

## SCHEME OF SPLITTING WITH RESPECT TO PHYSICAL PROCESSES FOR A MODEL OF HEAT AND MOISTURE TRANSFER

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A difference scheme of splitting with respect to physical processes for a model of heat and moisture transfer is proposed. The model involves three physical processes — heat, liquid and saturated vapor transfer in the porous material. The density of saturated vapor and the transfer coefficients of liquid and vapor moistures depend on the temperature. At the same time, the heat capacity and conductivity of the porous material depend on moisture. On the basis of the proposed scheme of the model, a numerical simulation of the heat and moisture transfer for a drying process has been performed.

В работе предложена схема расщепления, относящаяся к физическим процессам для модели передачи тепла и влаги. Предложенная модель включает три физических процесса — тепло, жидкость и передачу насыщенного пара в пористых материалах. Плотность пористого материала и коэффициенты передачи жидкости и пара зависят от температуры. В то же время теплоемкость и теплопроводимость пористого материала зависят от уровня влаги. На основе предложенной модели разработано численное моделирование передачи тепла и влаги для процесса сушки.

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### INTRODUCTION

We determined a moisture transfer coefficient in [1] by solving an inverse problem and using the experimental data [2]. At the same time, the temperature in the sample and its surrounding was assumed to be constant and equal to a room temperature during the experiment run. We solved more complex model in [3] for a temperature variation in the surrounding environment of a moist sample when the moisture transfer cannot be described by a single diffusion equation. A classical explicit difference scheme was used in [3].

We suggest new difference scheme of splitting with respect to physical processes [4, 5] for the model solution in this paper which contains temperature dependencies of the density of saturated vapor and the diffusion coefficients, and moisture dependencies of the heat

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capacity and conductivity. We study these corresponding dependencies in this paper because of increasing number of new applications [6–8]. We consider the heat and moisture transfer in the sample of a porous wet material with dimensions  $3 \times 9 \times 12$  cm as in [2,9]. The sample is sealed on by a self-adhesive aluminum tape from all sides except right side of  $3 \times 12$  cm which is open. As a consequence, the moisture diffusion occurs only in one direction  $x$  — along the width of the sample  $0 \leq x \leq l$ , where  $l = 9$  cm.

## 1. BASIC EQUATIONS

We describe the heat and moisture transfer by means of the following equations [10]:

$$\frac{\partial w_l}{\partial t} = Q_l - I, \quad Q_l = \frac{\partial}{\partial x} \left( D_l \frac{\partial w_l}{\partial x} \right), \quad 0 < x < l, \quad t > 0, \quad (1)$$

$$\frac{\partial w_v}{\partial t} = Q_v + I, \quad Q_v = \frac{\partial}{\partial x} \left( D_v \frac{\partial w_v}{\partial x} \right), \quad 0 < x < l, \quad t > 0, \quad (2)$$

$$C_s \frac{\partial T}{\partial t} = Q_T - rI, \quad 0 < x < l, \quad t > 0, \quad (3)$$

with additional condition [11] for material porosity

$$\Pi = \frac{w_l}{\rho_l} + \frac{w_v}{\rho_v}, \quad 0 \leq x \leq l, \quad t \geq 0,$$

$$Q_T = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \left( c_l D_l \frac{\partial w_l}{\partial x} + c_v D_v \frac{\partial w_v}{\partial x} \right) \frac{\partial T}{\partial x}, \quad (4)$$

$$C_s = c_d \rho_d + c_l w_l + c_v w_v, \quad \lambda = \lambda_{d0}(1 - \Pi) + c_l D_l w_l + c_v D_v w_v.$$

We have the following quantities in Eqs.(1)–(4):  $T = T(x, t)$  — temperature of the sample;  $\rho_i = \rho_i(T)$ ,  $D_i = D_i(T)$ ,  $c_i$ ,  $w_i = w_i(x, t)$  are correspondingly density, diffusion coefficient, heat capacity, and concentration for liquid ( $i = l$ ) and for vapor ( $i = v$ );  $\Pi$  — porosity;  $\rho_d$ ,  $c_d$ ,  $\lambda_{d0}(1 - \Pi)$ , and  $r$  are density, heat capacity, heat conductivity of the dry sample, and latent heat of vaporization;  $C_s$  and  $\lambda$  are volumetric heat capacity and heat conductivity of the wet sample correspondingly. The additional condition (4) used in [11] has a simple meaning. The pores volume is equal to the sum of the liquid and vapor volumes, which create a total pores volume. The function  $I$  in Eqs.(1)–(3) expresses a rate of variation of the moisture concentration that arises due to the evaporation ( $I > 0$ ) or condensation ( $I < 0$ ). In general, this is a function of  $I(w_l, w_v)$  and it should be written phenomenologically from a physical consideration. This is, however, not an easy task. That is why we suggest another way for determination of function  $I$ . The second term  $-rI$  in the right-hand side of Eq. (3) expresses a power density of the absorbed heat due to evaporation  $I > 0$  or a power density of the extracted heat due to condensation  $I < 0$ , and the last term in the equation  $Q_T$  expresses a power density of the heat that arises due to the mass transfer in the nonuniformly heated body.

We choose a temperature dependence of the diffusion coefficients  $D_l$  and  $D_v$  according to [12]

$$D_l(T) = D_{l0} \left( \frac{T}{T_0} \right)^{3/2}, \quad D_v(T) = D_{v0} \left( \frac{T}{T_0} \right)^{3/2}, \quad (5)$$

where  $D_{l0}$  and  $D_{v0}$  are diffusion coefficients for indoor temperature  $T = T_0 = 27^\circ\text{C}$ .

The dependence of liquid density  $\rho_l$  on temperature  $T$  is weak, so hereinafter we consider the density  $\rho_l$  as a constant ( $\partial\rho_l/\partial T = 0$ ). The dependence of the saturated vapor density is significant and can be found, for example, in [13] in tabulated form.

## 2. INITIAL AND BOUNDARY CONDITIONS

We solve the system (1)–(5) with the following initial and boundary conditions:

$$w_l(x, 0) = \rho_l\Pi, \quad w_v(x, 0) = 0, \quad T(x, 0) = T_0, \quad 0 \leq x \leq l, \quad (6)$$

$$-D_j(T) \frac{\partial w_j}{\partial x}(0, t) = 0, \quad j = l, v, \quad -\lambda \frac{\partial T}{\partial x}(0, t) = 0, \quad 0 \leq t \leq t_0, \quad (7)$$

$$w_l(l, t) = \rho_l\Pi + (v_0 - \rho_l\Pi)[1 - \exp(-\alpha t)], \quad \alpha \gg 1,$$

$$w_v(l, t) = \rho_v(T) \left[ \Pi - \frac{w_l(l, t)}{\rho_l} \right], \quad (8)$$

$$-\lambda \frac{\partial T}{\partial x}(l, t) = -rD_l(T) \frac{\partial w_l}{\partial x}(l, t) + \beta[T(l, t) - T_{1,\text{out}}],$$

for  $0 \leq t \leq t_0$ , where  $t_0 = 20$  days. The initial conditions (6) mean that in the beginning all pores of the sample are totally filled with liquid (no vapor is present) and the initial temperature is an indoor temperature. The mass and heat flux on the left boundary  $x = 0$  are absent (see conditions (7)). The boundary conditions (8) are written on the right side  $x = l$  of the sample. The first and second conditions describe the time dependences of the liquid and vapor concentrations. The last one expresses the heat flux. The  $\alpha$  parameter is introduced due to the consistency of the initial and boundary conditions and due to the smoothness with respect to the time of liquid and vapor on the boundary  $x = l$ .  $v_0$  represents a residual liquid concentration that we adopted from experiment [2].  $\beta$  is a heat transfer coefficient between the sample and the outer space.

## 3. SPLITTING SCHEME

In the beginning, we split the system (1)–(3) with unknowns  $(w_l, w_v) \equiv w$ , and  $I$  with respect to liquid transfer  $w_l$  and vapor transfer  $w_v$ , in the following way:

$$\frac{w^{k+1/2} - w^k}{\tau} + Lw^{k+1/2} + R w^k = 0, \quad (9)$$

$$\frac{w^{k+1} - w^{k+1/2}}{\tau} + Lw^{k+1/2} + R w^{k+1} = 0, \quad (10)$$

where

$$L = \begin{pmatrix} L_1 & 0 \\ 0 & L_2 \end{pmatrix}, \quad R = \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix}, \quad L_1 w_l = -\frac{\partial}{\partial x} \left( D_l \frac{\partial w_l}{\partial x} \right),$$

$$L_2 w_v = -\frac{\partial}{\partial x} \left( D_v \frac{\partial w_v}{\partial x} \right), \quad R_1 w_l = I, \quad R_2 w_v = -I.$$

Equation (9) represents two parabolic equations from which we determine  $w^{k+1/2}$  assuming that  $w^k$  is given, for example, by the initial conditions (6). Equation (10) in its component form can be written as

$$w_l^{k+1} + \tau I = w_l^{k+1/2} - \tau L_1 w_l^{k+1/2},$$

$$w_v^{k+1} - \tau I = w_v^{k+1/2} - \tau L_2 w_v^{k+1/2}. \quad (11)$$

If we add the last two equations, then we get

$$w_l^{k+1} + w_v^{k+1} = w_l^{k+1/2} + w_v^{k+1/2} - \tau(L_1 w_l^{k+1/2} + L_2 w_v^{k+1/2}).$$

Next, we rewrite Eq. (4) in the form

$$\frac{1}{\rho_l} w_l^{k+1} + \frac{1}{\rho_v} w_v^{k+1} = \Pi.$$

As  $\rho_v \neq \rho_l$ , the system of two last equations has a unique solution, and thus we determine  $w^{k+1}$ . Having  $w^{k+1}$ , we can determine the unknown  $I$ , for example, from (11) as

$$I = \frac{1}{\tau} (w_v^{k+1} - w_v^{k+1/2} + \tau L_2 w_v^{k+1/2}).$$

And finally, we split the scheme with respect to heat transfer. Namely, we write Eq. (3) in the difference form for inner points  $x_i = ih$ ,  $i = 1, 2, \dots, n-1$ ,  $h = 1/n$  in the interval  $[0, 1]$ :

$$C_s(w_i^{k+1}) \frac{T_i^{k+1} - T_i^k}{\tau} = \frac{1}{h} \left[ G_i^{k+1} \frac{T_{i+1}^{k+1} - T_i^{k+1}}{h} - G_{i-1}^{k+1} \frac{T_i^{k+1} - T_{i-1}^{k+1}}{h} \right] +$$

$$+ \left[ c_l D_l(T_i^k) \frac{w_{l,i+1}^{k+1} - w_{l,i-1}^{k+1}}{2h} + c_v D_v(T_i^k) \frac{w_{v,i+1}^{k+1} - w_{v,i-1}^{k+1}}{2h} \right] \frac{T_{i+1}^k - T_{i-1}^k}{2h} - r I_i,$$

$$G_i^{k+1} = [\lambda(T_{i+1}^{k+1}, w_{i+1}^{k+1}) + \lambda(T_i^{k+1}, w_i^{k+1})] / 2.$$

#### 4. RESULTS

The following additional parameters were set:  $D_{l0} = 4.68 \cdot 10^{-9}$  m<sup>2</sup>/s,  $D_{v0} = 2.34 \times 10^{-9}$  m<sup>2</sup>/s,  $\Pi = 0.2$ ,  $\rho_l = 1000$  kg/m<sup>3</sup>,  $r = 2.25 \cdot 10^6$  J/kg,  $c_d = 850$  J/(kg · K),  $c_l = 4200$  J/(kg · K),  $c_v = 1996$  J/(kg · K),  $\lambda_{d0} = 0.14$  W/(m · K),  $\beta = 23$  W/(m<sup>2</sup> · K),  $\alpha = 60$ ,  $v_0 = 3.928$  kg/m<sup>3</sup>, for calculation purposes. The constructed difference scheme has the approximation order  $O(\tau + h^2)$ , where the time step  $\tau$  was chosen as  $\tau = 0.432$  s and the space step  $h$  was chosen as  $h = 0.09$  cm.

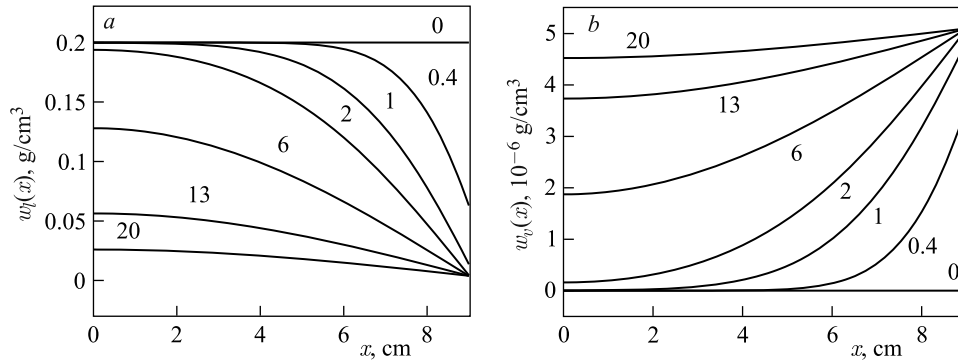


Fig. 1. Dynamics of the profiles of liquid  $w_l$  (a) and vapor  $w_v$  (b)

The numerical experiment was conducted in order to determine the dynamics of the profiles of liquid and vapor,  $w_l(x, t)$  and  $w_v(x, t)$ , temperature of the moist sample  $T(x, t)$ , and the dynamics of the temperature on the left and right sides of the sample  $T(0, t)$   $T(l, t)$ . Here, we provide the results of the drying process when the outer temperature is  $T_{1,\text{out}} = 27^\circ\text{C}$ .

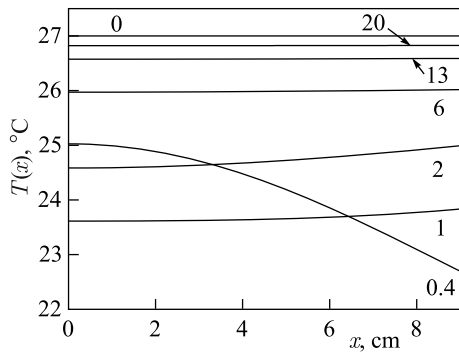


Fig. 2. Dynamics of the temperature  $T$

Figure 1 shows that the liquid concentration (plot a) decreases with time, while the vapor concentration (plot b) increases. This is explained by the fact that the removal of the liquid out of the pores of the sample leads to the growth of the pores volume occupied by the vapor, i.e., the liquid releases a space for the vapor in the pores. During the drying process we supposed that on the right boundary the liquid releases in the vapor form and due to the phase transition a heat absorption should take place. Figure 2 reflects this process. On the right boundary two kinds of the heat flux compete. The former reflects the heat that takes away moisture due to evaporation to the outer space. The latter reflects the heat so that the sample gets from the outer space. In the beginning, the first kind of the

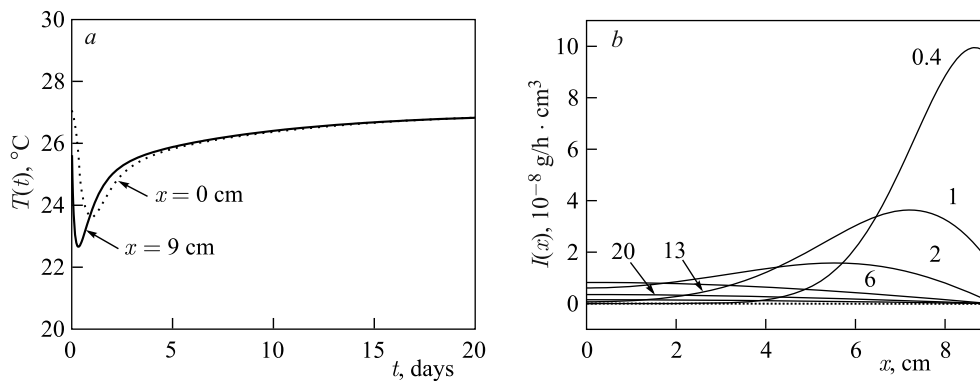


Fig. 3. Dynamics of the temperature  $T$  on the sides (a) and dynamics of the source  $I$  (b)

heat dominates the second kind, later on the first kind of the heat gets weaker. Figure 3, *a* displays a cooling and a heating of the sample on its sides. As can be seen, the temperature decreases from 27 °C of about three–four degrees and later increases up to 27 °C. Figure 3, *b* shows a source function  $I$ , the values of which in the beginning are rapidly increasing and later are slowly decreasing.

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