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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

DETECTION OF THE Z' BOSON AT THE ILC

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Integral one- or two-parameter observables for detecting the Abelian Z' gauge boson in $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ process at the ILC energies are proposed. They are based on the differential cross section of deviations from the Standard Model predictions calculated with a low-energy effective Lagrangian and taking into consideration the relations between the Z' couplings to the fermions derived before. The observables with 0.99 efficiency fit the axial-vector $a^2_{Z'}$, the product of vector couplings $v_e v_\mu (v_e v_\tau)$, and the mass $m_{Z'}$. Determination of the basic Z' model is discussed.

Предлагаются интегральные одно- и двухпараметрические наблюдаемые для детектирования абелева калибровочного Z'-бозона в процессе $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ при энергиях ускорителя ILC. Они основаны на дифференциальных сечениях отклонений от предсказаний Стандартной модели, рассчитанных с помощью эффективного низкоэнергетического лагранжиана, с учетом полученных ранее соотношений между константами связи Z' с фермионами. Наблюдаемые с эффективностью 0,99 фитируют квадрат аксиально-векторной константы связи $a^2_{Z'}$, произведение векторных констант связи $v_e v_\mu (v_e v_\tau)$ и массу $m_{Z'}$. Обсуждается также определение базовой модели для Z'-бозона.

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INTRODUCTION

Nowadays searching for new heavy particles beyond the energy scale of the Standard Model (SM) is established on the basis of experimental data accumulated at hadron colliders such as the Tevatron and the LHC. Further experiments will be continued at the ILC having energies of $\sim 500-1000$ GeV in the center-of-mass of beams, but much better precision of measurements due to point-like structure of leptons and experiments with polarized initial and final fermions.

One of the expected heavy particles is the Abelian Z' gauge boson which is related with an additional $\tilde{U}(1)$ group. A detailed description of the Z' is given in [1–4]. Searches for this particle have been already established within the data of the LEP experiments in either model-dependent [5] or model-independent [8] approaches and the Tevatron data [10, 11]. Modern model-dependent estimates predict that the mass $m_{Z'}$ is larger than $2.5-2.9$ TeV [12, 14]. So, at the future ILC experiments the Z' will be investigated as the virtual far from resonance state.

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At present about a hundred Z' models are discussed in the literature. They cover either the Abelian Z' or the other type of massive neutral vector particle. In the present paper, we deal only with the former case. In model-dependent searches noted, the most popular models, such as LR, ALR, χ, ψ, η, B–L, SSM, have been investigated and the Abelian Z' mass has been estimated. These models are also used as benchmarks in introducing the effective observables for future experiments at the ILC [15, 16].

Recent investigation of perspective variables for identification of the Z' models [16], in particular, allows us to conclude that a model-independent approach is also very desirable. An important feature of this method is that not only the Z' mass but also the couplings to the SM fermions are unknown parameters which must be fitted in experiments. These couplings are usually considered as independent arbitrary numbers. However, this is not the case and they are correlated parameters, if some requirements, which this model has to satisfy, are assumed. For instance, we could believe that the basic model is a renormalizable one. Hence, correlations follow and the amount of free low-energy parameters reduces. Moreover, after discovery of the Higgs boson with not heavy mass $m_H = 126$ GeV this requirement and hence the accounting for scalar particles became reasonable in experimental data analysis. This type of analysis is in between the standard model-dependent and model-independent methods. In what follows, we search for the Abelian Z' boson coming from an extended renormalizable model and containing one or two Higgs doublet SM as a subgroup. There are numerous models of such type. In particular, all E_6 motivated models and the ones mentioned above belong to this class. In general, the renormalizability requirement admits two sets of correlations between the low-energy couplings [18]. The first set is applied in what follows (Eq. (4)). The second one corresponds to a massive neutral vector particle interacting only with left-handed fermion species. It covers the other class of models where the SM is not a subgroup (see, for example, [20]). Searches for a particle of this type require other observables and separate analysis. For more details, see [18, 19].

In the present paper, we search for the Abelian Z' by analyzing the deviations of the differential cross sections for the process $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ from the SM predictions considered at center-of-mass energies 500–1000 GeV. We introduce the integral observables, $A(E, m_{Z'})$ (6), giving a possibility for estimating both the axial-vector coupling of the Abelian Z' to the SM fermions $a_{Z'}$ and the mass $m_{Z'}$, and the $V(E, m_{Z'})$ (9), for fitting the products of vector couplings $v_e v_\mu$, $v_e v_\tau$ and the mass $m_{Z'}$. Our analysis is carried out on the basis of the effective low-energy Lagrangian [6, 7] describing the interactions of the Z' with the SM fermions. At given energies and expected particle masses, distinguishable properties of the factors at couplings entering the cross section are observed that gives a possibility for introducing the announced observables. Their values can be used in subsequent determination of the basic Abelian Z' model.

1. CROSS SECTION FOR THE Z' DETECTIONS

In what follows, for definiteness, we concentrate on the case of one-doublet SM, as it is usually considered in the analysis of the present day experiment data. At low energies, the Z' boson can manifest itself as virtual intermediate state through couplings to the SM fermions and scalars. Moreover, the Z boson couplings are also modified due to a Z–Z' mixing. As is known (see reviews [2, 4, 8]), significant signals beyond the SM can be inspired by the couplings of renormalizable types. They can be described by adding new $\tilde{U}(1)$-terms to the electroweak covariant derivatives D^{ew} in an effective low-energy Lagrangian [6, 7] (see also [8]). In this approach, the Z–Z' mixing angle θ_0 is determined by the coupling $\tilde{Y}_\phi$ describing the coupling of the $\tilde{U}(1)$ gauge boson with the scalar doublet field as follows:

$$\theta_0 = \frac{\tilde{g}\sin\theta_W\cos\theta_W}{\sqrt{4\pi\alpha_{\text{em}}}}\frac{m_Z^2}{m_{Z'}^2}\tilde{Y}_\phi + \delta, \tag{1}$$

where θ_W is the SM Weinberg angle, α_{em} is an electromagnetic fine structure constant and $\delta \sim O(m_Z^4/m_{Z'}^4)$. Although the mixing angle is a small quantity of the order $m_{Z'}^{-2}$, it contributes to the Z boson exchange amplitude and has to be accounted for in general. The effective Lagrangian of interactions between the fermions and the Z and Z' mass eigenstates reads (see, for example, [8]):

$$\mathcal{L}_{Z\bar{f}f} = \frac{1}{2}Z_\mu\bar{f}\gamma^\mu\left[(v_{fZ}^{\text{SM}} + \gamma^5 a_{fZ}^{\text{SM}})\cos\theta_0 + (v_f + \gamma^5 a_f)\sin\theta_0\right]f, \tag{2}$$

$$\mathcal{L}_{Z'\bar{f}f} = \frac{1}{2}Z'_\mu\bar{f}\gamma^\mu\left[(v_f + \gamma^5 a_f)\cos\theta_0 - (v_{fZ}^{\text{SM}} + \gamma^5 a_{fZ}^{\text{SM}})\sin\theta_0\right]f, \tag{3}$$

where f is an arbitrary SM fermion state; v_{fZ}^{SM}, a_{fZ}^{SM} are the SM couplings of the Z boson.

As it occurs, if the extended model is renormalizable and contains the SM as a subgroup, the relations between the couplings hold ([8, 17, 18]):

$$v_f - a_f = v_{f^*} - a_{f^*}, \quad a_f = T_{3f}\tilde{g}\tilde{Y}_\phi. \tag{4}$$

Here f and f^* are the partners of the $SU(2)_L$ fermion doublet ($l^* = \nu_l$, $\nu^* = l$, $q_u^* = q_d$, and $q_d^* = q_u$), T_{3f} is the third component of weak isospin. They can be also derived the other way — by imposing the requirement of invariance of the SM Yukawa term with respect to the $\tilde{U}(1)$ gauge transformations [9]. Therefore, the relations (4) are independent of the number of scalar field doublets.

The couplings of the Abelian Z' to the axial-vector fermion current have a universal absolute value proportional to the Z' coupling to the scalar doublet. Then, the Z–Z' mixing angle (1) can be determined by the axial-vector coupling. Because of the universality, we will write in what follows a instead of a_f.

The relations (4) essentially influence the kinematics of scattering processes and give a possibility to uniquely detect the virtual Abelian Z' boson.

Let us consider the process $e^+e^- \to l^+l^-$ ($l = \mu, \tau$) with the nonpolarized initial and final fermions. The beam polarization is not very important for our consideration and will be discussed below. Two classes of diagrams have to be taken into consideration. The first one includes the pure SM graphs. This part should be calculated as accurately as possible. The second class includes heavy Z' boson as the virtual state described by the effective Lagrangian (2) and the scalar particle contributions. We assume that the Z' is decoupled and not excited inside loops at the ILC energies. The tree-level diagram $e^+e^- \to Z' \to l^+l^-$ defines a leading contribution to the cross section. It is enough to take into account this diagram to estimate the Z' signals. The cross section includes the contribution of the interference of the SM amplitudes with the Z' exchange amplitude (having the order $\sim a^2, v_f a$) and the squared of the latter one (of the order $\sim a^4, v_f^4$). Since the couplings of the Z' are small, the last contribution can be neglected at far from the Z' resonance energies. In our calculations, radiative corrections are incorporated with the Z'-exchange diagram in the improved Born approximation and the relations (4).

Within these assumptions, the deviation of the differential cross section for the process $e^+e^- \to \mu^+\mu^-$ $(\tau^+\tau^-)$ can be presented in the form

$$\Delta\sigma(z) = \frac{d\sigma}{dz} - \frac{d\sigma^{\rm SM}}{dz} = f_1^{\mu\mu}(z)\frac{a^2}{m_{Z'}^2} + f_2^{\mu\mu}(z)\frac{v_e v_\mu}{m_{Z'}^2} + f_3^{\mu\mu}(z)\frac{a v_e}{m_{Z'}^2} + f_4^{\mu\mu}(z)\frac{a v_\mu}{m_{Z'}^2}, \qquad (5)$$

where $z = \cos\theta$ of scattering angle θ. Equation (5) is our definition of the Z' signal. This cross section accounts for the relations (4) through the known dimensionless functions $f_1(z)$, $f_3(z)$, $f_4(z)$, since the mixing angle θ_0 is substituted by the axial-vector coupling a.

2. ESTIMATION OF a^2 AND $m_{Z'}$

According to Eq. (5), the deviation $\Delta\sigma(z)$ is described by four factors $f_i(z)$. Let us investigate their behavior assuming that couplings a, v_f have the same order of magnitude. In this case the kinematics properties of $f_i(z)$ can be elucidated. To realize that, we have used the packages FeynArts [21], FormCalc and LoopTools [22], and Mathematica. The results of calculations are gathered in Appendix.

For definiteness, in the Figure we show the behavior for energy $E = 500$ GeV in the e^+e^- center-of-mass and the mass $m_{Z'} = 2500$ GeV. Here, some remark is needed. As was reported in [12–14], the lower bound on the mass $m_{Z'}$ obtained from the data on the Drell–Yan process at the LHC is $m_{Z'} \geqslant 2.5-2.9$ TeV. It was estimated assuming the narrow resonances having the widths $\Gamma_{Z'}$ of the order $\Gamma_{Z'}/m_{Z'} \sim 0.01$. On the other hand, as was argued in [10], the resonances with not small $\Gamma_{Z'}$ are not excluded. They could considerably decrease the value of the lower bound on $m_{Z'}$. Below, to present the results, we take the ratio $\Gamma_{Z'}/m_{Z'} \sim 0.1$ (the results for narrow resonance are similar).

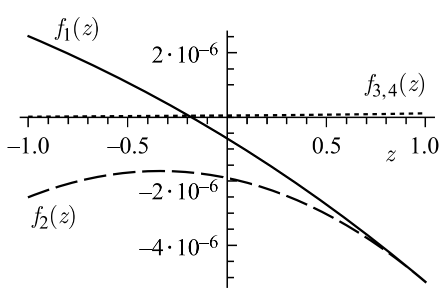

Fig. 1. Behavior of factors $f_1(z)$, $f_2(z)$, $f_{3,4}(z)$ for $m_{Z'} = 2500$ GeV, width $\Gamma_{Z'} = 250$ GeV for energy $E = 500$ GeV

In the plot, the function $f_1(z)$ is presented as solid line, the $f_2(z)$ is shown as dashed line, and the functions $f_{3,4}(z)$ are shown as dotted line. The $f_{3,4}(z)$ coincide at high energies when one can neglect fermion masses. The function $f_1(z)$ has opposite signs for the forward and backward beams. This is in contrast to the factors $f_2(z)$ and $f_{3,4}(z)$. The former is negative and the latter is positive. Moreover, the factors $f_{3,4}(z)$ are suppressed by two orders of magnitude as compared to the $f_1(z)$ and $f_2(z)$. The shown angular dependence is typical and takes place in the wide mass interval and for other energies, for example, 1000 GeV. The mass interval $1.5 \leqslant m_{Z'} \leqslant 4$ TeV was investigated. This behavior makes reasonable introducing the integral observable which picks out the contribution coming from the first a^2-dependent term in Eq. (5). Really, we can integrate $f_2(z)$ in the intervals $-1 < z < -0.2$ (where the function $f_1(z)$ is positive) and $-0.2 < z < z^*$ (where $f_1(z)$ is negative) and specify the limit z^* in such a way that the difference turns to zero: $\left(\int\limits_{-1}^{-0.2} - \int\limits_{-0.2}^{z^*}\right) f_2^{\mu\mu}(z)\, dz = 0$. Since $f_2(z)$ is a definite sign, this point always exists. At the same time, due to opposite signs of $f_1(z)$ in these intervals and sign definiteness of $f_{3,4}(z)$, the difference is mainly determined by the first term in (5). The partial cancelation of the contributions coming from $f_{3,4}(z)$ takes place. As a result, the value of the universal coupling constant a^2 can be estimated with high accuracy. As explicit calculations showed, the upper limit of integration equals to $z^* = 0.489$ for a wide interval of both the mass $m_{Z'}$ and beam energies E. It is also important that the function $f_1(z)$ changes its sign at the point $z = -0.2$ for all energies and masses investigated. On these grounds we introduce the observable for model-independent estimation of the a^2 and $m_{Z'}$:

$$A(E, m_{Z'}) = \left(\int\limits_{-1}^{-0.2} - \int\limits_{-0.2}^{z^*}\right) \left(\frac{d\sigma}{dz} - \frac{d\sigma^{\mathrm{SM}}}{dz}\right) dz. \tag{6}$$

Here, the lower and upper limits of integration are theoretical bounds. They can be substituted by others corresponding to the actual set up of the experiments. For example, for the lower limit $z_{\mathrm{lower}} = -0.9$, that is close to the values of measured scattering angles, $10 < \theta < 170°$, planned for the ILC detectors [23], the upper limit is $z^* = 0.406$. To complete this section, we adduce the values of the observable (6) for the number of the mass and energy values. In Table 1, in the first, second and third columns the energy, mass and width values (expressed in GeV) are given, correspondingly. In the fourth column the contribution coming from $f_1^{\mu\mu}(z)$ is adduced. In the fifth and sixth columns the values of the contributions coming from the factors $f_{3,4}^{\mu\mu}(z)$, $f_2^{\mu\mu}(z)$ of Eq. (5) are shown. As we see, the contributions coming from the factors $f_{2,3,4}^{\mu\mu}$ are two orders less compared to the contribution from $f_1^{\mu\mu}(z)$ and can be neglected. In this way we obtain the two-parameter observable for estimation of a^2 and $m_{Z'}$. Since the contribution of the factor $f_2(z)$ is chosen to be zero, $A(E, m_{Z'})$ is determined by two couplings: a^2 and av_μ. The efficiency of the observable is determined from the relation

$$\kappa_A = \frac{|f_1^{\mu\mu}|}{|f_1^{\mu\mu}| + |f_{3,4}^{\mu\mu}|}. \tag{7}$$

Here the quantities $|f_i^{\mu\mu}|, i = 1, 3, 4$, mark the integrals $\left(\int\limits_{-1}^{-0.2} - \int\limits_{-0.2}^{z^*}\right) f_i^{\mu\mu}(z)\, dz > 0$. From Table 1 it can be estimated as $\kappa_A = 0.9896$ for all the given energy and mass values.

Table 1. **Observable $A(E, m_{Z'})$ for the interval $[-1, 0.489]$. Energy, $m_{Z'}$, and $\Gamma_{Z'}$ are given in GeV**

Energy	$m_{Z'}$	$\Gamma_{Z'}$	$f_1(z)$	$f_{3,4}(z)$	$f_2(z)$
500	2500	250	$9.03452 \cdot 10^{-7}$	$-9.48604 \cdot 10^{-9}$	$4.93451 \cdot 10^{-10}$
500	3000	300	$8.55036 \cdot 10^{-7}$	$-8.97772 \cdot 10^{-9}$	$4.67504 \cdot 10^{-10}$
1000	2500	250	$1.96179 \cdot 10^{-6}$	$-2.058 \cdot 10^{-8}$	$-3.67104 \cdot 10^{-9}$
1000	3000	300	$1.3263 \cdot 10^{-6}$	$-1.39139 \cdot 10^{-8}$	$-2.4791 \cdot 10^{-9}$

The signature of the observable is also important. The coupling a^2 is positive and the integral $|f_1^{\mu\mu}|$ is also positive by construction. So, the positivity of the $A(E, m_{Z'})$ is the distinguishable signal of the Z' boson.

3. OBSERVABLE FOR ESTIMATION OF $m_{Z'}$

Interesting application of the observable $A(E, m_{Z'})$ (6) is related with the model-independent determination of the mass $m_{Z'}$. Really, the observable (6) includes the factor a^2 which is canceled in the ratio $R_A^{\rm exp} = \dfrac{A(E_1, m_{Z'})}{A(E_2, m_{Z'})}$, so that its behavior can be used in estimation of $m_{Z'}$. We can consider two cross sections with close energies E_1 and $E_2 = E_1 + \Delta E$ and write

$$R_A^{\rm exp} = \frac{A(E_1, m_{Z'})}{A(E_2, m_{Z'})} = 1 - \frac{\partial \ln A(E_1, m_{Z'})}{\partial E_1} \Delta E. \tag{8}$$

As a theoretical curve $R_A^{\rm th}$, the function $f_1^{\mu\mu}$ from Eq. (5) has to be substituted in Eq. (8) instead of $A(E_1, m_{Z'})$, because contributions of all other form-factors are suppressed in the difference. As a result, we obtain the observable dependent only on $m_{Z'}$. Hence, the value of the mass can be estimated by using a standard χ^2 method.

4. ESTIMATION OF $v_e v_\mu$ $(v_e v_\tau)$

The behavior of the factors shown in the Figure gives also a possibility of introducing the observable for model-independent determination of the product $v_e v_\mu (v_e v_\tau)$. As we see from the plots and Table 1, the contributions of the factors standing at $a v_\mu$ and $a v_e$ are suppressed and can be neglected. So, to exclude the contribution of the a^2-dependent term we have to integrate the differential cross section $\Delta\sigma(z)$ (5) over z in the interval $-1 \leqslant z \leqslant z^v$ and select the upper limit from the requirement $\int\limits_{-1}^{z^v} f_1^{\mu\mu}(z)\, dz = 0$. Hence, we obtain the observable $V_{e\mu}(E, m_{Z'})$ for estimation of $v_e v_\mu$ (or $v_e v_\tau$):

$$V_{e\mu}(E, m_{Z'}) = \int\limits_{-1}^{z^v} \left(\frac{d\sigma}{dz} - \frac{d\sigma^{\rm SM}}{dz} \right) dz, \tag{9}$$

where the limit z_v depends on the energy E and mass $m_{Z'}$.

Let us adduce the values of z^v and $V_{e\mu}(E, m_{Z'})$ for a number of energy and mass values.

In Table 2, the first three columns show the center-of-mass energy, mass, and width, as in Table 1. In the fourth column the cosine of boundary angles is adduced. In the last two columns the corresponding values of $V_{e\mu} m_{Z'}^2$ and the contributions of the factor at the product $a v_\mu$ are presented. As above, these limits can be substituted by other necessary ones. The contributions of the factors $\sim a v_e, a v_\mu$ are negligibly small and can be omitted in the total.

Table 2. **Upper limit z^v and the value $V_{e\mu}(E, m_{Z'})$. Energy, $m_{Z'}$, and $\Gamma_{Z'}$ are given in GeV**

Energy	$m_{Z'}$	$\Gamma_{Z'}$	z^v	$V_{e\mu} m_{Z'}^2$	$AV_{e\mu} m_{Z'}^2$
500	2500	250	0.567466	$-1.50333 \cdot 10^{-6}$	$1.65644 \cdot 10^{-8}$
500	3000	300	0.5675	$-1.42282 \cdot 10^{-6}$	$1.56777 \cdot 10^{-8}$
1000	2500	250	0.570118	$-3.31411 \cdot 10^{-6}$	$3.52717 \cdot 10^{-8}$
1000	3000	300	0.570115	$-2.24064 \cdot 10^{-6}$	$2.38447 \cdot 10^{-8}$

The efficiency of the observable $V(E, m_{Z'})$ is determined analogously to the κ_A (7), according to the condition

$$\kappa_V = \frac{|f_2^{\mu\mu}|}{|f_2^{\mu\mu}| + |f_{3,4}^{\mu\mu}|}, \tag{10}$$

where now the quantities $|f_i^{\mu\mu}|, i = 2, 3, 4$, mark the integrals over the interval $-1 < z < z^v$. The efficiency is estimated as $\kappa_V = 0.9891$. Again we obtain a very efficient observable.

Since the factor $f_2(z)$ is negative, the sign of the observable $V(E, m_{Z'})$ depends on the sign of the product $v_e v_\mu$. If this value is positive, we have a negatively defined observable.

As a result, we have obtained the two-parameter observable for fitting of the vector coupling products $v_e v_\mu, v_e v_\tau$, and the mass $m_{Z'}$.

CONCLUSIONS

We have investigated the process $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ for unpolarized initial and final fermions at the center-of-mass energies 500–1000 GeV and introduced the observables for model-independent detections of the Abelian Z' boson. In this procedure the relations (4) between couplings of the Z' to the SM fermions have been used. With accounting of (4), the factors entering the differential cross section (5) exhibit features giving a possibility for introducing the integral observables (6) and (9) dependent mainly on only one coupling a^2, or $v_e v_\mu$ ($v_e v_\mu$), correspondingly, and the mass $m_{Z'}$. Using them, all these parameters can be estimated within one- or two-parameter fits.

On the basis of these observables, the estimate of the Z' mass can be done. For this purpose the data on two differential cross sections with close energies are needed. Then the mass $m_{Z'}$ can be found from the observable $R_A^{\rm exp}$ (8) related to the observable (6) or from the similar observable $R_{e\mu}^{\rm exp}$ related to the $V_{e\mu}$ (9). To obtain the latter, we have to substitute the function $f_1^{\mu\mu}$ in the theoretical expression $R_A^{\rm th}$ by the $f_2^{\mu\mu}$ from (5) and make obvious modifications in Sec. 3. The same can be done for the τ-lepton final states. As a result, the mass $m_{Z'}$ can be fitted by using two different factor functions, so that there are two ways of measuring this parameter.

Let us consider the role of polarizations P_{e^+}, P_{e^-}. For the s-channel processes the cross section for polarized beams reads (see, for example, Eq. (3) in [24]):

$$\sigma_{P_{e^+}P_{e^-}} = (1 - P_{e^+}P_{e^-})[1 - P_{\rm eff} A_{\rm LR}]\sigma^{\rm un}, \tag{11}$$

where $A_{\rm LR}$ is a left-right asymmetry, $P_{\rm eff} = (P_{e^+} - P_{e^-})/(P_{e^+}P_{e^-} - 1)$ is an effective polarization, and $\sigma^{\rm un}$ is a contribution of unpolarized beams. As we see, the cross section $\sigma_{P_{e^+}P_{e^-}}$ is proportional to the unpolarized one. The polarization-dependent factors modify the effective luminosity of the process that does not change qualitatively the results obtained for unpolarized beams.

What can be verified next is the family independence of v_f couplings. Really, the ratio of the V_μ and V_τ observables

$$R_v = \frac{V(E, m_{Z'})_\mu}{V(E, m_{Z'})_\tau} = \frac{v_\mu}{v_\tau} \tag{12}$$

depends on the coupling values and has to be united in the case of the family independency. It can be simply checked.

Thus, the signature of the observables — positive sign of $A(E, m_{Z'})$ and negative sign of $V(E, m_{Z'})$ — is also the signal of the Abelian Z' boson.

The developed model-independent approach can be used as an additional way for detecting the Abelian Z' boson at the ILC, as well as determining the model which it has to belong. The found values of the couplings can be compared with the parameters entering specific Z' models. As a result, the number of perspective candidates can be considerably reduced. This is very important because the identification reach for the Z' models at the LHC is estimated as $m_{Z'} \leqslant 2.2{-}2.3$ TeV, whereas the nowadays lower bound is $\sim 2.5{-}2.9$ TeV [12, 14]. So, most probably, the basic model will not be identified at this collider at all. This problem must be attacked at the ILC.

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APPENDIX

In this appendix, we adduce the expression for the SM differential cross section and the factors $f_i(z, E)$ entering Eq. (5) and calculated in the improved Born approximation. To realize that, we have used the packages FeynArts [21], FormCalc and LoopTools [22], and Mathematica. The lepton masses are set to zero. For convenience, here we denote cosine of scattering angle as $x = z = \cos\theta$ and introduce the standard notations: $s_W = \sin\theta_W, c_W = \cos\theta_W$, where θ_W is the Weinberg angle, α is a fine structure constant.

The differential cross section reads:

$$\frac{\partial\sigma}{\partial x} = \sigma_{\mathrm{SM}} + a^2 f_1^{\mu\mu}(x) + v_e v_\mu f_2^{\mu\mu}(x) + a v_e f_3^{\mu\mu}(x) + a v_\mu f_4^{\mu\mu}(x). \tag{13}$$

In contrast to Eq. (5), the factor $m_{Z'}^{-2}$ is incorporated in the functions. The cross section is measured in GeV^{-2}.

The SM part is expressed in terms of the resonant functions f_Z and f_{ZE}:

$$\sigma_{\mathrm{SM}} = \frac{\alpha^2\pi}{32 s_W^4 c_W^4} \left\{(1+x^2)[4 s_W^4 c_W^4/E^2 + f_{ZE}(1 - 4s_W^2 + 8s_W^4)^2 + \right. \\ \left. + f_Z 2 s_W^2 c_W^2 (1 - 4s_W^2)^2] + x[2 f_{ZE}(1 - 4s_W^2)^2 + f_Z 4 c_W^2 s_W^2]\right\}.$$

The factors are expressed in terms of the functions f_Z, $f_{Z'}$, f_{ZE}, $f_{ZZ'}$:

$$f_1(x) = \frac{-\alpha}{64 s_W^4 c_W^4 m_{Z'}^4} \{(1+x^2)[f_{ZE} 4 c_W^2 s_W^2 m_Z^2 m_{Z'}^2 (1 - 4s_W^2 + 8s_W^4) - \\ - f_{Z'} c_W^4 s_W^4 m_Z^4 (1-4s_W^2)^2 - f_{ZE} f_{ZZ'} s_W^2 c_W^2 (m_{Z'}^2 + 2m_Z^2(1-4s_W^2+8s_W^4))^2] + \\ + x[f_Z 16 s_W^2 c_W^2 M_Z^2 M_{Z'}^2 - f_{Z'} 8 s_W^4 c_W^4 (m_Z^2 + m_{Z'}^2)^2 + f_{ZE} 8 s_W^2 c_W^2 m_Z^2 m_{Z'}^2 (1-4s_W^2)^2 - \\ - f_{ZE} f_{ZZ'} 2 s_W^2 c_W^2 (2m_Z^2 + m_{Z'}^2)(1-4s_W^2)^2]\}, \quad (14)$$

$$f_3(x), f_4(x) = \frac{-\alpha}{64 s_W^4 c_W^4 m_{Z'}^4} \{(1+x^2)[(f_Z - f_{Z'}) 4 c_W^4 s_W^4 m_Z^2 m_{Z'}^2 (1-4s_W^2) + \\ + f_{ZE} f_{ZZ'} s_W^2 c_W^2 (m_{Z'}^2 + 2m_Z^2(1-4s_W^2+8s_W^4))(1-4s_W^2) m_{Z'}^2 + \\ + f_{ZE} 2 s_W^2 c_W^2 m_Z^2 m_{Z'}^2 (-1 + 8s_W^2 - 24 s_W^4 + 32 s_W^6)] x[-f_{ZE} 4 s_W^2 c_W^2 m_Z^2 m_{Z'}^2 (1-4s_W^2) + \\ + f_{ZE} f_{ZZ'} 2 s_W^2 c_W^2 m_{Z'}^2 (2m_Z^2 + m_{Z'}^2)(1-4s_W^2)]\}, \quad (15)$$

$$f_2(x) = \frac{-\alpha}{64 s_W^4 c_W^4 m_{Z'}^4} \{(1+x^2)[-f_{Z'} 4 s_W^4 c_W^4 m_{Z'}^4 - \\ - f_{ZE} f_{ZZ'} s_W^2 c_W^2 m_{Z'}^4 (1-4s_W^2)^2] - x 2 f_{ZE} f_{ZZ'} s_W^2 c_W^2 m_{Z'}^4\}. \quad (16)$$

The resonant functions are

$$\begin{aligned} f_Z &= \frac{(4E^2 - m_Z^2)}{(4E^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}, \\ f_{Z'} &= \frac{(4E^2 - m_{Z'}^2)}{(4E^2 - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2}, \\ f_{ZE} &= \frac{E^2}{(4E^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}, \\ f_{ZZ'} &= \frac{(4E^2 - m_{Z'}^2)(4E^2 - m_Z^2) + m_{Z'} \Gamma_{Z'} m_Z \Gamma_Z}{(4E^2 - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2}, \end{aligned} \quad (17)$$

where $\Gamma_Z, \Gamma_{Z'}$ are the widths of the Z and Z' bosons.

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