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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ЭКСПЕРИМЕНТ

FRACTAL STRUCTURE OF HADRONS IN PROCESSES WITH POLARIZED PROTONS AT SPD NICA

(Proposal for Experiment)

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The concept of z-scaling previously developed for analysis of inclusive reactions in proton–proton collisions is applied for description of processes with polarized protons at the planned Spin Physics Detector (SPD) NICA in Dubna. A hypothesis of self-similarity and fractality of the proton spin structure is discussed. The possibilities to extract information on spin-dependent fractal dimensions of hadrons and fragmentation process from asymmetries and coefficients of polarization transfer are justified. The double longitudinal spin asymmetry A_{LL} of π^0-meson production and the coefficient of the polarization transfer D_{LL} of Λ-hyperon production in proton–proton collisions measured at RHIC are analyzed in the framework of z-scaling. The spin-dependent fractal dimensions of proton and fragmentation process with polarized Λ hyperon are estimated. A study of the spin-dependent constituent energy loss as a function of transverse momentum of the inclusive hadron and collision energy is suggested.

Концепция z-скейлинга, развитая ранее для анализа инклюзивных процессов в протон-протонных столкновениях, применяется для описания процессов с поляризованными протонами, эксперименты с которыми планируются на будущей установке SPD NICA. Обсуждается гипотеза самоподобия и фрактальности спиновой структуры протона. Обосновывается возможность получения информации о спин-зависимых фрактальных размерностях адронов и процесса фрагментации из асимметрий и коэффициентов передачи поляризации. Анализируются в рамках z-скейлинга двухспиновые продольные асимметрии рождения π^0-мезонов и коэффициент продольной передачи поляризации Λ-гиперона, измеренные в протон-протонных столкновениях на RHIC. Найдены спин-зависимые размерности протона и фрагментации Λ-гиперона. Предлагается изучение спин-зависимых потерь энергии конституента как функции поперечного импульса адрона и энергии столкновения.

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INTRODUCTION

Spin, along with the electric charge, mass and intrinsic symmetries, is one of the most fundamental properties of elementary particles. It is important to understand the spin of hadrons in terms of the underlying fundamental degrees of freedom, i.e., the spin of quarks

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and gluons and their orbital motion. The axial vector nature of spin has been useful in testing symmetries such as parity and time reversal invariance in fundamental interactions. The importance of physics with polarized proton program can be understood in two ways. One is elucidation of the spin structure of the nucleon and the other is utilizing the known spin structure to test symmetries in reactions. The spin structure of the nucleon has been studied for a long time in processes with longitudinally and transversely polarized leptons and protons [1–8]. The goal is to obtain a complete picture of the nucleon spin in terms of quark and gluon degrees of freedom.

In the framework of QCD, the basic elements of hadron structure are quarks and gluons. Non-linear Yang–Mills equations taking into account gauge invariance and Lorentz covariance regulate dynamics of constituent interactions both at hard and soft regimes. Problems of confinement and colorlessness of hadrons constructed from color constituents are not solved up to now. Microscopic scenario of hadron structure implies knowledge of momentum and spin distributions of hadron constituents at different scales. There are indications that the structure of unpolarized proton reveals self-similarity over a wide scale range [9, 10]. One could assume that spin content of proton has self-similar distribution in terms of polarized quarks and gluons as well. Both should reflect existence of subtle structure of a geometrical carrier in the momentum space. Anisotropy of the momentum space may occur due to spontaneous symmetry breaking at small scales [11]. This property can be connected with scale invariance of proton spin compositeness. The scale invariance of such properties as spin, mass, electric charge and other quantities implies existence of fractal topological invariants.

The idea of self-similarity of hadron interactions is a fruitful concept to study collective phenomena in the hadron and nuclear matter [12–17]. Important manifestation of such a concept is existence of a scaling itself [18–21]. Scaling in general means self-similarity at different scales. The physical content meant by behind it can be of different origin. Some of the scaling features constitute pillars of modern critical phenomena. Another category of scaling laws (self-similarity in point explosion, laminar fluid flow, etc.) reflects features not related to phase transitions. The z-scaling in inclusive reactions, which in a sense pertains to both the mentioned groups, has properties reviewed in [9, 10]. It is treated as manifestation of self-similarity of the structure of colliding objects (hadrons, nuclei), interaction mechanism of their constituents, and processes of fragmentation of the hadron constituents into real hadrons. The validity of the z-scaling is confirmed in the region which is far from the boundary of a phase transition. Nevertheless, parameters of the scaling can be sensitive to the vicinity of phase transitions [14]. There are three parameters, c, δ_A and ε_F, which have physical interpretation of heat capacity of the produced matter, fractal dimension of the structure of hadrons or nuclei and fractal dimension of the fragmentation process, respectively. The z-scaling approach shows itself as an effective tool for sophisticated data analysis in searching for new phenomena, verification of theoretical models, etc. Extension of the method for analysis of polarization phenomena and verification of self-similarity of spin-dependent inclusive cross sections for particle production in $p+p$ collisions is an interesting problem which could give new insight into the origin of proton spin at small scales. The spin-dependent fractal dimensions are new parameters of the z-scaling theory. They are new characteristics of polarization properties of proton structure, constituent interactions and hadronization process. The possibilities of using z-scaling for this type of investigations are discussed below.

1. SCALING AND UNIVERSALITY AS GENERAL CONCEPTS

The concepts developed to understand the critical phenomena are “scaling” and “universality”. Scaling means that the system near the critical point exhibiting self-similar properties is invariant under transformation of scale. According to universality, quite different systems behave in a remarkably similar way near the respective critical point [18–20]. The universality hypothesis reduces the great variety of critical phenomena to a small number of equivalence classes, the so-called “universality classes”, which depend only on few fundamental parameters (critical exponents). The universality has its origin in the long range character of the fluctuations. Close to the transition point the behavior of the cooperative phenomena becomes independent of the microscopic details of the considered system. The fundamental parameters determining the universality class are the symmetry of the order parameter and the dimensionality of space. The concept of universality remains the major tool to study the great variety of non-equilibrium phase transitions as well (see [21] and references therein). It is known that the scaling functions vary more widely between different universality classes than the exponents. Thus, universal scaling functions offer a sensitive and accurate test for the system universality class. We assume that polarization phenomena observed in collisions of polarized protons can be described by the new universal scaling functions with parameters which are spin-dependent fractal dimensions of proton and fragmentation process.

2. z-SCALING

The z-scaling belongs to the scaling laws with applications not limited to the regions near a phase transition. The scaling regularity concerns hadron production in the high-energy proton (antiproton) and nucleus collisions [9, 10, 12–17]. It manifests itself in the fact that the inclusive spectra of various types of particles are described with a universal scaling function. The function $\Psi(z)$ depends on single variable z in a wide range of the transverse momentum, registration angles, collision energies, and centralities. The scaling variable has the form

$$z = z_0\Omega^{-1}. \tag{1}$$

Here z_0 and Ω are functions of kinematic variables:

$$z_0 = \frac{\sqrt{s_\perp}}{(dN_{\mathrm{ch}}/d\eta|_0)^c\, m_N}, \tag{2}$$

$$\Omega = (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}(1 - y_a)^{\varepsilon_a}(1 - y_b)^{\varepsilon_b}. \tag{3}$$

The quantity z_0 is proportional to the transverse kinetic energy of the selected binary constituent subprocess required for the production of inclusive particle with mass m and its partner (antiparticle). The multiplicity density $dN_{\mathrm{ch}}/d\eta|_0$ of charged particles in the central region $\eta = 0$, the nucleon mass m_N and the parameter c completely determine the functional relationship of the dimensionless variable z_0. We introduce the following notation for (3):

$$\Omega \equiv \Omega_{0000} =: \{\delta_1, \delta_2, \varepsilon_a, \varepsilon_b\}. \tag{3'}$$

The low index (0000) corresponds to unpolarized particles in the initial and final states. The quantity Ω is proportional to a relative number of the configurations at the constituent level which include the binary subprocesses corresponding to the momentum fractions x_1 and x_2 of colliding hadrons (nuclei) and to the momentum fractions y_a and y_b of the secondary objects produced in these subprocesses. The parameters δ_1 and δ_2 are fractal dimensions of the colliding objects, whereas ε_a and ε_b stand for the fractal dimensions of the fragmentation process in the scattered and recoil direction, respectively. For unpolarized processes we have the same values $\delta_1 = \delta_2 = \delta$ and $\varepsilon_a = \varepsilon_b = \varepsilon_F$ which depend on the type of the inclusive particle. The selected binary subprocess, which results in production of the inclusive particle and its recoil partner (antiparticle), is defined by the maximum of $\Omega\ (x_1, x_2, y_a, y_b)$ with the kinematic constraint

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2. \tag{4}$$

Here $M_X = x_1 M_1 + x_2 M_2 + m/y_b$ is the mass of the recoil system in the subprocess. The 4-momenta of the colliding objects and the inclusive particle are P_1, P_2 and p, respectively. Equation (4) accounts for the locality of the interaction at the constituent level and sets a restriction on the momentum fractions x_1, x_2, y_a, y_b of particles via the kinematics of the constituent interactions. The microscopic scenario of constituent interactions developed in the framework of z-scaling is based on dependences of the momentum fractions on the collision energy, transverse momentum and centrality.

The scaling variable z has property of a fractal measure. It grows in a power-like manner with the increasing resolution Ω^{-1} defined with respect to the constituent subprocesses satisfying (4). The scaling function $\Psi(z)$ is expressed in terms of the inclusive cross section $Ed^3\sigma/dp^3$, multiplicity density $dN/d\eta$, and total inelastic cross section $\sigma_{\rm in}$ — measurable for the inclusive reaction $P_1 + P_2 \to p + X$. It is determined by the following expression:

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\sigma_{\rm in}} J^{-1} E \frac{d^3\sigma}{dp^3}. \tag{5}$$

Here J is Jacobian for the transition from the variables $\{p_T^2, y\}$ to $\{z, \eta\}$. The function $\Psi(z)$ satisfies the normalization condition:

$$\int\limits_0^\infty \Psi(z)\, dz = 1. \tag{6}$$

Equation (6) allows us to interpret $\Psi(z)$ as probability density of the production of the inclusive particle with the corresponding value of the variable z.

3. SCALING IN UNPOLARIZED *pp* COLLISIONS

Self-similarity property of hadron interactions at high energies provides the basis for analyzing inclusive spectra of particles produced in proton and nuclei collisions in the framework of z-scaling approach. Figure 1 shows spectra of hadrons produced in proton–proton interactions in z-presentation. The kinematic region covers a wide range of the collision energies,

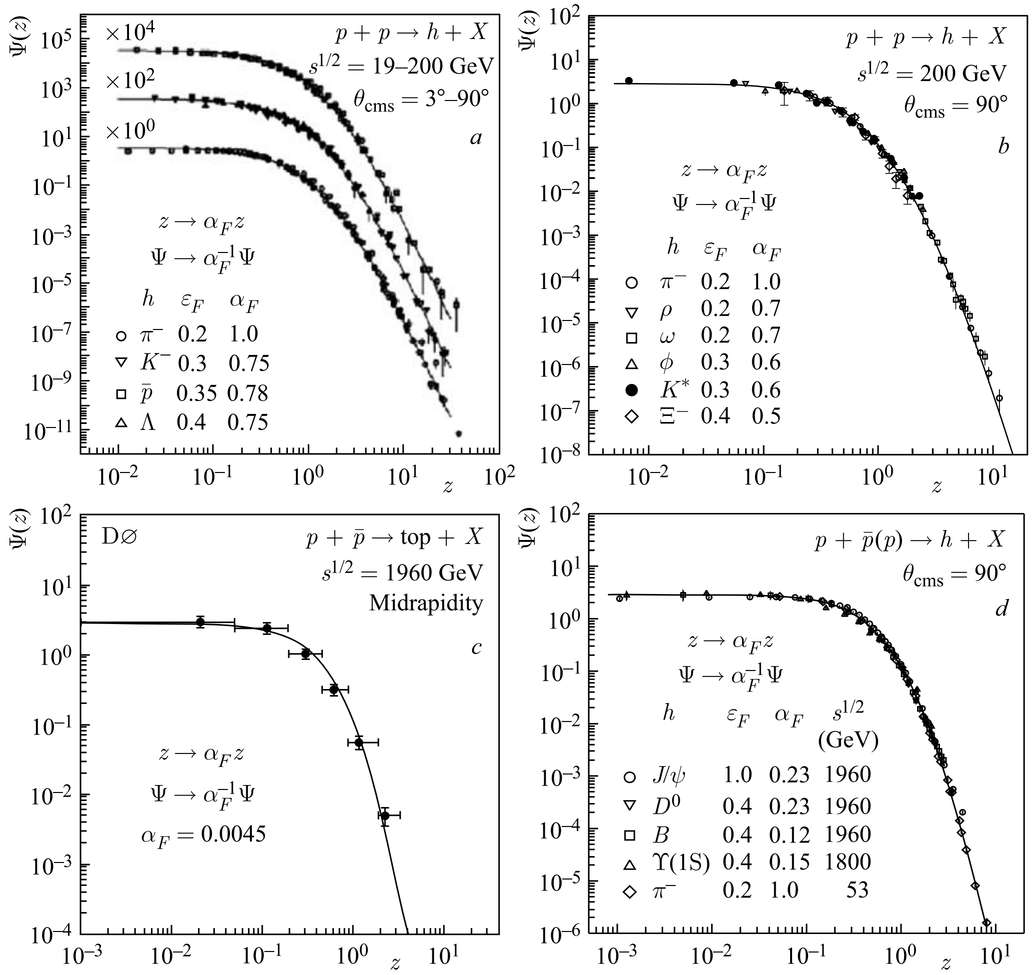

Fig. 1. Inclusive spectra of hadrons produced in proton–proton and proton–antiproton collisions in the z-presentation. The symbols denote the experimental data obtained in the experiments performed at CERN, FNAL, and BNL. Plots are taken from [9, 10, 16]

registration angles, and transverse momenta. The scale factors (10^0, 10^2, 10^4) are introduced to split the data into different groups. We observe a collapse of the experimental data points onto a single curve. The solid line is a fitting curve for these data. The derived representation shows the universality of the shape of the scaling curve $\Psi(z)$ for different types of hadrons. The regularity (universal shape of the function $\Psi(z)$ and its scaling behavior in the wide kinematic range found at constant values of the parameters δ, ε_F, and c) is treated as manifestation of the self-similarity of the structure of colliding objects, interaction mechanism of their constituents, and processes of fragmentation into registered particles. The scale transformation $z \to \alpha_F z$, $\Psi \to \alpha_F^{-1}\Psi$ results in compatibility of the corresponding scaling curves in the plane $\{z, \Psi\}$. The normalization condition (6) is conserved by the transformation.

As is seen from Fig. 1, the scaling function $\Psi(z)$ exhibits two kinds of behavior: one in the low-z and the other one in the high-z region. The low-z region corresponds to saturation of the scaling function with the typical flattening out. The behavior of $\Psi(z)$ at low z depends mainly on parameter c. This parameter is determined from the multiplicity dependence of the spectra. The region of low values of z (low transverse momenta) and high multiplicity density is preferable (even in proton–proton interactions) to study the collective effects and search for a phase transition in hadron matter. This region is best suited for studying collective phenomena in the systems of hadrons and their constituents. The scaling function at high z (high transverse momenta) is characterized by the power behavior $\Psi(z) \sim z^{-\beta}$ with a constant value of the slope β. At high z, the observed power shape of the scaling function reflects self-similarity of the constituent interactions at small scales. The asymptotic form of $\Psi(z)$ imposes restrictions on the cross sections at high p_T. It can be used to perform the global QCD fit for construction of quark and gluon distribution functions in the regions where the experimental data are still missing.

The parameters δ, ε_F and c introduced in the definition of the variable z are determined from analyses of many different sets of experimental data (see [9, 10] and references therein). They are found to be constant and independent of the multiplicity density and of the kinematic quantities such as collision energy, detection angle and transverse momentum of the inclusive particle. A possible change of the parameters can be used as a signature of new phenomena in the kinematic regions not yet explored experimentally. This is primarily true for the low ($z < 0.01$) and high ($z > 10$) regions of the variable z. In the intermediate region ($0.01 < z < 10$), the shape of $\Psi(z)$ is well determined from data obtained in the kinematic range which is now accessible for experiments at the current accelerators. Note that extension of the z-range does not require obligatory increase in the collision energy. This is possible when rare events are specially selected at low $z < 0.01$ or high $z > 30$. A more stringent restriction on the scaling behavior of $\Psi(z)$ at high z would bear witness to self-similarity at scales smaller than 10^{-4} fm related with the notion of fractal space-time.

New accelerator facilities NICA at JINR and FAIR at GSI are planned to be commissioned at the energy around $\sqrt{s} = 30$ GeV or below. In this range, extreme regime of hadron structure can be reached for $z > 30$. A test of the self-similarity of spin-dependent and spin-independent constituent interactions at small scales is of interest for the high values of z. New information expected from the experiments with polarized proton beams would be complementary to measurements with polarized particles in deep-inelastic, semi-inclusive deep-inelastic, deep-virtual Compton scattering.

4. SCALING HYPOTHESIS FOR POLARIZED pp COLLISIONS

In this section we discuss possibility to use z-scaling approach for study of spin-dependent inclusive cross sections of particle production in $\vec{p} + \vec{p}$ collisions to extract information on spin-dependent fractal dimensions of proton.

As pointed out in [22], the E581 and E704 collaborations at Fermilab reported data on hadron production in transversely polarized proton–proton scattering that showed large unanticipated transverse spin asymmetries up to 30–40%. Qualitatively, the asymmetries were consistent with zero for midrapidities, but increased rapidly in the forward scattering direction (Fig. 2, *a*).

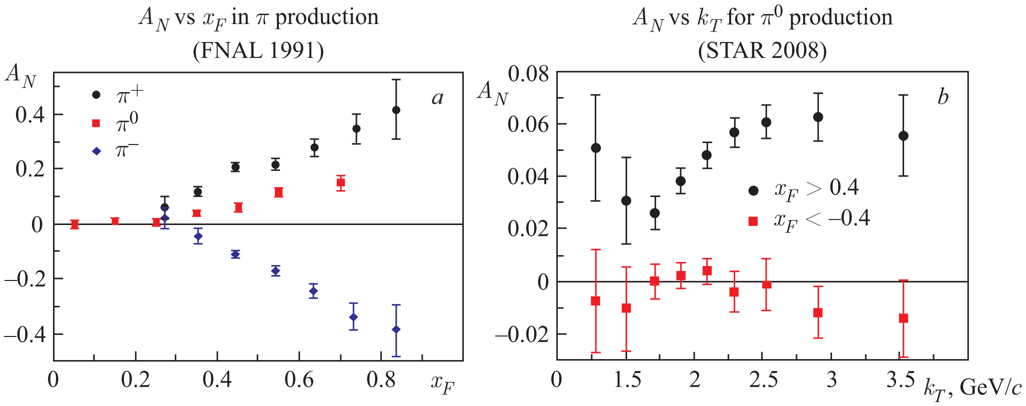

Fig. 2. The pion single transverse spin asymmetry A_N (*a*) as a function of Feynman x_F reported by the E581 and E704 collaborations for $0.7 < k_T < 2.0$ GeV/c, and (*b*) as a function of the pion transverse momentum k_T collected by the STAR collaboration. Plots are taken from [22]

More recently, the PHENIX, STAR, and BRAHMS collaborations at the Relativistic Heavy Ion Collider (RHIC) have studied the transverse spin asymmetries over a wide kinematic range at $\sqrt{s} = 200$ GeV. The obtained data confirmed and extended the Fermilab results and indicated nonmonotonic dependence of A_N on the transverse momentum k_T of the produced hadron (Fig. 2, *b*) (see also [7, 8, 23] and references therein). The measurements of the nonzero asymmetries give us strong motivation to study fractal properties of proton spin structure.

For processes with one polarized proton, the fractal dimensions should depend on the direction of the proton spin relative to its momentum (left ($\leftarrow$) and right ($\rightarrow$) helicity) or relative to the reaction plane (up ($\uparrow$) and down ($\downarrow$)). The opposite proton polarizations in the inclusive cross section for particle production in the reaction $\vec{p}+p \to h+X$ or $p\uparrow+p \to h+X$ are generally denoted by σ_+ and σ_-. The single longitudinal (A_L) or transverse (A_N) spin asymmetries of the processes are written as follows:

$$A_{L,N} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}. \tag{7}$$

The unpolarized cross section is equal to $\sigma_0 = (\sigma_+ + \sigma_-)/2$. Using the notation as in (3′), the corresponding quantities Ω can be expressed as follows:

$$\Omega_{+000} =: \{\delta - \Delta\delta/4, \delta + \Delta\delta/4, \varepsilon_F - \Delta_F, \varepsilon_F\}, \tag{8}$$

$$\Omega_{-000} =: \{\delta + \Delta\delta/4, \delta - \Delta\delta/4, \varepsilon_F + \Delta_F, \varepsilon_F\}. \tag{9}$$

We know from experiment that $\sigma_+ \neq \sigma_-$ when $x_1 \to 1$, $x_2 \to 0$ and $\sigma_+ = \sigma_-$ for $x_1 \to 0$, $x_2 \to 0$. This should result in $\Psi_{+000}(z) \neq \Psi_{-000}(z)$ and $\Psi_{+000}(z) = \Psi_{-000}(z)$ in these kinematic regions, respectively. The experimental observation represents restriction on interplay between the corrections $\Delta\delta/4$ and Δ_F which for various particles reflect different production and fragmentation mechanisms. This can give, e.g., zero asymmetry for $\pi^{0,\pm}$ mesons and nonzero one for $W^{\pm}$'s. The values of Ω_{+000} and Ω_{-000} are expected to be modified differently for transversely polarized protons generating the asymmetry A_N due to different spin-dependent corrections $\Delta\delta$ and Δ_F for longitudinal and transverse polarizations.

The reaction $\vec{p}+\vec{p}\to h+X$ with two longitudinal polarized protons in the initial state is described by spin-dependent cross sections $\sigma_{++},\sigma_{--},\sigma_{+-},\sigma_{-+}$. The symbols $(+)$ and $(-)$ denote positive and negative helicities of the protons, respectively. The double spin asymmetry A_{LL} of the process is expressed via combination of the cross sections in the form

$$A_{LL}=\frac{\sigma_{++}+\sigma_{--}-\sigma_{+-}-\sigma_{-+}}{\sigma_{++}+\sigma_{--}+\sigma_{+-}+\sigma_{-+}}. \tag{10}$$

The corresponding notation for the functions Ω is written as follows:

$$\Omega_{++00} =: \{\delta-\Delta\delta/2,\delta-\Delta\delta/2,\varepsilon_F,\varepsilon_F\}, \tag{11}$$

$$\Omega_{+-00} =: \{\delta,\delta+\Delta\delta,\varepsilon_F,\varepsilon_F\}, \tag{12}$$

$$\Omega_{--00} =: \{\delta-\Delta\delta/2,\delta-\Delta\delta/2,\varepsilon_F,\varepsilon_F\}, \tag{13}$$

$$\Omega_{-+00} =: \{\delta+\Delta\delta,\delta,\varepsilon_F,\varepsilon_F\}. \tag{14}$$

The measurements of A_{LL} at RHIC [1–4] performed by the STAR collaboration for jet production and by the PHENIX collaboration for π^0-meson production showed that the double spin asymmetry is small but nonzero in the central rapidity region and increases with the transverse momentum. Similar analysis is expected to be performed for the double transverse spin asymmetry A_{NN} in $p\uparrow+p\uparrow$ collisions. The measurements of A_{LL} and A_{NN} over a wide range of x_1 and x_2 could give more detailed information regarding polarized constituent interactions and provide complementary restriction on the parameters of the scaling variable z.

The inclusive reaction $\vec{p}+p\to\vec{h}+X$ with one longitudinal polarized proton in the initial state and one longitudinal polarized particle (e.g., lambda hyperon) in the final state is described by the transfer of polarization [4, 24]. The coefficient of polarization transfer is written in the following form:

$$D_{LL}=\frac{\sigma_{++}+\sigma_{--}-\sigma_{+-}-\sigma_{-+}}{\sigma_{++}+\sigma_{--}+\sigma_{+-}+\sigma_{-+}}. \tag{15}$$

Here the symbols $(+)$ and $(-)$ denote the cross sections corresponding to the parallel and antiparallel spin orientations relative to the respective momenta of the polarized particles (positive and negative helicities). The polarization in the initial state is related to the spin-dependent correction $(\Delta\delta)$ of the proton fractal dimension. Let us consider $(+)$ helicity in the initial state only and denote ε_F as the corresponding fractal dimension for hadronization of the unpolarized particle (h) in the final state. If the inclusive particle is polarized $(\vec{h})$, the spin-dependent correction $\Delta\varepsilon_F$ to the value of ε_F is included. The notation for Ω concerning the processes is as follows:

$$\Omega_{+000} =: \{\delta-\Delta\delta/4,\delta+\Delta\delta/4,\varepsilon_F,\varepsilon_F\}, \tag{16}$$

$$\Omega_{+0+0} =: \{\delta-\Delta\delta/4,\delta+\Delta\delta/4,\varepsilon_F-\Delta\varepsilon_F/2,\varepsilon_F\}, \tag{17}$$

$$\Omega_{+0-0} =: \{\delta-\Delta\delta/4,\delta+\Delta\delta/4,\varepsilon_F+\Delta\varepsilon_F/2,\varepsilon_F\}. \tag{18}$$

The coefficient of polarization transfer should be zero in the region where $\Omega_{+0+0}=\Omega_{+0-0}$ and nonzero in the region $x_1\to 1, y_a\to 1$, where $\Omega_{+0+0}\neq\Omega_{+0-0}$. In an analogous way, the transverse spin transfer coefficient D_{NN} for the production of vector mesons and baryons is of interest to verify the hypothesis of self-similarity for polarized meson and baryon production and to determine the transversely spin-dependent fractal dimensions.

Let us consider the process $\vec{p} + \vec{p} \to h + X$ in more detail. The inclusive particle can be a photon, pion, kaon, lambda, Drell–Yan pair, heavy quarkonium or jet. The measured cross sections for different polarizations of protons allow obtaining the double spin asymmetry A_{LL} and the unpolarized cross section σ_0 as a function of the transverse momentum p_T at some angle ϑ_{cms}. Using the information on the asymmetry and the unpolarized cross section, the spin-dependent functions $\Psi_{++}, \Psi_{--}, \Psi_{+-}, \Psi_{-+}$ can be constructed. The functions have different arguments which we denote as $z_{++}, z_{--}, z_{+-}, z_{-+}$, respectively. They depend on spin-dependent fractal dimensions in the way shown above. Based on the existence of z-scaling in unpolarized proton–proton collisions, we assume self-similarity of polarization processes at a constituent level expressed in the following form:

$$\Psi_{++} = \Psi(z_{++}), \quad \Psi_{+-} = \Psi(z_{+-}), \quad \Psi_{00} = \Psi(z_{00}). \tag{19}$$

The relation includes corrections $\Delta\delta$ and $\Delta\varepsilon_F$ to the fractal dimensions δ and ε_F established for the unpolarized reactions. The corrections can be determined using data on inclusive cross section and asymmetry of the processes under consideration. Information on both polarized and unpolarized cross sections are necessary to extract spin-dependent fractal dimensions from the polarization characteristics (A_{LL}, D_{LL}) of a given process. Such data allows us to obtain restrictions on the parameters $\Delta\delta$ and $\Delta\varepsilon_F$ of the model.

Figure 3 shows transverse momentum spectra of π^0 mesons produced in unpolarized $p+p$ collisions at $\sqrt{s} = 23{-}200$ GeV. One can see strong decrease of the cross sections as p_T increases. The high-p_T part of the spectra is transformed to the asymptotic power behavior of the z-scaling for large values of the variable z [12, 13]. New measurements of the cross section at high p_T are needed to verify the power law for the scaling function $\Psi(z)$. At high energies, the asymptotic behavior of $\Psi(z)$ is hard to reach.

It is therefore of interest to perform measurements at energies lower than typical $\sqrt{s} = 30$ GeV. On the other hand, if the energy is too low, the fractal dimensions δ_1 and δ_2 begin

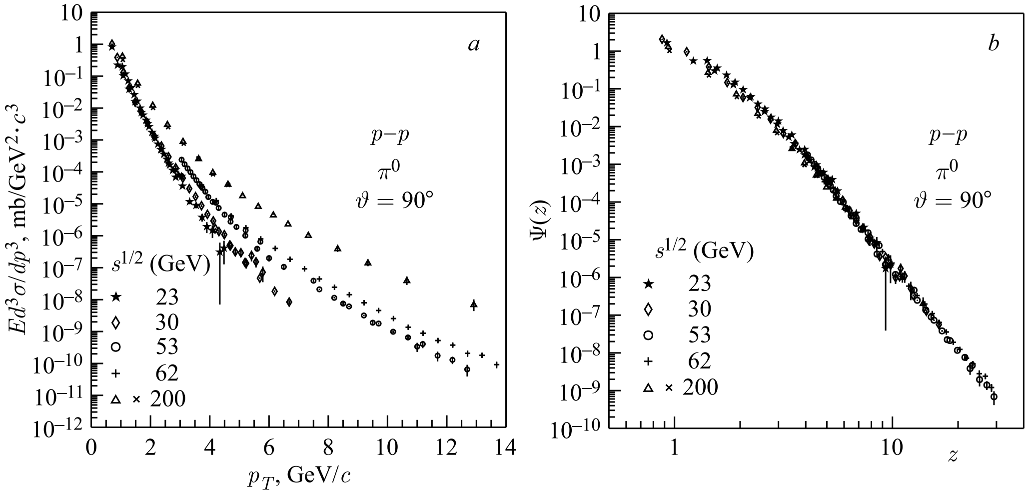

Fig. 3. Spectra of the pion production in the proton–proton collisions at $\sqrt{s} = 23{-}200$ GeV and $\vartheta_{\text{cms}} = 90°$ in (*a*) p_T- and (*b*) z-presentations [12, 13]

to decrease. Such behavior reflects smearing of the fractal structure of hadrons which are seen as structureless ($\delta_1 = \delta_2 = 0$) at very low $\sqrt{s}$. In this regard, it is desirable to perform measurements over a selected energy range, say $\sqrt{s} = 10{-}30$ GeV. In this range, spin effects are expected to be considerable.

In order to get some estimates concerning spin dependences of spectra in z-presentation at lower energies, we start with analysis of data on double longitudinal spin asymmetry of π^0 mesons measured by the STAR collaboration at higher energy. The data at $\sqrt{s} = 200$ GeV is shown in Fig. 4. One can see that the asymmetry is small and above some p_T increases with the transverse momentum.

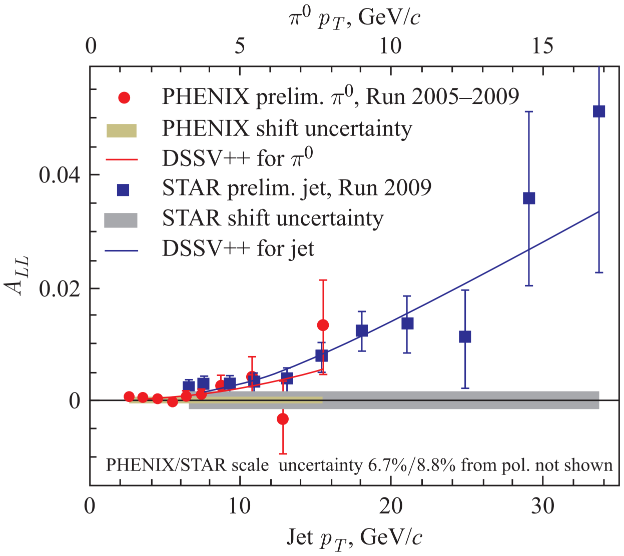

Fig. 4. The double longitudinal spin asymmetry A_{LL} of pion production in the proton–proton collisions at $\sqrt{s} = 200$ GeV and $\vartheta_{\rm cms} = 90^\circ$ as a function of transverse momentum [1–4]

The transverse momentum dependence of the asymmetry and corresponding unpolarized cross section were used in the analysis. We have constructed the functions $\Psi_{++}(z_{++})$ and $\Psi_{+-}(z_{+-})$ exploiting hypothesis on universality of their shape written in the form (19) to estimate spin-dependent correction $\Delta\delta$ to the proton fractal dimension δ from the asymmetry A_{LL} at $\sqrt{s} = 200$ GeV and $\vartheta_{\rm cms} = 90^\circ$. As follows from z-scaling for unpolarized interactions, the proton fractal dimension $\delta = 0.5$ does not depend on energy for $\sqrt{s} > 20$ GeV. This brings us to an assumption that the spin-dependent fractal dimensions are independent of the collision energy in this kinematic region as well. The hypothesis of self-similarity of spin-dependent structure of proton encoded in $\delta, \Delta\delta$ and in the functions Ψ_{++}, Ψ_{+-} is considered to play an important role to understand origin of the proton spin. New measurements of spin asymmetries are needed to verify the hypothesis at lower energies. Based on the functional form (19) and on the assumption of energy independence from the spin-dependent fractal dimensions, we have used data on π^+ production at $\sqrt{s} = 27.4$ GeV and $\vartheta_{\rm cms} = 90^\circ$ to estimate longitudinal double spin asymmetry at this energy. Figure 5 demonstrates the scaled spin-dependent functions $\Psi(z)$/fit for the reaction $\vec{p} + \vec{p} \to \pi^{+,0} + X$ at both energies

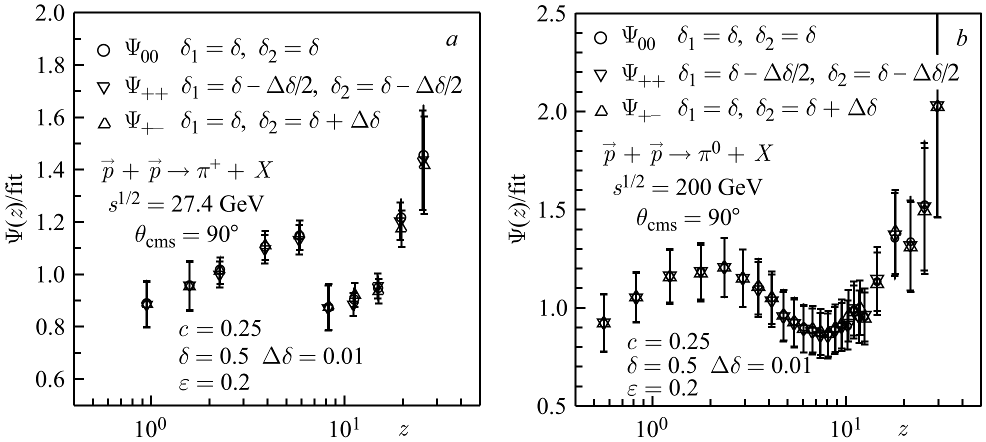

Fig. 5. The scaled spin-dependent Ψ_{++}, Ψ_{+-} and spin-independent Ψ_{00} functions of pion production in proton–proton collisions at $\sqrt{s} = 27.4, 200$ GeV and $\vartheta_{\text{cms}} = 90°$ in z-presentation

$\sqrt{s} = 27.4$ and 200 GeV. The self-similarity of spin processes expressed by these functions corresponds to Eq. (19). Accordingly, the correction to the fractal dimension δ of unpolarized proton is found to be $\Delta\delta = 0.01$.

Figure 6 shows longitudinal spin transfer coefficient D_{LL} for the process $\vec{p} + p \to \vec{\Lambda} + X$ at $\sqrt{s} = 200$ GeV and $\langle\eta\rangle = 0.5$. The data has been obtained by the STAR collaboration [4]. The coefficient can provide sensitivity to the spin-dependent hadronization of Λ hyperon. Though the coefficient D_{LL} from Fig. 6, *a* is rather small, the transverse coefficient D_{NN} shown in Fig. 6, *b* grows with x_F and reaches 30% at $x_F \approx 0.85$. We have studied corrections to the fractal dimensions δ and ε_F for the longitudinal spin transfer process as quoted in (17) and (18), exploiting the self-similarity condition (19). Using the value of $\Delta\delta = 0.01$ obtained from the analysis of double spin asymmetry mentioned above and approximating p_T-dependence of the coefficient D_{LL} from Fig. 6, *a*, the spin-dependent correction to ε_F is found to be $\Delta\varepsilon_\Lambda = 0.01$.

Relying on constancy of ε_F with $\sqrt{s}$ in unpolarized reactions, one can assume that the value of $\Delta\varepsilon_\Lambda$ is independent of the collision energy (similarly as $\Delta\delta$) in the considered

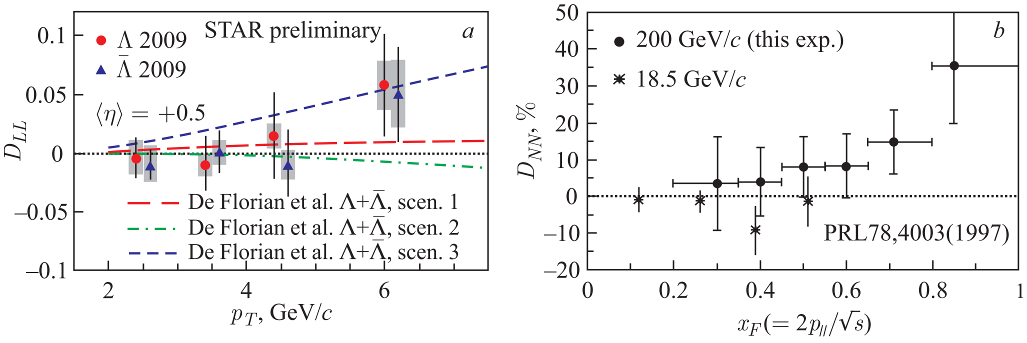

Fig. 6. Longitudinal D_{LL} and transverse D_{NN} transfer coefficients [4, 24]

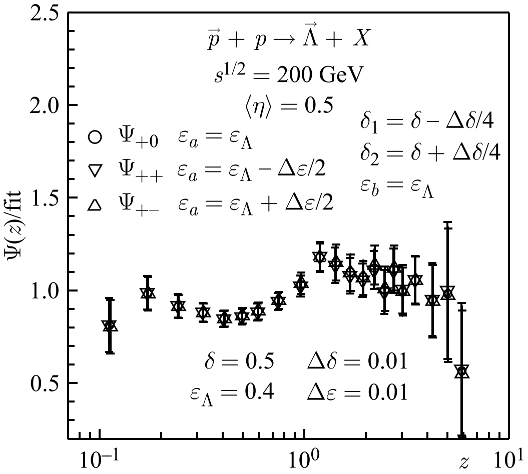

Fig. 7. The scaled spin-dependent Ψ_{+0}, Ψ_{++} and Ψ_{+-} functions for Λ hyperon production in proton–proton collisions at $\sqrt{s}=200$ GeV and $\langle\eta\rangle=0.5$ in z-presentation

kinematic range. Figure 7 demonstrates the corresponding scaled spin-dependent functions $\Psi(z)$/fit for the reaction $\vec{p}+p\to\vec{\Lambda}+X$ at $\sqrt{s}=200$ GeV (see Eqs. (16)–(18) and text above).

We consider that the self-similarity of spin-dependent structure of proton and lambda hyperon encoded in the parameters $\delta, \Delta\delta$ and $\varepsilon_\Lambda, \Delta\varepsilon_\Lambda$ and also in the functions Ψ_{++}, Ψ_{+-} plays an important role to understand the mechanism of spin polarization transfer. The suggested procedure of data analysis based on z-scaling can be applied to a wide class of polarization processes. Among them there are reactions with production of direct photons, J/ψ's, Drell–Yan pairs, pions, kaons, hyperons in $\vec{p}+\vec{p}$ collisions as well as production of polarized and unpolarized particles with different flavor content in $\vec{p}+p$ and $p+p$ interactions. Systematic experimental investigations of the processes with polarized protons will contribute to further development of theory and understanding of spin as one of the most important and basic properties of particles.

5. SPIN-DEPENDENT ENERGY LOSS

The concept of z-scaling allowed us to develop a microscopic scenario of the interaction between hadrons and nuclei at the level of interacting constituents. The microscopic picture of hadron collisions reflects fractal structure of the colliding objects, properties of constituent interactions and hadronization process characterized by the momentum fractions x_1, x_2, y_a, y_b and fractal dimensions $\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$. The scaling observed in many reactions under various (kinematic and multiplicity) conditions predicts values of energy losses during the process of inclusive production in dependence on the collision energy, centrality, transverse momentum and type of the inclusive particle [14, 15]. The energy loss is proportional to $1-y_a$.

Figure 8 illustrates dependence of the momentum fractions y_π and y_Λ of the scattered constituents carried away by the inclusive pion and Λ hyperon, respectively. The symbols correspond to data on production of both particles in the unpolarized $p+p$ collisions. As seen from Fig. 8, the energy losses decrease with increasing p_T. For pion production at

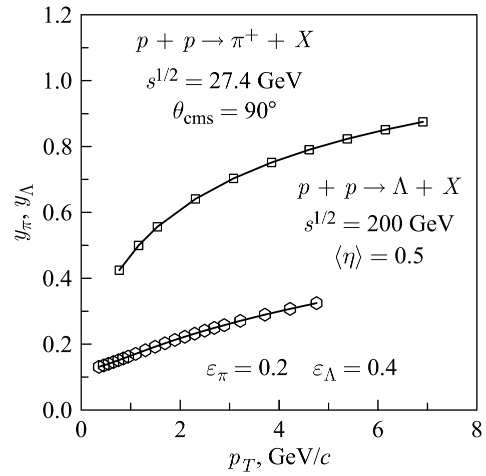

Fig. 8. Momentum fractions y_π and y_Λ as a function of the transverse momentum p_T and collision energy $\sqrt{s}$ for the processes $p{+}p \to \pi^+{+}X$ at $\sqrt{s} = 27.4$ GeV and $p{+}p \to \Lambda^0{+}X$ at $\sqrt{s} = 200$ GeV, respectively

$\sqrt{s} = 27.4$ GeV and $p_T = 7$ GeV/c the energy loss is found to be about 10%, whereas for Λ production at $\sqrt{s} = 200$ GeV and $p_T = 5$ GeV/c its value is significantly larger — 70%. As mentioned in [14, 15], smaller energy losses are preferable in dealing with interesting physical problems such as localization of critical point and detection of phase transitions in hadron and nuclei collisions.

For reactions with polarized protons, the energy loss depends on polarization of the incident protons or possibly on the spin state of the produced inclusive particle. Figure 9, *a*

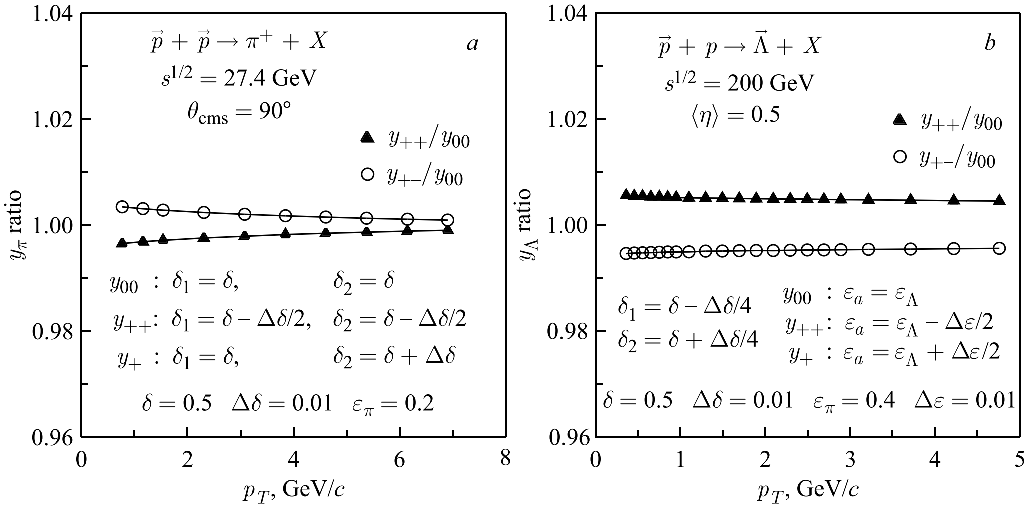

Fig. 9. Spin-dependent momentum fractions y_π, y_Λ as a function of the transverse momentum p_T and collision energy $\sqrt{s}$ for processes $\vec{p} + \vec{p} \to \pi^+ + X$ and $\vec{p} + p \to \vec{\Lambda} + X$

shows the ratio of spin-dependent y_{++}, y_{+-} and spin-independent y_{00} fractions for pion production in the reaction $\vec{p}+\vec{p} \rightarrow \pi^+ + X$ at $\sqrt{s} = 27.4$ GeV as a function of the transverse momentum p_T. The momentum fractions correspond to the same values of fractal dimensions as quoted in Fig. 5. One can see that the energy loss is slightly larger for the combination $(++)$ of proton polarizations than for the combination $(+-)$. The first one corresponds to the collisions with the opposite $(\rightarrow\leftarrow)$ and the second one with the same $(\rightarrow\rightarrow)$ spin orientations of the incident protons.

This feature is in accord with the behavior of the corresponding momentum ratio for the polarization transfer process in the reaction $\vec{p}+p \rightarrow \vec{\Lambda} + X$ at $\sqrt{s} = 200$ GeV. As seen from Fig. 9, *b*, the energy loss is smaller for the spin combination $(++)$ of polarized particles than for the combination $(+-)$. The difference between y_{++} and y_{+-} represents about 1% and is approximately independent of transverse momentum over the range $p_T = 0.5{-}4.5$ GeV/c. The momentum fractions correspond to the same values of fractal dimensions as quoted in Fig. 7. The scaling hypothesis (19) leads us to a conjecture that opposite spin orientation of proton spins (in reactions with longitudinal double spin asymmetries) and spin-flip process (in reactions with longitudinal polarization transfer) result in somewhat larger energy losses relative to the situation where the spins of both polarized particles are aligned in “the same direction”. Such a natural inference relies on the fractal structure of hadron constituents as implemented in the z-scaling formalism and, as we consider, reflects self-similarity of hadron interactions in spin-dependent processes at a constituent level.

CONCLUSIONS

Search for new features of spin structure in polarized proton–proton collisions has been discussed. We assume that one of the basic properties of spin as a quantum characteristic of particles is fractality. The hypothesis of self-similarity of proton structure, constituent interactions and hadronization process confirmed in unpolarized $p + p$ collisions over a wide kinematic range is extended for processes with polarized particles. The established properties of data z-presentation, like energy, angular and flavor independence from the scaling function give us basis to study the spin structure of proton in the framework of z-scaling theory. The requirement of a universal description of the hadron spectra in collisions at different energies gives restrictions on the values of spin-dependent parameters of the z-scaling and their dependences on proton polarization.

The parameters δ and ε_F interpreted as a fractal dimension of the proton structure and fractal dimension of the fragmentation process are modified for processes with polarized particles. The corresponding spin-dependent energy loss was studied as function of p_T and $\sqrt{s}$ in the framework of the microscopic scenario developed within the z-scaling approach. The analysis exploits results of measurements for the reactions $\vec{p}+\vec{p} \rightarrow \pi + X$ and $\vec{p}+p \rightarrow \vec{\Lambda} + X$. More accurate estimation of the energy loss needs new precise data with polarized particles and requires further detailed study.

We assume that the considered scaling property for polarization processes reflects self-similarity of the spin structure of the colliding objects, interaction mechanism of their constituents, and process of fragmentation of the polarized constituents in the final state. Study of fractality in the reactions with polarized particles can give a deeper understanding of origin of spin. The investigation is motivated by expectations that particle production in $\vec{p}+\vec{p}$ collisions in the energy range $\sqrt{s} = 10{-}30$ GeV is suitable for obtaining new information on fractal properties of proton spin. Such experiments are planned to be carried out at the future SPD NICA facility in Dubna.

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