\rm e}^{-nx}/x$, where n is an integer. Meanwhile, the SR repulsive part is given by an average of both very hard-core and (Yukawa) soft-core potentials. In 1981, Day extended the Reid68 potential roughly for the states with $J \geqslant 3$ up to $J = 5$ [31].

Now, for a structural comparison of the potentials, we reduce the mentioned potentials for all uncoupled and coupled states to the Reid potential structure. As the prime Reid potential includes three terms as central, tensor and spin-orbit (2.1), so after the reduction schema, all terms in the potentials reduce to these two central and noncentral parts. The most important reason for doing so is that because not only the main terms in a potential are these three terms but also, by having a similar operator form for all potentials, one can somehow compare the potential structures.

2.2. Reducing UrbanaV14 Potential into the Reid Potential. The UrbanaV14 potential [33] is still being used in some nuclear-structure calculations. Its two-nucleon interaction reads

$$V_{\mathrm{Urb}} = \sum_{i=1}^{14} \big(V_L^i(r) + V_M^i(r) + V_S^i(r)\big)\, O_i, \tag{2.3}$$

where V_L, V_M and V_S stand for the LR, MR and SR part potentials, and O_i's are 14 conveniently chosen operators that we indicate as c (for central), σ (for spin), τ (for isospin), $\sigma\tau$, t (for tensor), $t\tau$, ls (for spin-orbit), $ls\tau$, ll (for quadratic-orbit), $ll\sigma$, $ll\tau$, $ll\sigma\tau$, $ls2$ (for quadratic spin-orbit) and $ls2\tau$, respectively.

The LR part potential reads

$$V_L = V_\pi^{\sigma\tau}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + V_\pi^{t\tau}(r) S_{12}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2), \tag{2.4}$$

and for the MR part, the contribution is

$$\begin{aligned} V_M = T_\pi^2(r)\big(&I^c + I^\sigma(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + I^\tau(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + I^{\sigma\tau}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + \\ &+ I^t S_{12} + I^{t\tau} S_{12}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + I^{ll}L^2 + I^{ll\sigma}L^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + I^{ll\tau}L^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + \\ &+ I^{ll\sigma\tau}L^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + I^{ls2}(\mathbf{L}\cdot\mathbf{S})^2 + I^{ls2\tau}(\mathbf{L}\cdot\mathbf{S})^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\big), \end{aligned} \tag{2.5}$$

and also for the SR part, it becomes

$$\begin{aligned} V_S = W(r)\big(&S^c + S^\sigma(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + S^\tau(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + S^{\sigma\tau}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + \\ &+ S^{ls}(\mathbf{L}\cdot\mathbf{S}) + S^{ls\tau}(\mathbf{L}\cdot\mathbf{S})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + S^{ll}L^2 + S^{ll\sigma}L^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + \\ &+ S^{ll\tau}L^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + I^{ll\sigma\tau}L^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + I^{ls2}(\mathbf{L}\cdot\mathbf{S})^2 + \\ &+ I^{ls2\tau}(\mathbf{L}\cdot\mathbf{S})^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\big) + \grave{W}(r)\big(\grave{S}^{ls}(\mathbf{L}\cdot\mathbf{S}) + \grave{S}^{ls\tau}(\mathbf{L}\cdot\mathbf{S})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\big). \end{aligned} \tag{2.6}$$

One should note that in the above relations,

$$V_\pi^{\sigma\tau}(r) = 3.488\frac{\mathrm{e}^{-0.7r}}{0.7r}\left(1-\mathrm{e}^{-cr^2}\right), \tag{2.7}$$

$$V_\pi^{t\tau}(r) = 3.488\left(1+\frac{3}{0.7r}+\frac{3}{(0.7r)^2}\right)\frac{\mathrm{e}^{-0.7r}}{0.7r}\left(1-\mathrm{e}^{-cr^2}\right)^2 = 3.488T_\pi(r), \tag{2.8}$$

$$W(r) = \left(1+\exp\left(\frac{r-R}{a}\right)\right)^{-1}, \quad \grave{W}(r) = \left(1+\exp\left(\frac{r-\grave{R}}{\grave{a}}\right)\right)^{-1}, \tag{2.9}$$

where the cut-off parameter c and the strengths I^i, S^i, $\grave{S}^i$ are determined by fitting to the scattering data (phase shifts). Indeed, the parameter values of $c = 0.2$ fm^{-2}, $R = 0.5$ fm, $a = 0.2$ fm, $\grave{R} = 0.36$ fm, $\grave{a} = 0.17$ fm are used in the Urb84 potential.

In our reduction schema, we now estimate the expectation values for all operators in a particular state and so, as in the Reid potential, finally have just a radial function of r for uncoupled states. Therefore, for an uncoupled state, e.g., ${}^3P_0$, after a little calculation, we get

$$V({}^3P_0) = 2694.69W(r) + 4400\grave{W}(r) - 3.6T_\pi^2(r) + V_\pi^{\sigma\tau}(r) - 4V_\pi^{t\tau}(r), \tag{2.10}$$

and for a coupled state, e.g., ${}^3S_1 - {}^3D_1$, in the end, we get

$$V({}^3S_1 - {}^3D_1) = \big(2399.99W(r) - 6.8008T_\pi^2(r) - 2V_\pi^{\sigma\tau}(r)\big) + \\ + \big(0.75T_\pi^2(r) - 3V_\pi^{t\tau}(r)\big)\, S_{12} + (80W(r))\, \mathbf{L}\cdot\mathbf{S}. \tag{2.11}$$

It is noticeable that for the coupled states we consider $\ell = j - 1$, and that the coefficients in the resultant relations are coming from the expectation values of the operators during the reduction into the desired form.

2.3. Reducing ArgonneV18 Potential into the Reid Potential. The ArgonneV14 potential [34] has some improvements with respect to the Urb81 potential [33] in describing data as we hinted in Subsubsec. 1.2.1, and Arg94 is a real high-quality potential. Among 18 operators of Arg94, 14 operators are those of Urb81 and other four operators are three charge-asymmetry operators of T (for the isospin operator of $T_{12} = 3\tau_{z1}\tau_{z2} - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$), σT, tT and one charge-asymmetry operator of τz as $\tau_{z1} + \tau_{z2}$. In addition, a more complete electromagnetic interaction than that used in the Nijmegen PWA93 [53] is included.

The potential is written as a sum of an electromagnetic (EM) part, an OPE part and the remaining (R) MR and SR phenomenological parts. The EM part, in turn, for the pp, np and nn interactions, dependent on the case, includes one- and two-photon Coulomb terms, the Darwin–Foldy term, vacuum polarization and magnetic-moment interactions, each with a proper form factor. For the charge-dependent OPE part, the neutron–proton and neutral- and charged-pion mass differences, the same as in the Nijmegen PWA93, are included as well. For the SR and MR parts, the potential is similar to Urb81 but with the Yukawa and tensor functions and a Woods–Saxon function more improved than (2.7), (2.8), (2.9) next to four sets of the strengths to fit the scattering data and a regularization condition at the origin. In general, the Arg94 potential includes more intricacies and improvements than the Urb81 potential. For details see [35].

Therefore, the Arg84 reduction is similar to the Urb81 reduction. However, because of the four extra operators and a full electromagnetic interaction as well as further subtleties, a little more lengthy calculation is required. It is also necessary to mention that, as in the previous case, all terms including the operators, functions and constants, regardless of the meanings and implications of the individual terms, reduce or absorb into the chosen Reid form.

2.4. Reducing Nijmegen Potentials into the Reid Potential. *2.4.1. Nijm93, NijmI and NijmII Potentials.* All these potentials [32] are based on the Nijm78 potential [23] with some differences as we have mentioned in Subsubsec. 1.2.2 concisely. The basic potentials are

OBEs with the momentum-dependent central terms and exponential form factors. In general, for the LR part, OPEs with including the pion mass differences are considered. Indeed, the pp and np OPE potentials read

$$V_{\mathrm{OPE}}(pp) = f_\pi^2 V(m_{\pi_0}), \tag{2.12}$$

$$V_{\mathrm{OPE}}(np) = -f_\pi^2 V(m_{\pi_0}) \pm 2 f_\pi^2 V(m_{\pi_\pm}), \tag{2.13}$$

in which

$$V(m_{\mathrm{pi}}) = \left(\frac{m_{\mathrm{pi}}}{m_{\pi_\pm}}\right)^2 \frac{1}{3} m_{\mathrm{pi}} c^2 \left[\phi_C^1(m_{\mathrm{pi}}, r)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + 3\phi_T^0(m_{\mathrm{pi}}, r) S_{12}\right] \tag{2.14}$$

and

$$\phi_C^1(r) = \phi_C^0(r) - 4\pi\delta^3(m_{\mathrm{pi}}\mathbf{r}), \tag{2.15}$$

where, without the form factors, the latter is used instead of $\phi_C^0(r)$, and the tensor (spin-orbit) functions ϕ_T^0 (ϕ_{SO}^0) are written in derivatives of the central function ϕ_C^0. One should note that f_π^2 is for the pion–nucleon coupling constant and that the plus (minus) sign in (2.13) is for the total isospin of $T = 1(0)$.

For the remaining MR and SR parts, the potential's structure, in coordinate space, reads

$$V_{\mathrm{Nijm}} = V_C(r) + V_{SS}(r)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + V_T(r) S_{12} + V_{SO}(r)\, \mathbf{L}\cdot\mathbf{S} + \\ + V_{SOA}(r)\, \mathbf{L}\cdot\mathbf{A} + V_{Q_{12}}(r) Q_{12}, \tag{2.16}$$

where the potential forms are assumed to be the same in all partial waves; $\mathbf{L}\cdot\mathbf{A}$ with $\mathbf{A} = (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)/2$ is called the charge-symmetry operator and

$$Q_{12} = \frac{(\boldsymbol{\sigma}_1 . \mathbf{L})(\boldsymbol{\sigma}_2 \cdot \mathbf{L}) + (\boldsymbol{\sigma}_2 \cdot \mathbf{L})(\boldsymbol{\sigma}_1 \cdot \mathbf{L})}{2} = \left[(\mathbf{L}\cdot\mathbf{S})^2 - \delta_{LJ} L^2\right] \tag{2.17}$$

is the quadratic spin-orbit operator whose presence can be simulated by introducing nonlocal potentials. In fact, the Nijm93 and NijmI potentials have a little nonlocality in their central parts, which is

$$V_C(r) = V_C(r) - \frac{1}{2M_{\mathrm{red}}}\left[\nabla^2 V_P(r) + V_P(r)\nabla^2\right], \tag{2.18}$$

with $M_{\mathrm{red}} = (m_{\pi_0} + 2m_{\pi_\pm})/3 \equiv \bar{m}$ as the average pion mass, while in the NijmII potential, $V_P(r) \equiv 0$. It is notable that the antisymmetric spin-orbit part (SOA), in principle, is not used in these potentials.

Thus, in the reduction schema, for the Nijm93 and NijmI nonlocal potentials, we must add for the uncoupled states the expectation value of the second term in (2.18) as well. On the other hand, for the singlet-coupled states, the tensor and spin-orbit terms become zero, and in the uncoupled states, except for ${}^3P_0$, $\left\langle \delta_{LJ} L^2 \right\rangle$ is not zero.

Now, one can easily estimate the expectation values especially for the second term of (2.18) by having $V_P(r)$ and using the direct Laplacian in the spherical coordinate for a state with definite angular momentum.

For reducing the potentials to the three terms of (2.1), we note that since for all coupled states, $L \neq J$, therefore $\left\langle L^2 \delta_{LJ} \right\rangle$ is zero. So, in general, for reduction we write

$$V_{\mathrm{centr}} = V_C(r) + V_{SS}(r)\left\langle(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\right\rangle - \frac{1}{2M_{\mathrm{red}}}\left\langle[\nabla^2 V_P(r) + V_P \nabla^2]\right\rangle, \tag{2.19}$$

$$V_{\text{tensor}} = V_T(r), \tag{2.20}$$

$$V_{\text{spin-orbit}} = V_{SO}(r) + V_{Q_{12}}(r)\langle \mathbf{L}\cdot\mathbf{S}\rangle. \tag{2.21}$$

2.4.2. Reid93 Potential. For the Reid93 potential [32], the OPE tail is the same as in (2.13), while in (2.12) ϕ_C^0 is used instead of ϕ_C^1 except for S-waves. The potential, for each partial wave, is parameterized with the same central, tensor and spin-orbit operators as Reid68 [30] besides some combinations of the following functions with arbitrary masses and cut-off parameters:

$$\bar{Y}(p) = p\bar{m}\phi_c^0(p\bar{m}, r), \quad \bar{Z}(p) = p\bar{m}\phi_T^0(p\bar{m}, r), \quad \bar{W}(p) = p\bar{m}\phi_{SO}^0(p\bar{m}, r), \tag{2.22}$$

where p is an integer. There are also some coefficients multiplying the linear combinations of the above functions in each partial wave to fit the scattering data. The potential that is now regularized at the origin, has a momentum-space version and is extended for the high partial waves.

3. DISCUSSIONS AND RESULTS

In the Table, some two-nucleon states considered here with their quantum numbers are given. In our reduction schema, there are three potential types, i.e., the central (for all states), tensor and spin-orbit, where the last two are only present in the coupled states. In the CI Reid68 potential [30], just the states up to $J \leqslant 2$ are included and for the $J > 2$ states, only in the tensor part OPEP is used. B. D. Day extended the Reid68 potential up to the $J \leqslant 5$ states [31]; and for the $J > 5$ states, he took the tensor part of OPEP; and for the spin-orbit

Two-nucleon states from $J = 0$ up to $J = 9$ and potential types in our reduction plan. For other higher states, the process is similar with $J = 5$ states and shown by the Latin letter H

Potential type (state)	Central	Tensor and spin-orbit
$J=0,\ S=0,\ T=1,\ L=0$	${}^1S_0\,(pp,np,nn)$	—
$J=0,\ S=1,\ T=1,\ L=1$	${}^3P_0\,(pp,np,nn)$	—
$J=1,\ S=0,\ T=0,\ L=1$	${}^1P_1\,(np)$	—
$J=1,\ S=1,\ T=1,\ L=1$	${}^3P_1\,(pp,np,nn)$	—
$J=1,\ S=1,\ T=0,\ L=0, L=2$	${}^3S_1{-}{}^3D_1\ (np)$	${}^3S_1 - {}^3D_1\ (np)$
$J=2,\ S=0,\ T=1,\ L=2$	${}^1D_2\,(pp,np,nn)$	—
$J=2,\ S=1,\ T=0,\ L=2$	${}^3D_2\,(np)$	—
$J=2,\ S=1,\ T=1,\ L=1, L=3$	${}^3P_2{-}{}^3F_2\ (pp,np,nn)$	${}^3P_2 - {}^3F_2\ (pp,np,nn)$
$J=3,\ S=0,\ T=0,\ L=3$	${}^1F_3\,(np)$	—
$J=3,\ S=1,\ T=1,\ L=3$	${}^3F_3\,(pp,np,nn)$	—
$J=3,\ S=1,\ T=0,\ L=2, L=4$	${}^3D_3{-}{}^3G_3\ (np)$	${}^3D_3 - {}^3G_3\ (np)$
$J=4,\ S=0,\ T=1,\ L=4$	${}^1G_4\,(pp,np,nn)$	—
$J=4,\ S=1,\ T=0,\ L=4$	${}^3G_4\,(np)$	—
$J=4,\ S=1,\ T=1,\ L=3, L=5$	${}^3F_4{-}{}^3H_4\ (pp,np,nn)$	${}^3F_4 - {}^3H_4\ (pp,np,nn)$

part from $J \geqslant 5$ on, he set zero. The CD Reid93 potential has the states up to $J = 9$ in the central and tensor parts and for the spin-orbit potentials in the states from $J \geqslant 5$ on, he set zero as was done also by him when he extended the Reid68 potential to the higher partial waves. The CD Nijm93, NijmI and NijmII potentials [32] have the same states as the Reid93 potential. The CI UrbanaV14 potential [33] has the states up to $F(J = 3)$; and the CD ArgonneV18 potential [35] has all three potential types up to the higher states.

The Arg94, Reid93 and Urb84 potentials do not use the direct meson exchanges for MRs and SRs but the phenomenological parameterization is chosen. Arg94 uses the local functions of the Woods–Saxon type and special Yukawa's with the exponential cut-offs; whereas Reid93 employs local Yukawa's with multiples of the pion masses similar to the original Reid68 potential. The new feature of the Reid93 potential with respect to the Reid68 potential is that in Reid93 the fitting is to the new data of Nijmegen group [53] and $1/r$ singularity in all partial waves is removed by introducing a dipole form factor. In the Urb81 potential, for the MR part, the local functions are the usual Yukawa's with exponential cut-offs, where the cut-off parameters are determined by fitting to the scattering data, and in the SR part, the special Woods–Saxon potentials are used. Still, at the very short distances, the potentials are regularized by the exponential (Arg94, Nijm93, NijmI, NijmII) or by the dipole (Reid93) form factors that are all local functions. The three Nijmegen potentials are based on the Nijm78 potential, which is framed from the estimated OBE amplitudes next to the contributions of the Pomeron and some tensor Regge trajectories. In fact, the NijmII potential uses the totally local approximations for all OBE contributions, and the Nijm93 and NijmI potentials keep some nonlocal terms in the central force components, while their tensor forces are local totally. The nonlocalities in the central force have only a very moderate impact on the nuclear-structure calculations compared with the nonlocalities in the tensor force.

According to the discussions so far, clearly the Reid68 and Reid93 potential forms are similar and also the Urb81 and Arg94 potential forms are alike, as well as three Nijm93, NijmI, NijmII potential forms together. In the second step, the Reid potentials (especially Reid93) have more likenesses with the Urb81 and Arg94 potentials. It is also notable that the LR OPEPs are almost the same for all potentials, except for few subtleties as taking the pion-mass differences.

In Figs. 1, 2 and 3, the central, tensor and spin-orbit parts of the considered potentials, reduced into the Reid potential, are respectively given for some np states (without any preference) between $J = 0$ and $J = 8$, the latter of which is almost the highest fitted wave for the potentials here. The figures are plotted in the range that the potentials have definite values and so, the ranges, in which the differences are not clear, are neglected. In the CI potentials, we only set the present potential in that special case. Although reproducing of the phase shifts and some other results from the calculations with these potentials are almost similar, the potentials are largely different.

With a glance to the figures, a close likeness of the Reid68 potential to the Reid93 potential, the Urb81 potential to the Arg94 potential, and the Nijm93, NijmI, NijmII potentials together is obvious. That is, of course, reasonable and predictable mainly from their structural similarities. It is obvious from the figures that for the LR part almost all potential shapes converge as it is, of course, predictable from their almost similar OPEPs. One can simply see which potentials are «softer» in the MR and SR parts. So, we note that the Nijm93 and NijmI potentials, which have some nonlocalities, are softer.

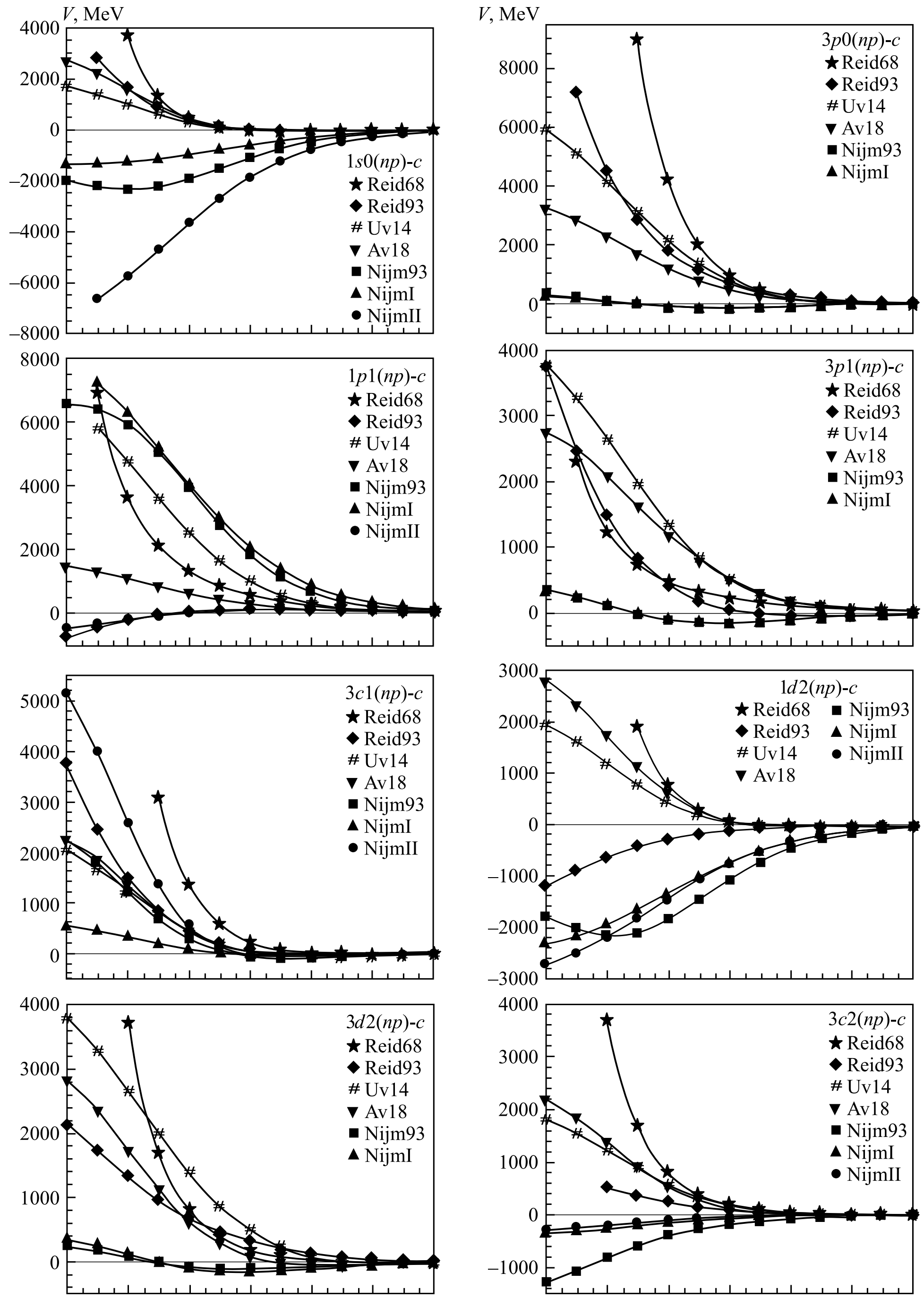

Fig. 1. The central potentials of various potential forms in the states from $J = 0$ up to $J = 8$, for the np system

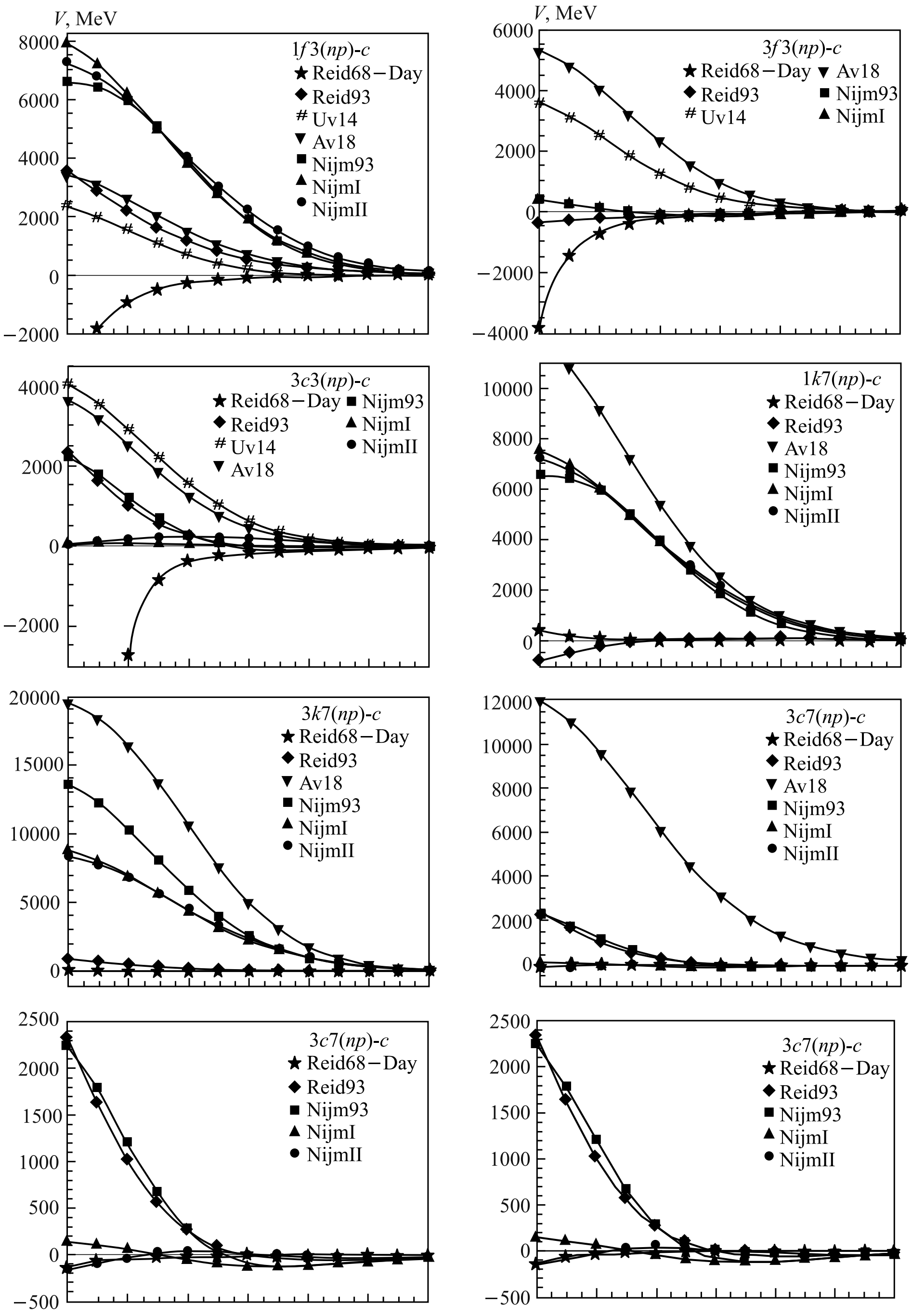

Fig. 1 (*continuation*)

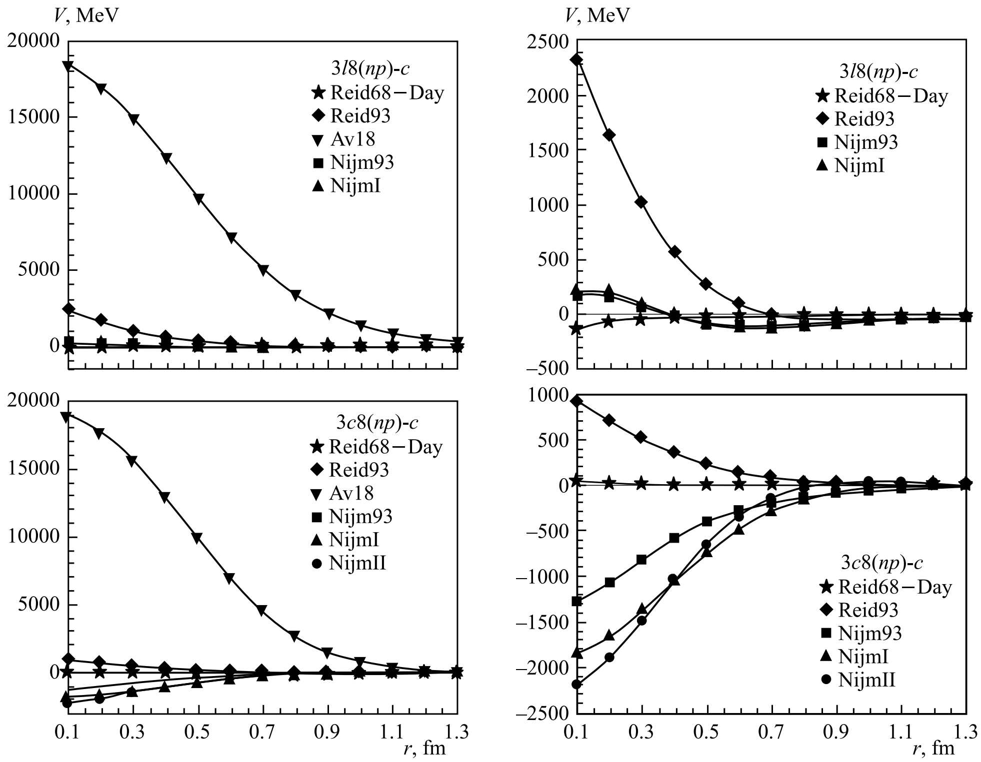

Fig. 1 (*the end*)

The looseness of the expansion by B. D. Day from the Reid68 potential is clear from the figures in that the Day expansion was to hold only satisfactory results in the nuclear calculations and was not based on any tight physical ground. The degree of the potential softness is obvious from the plots as well. The dependence on the even or odd values of the NN relative angular momentum, which is a space-exchange marker, is also clear from the figures. For instance, in the ${}^1D_2$ channel with an even L and in the ${}^1F_3$ channel with an odd L, one can easily see from Fig. 1 that the Reid68 and Reid93 potentials have a tendency to oppose each other. The same is valid for the three Nijmegen potentials, which in turn means that the spatial exchanges are strong. For the tensor and spin-orbit potentials in Figs. 2 and 3, one can also see that for each state with either an even or an odd J, a special procedure is dominant and the differences are discussable from the various points of view.

In Figs. 4, 5 and 6, three groups of the similar potentials, for the np states, from $J = 0$ up to $J = 2$, are compared. In Fig. 4, the Reid potentials (Day81, Reid93) are pictured for some channels. In general, the differences between these two potentials return to two important adjustments mentioned above. The presence of a softer core in the Reid93 potential is obvious compared with the singular SR part of the Reid68 potential. The small differences in Fig. 5 are also expectable because of the small differences in the structure of the Urb81 and Arg94 potentials. The same is true for the three Nijmegen potentials in Fig. 6. The charge-dependence of the CD potentials is also showed in Fig. 7 for the ${}^1S_0$ central potential and the tensor potential in the ${}^3P_2 - {}^3F_3$ state, as well as for the spin-orbit potential in the

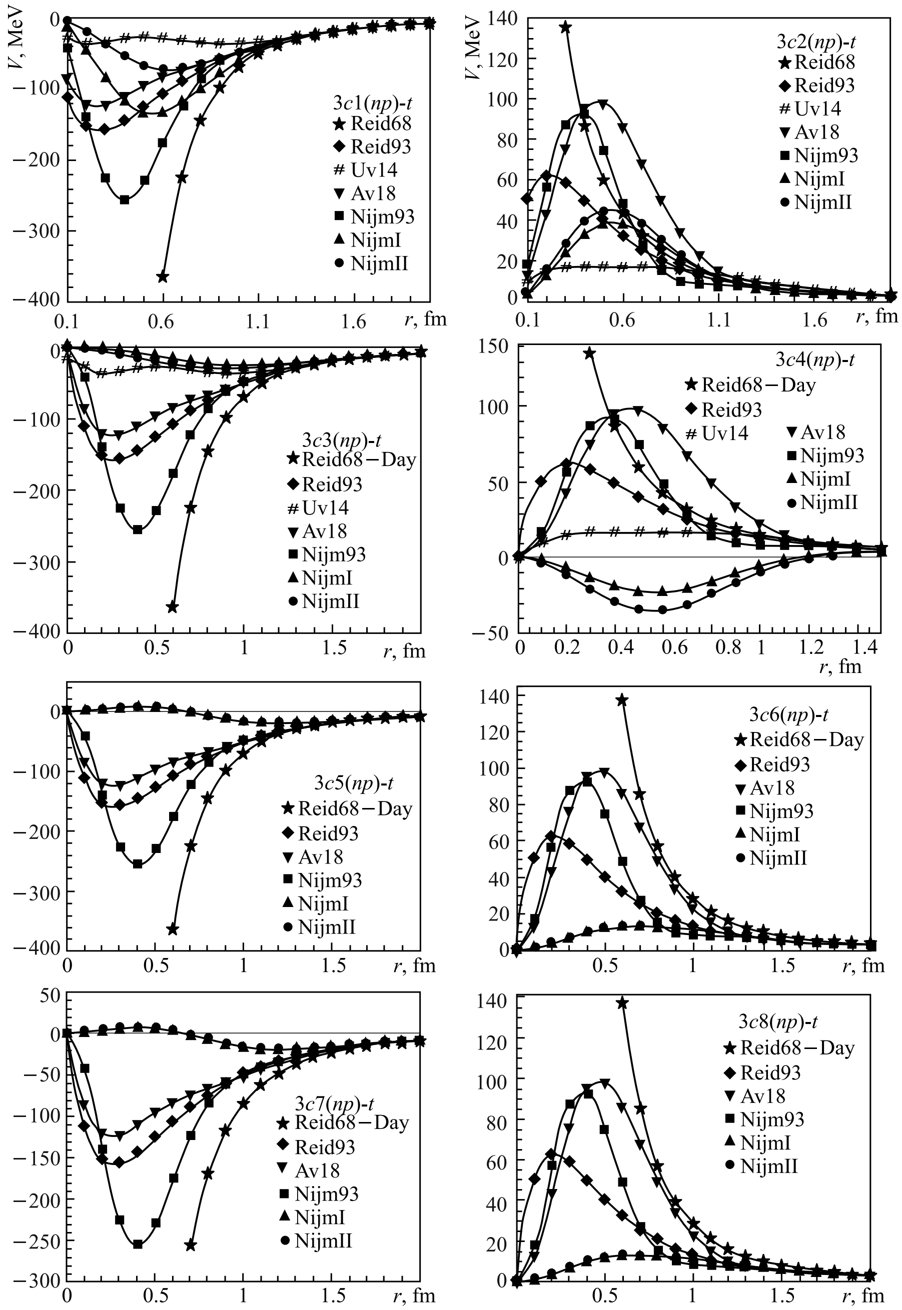

Fig. 2. The tensor potentials of various potential forms in the states from $J = 1$ up to $J = 8$, for the np system

$^3P_2$ channel. It is obvious from the figures that the charge-dependence is for the pp and np systems, and for the nn system is almost the same with pp. As a final illustration, in Fig. 8, the dependence on the orbital angular momentum for $^3S_1$, $^3D_1$, $^3P_2$ and $^3F_2$ states, for the np

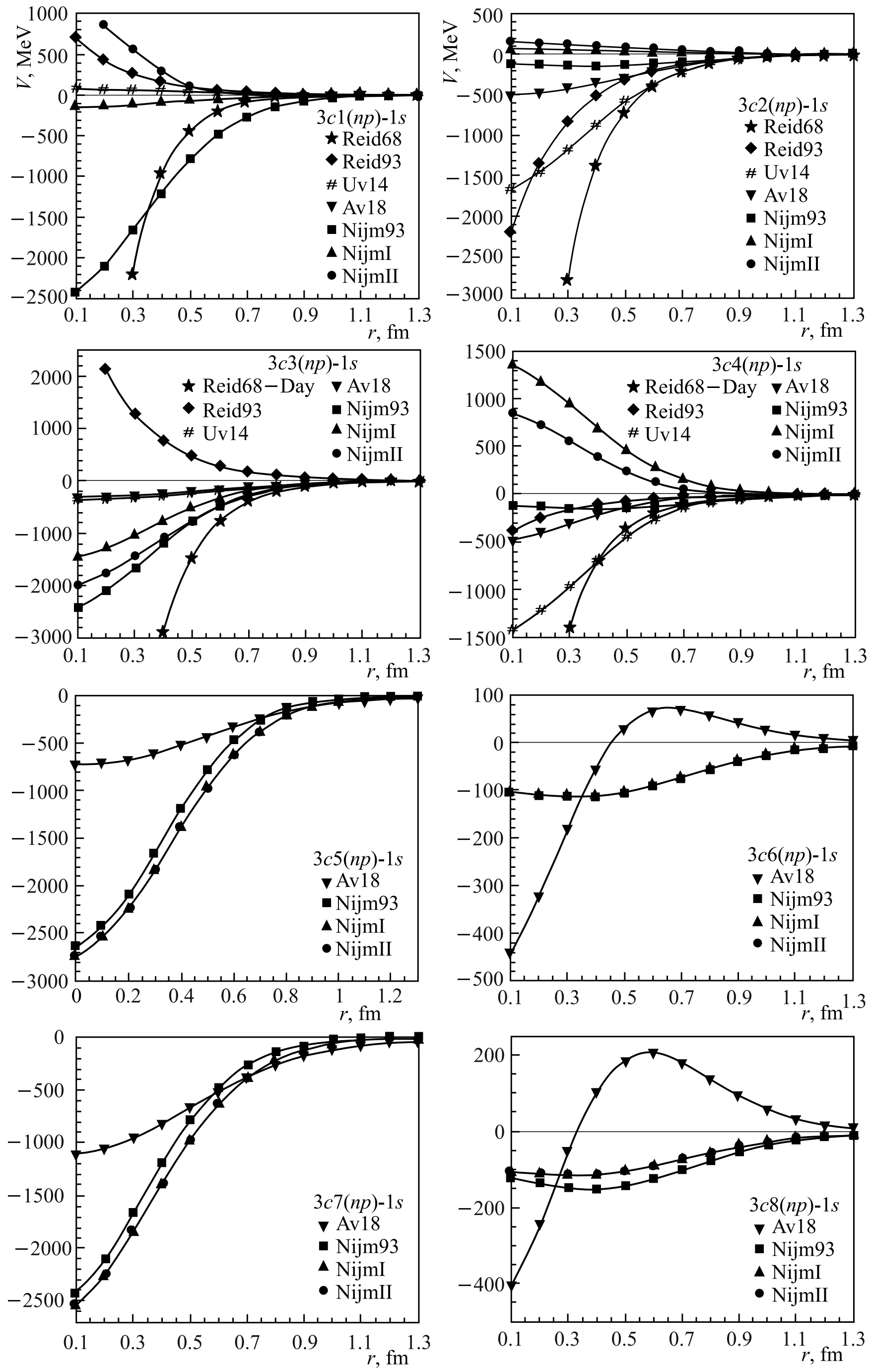

Fig. 3. The spin-orbit potentials of various potential forms in the states from $J = 1$ up to $J = 8$, for the np system

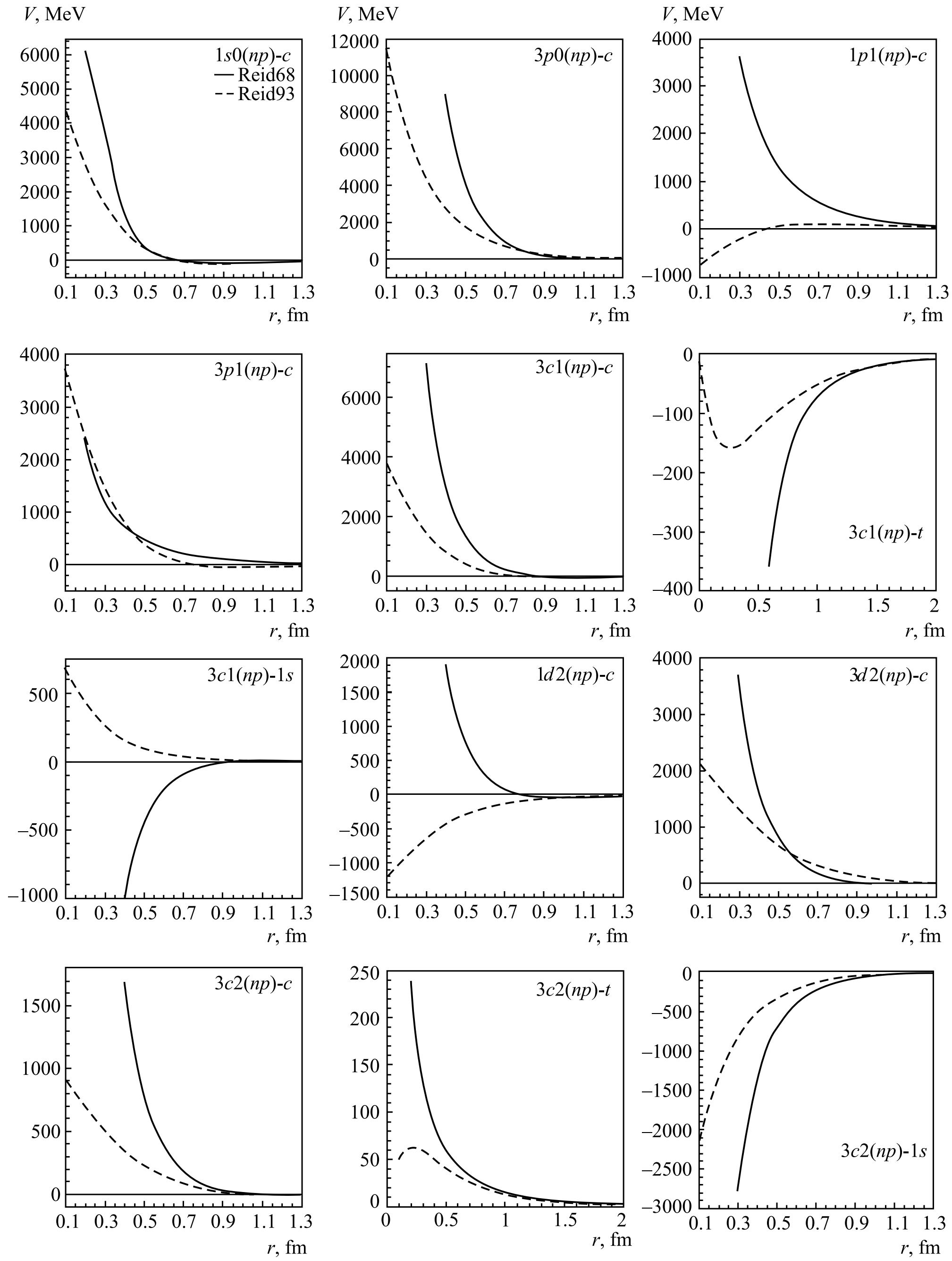

Fig. 4. The comparison of the central, tensor and spin-orbit potentials of Reid68 and Reid93, in the states from $J = 0$ up to $J = 2$, for the np system

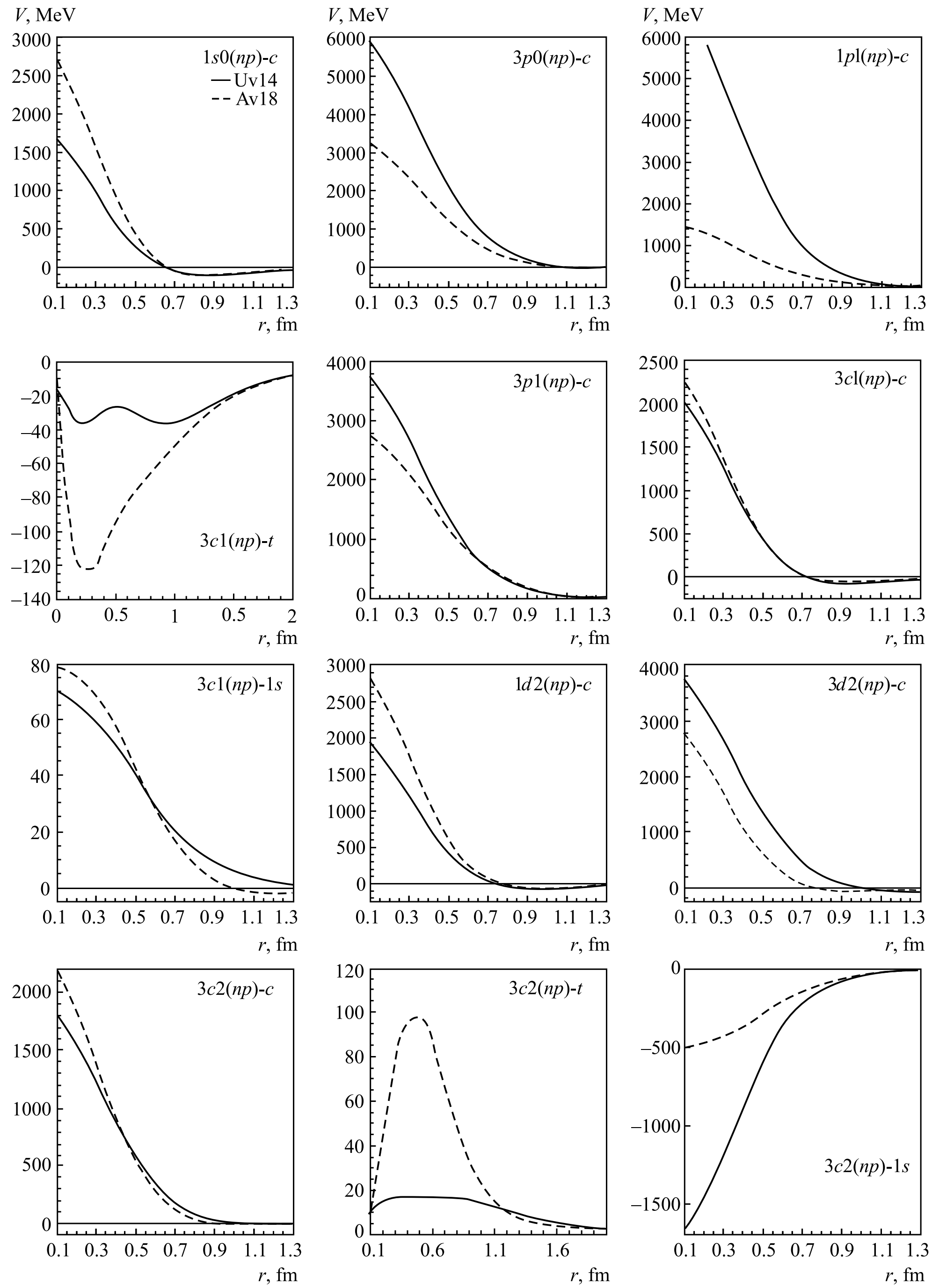

Fig. 5. The comparison of the central, tensor and spin-orbit potentials of Urb81 (Uv14) and Arg94 (Av18), in the states from $J = 0$ up to $J = 2$, for the np system

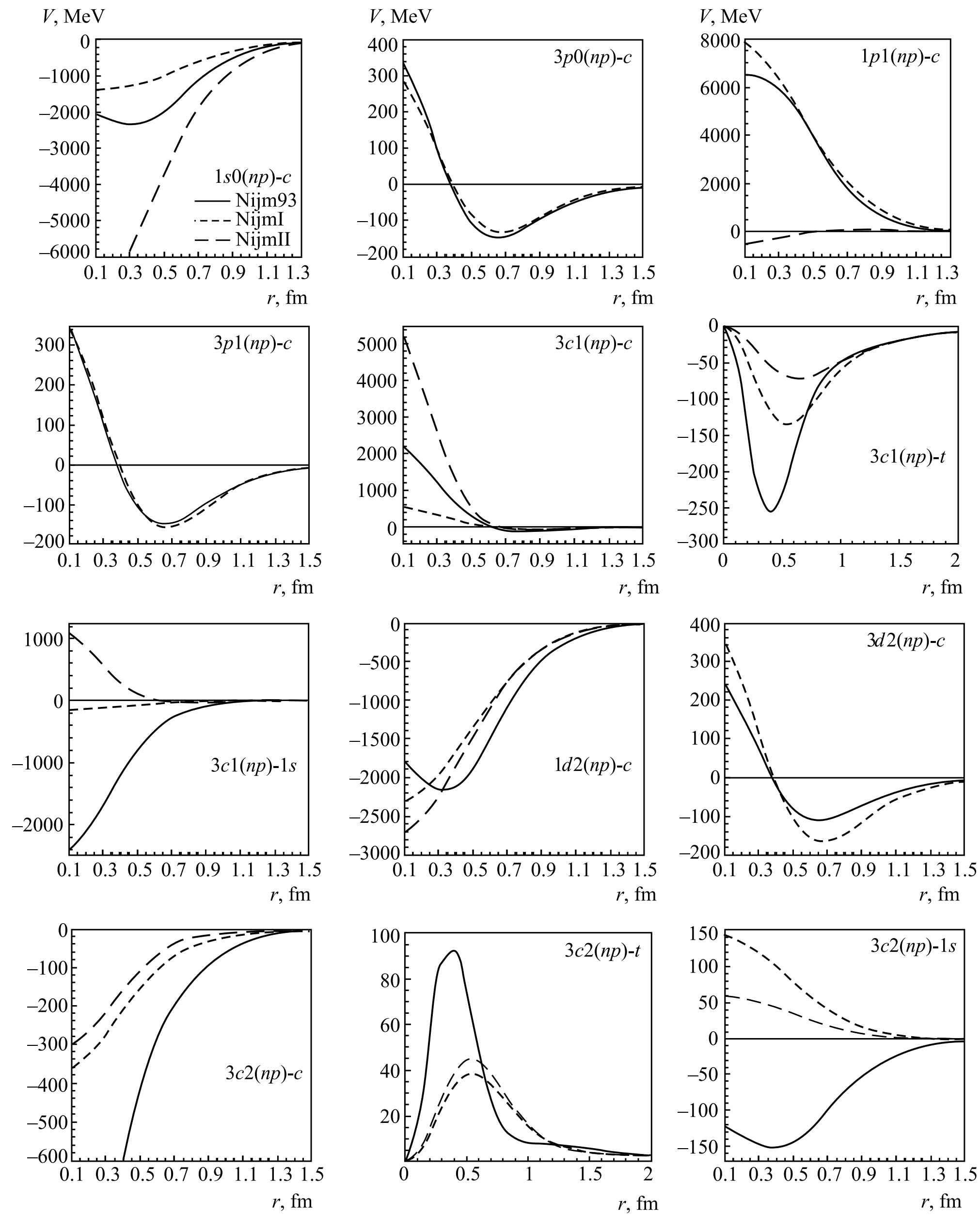

Fig. 6. The comparison of the central, tensor and spin-orbit potentials of Nijm93, NijmI, and NijmII reduced into the Reid potential, in the states from $J = 0$ up to $J = 2$, for the np system

system, is pictured. The plots demonstrate an explicit dependence on L or, in other words, the spatial exchanges in the potentials, and so on.

In summary, the likenesses and differences in the figures are related to the structural and theoretical bases of the potentials as well as the external conditions such as fitting to the

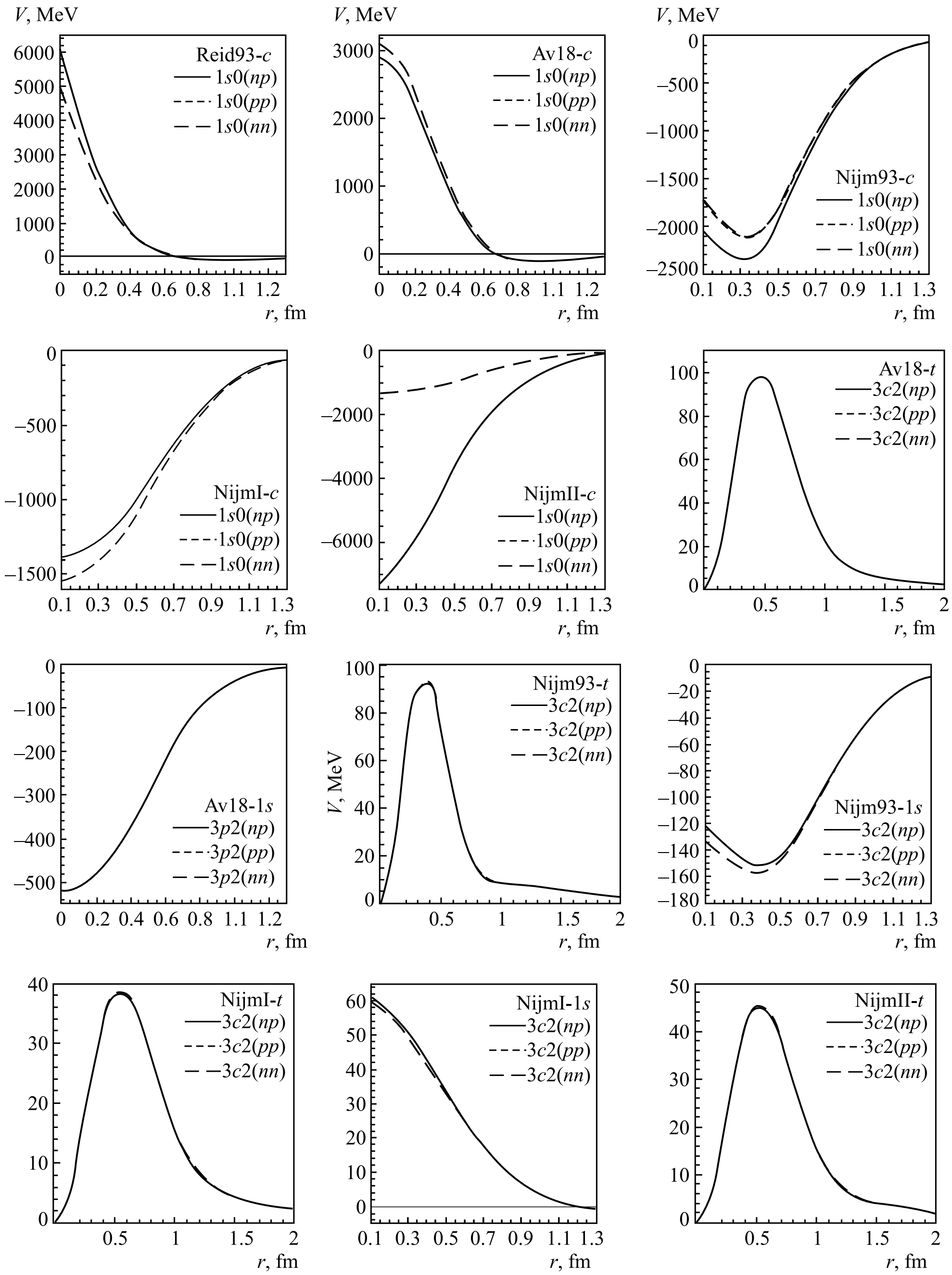

Fig. 7. The charge-dependence of the CD potentials reduced to the Reid potential, in the ${}^1S_0$ (central) and ${}^3P_2$–${}^3F_2(3C2)$ (tensor and spin-orbit) states

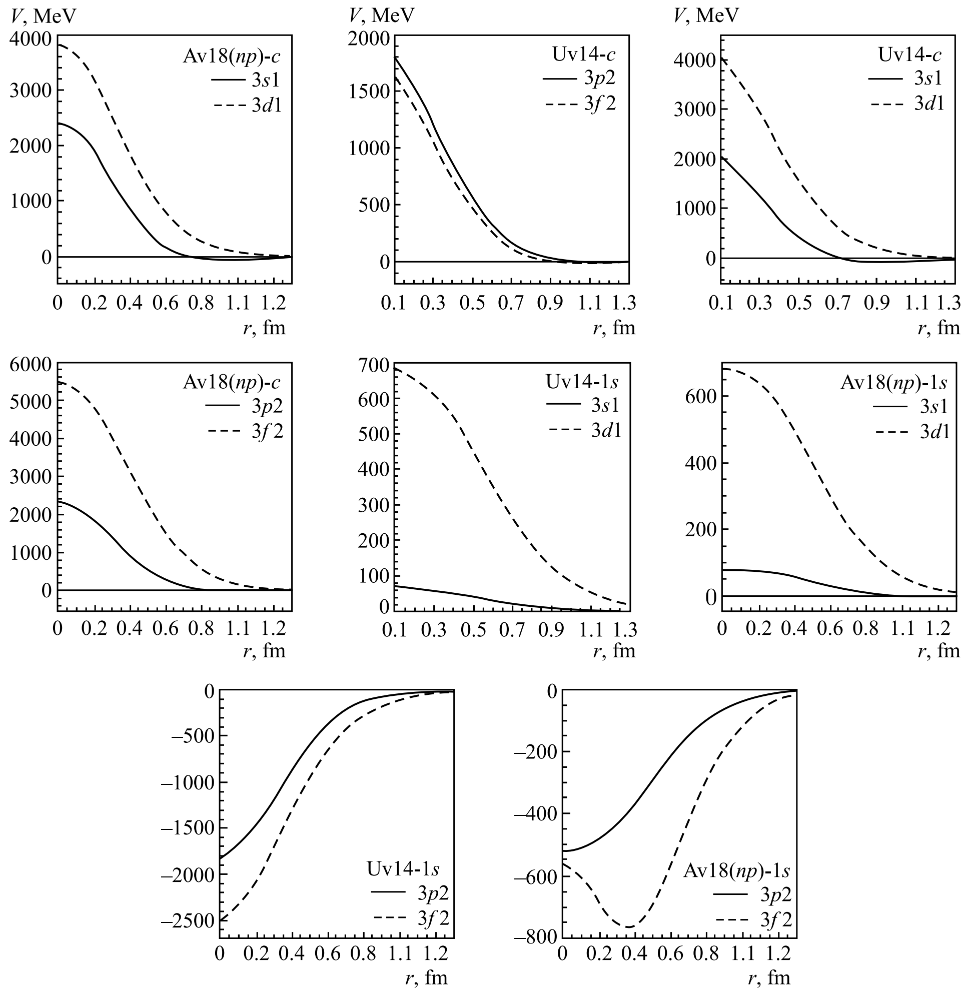

Fig. 8. The comparison of $^3S_1(\ell = 0)$, $^3D_1(\ell = 2)$ and also $^3P_2(\ell = 1)$, $^3F_2(\ell = 3)$ central and spin-orbit potentials of the Urb81 (Uv14) and Arg94 (Av18) potentials reduced into the Reid potential, for the np system

special scattering database. The almost different figures, although slightly show that at least some basic physical assumptions of these models and potentials, which give almost similar results, should be wrong. That is because the similar forms reproducing the similar results for the nuclear force are somewhat ambiguous.

4. CONCLUDING COMMENTS

In the recent decades, many NN potentials have been presented. Most potentials use OPEs for the LR part, while the TPEs and special OBEs are used in the MR part. For the SR part, the heavy-meson exchanges, the QCD effects and the phenomenological procedures are often used. The potential's precision and quality are explored through various methods. The most important method for the Potential-Quality Measurement (PQM) is giving satisfactory results in nuclear-structure calculations. Finding out $\chi^2/N_{\rm data}$ is another usual method that, as already discussed, has its own problems and difficulties. Based on these standards, several high-precision charge-dependent NN potentials are built as we have mentioned some of them.

A main conclusion that one can deduce from the comparison done here is that a definite and fixed form for the NN interaction is still a critical challenge. In fact, as we have different models and forms for the strong nuclear force, which almost all give similar results while having different structures, the nuclear force will obviously become meaningless. A certain statement is that although some quantitative correspondences are present among the potentials, there are some other quantitative differences. Generally speaking, one can assign the quantitative differences to the theoretical and structural differences of the potentials. Various interaction ingredients such as the meson and/or quark and gluon exchanges, various phenomenological parts, and mainly the base employed models, result in partially different results. For example, using different Yukawa and Woods–Saxon functions, the form factors to regularize the potentials at the origin and in general the functions used in the various parts of the potentials, make many differences explainable. Meanwhile, the likening features are fitting the scattering data and deuteron properties that in turn make the closeness more reasonable. Although the difficulties are important in their place, however, they are not so big to stop applying some potentials in the nuclear-structure calculations. The people, who use the potentials so, by noting the comparison sketched here, may find more satisfactory reasons to the present discrepancies in the potential shapes and forms.

In summary, the differences could be arisen from the involved approximations and the failures of our knowledge on the nuclear forces. Therefore, it seems that the models and potentials in which many guesses (such as selecting the special potentials, merely fitting to the data, and so on) are used, are only a temporary way for solving the NN interaction problem. Efforts for discovering a more fundamental theory and a probable definite and fixed form for the potential, as an important question in nuclear physics, are in progress yet.

By the way, although nowadays the chiral effective field-theory potentials describe almost well the two- and few-nucleon systems both quantitatively and qualitatively and stand as the best so far candidates to describe this long-standing issue in nuclear physics, there are some unsolved problems even in these conventional frameworks. The perturbative character and proper renormalization of the chiral potentials as well as the three- and few-nucleon forces wait to be addressed carefully. Fortunately, nowadays and for future, holographic QCD as born from the string/gauge, AdS/CFT, correspondence, seems to be a new promising viewpoint to the nuclear physics problems and especially the nuclear force issue [66, 67].

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