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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

HIGH-SPIN STRUCTURE OF ^{80}Kr

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We have made theoretical investigations on the structure of high-spin states of ^{80}Kr within the framework of cranked Hartree–Fock–Bogoliubov theory employing a pairing + quadrupole + hexadecapole model interaction. Dependence of shape on the spin, g factor, alignment of proton as well as neutron $0g_{9/2}$ orbital along with backbending phenomenon are investigated up to a high spin $J = 24$. We have found excellent agreement with the experimental values and other theoretical calculations.

В работе представлено теоретическое исследование структуры состояний с большим спином для ядра ^{80}Kr в рамках теории Хартри–Фока–Боголюбова, когда используется модель взаимодействия спаривание + квадрупольное + гексадекапольное. Зависимость формы от спина, g-фактор, выравнивание протонной, а также нейтронной $0g_{9/2}$-орбитали наряду с эффектом обратного изгиба рассматриваются вплоть до значений спина $J = 24$. Получено прекрасное согласие теоретического описания с экспериментальными значениями и результатами других теоретических моделей.

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INTRODUCTION

The nuclear structure in mass $A = 80$ region is characterized by shape coexistence, softness, interaction of neutron and proton in $g_{9/2}$ state, strong dependence on spin and particle number, etc. [1]. The study in this particular region has been motivated by the new experimental facilities. In particular, the newly constructed detector arrays, the domain of nuclides accessible for spectroscopic studies have increased drastically during the past decade. For example, some extensive measurements [1–11] of the transition quadrupole moments, extracted from the level lifetimes and excitation energy of 2^+ state, have been carried out for Kr, Sr and Zr isotopes. These measurements have revealed large variations in nuclear structure of these isotopes with respect to angular momentum and particle number. It has been proved that alignment of proton and neutron pairs at higher angular momenta can change the nuclear shape from prolate to triaxial and to oblate [12–15].

First experimental evidence for ground-state deformation in neutron-deficient Kr isotopes was found by Piercey et al. in 1981 [16]. Subsequent experiments revealed the rich structure of the low-lying excitation spectrum in these nuclei, with coexisting and mixed bands in $^{72-78}$Kr [17–19]. Recently, ^{80}Kr has been predicted with large γ deformation and large

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quadrupole deformation and found to possess triaxial superdeformed (TSD) in mass $A \sim 80$ region [20]. Moreover, the yrast spectra of $^{78-82}$Kr are studied by using the projected shell model (PSM) approach [21]. The theoretical results predict low-lying states in $^{78-82}$Kr to be oblate and coexistence of oblate-prolate shapes for ^{80}Kr [21].

More recently in 2010, shape mixing dynamics in the low-lying states of proton-rich ^{72,74,76}Kr isotopes has been studied by Koichi Sato and Nobuo Hinohara using collective Hamiltonian, which is derived microscopically by means of the CHFB (constrained Hartree–Fock–Bogoliubov) + Local QRPA (quasiparticle random phase approximation) method [22]. There are a few more efforts made by physicists to understand the shape coexistence phenomenon in Kr isotope specially in ^{76,78}Kr. The simultaneous alignments of proton and neutron pairs in ^{76}Kr could be explained by assuming a prolate shape that changes over to a triaxial shape with positive g due to the first proton alignment, thereby reducing the neutron alignment frequency [23, 24]. This change in shape was experimentally confirmed from the lifetime measurements of high-spin states in this nucleus [25]. Further, using cranked shell model and shell correction method with the Woods–Saxon average field and pairing term, Gross et al. [24] concluded that the first backbending in the ^{78}Kr yrast line is due to the alignment of a pair of $g_{9/2}$ protons, while the second irregularity is interpreted in terms of the $g_{9/2}$ neutron alignment with the change in $\gamma = 15$ to $\gamma = -30$. The same was supported by Billowes et al. [26] by measuring average g factors in ^{78}Kr. Jakhar et al. [23] have studied the yrast band of ^{76,78}Kr with the conclusion that the shape ^{76}Kr becomes triaxial at high spin and the shape of ^{78}Kr remains prolate all through up to $J = 24$.

Encouraged by the above studies on Kr isotopes near $A = 80$ region, we would like to investigate structure of ^{80}Kr at high spins using our cranked Hartree–Fock–Bogoliubov theory employing a pairing + quadrupole + hexadecapole model interaction [23, 27].

1. FORMALISM AND CALCULATIONAL DETAILS

We employ a pairing + quadrupole + hexadecapole model interaction Hamiltonian,

$$H = H_0 - \frac{1}{2} \sum_{\lambda=2,4} \chi_\lambda \sum_\mu \hat{Q}_{\lambda\mu} (-1)^\mu \hat{Q}_{\lambda-\mu} - \frac{1}{4} \sum_{\tau=p,n} G_\tau \hat{P}_\tau^\dagger \hat{P}_\tau, \tag{1}$$

where H_0 stands for the one-body spherical part, χ_λ term represents the quadrupole and hexadecapole terms with $\lambda = 2$, 4, and G_τ term is the proton and neutron monopole pairing interaction. Explicitly we have

$$\hat{Q}_{\lambda\mu} = \left(\frac{r^2}{b^2}\right) Y_{\lambda\mu}(\theta, \phi), \tag{2}$$

$$\hat{P}_\tau^\dagger = \sum_{\alpha_\tau, \bar{\alpha_\tau}} c_{\alpha_\tau}^\dagger c_{\bar{\alpha_\tau}}^\dagger. \tag{3}$$

In the above equation, $c^\dagger$ are the creation operators with $\alpha \equiv (n_\alpha l_\alpha j_\alpha m_\alpha)$, as the spherical basis states quantum numbers with $\bar{\alpha}$ denoting the conjugate time-reversed orbital. The standard mean field cranked Hartree–Fock–Bogoliubov equations [8] are solved self-consistently for the quadrupole, hexadecapole and pairing gap parameters. The deformation parameters are defined in terms of the following expectation values:

$$D_{2\mu} = \chi_2 \langle \hat{Q}_{2\mu} \rangle, \quad D_{4\mu} = \chi_4 \langle \hat{Q}_{4\mu} \rangle, \tag{4}$$

$$\hbar\omega\beta\cos\gamma = D_{20}, \quad \hbar\omega\beta\sin\gamma = \sqrt{2}D_{22}, \quad \hbar\omega\beta_{40} = D_{40}, \tag{5}$$

$$\Delta_\tau = \frac{1}{2}G_\tau\langle\hat{P}_\tau\rangle. \tag{6}$$

The oscillator frequency $\hbar\omega = 41.0A^{-1/3}$ (MeV), β, γ, and β_{40} are the usual deformation parameters, while Δ_p and Δ_n are the pairing gap parameters. The basis space consists of $N = 4, 5$ harmonic oscillator major shells $+\, 0i_{13/2}$ orbitals for protons, and of $N = 5, 6$ major shells $+\, 0j_{15/2}$ orbitals for neutrons with the assumption of an inert core $Z = 40$ and $N = 70$. The spherical single-particle energies are taken as the spherical Nilsson model single-particle energies with A-dependent Nilsson parameters [9]. The upper shell radial matrix elements are reduced by some factors, $(N + 3/2)/(N_0 + 3/2)$, as discussed in [3], where N_0 takes the value 4 for protons and 5 for neutrons. Finally, the interaction strengths are chosen such that reasonable values of the ground-state shape parameters, the first 2^+ excitation energy ($\sim$ 100 keV), and the spin-dependent (up to $J = 10$) g factors of ^{164}Dy are obtained. We have taken (all in MeV) the following values of the interaction strengths:

$$\chi_2 = 60/A^{1.4}, \quad \chi_4 = 55/A^{1.4}, \quad G_p = 25.3/A, \quad G_n = 21.5/A. \tag{7}$$

2. RESULTS AND DISCUSSIONS

With the introductory comments in Introdiction, it would be interesting to examine the structure of ^{80}Kr at high spins, for which we present in Fig. 1 the values of the deformation parameters, i.e., quadrupole deformation parameter (β), hexadecapole deformation parameter (β_4) and asymmetric deformation (γ) along with pairing gaps as a function of spin J for ^{80}Kr. It is clear from the figure that the shape of ^{80}Kr is prolate at ground state, which is in accordance with experimental data [28]. For higher spins the shape of ^{80}Kr remains prolate with lower value of quadrupole deformation parameter β. It is gratifying to note that this change in the value of β at higher spin is similar to the study using cranked Hartree–Fock–Bogoliubov model calculations with the Woods–Saxon potential and monopole pairing, and to the experimental study done in [29]. Figure 1 also reflects that at the highest spin ($J = 24$) considered, ^{80}Kr becomes very soft leading to a triaxial shape with $\gamma = 17$.

Moreover, in Fig. 1, *a, b*, we have displayed variation of pairing gaps for protons and neutrons, respectively. It is seen that with the increase of spin the proton pairing collapses at $J = 20$, though it is quite sizeable for other spins. Whereas the neutron pairing persists for much higher spin values in ^{80}Kr.

Besides the above-mentioned deformation parameters and pairing gaps, study of g factor has been extensively employed in the past as a sensitive probe for a better understanding of the structure of ground state as well as excited states up to very high angular momentum in stable nuclei in different mass regions. Nuclear structure at high spins in a mid-weight mass region of $A = 80$ possesses many interesting features. Among them, the quasiparticle alignment (QPA) is a significant feature. The g-factor measurement of intra-band states can provide unique information on QPA, since g factors of $g_{9/2}$ protons are positive and large ($g_p = 1.38$) and g factors of $g_{9/2}$ neutrons are negative and small ($g_n = -0.24$). The neutron and/or proton alignments govern the value and variation of g factors with spin. With this in view, we have shown comparison of calculated g factor and experimental g-factor value as a function of angular momentum in Fig. 2, *b*.

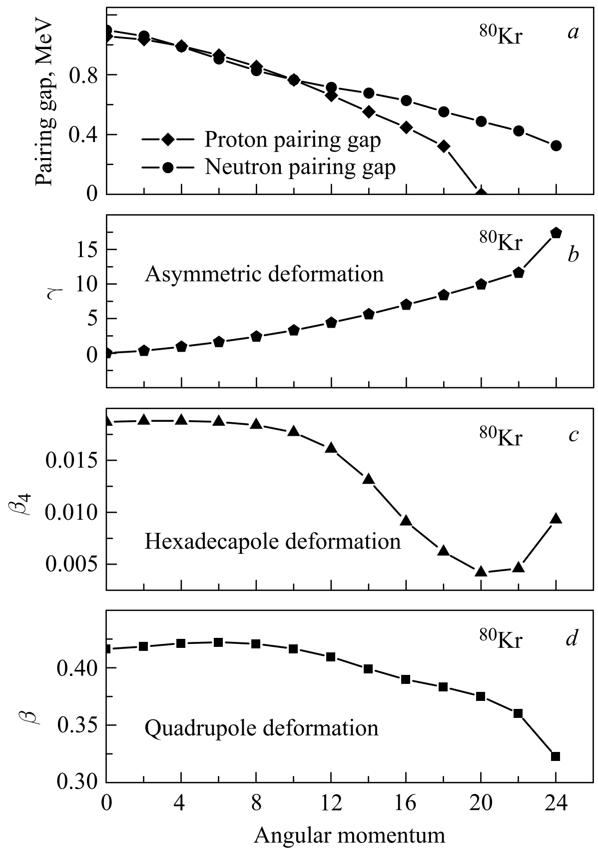

Fig. 1. Results of cranked Hartree–Fock–Bogoliubov calculations for the angular momentum dependence of shape parameters β, β_4, γ and pairing gaps for the yrast levels in ^{80}Kr

From Fig. 2, it is important to note that our results of g factor are in good agreement with experimental data known so far [28]. It is also clear that g factor increases with the spin and, for higher value of spin it decreases. Since, proton alignment causes increasing of g factor and neutron alignment provides decreasing of g factor, therefore this type of trend of g factor can be attributed as proton alignment followed by neutron alignment.

To get into more insight, we have also plotted contributions of protons and neutrons in $0g_{9/2}$ orbital to total angular momenta in Fig. 2, *a*. In the low-spin part, the neutron and proton alignments are almost the same, but for higher spin the proton alignment dominates, which actually results in increased g factor as can be seen in Fig. 2, *b*. It is important to point out here that proton pairing gap vanishes at $J = 20$ as shown already in Fig. 1, *a*, whereas neutron pairing persists for much higher spin values, which, in turn, decreases g factor at high spins as reflected in Fig. 2, *b* for the value of spin $J = 20{-}24$.

With the above dependence of the shape parameters on the spin, now we investigate the energies of the yrast levels with comparison to the experimental values, which are plotted in Fig. 3, *b*. The parabolic shape is quite well reproduced, but calculated numbers are found larger as compared to the experimental values toward higher angular momentum. In order

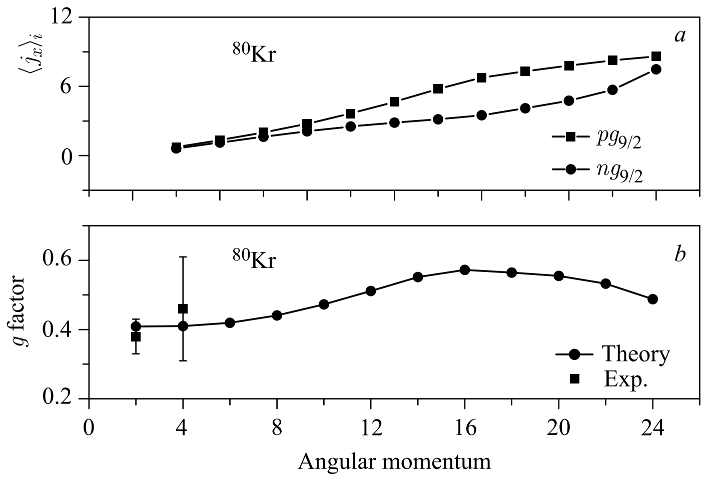

Fig. 2. The angular momentum dependence of g factor and contributions of protons and neutrons in $g_{9/2}$ orbital for the yrast levels in ^{80}Kr

to demonstrate the change in structure as a function of angular momentum, we display a plot of J vs. rotational frequency ω in Fig. 3, *a*, depicting the variation of the moment of inertia ($J = I\omega$). It is clear from Fig. 3, *a* that experimental results exhibit upbends at $J = 8$, at frequency $\omega = 0.5$ MeV like a sharp discontinuity at a point. In the theoretical curve, this small sudden jump is smoothed out, but it does exhibit the gross features similar to the measurements.

It is here worth pointing out that a conclusion has been drawn already in [30, 31] that in ^{80}Kr at spins $J = 8$ and $J = 10$, the positive-parity ground-state band is crossed by an

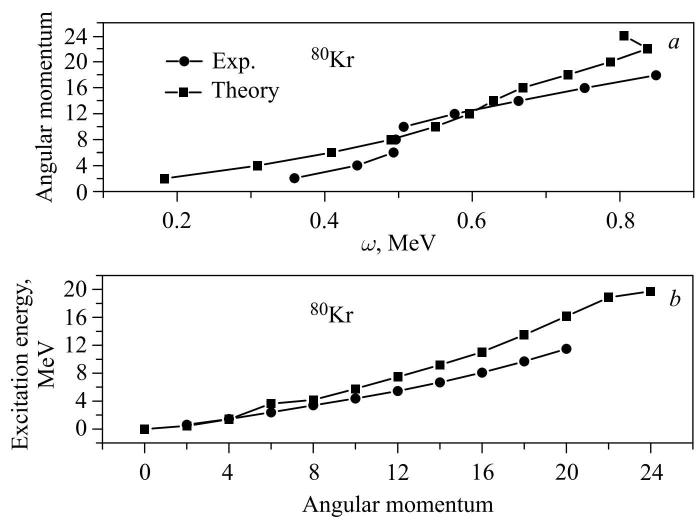

Fig. 3. *a*) Variation of angular momentum as a function of rotational frequency in ^{80}Kr. *b*) Energies of the yrast states of ^{80}Kr compared with the experimental values

aligned two-quasiparticle $g_{9/2}$ proton band. Doring et al. [32] showed that the ground-state positive-parity band is crossed by a two-quasiproton (2-qp) band at a rotational frequency $\omega = 0.5$ MeV and becomes yrast above the 8^+ state. Verma et al. [21] have also concluded recently that the observed backbendings in ^{80}Kr around spins $J = 8$ are reproduced around spins $J = 6$, with the result indicating the crossing of both oblate and prolate $g_{9/2}$ 2-qp bands.

Our results (Fig. 3, *a*), however, do not show very sharp backbendings, but the results are very much close to experimental results specially in the region of interest at $J = 8$, 10, 12, and 14. A very sharp backbending can be seen though at a very high spin at $J = 24$.

3. SUMMARY

We have performed a fully self-consistent cranked Hartree–Fock–Bogoliubov calculation to examine high-spin structure of ^{80}Kr up to $J = 24$. From our calculations ^{80}Kr was found to possess prolate deformation for all the values of spins. Experimental value of g factor is well reproduced in our calculation. Variation of the g factors as well as the rotation alignment of $0g_{9/2}$ orbital as a function of spin show that the alignment of proton dominates over the neutron alignment in the higher spin region. At very high spins it goes the other way. The angular momentum vs. rotational frequency plot shows a good qualitative agreement between theory and experiment and we conclude that change in structure with spin is qualitatively correctly described in this microscopic many-body approach.

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