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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

YUKAWA SECTOR IN MINIMAL D-BRANE MODELS

Salah Eddine Ennadifi [1]

Laboratory of High Energy Physics, Modeling and Simulation, Mohammed V University, Faculty of Sciences, Rabat, Morocco

We investigate the Yukawa couplings sector in the minimal gauge theory $U(3) \times U(2) \times U(1)$ with the Standard Model chiral and Higgs spectrum based on three stacks of intersecting D-branes. In this model, stringy corrections are required to induce the missing Yukawa couplings and generate hierarchical pattern. Under the known data, we assign the realistic Yukawa texture and then bound its strengths.

В представленной работе исследуется сектор связей Юкавы в минимальной калибровочной теории $U(3) \times U(2) \times U(1)$ с киральным и хиггсовским спектрами Стандартной модели, основанной на трех множествах пересекающихся D-бран. В рамках такой модели необходимо ввести струнные поправки, чтобы обеспечить появление связей Юкавы и сгенерировать иерархическую структуру. Используя известные данные, мы задаем реалистичную текстуру Юкавы и затем связываем ее напряженности.

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INTRODUCTION

The Standard Model of particle physics (SM) has been the fruit of our understanding of Quantum Field Theory over the last fifty years. It appears to be the correct low-energy effective field theory for all particle interactions below the weak scale [1, 2]. However, is seems to be rather complicated and incomplete. There are three families of quarks and leptons which transform under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and 26 parameters making up the particle masses, mixing angles, and gauge coupling constants. There are large hierarchies in the parameters, as the masses of fermions where the transformation way, with respect to the symmetries of the quantum theory, does not provide an explanation. This makes it obscure since the Yukawa couplings do not have a predicted structure within the SM.

Superstring theory seems to offer a framework which gives rise to all of these ingredients explaining the origin of the SM gauge group, particle representations, and parameters within an identified particular string vacua.

In the quest for obtaining a realistic string-based model, generic properties of the low-energy effective Lagrangian, such as $D = 4$ chirality and unitary gauge groups, are of fundamental importance [3–5]. Once these have been found in a particular setup of string theory, there are still many other issues to face in order to reproduce some realistic physics

[1] E-mail: ennadifis@gmail.com

at low energies [4–7]. In particular, even if one manages to obtain a massless spectrum quite close to the SM (or some extension of it) [8, 9], one is eventually faced with the problem of computing some finer data defining the Quantum Field Theory. These data may tell us how close we are to reproducing the SM which, as we know, is not a bunch of chiral fermions with appropriate quantum numbers, but an intricate theory with lots of well-measured parameters. In this section, we review intersecting D-brane models from a general viewpoint, collecting the necessary information for addressing the problem of Yukawa couplings. Most part of the effort on constructing phenomenologically appealing intersecting brane configurations has centered on simple toroidal and orbifold/orientifold compactifications. Nevertheless, important issues, such as massless chiral spectrum and tadpole cancellation conditions, are of topological nature, thus, easily tractable in more general compactifications where the metric cannot be known explicitly. Following this general philosophy, we will introduce Yukawa couplings as arising from worldsheet instantons in a generic compactification. Although the specific computation of these worldsheet instantons needs the knowledge of the target space metric, many important features, i.e., textures and strengths, can be discussed in a more general level.

The aim of this paper is to address the Yukawa sector in the context of Intersecting Brane Worlds. In particular, we discuss the Yukawa couplings structure from a general view point within a minimal D-brane orientifold models of three stacks of intersecting D-branes with $U(3)_a \times U(1)_b \times U(1)_c$ gauge symmetry and SM spectrum. With the minimal chiral spectrum, the corresponding effective Yukawa sector requires stringy corrections in terms of Euclidean instantons to induce the missing couplings by way of their fermionic charged zero modes $\lambda_{a,b,c}, \overline{\lambda}_{a,b,c}$. According to the known SM fermion mass scales, we set down the corresponding texture of the Yukawa constants and then bound their strengths.

YUKAWA SECTOR: STRUCTURE AND STRENGTHS

In the context of intersecting brane worlds [10], Yukawa couplings arise from open string worldsheet instantons connecting three D-brane intersections, in such a way that the open string states located there have suitable Lorentz and gauge quantum numbers to build up an invariant in the effective Lagrangian. This will usually involve the presence of three different D-branes, which determine the boundary conditions of the worldsheet instanton contributing to this Yukawa coupling, as in Fig. 1, where F_i, f_j denote the three family $i,\ j = 1, 2, 3$ fermion (quarks and leptons) doublets and singlets, respectively, $F_i \equiv Q_i$, L_i, $f_j \equiv q_j$, e_j, and h denotes the Higgs boson. In this picture, the Yukawa couplings

$$\zeta_{\rm Yuk} = y_{ij} F_i f_j h \tag{1}$$

between the fields F_i, f_j and h living at brane intersections will arise from worldsheet instantons involving three different boundary conditions. Roughly speaking, the instanton contribution to the Yukawa coupling will be given by evaluating the classical action $\mathrm{e}^{-S_{\rm cl}}$ on the surface of minimal area connecting the three

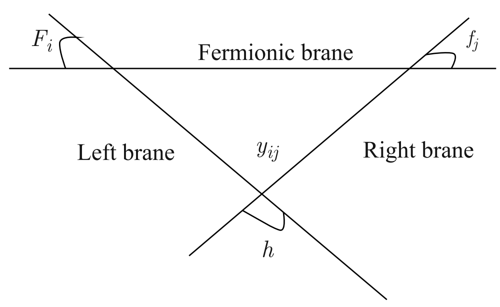

Fig. 1. Yukawa coupling between two fermions of opposite chirality and the Higgs boson

intersections. As a result, the Yukawa constants y_{ij} will depend on several moduli of the theory, such as D-brane positions (open string moduli) and the compact manifold metric (closed string moduli). More concretely, we expect the Yukawa constants y_{ij} to be roughly of the form

$$y_{ij} \sim \exp\left(-\frac{A_{ij}}{2\pi\alpha'}\right), \tag{2}$$

where $1/2\pi\alpha'$ is the string tension, and the exponentiation A_{ij} is the main contribution to the Yukawa constants, which is the target-area of such above triangular surface. Thus, the specific computation of these constants depends on the internal space. However, some important features can be derived without the specific details of the underlying geometry giving a hint of how these quantities may behave in a more general setup.

Large ingredients, such as gauge groups and chiral matter, that we can use to build up the SM-like gauge theories, are offered by D-brane constructions with orientifold configurations [11–14]. The gauge symmetry and the matter content of the SM in this framework can be accommodated in the three-stack model with the gauge symmetry,

$$U(3)_a \times U(2)_b \times U(1)_c. \tag{3}$$

Consistency conditions, such as tadpole cancellation and the presence of a massless hypercharge, constrain the chiral content and the transformation behavior [15]. The Abelian and mixed anomalies are canceled by the Green–Schwarz mechanism and promoted to global symmetries which are respected by all perturbative couplings, and a linear combination $U(1)_Y = \sum_{\alpha_{k=a,b,c}} \alpha_k U(1)_k$ of them does not acquire a Stueckelberg mass and remains massless to be identified as the hypercharge. Tadpole cancellations, which are conditions on the cycles the D-branes wrap, imply restrictions on the transformation properties of the chiral spectrum and guarantees the cancellation of gauge anomalies $U(N_{\alpha=a,b,c})$. These conditions are used to fit the $U(1)_{a,b,c}$ charged SM particles to the following intersection numbers[1]:

$$\begin{aligned} I_{ab} &= 1, & I_{ab^*} &= 2, \\ I_{ac} &= -3, & I_{ac^*} &= -3, \\ I_{bc} &= -3, & I_{cc^*} &= -3. \end{aligned} \tag{4}$$

The other intersection numbers are set to zero as we discussed. From these intersection numbers, we summarize in the following table the fields content and the corresponding charges, which depend on the anomaly-free hypercharge linear combinantion for which all the matter particles have the proper electroweak hypercharge. Roughly, Abelian and mixed anomalies are canceled via the Green–Schwarz mechanism, and non-Abelian anomalies are vanished by tadpole conditions [4–7]. The anomalous $U(1)_{a,b,c}$ acquire masses and survive only as global symmetries and forbid various couplings at the perturbative level. A linear combination of these global symmetries remains massless to be identified with the hypercharge in the resulting four-dimensional spacetime gauge group. Vanishing of anomalies which we require to be satisfied are used to fit the SM fermions [15]. The chiral spectrum, including the Higgs

[1] We have not included those involving $b^* = b$.

Table 1. **The fields content corresponding to the free anomaly linear combination $\Upsilon = (1/6)U(1)_a - (1/2)U(1)_c$. The factors 1, 2, 3 denote the field multiplicity**

Fields	$1Q$	$2Q'$	$3u^c$	$3d^c$	$3L$	$3e^c$	h
C_a	1	1	–1	–1	0	0	0
C_b	–1	1	0	0	–1	0	–1
C_c	0	0	1	–1	1	–2	–1
Υ	1/6	1/6	–2/3	1/3	–1/2	1	1/2

doublets and the gauge symmetry together with their identification with SM matter fields, is given in Table 1.

The model with three stacks can be encoded in a quiver where each node represents a D6-brane and the links between them indicate their chiral intersections. The quiver summarizing the above spectrum with the two Higgses is shown in Fig. 2. In this setup, only the three-quark doublets arise from two different intersections. This feature will be addressed at the level of the perturbative Yukawa Lagrangian where we require the presence of the phenomenologically desired terms involoving all the SM Yukawa couplings and the absence of the phenomenologically undesired coupling terms, such as R-parity violating or proton decay terms. According to the charges presented in Table 1 and the field multiplicities assignment of this minimal chiral and Higgs spectrum which is consistent with the above hypercharge, only the following Yukawa terms

$$\zeta_{\rm Yuk} = y^u_{nj} Q^{'}_n u^c_j h + y^d_{mj} Q_m d^c_j h^\dagger + y^e_{ij} L_i e^c_j h^\dagger \tag{5}$$

are perturbatively allowed. y's are the Yukawa coupling matrices, where the indices i, j run over all three-fermion generations, while n and m take only two and one values, respectively. Depending on the particular n, m assignment of the quark doublet, three possible up and down Yukawa matrix y^u_{nj} and y^d_{mj} textures arise with a complementary texture zero structure at the perturbative level, in the sense that the zero entries of the former are nonzero in the

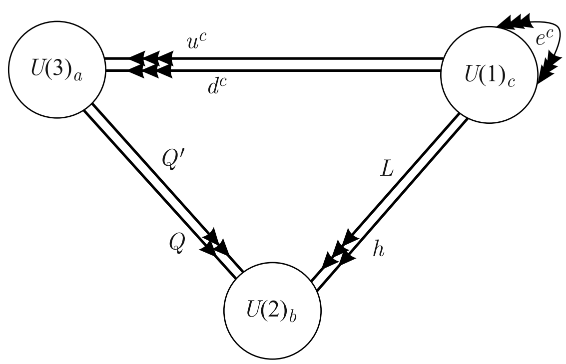

Fig. 2. Three-stack quiver. Circles denote D6/D6*-branes, bold lines denote chiral spectrum and the scalar Higgs. Arrow directions indicate fundamental (antifundamental) representations of $U(N)$ gauge group

latter, and vice versa,

$$\begin{aligned} &\left[y^{u}_{(1,2,0)j},\ y^{d}_{(0,0,3)j}\right] \quad (n=1,2,\ m=3), \\ &\left[y^{u}_{(1,0,3)j},\ y^{d}_{(0,2,0)j}\right] \quad (n=1,3,\ m=2), \\ &\left[y^{u}_{(0,2,3)j},\ y^{d}_{(1,0,0)j}\right] \quad (n=2,3,\ m=1). \end{aligned} \tag{6}$$

The absent but phenomenologically desired matrix entries violate the $U(1)_b$ symmetry,

$$C_b(Q_m u^c_j h) = 2, \quad C_b(Q^{'}_n d^c_j h^{\dagger}) = -2. \tag{7}$$

Within this view, the missing superpotential terms are not sensitive to the electroweak symmetry breaking. The most exciting and economic mechanism to communicate the electorweak symmetry breaking and produce these missing terms without extending the field spectrum is that of invoking stringy instanton nonperturbative effects. In this minimal D6-branes configuration, such nonperturbative effects are generated by $O(1)$ instantons realized from D2-branes wrapped appropriately on rigid orientifold-invariant three-cycles in the internal space. Indeed, this will give rise only to the charged fermionic zero modes λ_b, $\overline{\lambda}_b$ at the intersection with the relevant D6$_b$-brane and carrying suitable charges required to cancel the $U(1)_b$ charges excess of the missing superpotential quark and the possible neutrino coupling terms,

$$C_b(L_i L_i h h) = -4. \tag{8}$$

These charge excess (7) and (8) could be compensated by the following $E2$-instantons (see Table 2).

The quiver part illustrating this stringy correction pattern with its appropriate charged fermionic zero modes is depicted in Fig. 3. Performing the Grassmann path integral over all the fermionic zero modes for each missing terms, one gets the nonperturbative stringy correction to the low-energy effective theory,

$$\zeta^{'}_{\mathrm{Yuk}} = \mathrm{e}^{-S^{\mathrm{cl}}_{u}} y^{u}_{nj} Q_n u^c_j h + \mathrm{e}^{-S^{\mathrm{cl}}_{d,s}} y^{d}_{mj} Q^{'}_m d^c_j h^{\dagger} + \mathrm{e}^{-S^{\mathrm{cl}}_{v_i}} M_s^{-1} y_{v_i} (L_i h)^2, \tag{9}$$

where the exponential factors are the remaining charged classical parts of the $E2_{u,d,v_i}$ instanton actions absoring the above $U(1)_b$ charges excess through the charged fermionic zero modes λ_b, $\overline{\lambda}_b$ and the high suppressing mass scale M_s taken as a lower string scale at which

Table 2. **The stringy instanton corrections with their suitable $U(1)_{a,b,c}$ charges**

$E2$-modes	$E_u(\lambda^u_b)$	$E_{d,s}(\lambda^d_b)$	$E_{v_i}(\lambda^{v^i}_b)$
C_a	0	0	0
C_b	2	–2	4
C_c	0	0	0
Υ	0	0	0

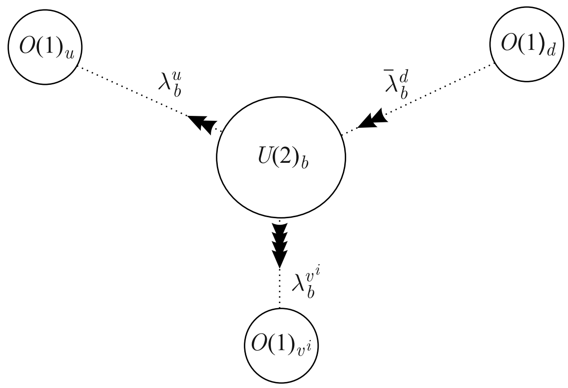

Fig. 3. The correcting intersections of the $E2$-instantons and D6_b-brane: dotted lines indicate their chiral intersections with the D6_b-brane

neutrino masses have origin. This induces the missing desired matrix entries,

$$\begin{aligned}
&\left[y^u_{(0,0,3)j},\ y^d_{(1,2,0)j}\right] \quad (n=3,\ m=1,2),\\
&\left[y^u_{(0,2,0)j},\ y^d_{(1,0,3)j}\right] \quad (n=2,\ m=1,3),\\
&\left[y^u_{(1,0,0)j},\ y^d_{(0,2,3)j}\right] \quad (n=1,\ m=2,3).
\end{aligned} \tag{10}$$

At this stage, we need to determine the semirealistic Yukawa texture among the three possible up and down complementary textures. For that, we can refer to the known quark masses. Indeed, by imposing the heavy top quark t to be realized perturbatively and the light up quark u to be realized nonperturbatively, we can end with the last Yukawa texture,

$$\begin{aligned}
&\left[y^u_{(0,2,3)j},\ y^d_{(1,0,0)j}\right] \quad (n=2,3,\ m=1),\\
&\left[y^u_{(1,0,0)j},\ y^d_{(0,2,3)j}\right] \quad (n=1,\ m=2,3).
\end{aligned} \tag{11}$$

In this texture, the top t, charm c, and down d quarks are realized perturbatively, while the quarks strange s, bottom b, and up u are realized nonperturbatively,

$$\begin{aligned}
\zeta_{\rm Yuk} = {}& y_t Q_3^{'} t h + y_c Q_2^{'} c h + y_d Q_1 d h^\dagger + \mathrm{e}^{-S^{\rm cl}_b}\, y_b Q_3^{'} b h^\dagger + \\
&+ \mathrm{e}^{-S^{\rm cl}_u} y_u Q_1 u h + \mathrm{e}^{-S^{\rm cl}_s} y_s Q_2^{'} s h^\dagger + y^e_{ij} L_i\, \mathrm{e}^c_j h^\dagger + \mathrm{e}^{-S^{\rm cl}_{v_i}} M_s^{-1} y_{v_i} \left(L_i h\right)^2.
\end{aligned} \tag{12}$$

The exponential suppressions $\mathrm{e}^{-S^{\rm cl}_{f=u,s,b,v_i}} \leqslant 1$ depend on the internal geometry of the model. In particular, they depend on the volume of the three-cycles wrapped by the D2-branes to make the stringy correction effects, the leading one comes from instantons having minimal

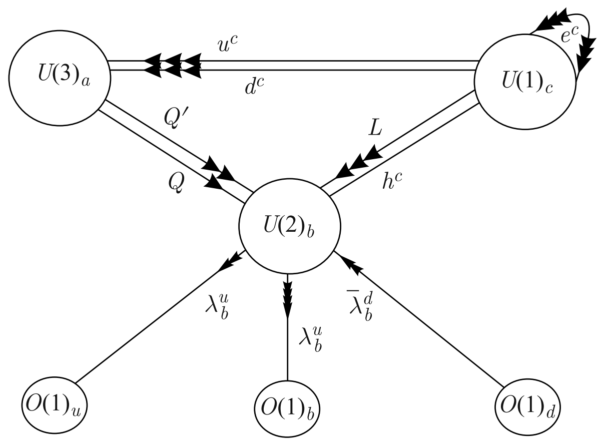

Fig. 4. The minimal stringy SM quiver

volume [12–14]. According to their values and to quark mass scales,

$$m_t = y_t \langle h \rangle \sim \langle h \rangle \sim 10^2 \text{ GeV}, \quad m_c = y_c \langle h \rangle \sim 1 \text{ GeV}, \quad m_d = y_d \langle h \rangle \sim 10^{-2} \text{ GeV},$$
$$m_s = \mathrm{e}^{-S_s^{\mathrm{cl}}} y_s \langle h \rangle \sim 10^{-1} \text{ GeV}, \quad m_b = \mathrm{e}^{-S_b^{\mathrm{cl}}} y_b \langle h \rangle \sim 1 \text{ GeV}, \tag{13}$$
$$m_u = \mathrm{e}^{-S_u^{\mathrm{cl}}} y_u \langle h \rangle \sim 10^{-2} \text{ GeV}, \quad M_s \leqslant 10^{14} \text{ GeV}.$$

We can now approach the strengths of the Yukawa coupling constants and the string scale upper bound,

$$y_t \simeq 1, \quad y_c \simeq 10^{-2}, \quad y_d \simeq 10^{-4}, \quad y_s \geqslant 10^{-3},$$
$$y_b \geqslant 10^{-2}, \quad y_u \geqslant 10^{-4}, \quad M_s \leqslant 10^{14} \text{ GeV}. \tag{14}$$

We end up finally with the following quiver containing the chiral spectrum, the gauge symmetry, and the stringy corrections (see Fig. 4).

CONCLUSIONS

In this work, we have investigated the Yukawa sector of the SM structure within a stringy framework. In particular, we have considered the Yukawa couplings in a minimal D-brane model consisting of three intersecting stacks of D-branes in general orientifolded geometries and illustrated the corresponding Yukawa sector. In the emerging effective field theory with the exact SM spectrum, some quark couplings arised with a complementary texture zero matrices structure at the perturbative level while the others are missing by the fact that they violate the survival of global $U(1)$ symmetry. This has led to consideration of stringy corrections arising from $E2$-branes to induce the absent Yukawa terms for quarks and leptons. Compared to perturbative allowed ones, these induced Yukawa terms are exponentially suppressed by the stringy instanton effects, reflecting thereby a hierarchical fermionic structure. Among the three possible realized Yukawa textures, we have ended with the texture where the top quark t arises perturbatively and the up quark u arises nonperturbatively and then bounded the strengths of the corresponding coupling constants and the involved string scale according to the known fermion mass scales.

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