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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

REDUCTION OF COUPLINGS IN A FINITE GUT AND THE MSSM

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We apply the method of reduction of couplings in a Finite Unified Theory and in the MSSM. The method consists in searching for renormalization group invariant relations among couplings of a renormalizable theory holding to all orders in perturbation theory. In both cases, we predict the masses of the top and bottom quarks and the light Higgs in remarkable agreement with the experiment. Moreover, we predict the masses of the other Higgses too, as well as the supersymmetric spectrum, the latter being in very comfortable agreement with the LHC bounds on supersymmetric particles.

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INTRODUCTION

The main goal expected from a unified description of interactions is to understand the large number of free parameters of the Standard Model (SM) in terms of a few fundamental ones. In other words, to achieve *reduction of couplings* at a more fundamental level. To reduce the number of free parameters of a theory, and thus render it more predictive, one is usually led to introduce more symmetry. Supersymmetric Grand Unified Theories (GUTs) are very good examples of such a procedure [1].

A complementary strategy in searching for a more fundamental theory, consists in looking for all-loop renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale [2–5]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all orders in perturbation theory [6]. Through the method of reduction of couplings it is possible to relate the gauge and Yukawa sectors of a theory, that is to achieve Gauge-Yukawa Unification (GYU) [3].

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1. THE METHOD OF REDUCTION OF COUPLINGS

In this section we will briefly outline the reduction of couplings method. Any RGI relation among couplings (i.e., which does not depend on the renormalization scale μ explicitly) can be expressed in the implicit form $\Phi(g_1, \ldots, g_A) = \text{const.}$, which has to satisfy the partial differential equation (PDE)

$$\frac{d\Phi}{dt} = \sum_{a=1}^{A} \frac{\partial \Phi}{\partial g_a} \frac{dg_a}{dt} = \sum_{a=1}^{A} \frac{\partial \Phi}{\partial g_a} \beta_a = \boldsymbol{\nabla}\Phi \cdot \boldsymbol{\beta} = 0, \tag{1}$$

where $t = \ln \mu$ and β_a is the β-function of g_a. This PDE is equivalent to a set of ordinary differential equations, the so-called reduction equations (REs) [5],

$$\beta_g \frac{dg_a}{dg} = \beta_a, \quad a = 1, \ldots, A, \tag{2}$$

where g and β_g are the primary coupling and its β-function, and the counting on a does not include g. Since maximally $(A-1)$-independent RGI "constraints" in the A-dimensional space of couplings can be imposed by the Φ_a's, one could, in principle, express all the couplings in terms of a single coupling g. The strongest requirement is to demand power series solutions to the REs,

$$g_a = \sum_{n=0} \rho_a^{(n)} g^{2n+1}, \tag{3}$$

which formally preserve perturbative renormalizability. Remarkably, the uniqueness of such power series solutions can be decided already at the one-loop level [5].

Searching for a power series solution of the form (3) to the REs (2) is justified since various couplings in supersymmetric theories have the same asymptotic behaviour, thus one can rely that keeping only the first terms in the expansion is a good approximation in realistic applications.

2. REDUCTION OF COUPLINGS IN SOFT BREAKING TERMS

The method of reducing the dimensionless couplings was extended [4, 7] to the soft supersymmetry breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. In addition, it was found [8, 9] that RGI SSB scalar masses in gauge-Yukawa unified models satisfy a universal sum rule.

Consider the superpotential given by

$$W = \frac{1}{2}\mu^{ij}\Phi_i\Phi_j + \frac{1}{6}C^{ijk}\Phi_i\Phi_j\Phi_k, \tag{4}$$

where μ^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge-invariant tensors, and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G. The Lagrangian for SSB terms is

$$-\mathcal{L}_{\text{SSB}} = \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}(m^2)^j_i\phi^{*i}\phi_j + \frac{1}{2}M\lambda\lambda + \text{h.c.}, \tag{5}$$

where ϕ_i are the scalar parts of the chiral superfields Φ_i; λ are the gauginos and M is their unified mass; h^{ijk} and b^{ij} are the trilinear and bilinear dimensionful couplings, respectively, and $(m^2)^j_i$ are the soft scalars masses.

Let us recall that the one-loop β-function of the gauge coupling g is given by [10]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i T(R_i) - 3C_2(G)\right], \tag{6}$$

where $C_2(G)$ is the quadratic Casimir of the adjoint representation of the associated gauge group G. $T(R)$ is given by the relation Tr $[T^aT^b] = T(R)\delta^{ab}$, where T^a is the generator of the group in the appropriate representation. Similarly, the β-functions of C_{ijk}, by virtue of the nonrenormalization theorem, are related to the anomalous dimension matrix γ^i_j of the chiral superfields as

$$\beta_C^{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma^l_k + C_{ikl}\gamma^l_j + C_{jkl}\gamma^l_i. \tag{7}$$

At the one-loop level, the anomalous dimension, $\gamma^{(1)}{}^i_j$, of the chiral superfield is [10]

$$\gamma^{(1)}{}^i_j = \frac{1}{32\pi^2}[C^{ikl}C_{jkl} - 2g^2C_2(R_i)\delta_{ij}], \tag{8}$$

where $C_2(R_i)$ is the quadratic Casimir of the representation R_i, and $C^{ijk} = C^*_{ijk}$. Then, the $N = 1$ nonrenormalization theorem [11, 12] ensures that there are no extra mass and cubic-interaction-term renormalizations, implying that the β-functions of C_{ijk} can be expressed as linear combinations of the anomalous dimensions γ^i_j.

Here we assume that the reduction equations admit power series solutions of the form

$$C^{ijk} = g\sum_{n=0}\rho^{ijk}_{(n)}g^{2n}. \tag{9}$$

In order to obtain higher-loop results instead of knowledge of explicit β-functions, which anyway are known only up to two loops, relations among β-functions are required.

The progress made using the spurion technique [12–14], leads to all-loop relations among SSB β-functions [15–17]. The assumption, following [16], that the relation among couplings

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g} \tag{10}$$

is RGI and, furthermore, the use of the all-loop gauge β-function of Novikov et al. [18]

$$\beta_g^{\mathrm{NSVZ}} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C_2(G)}{1 - g^2C_2(G)/8\pi^2}\right] \tag{11}$$

lead to the all-loop RGI sum rule [19] (assuming $(m^2)^i_j = m^2_j\delta^i_j$),

$$\begin{aligned} m_i^2 + m_j^2 + m_k^2 = |M|^2\left\{\frac{1}{1-g^2C_2(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2}\right\} + \\ + \sum_l \frac{m_l^2T(R_l)}{C_2(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g}. \end{aligned} \tag{12}$$

The all-loop results on the SSB β-functions lead to all-loop RGI relations (see, e.g., [20]). If we assume:

(a) the existence of a RGI surface on which $C = C(g)$, or, equivalently, that

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} \tag{13}$$

holds, i.e., reduction of couplings is possible, and

(b) the existence of a RGI surface on which

$$h^{ijk} = -M\frac{dC(g)^{ijk}}{d\ln g} \tag{14}$$

holds, too, in all orders, then one can prove [21, 22] that the following relations are RGI to all-loops (note that in both (a) and (b) assumptions above we do not rely on specific solutions of these equations)

$$M = M_0\frac{\beta_g}{g}, \tag{15}$$

$$h^{ijk} = -M_0\beta_C^{ijk}, \tag{16}$$

$$b^{ij} = -M_0\beta_\mu^{ij}, \tag{17}$$

$$(m^2)^i_j = \frac{1}{2}|M_0|^2\mu\frac{d\gamma^i_j}{d\mu}, \tag{18}$$

where M_0 is an arbitrary reference mass scale to be specified shortly.

Finally, we would like to emphasize that under the same assumptions (a) and (b), the sum rule given in Eq. (12) has been proven [19] to be all-loop RGI, which gives us a generalization of Eq. (18) to be applied in considerations of nonuniversal soft scalar masses, which are necessary in many cases including the MSSM.

As was emphasized in [21], the set of the all-loop RGI relations (15)–(18) is the one obtained in the *Anomaly Mediated SB Scenario* [23] by fixing M_0 to be $m_{3/2}$, which is the natural scale in the supergravity framework. A final remark concerns the resolution of the fatal problem of the anomaly-induced scenario in the supergravity framework, which is here solved thanks to the sum rule (12). Other solutions have been provided by introducing Fayet–Iliopoulos terms [24].

3. APPLICATIONS OF THE REDUCTION OF COUPLINGS METHOD

In this section, we show how to apply the reduction of couplings method in two scenarios, the MSSM and Finite Unified Theories (FUT). In both cases, the reduction of couplings is assumed to take place at the unification scale, and we will apply it only to the third generation of fermions and in the soft supersymmetry breaking terms. After the reduction of couplings takes place, we are left with relations at the unification scale for the Yukawa couplings of the quarks in terms of the gauge coupling according to (9), for the trilinear terms in terms of the Yukawa couplings and the unified gaugino mass (14), and a sum rule for the soft scalar masses also proportional to the unified gaugino mass (12), as applied in each model.

3.1. RE in the MSSM. We will examine here the reduction of couplings method applied to the MSSM, which is defined by the superpotential,

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2, \tag{19}$$

with soft breaking terms,

$$\begin{aligned} -\mathcal{L}_{\rm SSB} = \sum_\phi m_\phi^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^3 \frac{1}{2} M_i \lambda_i \lambda_i + \text{h.c.} \right] + \\ + [h_t H_2 Q t^c + h_b H_1 Q b^c + h_\tau H_1 L \tau^c + \text{h.c.}], \end{aligned} \tag{20}$$

where the last line refers to the scalar components of the corresponding superfield. In general, $Y_{t,b,\tau}$ and $h_{t,b,\tau}$ are 3×3 matrices, but we work throughout in the approximation that the matrices are diagonal, and neglect the couplings of the first two generations.

Assuming perturbative expansion of all three Yukawa couplings in favour of g_3 satisfying the reduction equations, we find that the coefficients of the Y_τ coupling turn imaginary. Therefore, we take Y_τ at the GUT scale to be an independent variable. Thus, in the application of the reduction of couplings in the MSSM that we examine here, in the first stage we neglect the Yukawa couplings of the first two generations, while we keep Y_τ and the gauge couplings g_2 and g_1, which cannot be reduced consistently, as corrections. This "reduced" system holds at all scales, and thus serves as boundary conditions of the RGEs of the MSSM at the unification scale, where we assume that the gauge couplings meet [20].

In that case, the coefficients of the expansions (again at the GUT scale)

$$\frac{Y_t^2}{4\pi} = c_1 \frac{g_3^2}{4\pi} + c_2 \left(\frac{g_3^2}{4\pi}\right)^2, \quad \frac{Y_b^2}{4\pi} = p_1 \frac{g_3^2}{4\pi} + p_2 \left(\frac{g_3^2}{4\pi}\right)^2 \tag{21}$$

are given by

$$\begin{aligned} c_1 &= \frac{157}{175} + \frac{1}{35} K_\tau = 0.897 + 0.029 K_\tau, \\ p_1 &= \frac{143}{175} - \frac{6}{35} K_\tau = 0.817 - 0.171 K_\tau, \\ c_2 &= \frac{1}{4\pi} \frac{1457.55 - 84.491 K_\tau - 9.66181 K_\tau^2 - 0.174927 K_\tau^3}{818.943 - 89.2143 K_\tau - 2.14286 K_\tau^2}, \\ p_2 &= \frac{1}{4\pi} \frac{1402.52 - 223.777 K_\tau - 13.9475 K_\tau^2 - 0.174927 K_\tau^3}{818.943 - 89.2143 K_\tau - 2.14286 K_\tau^2}, \end{aligned} \tag{22}$$

where

$$K_\tau = Y_\tau^2 / g_3^2. \tag{23}$$

The couplings Y_t, Y_b, and g_3 are not only reduced, but they provide predictions consistent with the observed experimental values. According to the analysis presented in Sec. 2, the RGI relations in the SSB sector hold, assuming the existence of RGI surfaces, where Eqs. (13) and (14) are valid.

Since all gauge couplings in the MSSM meet at the unification point, we are led to the following boundary conditions at the GUT scale:

$$Y_t^2 = c_1 g_U^2 + c_2 g_U^4/(4\pi) \quad \text{and} \quad Y_b^2 = p_1 g_U^2 + p_2 g_U^4/(4\pi), \tag{24}$$

$$h_{t,b} = -M_U Y_{t,b}, \tag{25}$$

$$m_3^2 = -M_U \mu, \tag{26}$$

where M_U is the unification scale; $c_{1,2}$ and $p_{1,2}$ are the solutions of the algebraic system of the two reduction equations taken at the GUT scale (while keeping only the first term[1] of the perturbative expansion of the Yukawas in favour of g_3 for Eqs. (25) and (26)), and to a set of equations resulting from the application of the sum rule

$$m_{H_2}^2 + m_Q^2 + m_{t^c}^2 = M_U^2, \quad m_{H_1}^2 + m_Q^2 + m_{b^c}^2 = M_U^2, \tag{27}$$

noting that the sum rule introduces four free parameters.

3.2. Predictions of the Reduced MSSM. With these boundary conditions, we run the MSSM RGEs down to the SUSY scale, which we take to be the geometrical average of the stop masses, and then run the SM RGEs down to the electroweak scale (M_Z), where we compare them with the experimental values of the third generation quark masses. The RGEs are taken at two loops for the gauge and Yukawa couplings and at one loop for the soft breaking parameters. We let M_U and $|\mu|$ at the unification scale to vary between $\sim 1-11$ TeV, for the two possible signs of μ. In evaluating the τ and bottom masses, we have taken into account the one-loop radiative corrections that come from the SUSY breaking [25]; in particular, for large $\tan\beta$ they can give sizeable contributions to the bottom quark mass.

Recall that Y_τ is not reduced and is a free parameter in this analysis. The parameter K_τ, related to Y_τ through Eq. (23), is further constrained by allowing only the values that are also compatible with the top and bottom quark masses simultaneously within 1 and 2σ of their central experimental value. In the case that $\mathrm{sign}\,(\mu) = +$, there is no value for K_τ where both the top and the bottom quark masses agree simultaneously with their experimental value, therefore we only consider the negative sign of μ from now on. We use the experimental value of the top quark pole mass as

$$M_t^{\mathrm{exp}} = (173.2 \pm 0.9)\ \text{GeV}. \tag{28}$$

The bottom mass is calculated at M_Z to avoid uncertainties that come from running down to the pole mass, and, as was previously mentioned, the SUSY radiative corrections both to the tau and to the bottom quark masses have been taken into account [26]

$$M_b(M_Z) = (2.83 \pm 0.10)\ \text{GeV}. \tag{29}$$

The variation of K_τ is in the range $\sim 0.37-0.49$ in order to agree with the experimental values of the bottom and top masses at 1σ, and in $\sim 0.34-0.49$ if the agreement is at the 2σ level.

Finally, assuming the validity of Eq. (14) for the corresponding couplings to those that have been reduced before, we calculate the Higgs mass as well as the whole Higgs and

[1] The second term can be determined once the first term is known.

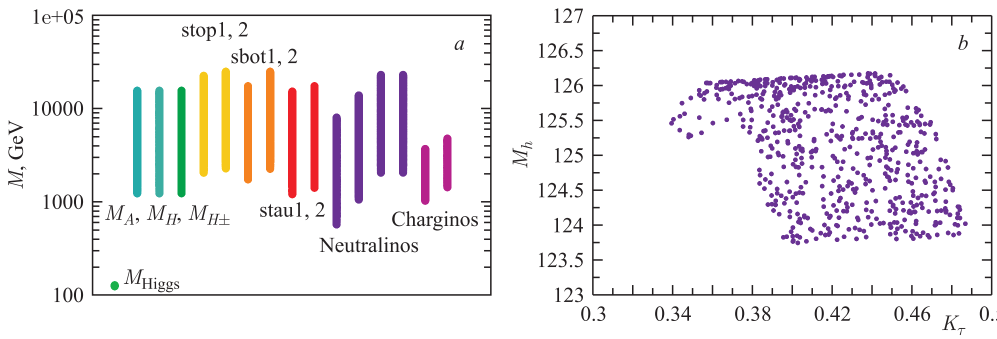

Fig. 1. *a*) The SUSY spectrum as a function of the reduced MSSM. From left to right are shown: the lightest Higgs, the pseudoscalar one M_A, the heavy neutral one M_H, two charged Higgses $M_{H\pm}$; then come two stops, two sbottoms and two staus, four neutralinos, and at the end two charginos. *b*) The lightest Higgs boson mass as a function of $K_\tau = Y_\tau^2/g_3^2$

sparticle spectrum using Eqs. (24)–(27), and we present them in Fig. 1. The Higgs mass was calculated using a "mixed-scale" one-loop RG approach, which is known to include the leading two-loop corrections as evaluated by the full diagrammatic calculation [27]. However, more refined Higgs mass calculations, and, in particular, the results of [28], are not (yet) included.

In Fig. 1, *a*, we show the full mass spectrum of the model. We find that the masses of the heavier Higgses have relatively high values, above the TeV scale. In addition, we find a generally heavy supersymmetric spectrum, starting with a neutralino as LSP at ~ 500 GeV, and comfortable agreement with the LHC bounds due to the nonobservation of coloured supersymmetric particles [29]. Finally, note that although the $\mu < 0$ found in our analysis would disfavour the model in connection with the anomalous magnetic moment of the muon, such a heavy spectrum gives only a negligible correction to the SM prediction. We plan to extend our analysis by examining the restrictions that will be imposed in the spectrum by the B-physics and CDM constraints.

In Fig. 1, *b*, we show the results for the light Higgs boson mass as a function of K_τ. The results are in the range $123.7-126.3$ GeV, where the uncertainty is due to the variation of K_τ, the gaugino mass M_U and the variation of the scalar soft masses, which are, however, constrained by the sum rules (27). The gaugino mass M_U is in the range $\sim 1.3-11$ TeV, the lower values having been discarded since they do not allow for radiative electroweak symmetry breaking. To the lightest Higgs mass value, one has to add at least ± 2 GeV coming from unknown higher-order corrections [30]. Therefore it is in excellent agreement with the experimental results of ATLAS and CMS [31].

3.3. Finiteness. Finiteness can be understood by considering a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. Consider the superpotential Eq. (4) together with the soft supersymmetry breaking Lagrangian Eq. (5). All the one-loop β-functions of the theory vanish if the β-function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions of the Yukawa couplings $\gamma_i^{j(1)}$, vanish, i.e.,

$$\sum_i \ell(R_i) = 3C_2(G), \quad \frac{1}{2}C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R_i), \tag{30}$$

where $\ell(R_i)$ is the Dynkin index of R_i, and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of G. These conditions are also enough to guarantee two-loop finiteness [32]. A striking fact is the existence of a theorem [6], that guarantees the vanishing of the β-functions to all orders in perturbation theory. This requires that, in addition to the one-loop finiteness conditions (30), the Yukawa couplings are reduced in favour of the gauge coupling to all orders (see [33] for details). Alternatively, similar results can be obtained [34] using an analysis of the all-loop NSVZ gauge β-function [18, 35].

Since we would like to consider only finite theories here, we assume that the gauge group is a simple group and the one-loop β-function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the Eq. (9) form. According to the finiteness theorem of [6, 36], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved through the relation [37]

$$h^{ijk} = -MC^{ijk} + \ldots = -M\rho^{ijk}_{(0)} g + O(g^5), \tag{31}$$

where ... stand for higher-order terms.

In addition, it was found that the RGI SSB scalar masses in gauge-Yukawa unified models satisfy a universal sum rule at one loop [8]. This result was generalized to two loops for finite theories [9], and then to all loops for general gauge-Yukawa and finite unified theories [19]. From these latter results, the following soft scalar-mass sum rule is found [9]:

$$\frac{(m_i^2 + m_j^2 + m_k^2)}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4) \tag{32}$$

for i, j, k with $\rho^{ijk}_{(0)} \neq 0$, where $m^2_{i,j,k}$ are the scalar masses and $\Delta^{(2)}$ is the two-loop correction which vanishes for the universal choice, i.e., when all the soft scalar masses are the same at the unification point, as well as in the model considered here.

3.4. $SU(5)$ **Finite Unified Theories.** We examine an all-loop Finite Unified Theory (FUT) with $SU(5)$ as gauge group, where the reduction of couplings has been applied to the third generation of quarks and leptons. The particle content of the model we will study, which we denote **FUT**, consists of the following supermultiplets: three $(\overline{\mathbf{5}} + \mathbf{10})$, needed for each of the three generations of quarks and leptons, four $(\overline{\mathbf{5}} + \mathbf{5})$ and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken, the theory is no longer finite, and we will assume that we are left with the MSSM [2, 3].

A predictive gauge-Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)\,j} \propto \delta^j_i$.

2. Three fermion generations, in the irreducible representations $\overline{\mathbf{5}}_i, \mathbf{10}_i$ $(i = 1, 2, 3)$, which obviously should not couple to the adjoint $\mathbf{24}$.

3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and antiquintet, which couple to the third generation.

After the reduction of couplings the symmetry is enhanced, leading to the following superpotential [38]:

$$W = \sum_{i=1}^{3} \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \overline{\mathbf{5}}_i \overline{H}_i \right] + g_{23}^u \mathbf{10}_2 \mathbf{10}_3 H_4 +$$
$$+ g_{23}^d \mathbf{10}_2 \overline{\mathbf{5}}_3 \overline{H}_4 + g_{32}^d \mathbf{10}_3 \overline{\mathbf{5}}_2 \overline{H}_4 + g_2^f H_2 \mathbf{24} \overline{H}_2 + g_3^f H_3 \mathbf{24} \overline{H}_3 + \frac{g^\lambda}{3} (\mathbf{24})^3. \quad (33)$$

The nondegenerate and isolated solutions to $\gamma_i^{(1)} = 0$ give us

$$\begin{gathered} (g_1^u)^2 = \frac{8}{5} g^2, \quad (g_1^d)^2 = \frac{6}{5} g^2, \quad (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5} g^2, \\ (g_2^d)^2 = (g_3^d)^2 = \frac{3}{5} g^2, \quad (g_{23}^u)^2 = \frac{4}{5} g^2, \quad (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5} g^2, \\ (g^\lambda)^2 = \frac{15}{7} g^2, \quad (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2} g^2, \quad (g_1^f)^2 = 0, \quad (g_4^f)^2 = 0, \end{gathered} \quad (34)$$

and from the sum rule we obtain

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2, \quad m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}, \quad m_{\mathbf{\overline{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}, \quad (35)$$

i.e., in this case we have only two free parameters $m_{\mathbf{10}}$ and M for the dimensionful sector.

As already mentioned, after the $SU(5)$ gauge-symmetry breaking we assume, we have the MSSM, i.e., only two Higgs doublets. This can be achieved by introducing appropriate mass terms that allow one to perform a rotation of the Higgs sector [2, 39], in such a way that only one pair of Higgs doublets, coupled mostly to the third family, remains light and acquires vacuum expectation values. To avoid fast proton decay, the usual fine tuning to achieve doublet-triplet splitting is performed, although the mechanism is not identical to minimal $SU(5)$, since we have an extended Higgs sector.

Thus, after the gauge symmetry of the GUT theory is broken, we are left with the MSSM, with the boundary conditions for the third family given by the finiteness conditions, while the other two families are not restricted.

3.5. Predictions of the Finite Model. Since the gauge symmetry is spontaneously broken below $M_{\rm GUT}$, the finiteness conditions do not restrict the renormalization properties at low energies, and all that remains are boundary conditions on the gauge and Yukawa couplings (34), the $h = -MC$ (31) relation, and the soft scalar-mass sum rule at $M_{\rm GUT}$. The analysis follows along the same lines as in the MSSM case.

In Fig. 2, we show the **FUT** predictions for m_t and $m_b(M_Z)$ as a function of the unified gaugino mass M, for the two cases $\mu < 0$ and $\mu > 0$. The bounds on the $m_b(M_Z)$ and the m_t mass clearly single out $\mu < 0$, as the solution most compatible with these experimental constraints.

We now analyze the impact of further low-energy observables on the model **FUT** with $\mu < 0$. As additional constraints, we consider the rare b decays ${\rm BR}(b \to s\gamma)$ and ${\rm BR}(B_s \to \mu^+\mu^-)$.

For the branching ratio ${\rm BR}(b \to s\gamma)$, we take the value given by the Heavy Flavour Averaging Group (HFAG) to be [40]

$${\rm BR}(b \to s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \cdot 10^{-4}. \quad (36)$$

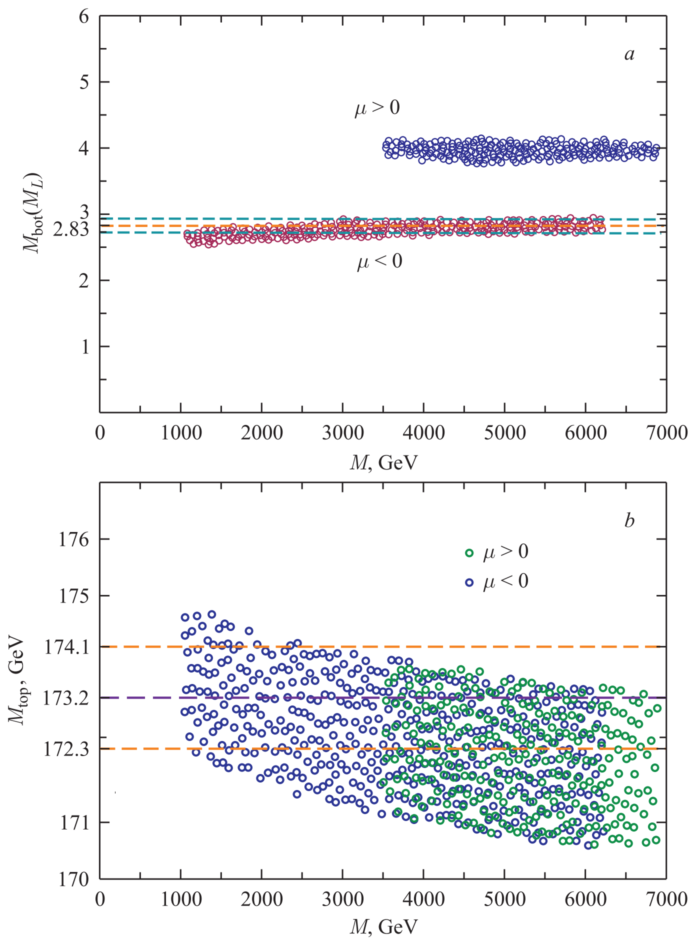

Fig. 2. The bottom quark mass at the Z boson scale (*a*) and top quark pole mass (*b*) are shown as a function of M, the unified gaugino mass

For the branching ratio $\mathrm{BR}(B_s \to \mu^+\mu^-)$, the SM prediction is at the level of 10^{-9}, while the present experimental upper limit is

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = 4.5 \cdot 10^{-9} \tag{37}$$

at the 95% C.L. [41]. This is in relatively good agreement with the recent direct measurement of this quantity by CMS and LHCb [42]. As we do not expect a sizable impact of the new measurement on our results, we stick for our analysis to the simple upper limit.

For the lightest Higgs mass prediction we used the code FeynHiggs [30,43]. The prediction for M_h of **FUT** with $\mu < 0$ is shown in Fig. 3, where the constraints from the two B-physics observables are taken into account. The lightest Higgs mass ranges in

$$M_h \sim 121\text{–}126 \text{ GeV}, \tag{38}$$

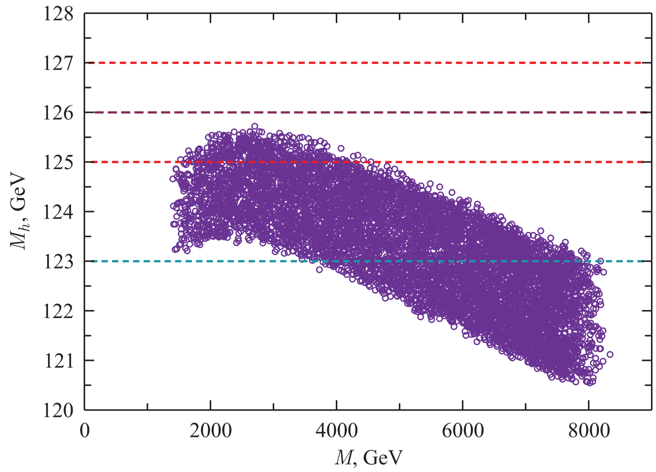

Fig. 3 (color online). The lightest Higgs mass, M_h, as a function of M for the model **FUT** with $\mu < 0$

where the uncertainty comes from variations of the soft scalar masses. To this value one has to add at least ± 2 GeV coming from unknown higher-order corrections [30][1]. We have also included a small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions. The masses of the heavier Higgs bosons are found at higher values in comparison with our previous analyses [28,45]. This is due to the more stringent bound on $\mathrm{BR}(B_s \to \mu^+\mu^-)$, which pushes the heavy Higgs masses beyond ~ 1 TeV, excluding their discovery at the LHC.

We impose now a further constraint on our results, which is the value of the Higgs mass

$$M_h \sim (126.0 \pm 1 \pm 2) \text{ GeV}, \tag{39}$$

where ± 1 comes from the experimental error and ± 2 corresponds to the theoretical error, and see how this affects the SUSY spectrum[2]. We find that constraining the allowed values of the Higgs mass puts a limit on the allowed values of the unified gaugino mass, as can be seen from Fig. 3. The red lines correspond to the pure experimental uncertainty and restrict $2 \lesssim M \lesssim 5$ TeV. The blue line includes the additional theory uncertainty of ± 2 GeV. Taking this uncertainty into account no bound on M can be placed.

The full particle spectrum of model **FUT** with $\mu < 0$, compliant with quark mass constraints and the B-physics observables is shown in Fig. 4. It can be seen from the figure that the lightest observable SUSY particle (LOSP) is the light scalar tau. In plots *a* and *b*, we impose $M_h = (126 \pm 3(1))$ GeV. Without any M_h restrictions, the coloured SUSY particles have masses above ~ 1.8 TeV in agreement with the nonobservation of those particles at the LHC [29]. Including the Higgs mass constraints, in general, favours the lower part of

[1]We have not yet taken into account the improved M_h prediction presented in [44] (and implemented into the most recent version of FeynHiggs), which will lead to an upward shift in the Higgs boson mass prediction.

[2]In this analysis, the new M_h evaluation [44] may have a relevant impact on the restrictions on the allowed SUSY parameter space shown below.

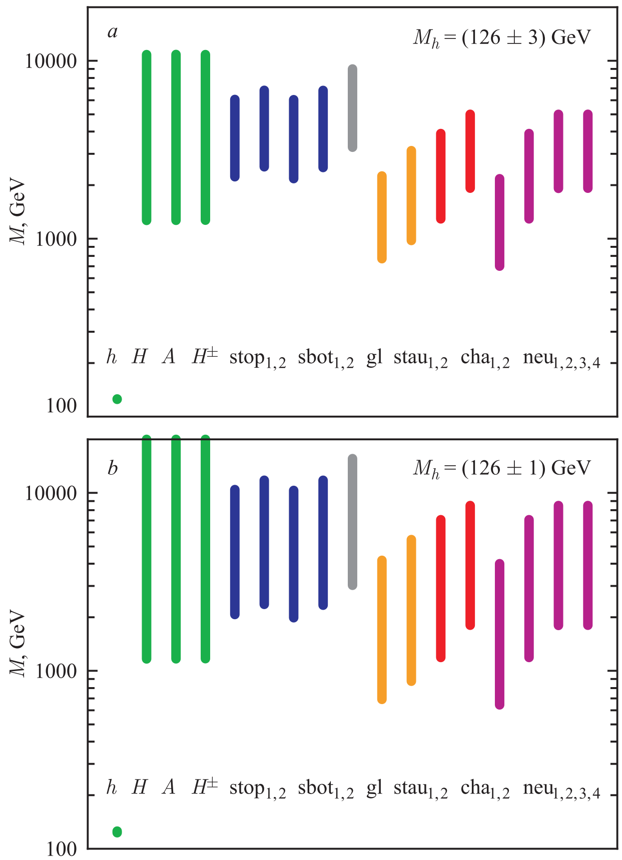

Fig. 4 (color online). The plots *a* and *b* show the spectrum after imposing the constraint $M_h = (126 \pm 3(1))$ GeV. The light (green) points are the various Higgs boson masses, the dark (blue) points following are the two scalar top and bottom masses, the gray ones are the gluino masses, then come the scalar tau masses in orange (light gray), the darker (red) points to the right are the two chargino masses followed by the lighter shaded (pink) points indicating the neutralino masses

the SUSY particle mass spectra, but also cuts away the very low values. Neglecting the theory uncertainties of M_h (as shown in Fig. 4, *b*) permits SUSY masses which would remain unobservable at the LHC, the ILC or CLIC. On the other hand, large parts of the allowed spectrum of the lighter scalar tau or the lighter neutralinos might be accessible at CLIC with $\sqrt{s} = 3$ TeV.

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