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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

SOME COMMENTS ON HIGH-PRECISION STUDY OF NEUTRINO OSCILLATIONS

S. M. Bilenky [1]

Joint Institute for Nuclear Research, Dubna

Some problems connected with the high-precision study of neutrino oscillations are discussed. In the general case of n-neutrino mixing, a convenient expression for transition probability, in which only independent terms (and mass-squared differences) enter, is derived. For three-neutrino mixing, a problem of a definition of a large (atmospheric) neutrino mass-squared difference is discussed. The possibilities to reveal the character of neutrino mass spectrum in future reactor neutrino experiments are commented on as well.

Обсуждаются проблемы, связанные с изучением осцилляций нейтрино в экспериментах с высокой точностью. В общем случае смешивания нейтрино получено удобное выражение для вероятности перехода, в которое входят только независимые члены (включая разности квадратов масс). Для случая смешивания трех нейтрино обсуждается проблема определения большой (атмосферной) разности квадратов масс. Рассматриваются возможности изучения спектра масс нейтрино в будущих реакторных нейтринных экспериментах.

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INTRODUCTION

The observation of neutrino oscillations in the atmospheric Super-Kamiokande [1], solar SNO [2], reactor KamLAND [3], and solar neutrino oscillation experiments [4–6] is one of the most important recent discoveries in particle physics.

Small neutrino masses, many orders of magnitude smaller than those of other fundamental fermions, are an evidence of a beyond the Standard Model physics. One of plausible scenarios which allows one to explain the smallness of neutrino masses is based on the assumption that small (Majorana) neutrino masses are generated by the lepton-number violating the dimension of five effective Lagrangians [7]. In this case neutrino masses are suppressed with respect to masses of leptons and quarks by the ratio of the electroweak scale $v = (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV and a scale $\Lambda \gg v$ of a new lepton-number violating physics.

Neutrino oscillation data can be described by the three-neutrino mixing

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x) \quad (l = e, \mu, \tau). \tag{1}$$

Here $\nu_i(x)$ is the field of neutrinos (Dirac or Majorana) with mass m_i, and U is the unitary 3×3 PMNS [8–10] mixing matrix.

[1] E-mail: bilenky@gmail.com

In the framework of the three-neutrino mixing, neutrino oscillations are characterized by two neutrino mass-squared differences Δm^2_{23} and Δm^2_{12}, three mixing angles θ_{12}, θ_{23}, θ_{13}, and one CP phase δ. From the analysis of the data of neutrino oscillation experiments, it was established that $\Delta m^2_{12} \ll \Delta m^2_{23}$, mixing angles θ_{23} and θ_{12} are large and mixing angle θ_{13} is small. The first information about the angle θ_{13} was obtained from the reactor CHOOZ experiment [11], in which only the upper bound $\sin^2 2\theta_{13} \leqslant 1 \cdot 10^{-1}$ was found.

The first data of neutrino oscillation experiments were described by expressions for neutrino transition probabilities in the leading approximation which was based on the assumption that $\sin^2\theta_{13} = 0$. In this approximation, oscillations in atmospheric and KamLAND (solar) regions are decoupled (see [12]): in the atmospheric region (atmospheric and long-baseline accelerator neutrino oscillation experiments), neutrino oscillations are two-neutrino $\nu_\mu \rightleftarrows \nu_\tau$ oscillations; in the solar region (the reactor KamLAND experiment), neutrino oscillations are $\bar{\nu}_e \rightleftarrows \bar{\nu}_{\mu,\tau}$ oscillations. From analysis of the atmospheric and long-baseline accelerator oscillation experiments, parameters Δm^2_{23} and $\sin^2 2\theta_{23}$ were determined. From analysis of the data of the KamLAND and solar experiments, the other two neutrino oscillation parameters Δm^2_{12} and $\sin^2 2\theta_{12}$ were inferred. In the leading approximation, the character of the neutrino mass spectrum and such an important effect of the three-neutrino mixing as CP violation in the lepton sector cannot be revealed.

With the measurement of the mixing angle θ_{13} in the reactor Daya Bay [13], RENO [14], and Double CHOOZ [15] experiments, the situation with the study of neutrino oscillations drastically changed. The investigation of neutrino oscillations entered into a high-precision era, the era of measurements of small, beyond the leading approximation effects, which could allow one to determine the character of the neutrino mass spectrum and to measure CP phase δ.

In this paper, for the general case of the n-neutrino mixing, we derive a convenient expression for the neutrino transition probability in vacuum, in which only independent terms (and mass-squared differences) enter.

In various papers on neutrino oscillations large (atmospheric) neutrino mass-squared difference is determined differently. Difference between these definitions is small (a few percent), but in the era of precision measurements, apparently, it is desirable to have one unified definition. The expression for transition probability presented here provides a natural framework for introduction of two independent neutrino mass-squared differences in the case of the three-neutrino mixing.

Determination of the neutrino mass spectrum character is one of the major aims of future reactor neutrino experiments JUNO [16] and RENO-50 [17]. On the basis of the proposed expression for the transition probability, we comment on this possibility.

1. GENERAL EXPRESSION FOR NEUTRINO TRANSITION PROBABILITY IN VACUUM

For the general case of the neutrino mixing

$$\nu_{\alpha L}(x) = \sum_{i=1}^{3+n_s} U_{\alpha i}\ \nu_{iL}(x) \quad (\alpha = e, \mu, \tau, s_1, \ldots, s_{n_s}), \tag{2}$$

we derive here an expression for $\nu_\alpha \to \nu_{\alpha'}$ transition probability alternative to the standard one. Here n_s is the number of sterile neutrino fields, U is an unitary $(3+n_s) \times (3+n_s)$ mixing matrix, $\nu_i(x)$ is the field of neutrino with mass m_i.

From (2) and Heisenberg uncertainty relation it follows that normalized states of flavor ν_e, ν_μ, ν_τ and sterile $\nu_{s_1}, \nu_{s_2}, \ldots$ neutrinos are described by *coherent superpositions* of the states of neutrinos with definite masses (see, for example, [12, 18, 19]):

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U^*_{\alpha i} |\nu_i\rangle. \tag{3}$$

Here $|\nu_i\rangle$ is the state of the left-handed neutrino with mass m_i, momentum $\vec{p}$, and energy $E_i = \sqrt{p^2 + m_i^2} \simeq E + m_i^2/2E$ ($E = p$ is the energy of neutrino at $m_i \to 0$).

If at $t = 0$ flavor neutrino ν_α is produced, at the time t we have

$$|\nu_\alpha\rangle_t = \sum_{\alpha'} |\nu_{\alpha'}\rangle \langle \nu_{\alpha'}|\, \mathrm{e}^{-iH_0 t} |\nu_\alpha\rangle = \sum_{\alpha'} |\nu_{\alpha'}\rangle \left(\sum_i U_{\alpha' i}, \mathrm{e}^{-iE_i t}\, U^*_{\alpha i} \right), \tag{4}$$

where H_0 is the free Hamiltonian.

From (4) for the normalized probability of the $\nu_\alpha \to \nu_{\alpha'}$ transition, we find the following expression:

$$P(\nu_\alpha \to \nu_{\alpha'}) = \left| \sum_i U_{\alpha' i},\, \mathrm{e}^{-iE_i t}\, U^*_{\alpha i} \right|^2 = \\ = \sum_i |U_{\alpha' i}|^2 |U_{\alpha i}|^2 + 2 \sum_{i>k} \mathrm{Re}\, (U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k}\, \mathrm{e}^{-2i\Delta_{ki}}). \tag{5}$$

Here

$$\Delta_{ki} = \frac{\Delta m^2_{ki} L}{4E}, \tag{6}$$

where $\Delta m^2_{ki} = m_i^2 - m_k^2$ and $L \simeq t$ is the neutrino source-detector distance.

Taking into account the unitarity of the mixing matrix U for the first term of the probability (5), we have

$$\sum_i |U_{\alpha' i}|^2 |U_{\alpha i}|^2 = \delta_{\alpha' \alpha} - 2 \sum_{i>k} \mathrm{Re}\, (U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k}). \tag{7}$$

From (5) and (7) for the $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_{\alpha'}$ transition probability, we obtain the following *standard expression* (see [20–22]):

$$P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) = \delta_{\alpha' \alpha} - 4 \sum_{i>k} \mathrm{Re}\, (U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k}) \sin^2 \Delta_{ki} \pm \\ \pm 2 \sum_{i>k} \mathrm{Im}\, (U_{\alpha' i} U^*_{\alpha i} U^*_{\alpha' k} U_{\alpha k}) \sin 2\Delta_{ki}. \tag{8}$$

Let us stress that not all quantities in (8) are independent. For example, in the case of the three-neutrino mixing, three mass-squared differences in (8) are connected by the relation $\Delta m^2_{13} = \Delta m^2_{12} + \Delta m^2_{23}$. For $\alpha' \neq \alpha$ the quantities in the last term of (8) are connected by the relations $\mathrm{Im}\,(U_{\alpha' 2}U^*_{\alpha 2}U^*_{\alpha' 1}U_{\alpha 1}) = \mathrm{Im}\,(U_{\alpha' 3}U^*_{\alpha 3}U^*_{\alpha' 2}U_{\alpha 2}) = -\mathrm{Im}\,(U_{\alpha' 3}U^*_{\alpha 3}U^*_{\alpha' 1}U_{\alpha 1})$ which follow from the unitarity of the mixing matrix (see [21, 22]).

We obtain here a simple expression for the neutrino transition probability in vacuum, in which

- we take into account that there is one arbitrary common phase in the transition amplitude;
- we use the unitarity of the mixing matrix in the transition amplitude.

We have

$$P(\nu_\alpha \to \nu_{\alpha'}) = \left|\sum_i U_{\alpha' i},\, \mathrm{e}^{-i(E_i - E_p)t}\, U^*_{\alpha i}\right|^2 = \left|\delta_{\alpha'\alpha} + \sum_{i\neq p} U_{\alpha' i}(\mathrm{e}^{-2i\Delta_{pi}} - 1)\, U^*_{\alpha i}\right|^2 = \\ = \left|\delta_{\alpha'\alpha} - 2i \sum_{i\neq p} U_{\alpha' i}\, U^*_{\alpha i}\, \mathrm{e}^{-i\Delta_{pi}}\, \sin\Delta_{pi}\right|^2, \quad (9)$$

where p is an arbitrary fixed index.

From (9) we find

$$P(\nu_\alpha \to \nu_{\alpha'}) = \delta_{\alpha'\alpha} - 4\sum_{i\neq p} |U_{\alpha i}|^2 (\delta_{\alpha'\alpha} - |U_{\alpha' i}|^2) \sin^2\Delta_{pi} + \\ + 8 \sum_{i>k;\, i,k\neq p} \mathrm{Re}\,(U_{\alpha' i}U^*_{\alpha i}U^*_{\alpha' k}U_{\alpha k}\, \mathrm{e}^{-i(\Delta_{pi} - \Delta_{pk})}) \sin\Delta_{pi} \sin\Delta_{pk}. \quad (10)$$

Finally, we obtain the following general expression for $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_{\alpha'}$ transition probability [23]:

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_{\alpha'}) = \delta_{\alpha'\alpha} - 4\sum_{i\neq p} |U_{\alpha i}|^2 (\delta_{\alpha'\alpha} - |U_{\alpha' i}|^2) \sin^2\Delta_{pi} + \\ + 8 \sum_{i>k;\, i,k\neq p} \mathrm{Re}\,(U_{\alpha' i}U^*_{\alpha i}U^*_{\alpha' k}U_{\alpha k}) \cos\,(\Delta_{pi} - \Delta_{pk}) \sin\Delta_{pi} \sin\Delta_{pk} \pm \\ \pm 8 \sum_{i>k;\, i,k\neq p} \mathrm{Im}\,(U_{\alpha' i}U^*_{\alpha i}U^*_{\alpha' k}U_{\alpha k}) \sin\,(\Delta_{pi} - \Delta_{pk}) \sin\Delta_{pi} \sin\Delta_{pk}, \quad (11)$$

where sign $+$ $(-)$ refers to $\nu_\alpha \to \nu_{\alpha'}$ $(\bar\nu_\alpha \to \bar\nu_{\alpha'})$ transition.

In (11) only independent terms (and mass-squared differences) enter. For example, for the three-neutrino mixing there are only two independent mass-squared differences and one $i > k$ term in the transition probability (because $i, k \neq p$).

2. THREE-NEUTRINO OSCILLATIONS

2.1. Atmospheric Neutrino Mass-Squared Difference. Flavor Neutrino Transition Probability. From analysis of the neutrino oscillation data it follows that one mass-squared difference (atmospheric) is much larger than the other one (solar). Two neutrino mass spectra are possible in such a situation[1]:

[1]Usually neutrinos with small mass-squared difference are called ν_1 and ν_2. It is also assumed that $m_2 > m_1$, i.e., $\Delta m^2_{12} > 0$.

1. Neutrino spectrum with small mass-squared difference between the lightest neutrinos (normal spectrum, NS)

$m_1 < m_2 < m_3, \quad \Delta m_{12}^2 \ll \Delta m_{23}^2;$

2. Neutrino spectrum with small mass-squared difference between the heaviest neutrinos (inverted spectrum, IS)

$m_3 < m_1 < m_2, \quad \Delta m_{12}^2 \ll |\Delta m_{13}^2|.$

There are only two possibilities to introduce small (solar) Δm_S^2 and large (atmospheric) Δm_A^2 mass-squared differences in the framework of the approach advocated here[1]:

1.

$$\text{NS.} \quad \Delta m_{21}^2 = -\Delta m_S^2, \quad \Delta m_{23}^2 = \Delta m_A^2 \quad (p=2), \tag{12}$$

$$\text{IS.} \quad \Delta m_{12}^2 = \Delta m_S^2, \quad \Delta m_{13}^2 = -\Delta m_A^2 \quad (p=1); \tag{13}$$

2.

$$\text{NS.} \quad \Delta m_{12}^2 = \Delta m_S^2, \quad \Delta m_{13}^2 = \Delta m_A^2 \quad (p=1), \tag{14}$$

$$\text{IS.} \quad \Delta m_{21}^2 = -\Delta m_S^2, \quad \Delta m_{23}^2 = -\Delta m_A^2 \quad (p=2). \tag{15}$$

In all papers on neutrino oscillations mixing angles, CP phase and solar mass-squared difference are determined in the same way. However, atmospheric mass-squared difference in various papers is determined differently. For example, (in terms of parameters introduced in 1 above)

1. The Bari group determines large neutrino mass-squared difference as follows (see [24]):

$$\Delta m^2 = \frac{1}{2}|\Delta m_{13}^2 + \Delta m_{23}^2| = \Delta m_A^2 + \frac{1}{2}\Delta m_S^2. \tag{16}$$

2. The NuFit group determines the atmospheric mass-squared difference as in 2 (see [25]):

$$\Delta m_{13}^2 = \Delta m_A^2 + \Delta m_S^2 \quad (\text{NS}), \quad \Delta m_{23}^2 = -(\Delta m_A^2 + \Delta m_S^2) \quad (\text{IS}). \tag{17}$$

3. In the T2K paper [26], the atmospheric mass-squared difference is determined as in 1.

4. In the MINOS paper [27], large mass-squared difference is determined as $|\Delta m_{23}^2|$ for both mass spectra. It is obvious, however, that Δm_{23}^2 for NS and $|\Delta m_{23}^2|$ for IS are different quantities.

The difference between different "atmospheric neutrino mass-squared differences" is a few percent. It is determined by the ratio $\Delta m_S^2/\Delta m_A^2 \simeq 3 \cdot 10^{-2}$ and cannot be neglected in the precision era. Apparently, one definition is desirable.

We choose here the option 1. From (11) in the case of normal and inverted neutrino mass spectra, we have, respectively,

$$\begin{aligned} P^{\text{NS}}(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) &= \delta_{l'l} - 4|U_{l3}|^2(\delta_{l'l} - |U_{l'3}|^2)\sin^2\Delta_A - \\ &- 4|U_{l1}|^2(\delta_{l'l} - |U_{l'1}|^2)\sin^2\Delta_S - 8\,\mathrm{Re}\,(U_{l'3}U_{l3}^*U_{l'1}^*U_{l1})\cos(\Delta_A+\Delta_S)\sin\Delta_A\sin\Delta_S \mp \\ &\mp 8\,\mathrm{Im}\,(U_{l'3}U_{l3}^*U_{l'1}^*U_{l1})\sin(\Delta_A+\Delta_S)\sin\Delta_A\sin\Delta_S \end{aligned} \tag{18}$$

[1]Notice that the first option corresponds to extraction of the phase connected with the intermediate neutrino mass in the expression (9) and the second one, to extraction of the phase connected with the last mass.

and

$$P^{\mathrm{IS}}(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) = \delta_{l'l} - 4|U_{l3}|^2(\delta_{l'l} - |U_{l'3}|^2)\sin^2\Delta_A -$$
$$- \; 4|U_{l2}|^2(\delta_{l'l} - |U_{l'2}|^2)\sin^2\Delta_S - 8\,\mathrm{Re}\,(U_{l'3}U^*_{l3}U^*_{l'2}U_{l2})\cos(\Delta_A + \Delta_S)\sin\Delta_A\sin\Delta_S \pm$$
$$\pm\, 8\,\mathrm{Im}\,(U_{l'3}U^*_{l3}U^*_{l'2}U_{l2})\sin(\Delta_A + \Delta_S)\sin\Delta_A\sin\Delta_S. \quad (19)$$

Here

$$\Delta_{A,S} = \frac{\Delta m^2_{A,S} L}{4E}. \quad (20)$$

Thus, transition probabilities depend on "extreme values" of the elements of neutrino mixing matrix: $U_{l'1(3)}$ and $U_{l1(3)}$ in the NS case ($p = 2$); $U_{l'2(3)}$ and $U_{l2(3)}$ in the IS case ($p = 1$). Difference in signs of the last terms of (18) and (19) is connected with signs in (12) and (13).

If CP is violated in the lepton sector, we have

$$P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'}) \quad (l' \neq l). \quad (21)$$

Let us determine the CP asymmetry

$$A^{\mathrm{CP}}_{l'l} = P(\nu_l \to \nu_{l'}) - P(\bar{\nu}_l \to \bar{\nu}_{l'}). \quad (22)$$

The CP asymmetry satisfies the following general conditions:

$$A^{\mathrm{CP}}_{l'l} = -A^{\mathrm{CP}}_{ll'} \quad (23)$$

and

$$\sum_{l'} A^{\mathrm{CP}}_{l'l} = 0. \quad (24)$$

The first condition follows from the relation

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l), \quad (25)$$

which is a consequence of the CPT invariance. The second condition follows from the conservation of the probability

$$\sum_{l'} P(\nu_l \to \nu_{l'}) = \sum_{l'} P(\bar{\nu}_l \to \bar{\nu}_{l'}) = 1. \quad (26)$$

From (23) and (24) it follows that in the case of the three-neutrino mixing CP asymmetries in different flavor channels are connected by the following relations [28]:

$$A^{\mathrm{CP}}_{\mu e} = A^{\mathrm{CP}}_{e\tau} = -A^{\mathrm{CP}}_{\mu\tau}. \quad (27)$$

From (18) in the case NS, we have

$$A^{\mathrm{CP}}_{l'l} = -16\,\mathrm{Im}\,U_{l'3}U^*_{l3}U^*_{l'1}U_{l1}\sin(\Delta_A + \Delta_S)\sin\Delta_A\sin\Delta_S. \quad (28)$$

For IS from (19) we find

$$A^{\mathrm{CP}}_{l'l} = 16\,\mathrm{Im}\,U_{l'3}U^*_{l3}U^*_{l'2}U_{l2}\sin(\Delta_A + \Delta_S)\sin\Delta_A\sin\Delta_S. \quad (29)$$

In the next subsections, we present expressions for transition probabilities which are of experimental interest. For that we use the standard parameterization of the PMNS mixing matrix

$$U=\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}\,\mathrm{e}^{-i\delta} \\ -c_{23}s_{12}-s_{23}c_{12}s_{13}\,\mathrm{e}^{i\delta} & c_{23}c_{12}-s_{23}s_{12}s_{13}\,\mathrm{e}^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12}-c_{23}c_{12}s_{13}\,\mathrm{e}^{i\delta} & -s_{23}c_{12}-c_{23}s_{12}s_{13}\,\mathrm{e}^{i\delta} & c_{13}c_{23} \end{pmatrix}. \quad (30)$$

Here $c_{12}=\cos\theta_{12}$, $s_{12}=\sin\theta_{12}$, etc.

2.2. $\bar\nu_e\to\bar\nu_e$ **Survival Probability.** Expressions for the three-neutrino $\bar\nu_e$ survival probabilities are important for the analysis of the data of the reactor neutrino experiments. From (18) and (19) for normal and inverted mass ordering, we have, respectively,

$$P^{\mathrm{NS}}(\bar\nu_e\to\bar\nu_e)=1-4\,|U_{e3}|^2(1-|U_{e3}|^2)\,\sin^2\Delta_A-4\,|U_{e1}|^2(1-|U_{e1}|^2)\,\sin^2\Delta_S- \\ -\;8\,|U_{e3}|^2|U_{e1}|^2\,\cos{(\Delta_A+\Delta_S)}\,\sin\Delta_A\sin\Delta_S \quad (31)$$

and

$$P^{\mathrm{IS}}(\bar\nu_e\to\bar\nu_e)=1-4\,|U_{e3}|^2(1-|U_{e3}|^2)\,\sin^2\Delta_A-4\,|U_{e2}|^2(1-|U_{e2}|^2)\,\sin^2\Delta_S- \\ -\,8\,|U_{e3}|^2|U_{e2}|^2\,\cos{(\Delta_A+\Delta_S)}\,\sin\Delta_A\sin\Delta_S. \quad (32)$$

Using the standard parameterization of the PMNS mixing matrix (30), from (18) and (19) for NS and IS, we have, respectively,

$$P^{\mathrm{NS}}(\bar\nu_e\to\bar\nu_e)=1-\sin^2 2\theta_{13}\sin^2\Delta_A-(\sin^2 2\theta_{12}c_{13}^2+\sin^2 2\theta_{13}c_{12}^4)\,\sin^2\Delta_S- \\ -\;2\sin^2 2\theta_{13}c_{12}^2\,\cos{(\Delta_A+\Delta_S)}\,\sin\Delta_A\sin\Delta_S \quad (33)$$

and

$$P^{\mathrm{IS}}(\bar\nu_e\to\bar\nu_e)=1-\sin^2 2\theta_{13}\sin^2\Delta_A-(\sin^2 2\theta_{12}c_{13}^2+\sin^2 2\theta_{13}s_{12}^4)\,\sin^2\Delta_S- \\ -\;2\sin^2 2\theta_{13}s_{12}^2\,\cos{(\Delta_A+\Delta_S)}\,\sin\Delta_A\sin\Delta_S. \quad (34)$$

Notice that $P^{\mathrm{IS}}(\bar\nu_e\to\bar\nu_e)$ can be obtained from $P^{\mathrm{NS}}(\bar\nu_e\to\bar\nu_e)$ by the change $c_{12}^2\to s_{12}^2$.

2.3. $\nu_\mu\to\nu_e$ **(**$\bar\nu_\mu\to\bar\nu_e$**) Appearance Probability.** Vacuum three-neutrino expressions for $\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e$ transition probabilities are important for the analysis of the data of long-baseline accelerator experiments, in which matter effects are negligible. From (18) and (19), we have

$$P^{\mathrm{NS}}(\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e)=4\,|U_{e3}|^2|U_{\mu3}|^2\,\sin^2\Delta_A+4\,|U_{e1}|^2|U_{\mu1}|^2\sin^2\Delta_S- \\ -\;8\,\mathrm{Re}\,(U_{e3}U_{\mu3}^*U_{e1}^*U_{\mu1})\,\cos{(\Delta_A+\Delta_S)}\sin\Delta_A\sin\Delta_S\mp \\ \mp\;8\,\mathrm{Im}\,(U_{e3}U_{\mu3}^*U_{e1}^*U_{\mu1})\,\sin{(\Delta_A+\Delta_S)}\sin\Delta_A\sin\Delta_S \quad (35)$$

and

$$P^{\mathrm{IS}}(\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e)=4\,|U_{e3}|^2|U_{\mu3}|^2\,\sin^2\Delta_A+4\,|U_{e2}|^2|U_{\mu2}|^2\,\sin^2\Delta_S- \\ -\,8\,\mathrm{Re}\,(U_{e3}U_{\mu3}^*U_{e2}^*U_{\mu2})\,\cos{(\Delta_A+\Delta_S)}\sin\Delta_A\sin\Delta_S\pm \\ \pm\;8\,\mathrm{Im}\,(U_{e3}U_{\mu3}^*U_{e2}^*U_{\mu2})\,\sin{(\Delta_A+\Delta_S)}\sin\Delta_A\sin\Delta_S. \quad (36)$$

Using the standard parameterization of the PMNS mixing matrix in the case of NS, we have

$$\begin{aligned} P^{\mathrm{NS}}(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) = \sin^2 2\theta_{13} s_{23}^2 \sin^2 \Delta_A + \\ + (\sin^2 2\theta_{12} c_{13}^2 c_{23}^2 + \sin^2 2\theta_{13} c_{12}^4 s_{23}^2 + K c_{12}^2 \cos\delta)\, \sin^2 \Delta_S + \\ + (2\sin^2 2\theta_{13} s_{23}^2 c_{12}^2 + K\cos\delta)\, \cos(\Delta_A + \Delta_S)\, \sin\Delta_A \sin\Delta_S \mp \\ \mp 8 J_{\mathrm{CP}}\, \sin(\Delta_A + \Delta_S) \sin\Delta_A \sin\Delta_S. \end{aligned} \quad (37)$$

Here

$$K = \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} c_{13}, \quad (38)$$

and

$$J_{\mathrm{CP}} = \frac{1}{8} K \sin\delta \quad (39)$$

is the Jarlskog invariant [29].

In the case of the inverted neutrino mass spectrum, we find

$$\begin{aligned} P^{\mathrm{IS}}(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) = \sin^2 2\theta_{13} s_{23}^2 \sin^2 \Delta_A + \\ + (\sin^2 2\theta_{12} c_{13}^2 c_{23}^2 + \sin^2 2\theta_{13} s_{12}^4 s_{23}^2 - K s_{12}^2 \cos\delta)\, \sin^2 \Delta_S + \\ + (2\sin^2 2\theta_{13} s_{23}^2 s_{12}^2 - K\cos\delta)\, \cos(\Delta_A + \Delta_S)\, \sin\Delta_A \sin\Delta_S \mp \\ \mp 8 J_{\mathrm{CP}}\, \sin(\Delta_A + \Delta_S) \sin\Delta_A \sin\Delta_S. \end{aligned} \quad (40)$$

For the CP asymmetry in the case of NS (IS), we have

$$A_{e\mu}^{\mathrm{CP}} = -16 J_{\mathrm{CP}}\, \sin(\Delta_A + \Delta_S)\, \sin\Delta_A \sin\Delta_S. \quad (41)$$

2.4. $\nu_\mu \to \nu_\mu$ $(\bar{\nu}_\mu \to \bar{\nu}_\mu)$ **Survival Probability.** From (11) for $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$ survival probability in the case of the normal and inverted mass ordering, we have, correspondingly,

$$\begin{aligned} P^{\mathrm{NS}}(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu) = 1 - 4\,|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2)\, \sin^2 \Delta_A - 4\,|U_{\mu 1}|^2 (1 - |U_{\mu 1}|^2)\, \sin^2 \Delta_S - \\ - 8\,|U_{\mu 3}|^2 |U_{\mu 1}|^2 \cos(\Delta_A + \Delta_S)\, \sin\Delta_A \sin\Delta_S \end{aligned} \quad (42)$$

and

$$\begin{aligned} P^{\mathrm{IS}}(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu) = 1 - 4\,|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2)\, \sin^2 \Delta_A - 4\,|U_{\mu 2}|^2 (1 - |U_{\mu 2}|^2)\, \sin^2 \Delta_S - \\ - 8\,|U_{\mu 3}|^2 |U_{\mu 2}|^2 \cos(\Delta_A + \Delta_S)\, \sin\Delta_A \sin\Delta_S. \end{aligned} \quad (43)$$

Using standard parameterization of the PMNS matrix, we find

$$\begin{aligned} P^{\mathrm{NS}}(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu) = 1 - (\sin^2 2\theta_{23} c_{13}^2 + \sin^2 2\theta_{13} s_{23}^4)\, \sin^2 \Delta_A - \\ - 4\left(c_{23}^2 s_{12}^2 + s_{23}^2 c_{12}^2 s_{13}^2 + \frac{K\cos\delta}{4c_{13}^2}\right)\left(1 - c_{23}^2 s_{12}^2 - s_{23}^2 c_{12}^2 s_{13}^2 - \frac{K\cos\delta}{4c_{13}^2}\right) \sin^2 \Delta_S - \\ - 2(\sin^2 2\theta_{23} c_{13}^2 s_{12}^2 + \sin^2 2\theta_{13} c_{12}^2 s_{23}^4 + \\ + K s_{23}^2 \cos\delta)\, \cos(\Delta_A + \Delta_A) \sin\Delta_A \sin\Delta_S \end{aligned} \quad (44)$$

and

$$P^{\mathrm{IS}}(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu) = 1 - (\sin^2 2\theta_{23} c_{13}^2 + \sin^2 2\theta_{13} s_{23}^4) \sin^2 \Delta_A - \\ - 4 \left(c_{23}^2 c_{12}^2 + s_{23}^2 s_{12}^2 s_{13}^2 - \frac{K \cos\delta}{4 c_{13}^2} \right) \left(1 - c_{23}^2 c_{12}^2 - s_{23}^2 s_{12}^2 s_{13}^2 + \frac{K \cos\delta}{4 c_{13}^2} \right) \sin^2 \Delta_S - \\ - 2(\sin^2 2\theta_{23} c_{13}^2 c_{12}^2 + \sin^2 2\theta_{13} s_{12}^2 s_{23}^4 - K s_{23}^2 \cos\delta) \cos(\Delta_A + \Delta_A) \sin\Delta_A \sin\Delta_S, \quad (45)$$

where K is given by the relation (38). Notice that in the case of long-baseline experiments with $\Delta_A \simeq 1$ (MINOS, T2K), the term proportional to $\sin^2 \Delta_S$ gives a very small contribution to the probability ($\sin^2 \Delta_S \simeq 10^{-3}$).

2.5. A Comment on the Possibility to Reveal the Character of Neutrino Mass Spectrum in Future Reactor Experiments. Dependence on the neutrino mass ordering of the probability of reactor ν_e's to survive was noticed in [30], in which reactor CHOOZ data were analyzed in the framework of three-neutrino mixing. A reactor experiment with reactor-detector distance of 20–30 km, which could reveal the character of neutrino mass spectrum, was proposed in [31, 32]. Later in numerous papers a possibility to determine the neutrino mass ordering in an intermediate-baseline reactor experiment ($\sim$ 50 km) was analyzed in detail (see [33] and references therein). Two reactor experiments JUNO [16] and RENO-50 [17], in which the neutrino mass ordering is planned to be determined, are under preparation at present.

The $\bar{\nu}_e \to \bar{\nu}_e$ survival probability (expressions (18) and (19)) can be written in the form

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_A - 4X(1-X) \sin^2 \Delta_S - \\ - 8 \sin^2 \theta_{13} X \cos(\Delta_A + \Delta_S) \sin\Delta_A \sin\Delta_S. \quad (46)$$

In the case of the normal and inverted mass spectra, we have, respectively,

$$X = X_{\mathrm{NS}} = \cos^2 \theta_{13} \cos^2 \theta_{12} \quad (47)$$

and

$$X = X_{\mathrm{IS}} = \cos^2 \theta_{13} \sin^2 \theta_{12}. \quad (48)$$

From the fit of the data that will be obtained in the reactor JUNO experiment after six years of data taking, the parameters Δm_S^2, Δm_A^2, and $\sin^2 2\theta_{12}$ will be determined with accuracy better than 1% (see, for example, [34]). In the Daya Bay experiment the parameter $\sin^2 \theta_{13}$ can be determined with accuracy $\sim$ 4%. Such a precision will, apparently, allow one to distinguish the value $X \simeq 0.682$ (NS) from the value $X \simeq 0.295$ (IS) (we used best-fit values $\sin^2 \theta_{12} = 0.302$, $\sin^2 \theta_{13} = 0.0227$).

3. TRANSITIONS OF FLAVOR NEUTRINOS INTO STERILE STATES

Data of atmospheric, solar, reactor, and accelerator neutrino oscillation experiments are described by the three-neutrino mixing with two-neutrino mass-squared differences $\Delta m_S^2 \simeq 7.5 \cdot 10^{-5}$ eV^2 and $\Delta m_A^2 \simeq 2.4 \cdot 10^{-5}$ eV^2. There exist, however, indications in favor of neutrino oscillations with mass-squared difference(s) about 1 eV^2. These indications were obtained in the following short-baseline neutrino experiments (with L ranging from a few meters to about 500 m):

1. In the LSND experiment [35]. In this experiment neutrinos were produced in decays of π^+'s and μ^+'s. Appearance of $\bar{\nu}_e$'s (presumably produced in the transition $\bar{\nu}_\mu \to \bar{\nu}_e$) was detected. In the MiniBooNE experiment [36,37]. In this experiment an excess of low energy ν_e's ($\bar{\nu}_e's$) was observed.

2. In the old reactor neutrino experiments. Data of these experiments were reanalyzed in [38]. In this new analysis recent calculations of the reactor neutrino flux [39,40] were used.

3. In the calibration experiments, performed with radioactive sources by the GALLEX [41] and SAGE [42] collaborations. In these experiments a deficit of ν_e's was observed.

In order to interpret these data in terms of neutrino oscillations, it is necessary to assume, that, in addition to the flavor neutrinos ν_e, ν_μ, ν_τ, sterile neutrinos exist as well.

Let us first consider $3+1$ scheme with three close neutrino masses m_i ($i = 1, 2, 3$) and the forth mass m_4 separated from m_i by about 1 eV gap. We choose $p = 1$. In the region of L/E sensitive to large neutrino mass-squared difference $\left(\dfrac{\Delta m_{14}^2 L}{4E} \gtrsim 1\right)$, we have $\Delta_{12} \simeq \Delta_{13} \simeq 0$. From (11) we find in this case

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_{\alpha'}) = \delta_{\alpha'\alpha} - 4|U_{\alpha 4}|^2(\delta_{\alpha'\alpha} - |U_{\alpha' 4}|^2)\sin^2\Delta_{14}. \tag{49}$$

From this expression for $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ appearance probability, $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$ disappearance probabilities, we have, respectively, the following expressions:

$$P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) = \sin^2 2\theta_{e\mu}\sin^2\Delta_{14}, \tag{50}$$

$$P(\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e) = 1 - \sin^2 2\theta_{ee}\sin^2\Delta_{14}, \tag{51}$$

$$P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu) = 1 - \sin^2 2\theta_{\mu\mu}\sin^2\Delta_{14}. \tag{52}$$

Here

$$\begin{aligned} \sin^2 2\theta_{e\mu} &= 4|U_{e4}|^2|U_{\mu 4}|^2, \\ \sin^2 2\theta_{ee} &= 4|U_{e4}|^2(1 - |U_{e4}|^2), \\ \sin^2 2\theta_{\mu\mu} &= 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2). \end{aligned} \tag{53}$$

Notice that the global analysis of all short-baseline neutrino data [43,44] revealed the inconsistency (tension) of existing short-baseline data.

Let us consider a more complicated $3+2$ scheme with two masses m_4 and m_5 separated from three close masses m_i ($i = 1, 2, 3$) by about 1 eV gaps. We choose $p = 1$. In the region of L/E sensitive to large neutrino mass-squared differences Δm_{14}^2 and Δm_{15}^2, we have $\Delta_{12} \simeq \Delta_{13} \simeq 0$. From (11) we find the following expression for $\overset{(-)}{\nu}_l$ ($l = e, \mu$) survival probability:

$$\begin{aligned} P(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_l) = 1 - 4\,|U_{l4}|^2(1 - |U_{l4}|^2)\,\sin^2\Delta_{14} - 4\,|U_{l5}|^2(1 - |U_{l5}|^2)\,\sin^2\Delta_{15} + \\ + 8\,|U_{l5}|^2|U_{l4}|^2\cos(\Delta_{15} - \Delta_{14})\,\sin\Delta_{15}\sin\Delta_{14}. \end{aligned} \tag{54}$$

For the probability of the transitions $\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}$, $\;l' \neq l$, we find

$$P(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) = 4|U_{l'4}|^2|U_{l4}|^2 \sin^2\Delta_{14} + 4|U_{l'5}|^2|U_{l5}|^2 \sin^2\Delta_{15} + \\ + 8\,\mathrm{Re}\,(U_{l'5}U^*_{l5}U^*_{l'4}U_{l4})\,\cos(\Delta_{15}-\Delta_{14})\sin\Delta_{15}\sin\Delta_{14} \pm \\ \pm 8\,\mathrm{Im}\,(U_{l'5}U^*_{l5}U^*_{l'4}U_{l4})\,\sin(\Delta_{15}-\Delta_{14})\sin\Delta_{15}\sin\Delta_{14}. \quad (55)$$

CONCLUSIONS

Discovery of neutrino oscillations is one of the most important recent discoveries in particle physics. After the first stage of investigation of this new phenomenon, now with the measurement of the small parameter $\sin^2\theta_{13} \simeq 2.5\cdot 10^{-2}$ the era of precision study has started. Such fundamental problems of neutrino masses and mixing as

- what is the ordering of neutrino masses (normal or inverted),
- what is the value of the CP phase δ,
- what are precise values (with accuracies better than 1%) of other oscillation parameters,
- is the number of massive neutrinos equal to the number of flavor neutrinos (three) or larger than three (do sterile neutrinos exist)

are planned to be solved by future neutrino oscillation experiments.

At the moment there is no consensus in definition of the large (atmospheric) neutrino mass-squared difference: in various experimental and theoretical papers this parameter is defined differently. Today it is not so important but with future precision different “atmospheric mass-squared differences” will be distinguishable. We believe that a universal definition must be accepted.

In this paper, for the general case of n-neutrino mixing we propose a convenient expression for neutrino transition probability in vacuum, in which the unitarity of the mixing matrix is fully employed and freedom of the common phase is used. As a result, only independent quantities (including mass-squared differences) enter into expression for the transition probability.

On the basis of the proposed expression, we discuss the problem of the atmospheric neutrino mass-squared difference and comment on the possibility to reveal the character of the neutrino mass spectrum in future reactor neutrino experiments.

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