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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

INVESTIGATING THE CHERENKOV LIGHT LATERAL DISTRIBUTION FUNCTION FOR PRIMARY PROTON AND IRON NUCLEI IN EXTENSIVE AIR SHOWERS

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The lateral distribution function (LDF) of Cherenkov radiation in extensive air showers (EAS) was simulated by CORSIKA program for the conditions of Yakutsk Cherenkov array at high energy range (10^{13}–10^{16} eV) for two primary particles (p and Fe) for different zenith angles. Using Breit–Wigner function for analyzing Cherenkov light LDF, a parameterization of Cherenkov light LDF was reconstructed by depending on CORSIKA simulation as a function of primary energy. The comparison between the estimated Cherenkov light LDF and the LDF that was measured on the Yakutsk EAS array gives the ability of particle identification that initiated the shower and determination of particle's energy around the knee region. The extrapolation of approximated Cherenkov light LDF for energies 20 and 30 PeV was obtained for primary particles (p and Fe).

Представлена симуляция функции пространственного распределения (ФПР) черенковского излучения в широких атмосферных ливнях (ШАЛ) с помощью программы CORSIKA для условий якутского черенковского массива при больших энергиях (10^{13}–10^{16} эВ) двух первичных частиц (p и Fe) при различных значениях зенитного угла. Параметризация ФПР черенковского излучения восстановлена с помощью функции Брейта–Вигнера на основе симуляции с помощью CORSIKA в зависимости от энергии первичных частиц. Сравнение полученной ФПР черенковского излучения и ФПР, измеренной в ШАЛ на установке в Якутске, дает возможность оценить качество идентификации частицы, которая инициирует образование ливня, а также определить энергию этой частицы в области колена. Путем экстраполяции получена приближенная ФПР черенковского излучения для энергий 20 и 30 ПэВ первичных частиц (p и Fe).

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INTRODUCTION

One of the fundamental problems of the primary cosmic radiation study is the accurate investigation of energy spectrum and chemical composition. One of the probable techniques of the investigation depends on Cherenkov light measurements in EAS. The interpretation of these measurements requires a comparison with EAS simulations in the atmosphere [1, 2]. One of the necessary tools of numerical simulation of Cherenkov radiation is the CORSIKA code method for investigation of EAS characteristics and experimental data analysis.

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The CORSIKA software package [3, 4] is one of the numerical methods that simulate the Cherenkov light LDF emitted in EAS that were produced by high energy cosmic ray (CR) protons and nuclei. This simulation requires a long time for commutating one shower with high energies (greater than 100 PeV) for a few GHz processors. Accordingly, developing of a fast modeling algorithms and searching for parameterizations of the numerical modeling results are important practical problems. In [5, 6], a function for Cherenkov radiation as a function of the core shower in EAS was proposed. This function was developed by parameterizing the CORSIKA simulation results of Cherenkov radiation of emitted photons by EAS that were produced in the Earth's atmosphere by CR particles as a function of the distance from the core showers and the energy of the primary particles [7].

Korosteleva et al. [8] have developed a new method for obtaining the Cherenkov light LDF in EAS by exploiting the simulations based on CORSIKA program using QGSJET hadronic model for high energies. This method depends on the relation between the ratio of shower size upon energy and the steepness of the LDF of Cherenkov radiation. On the other hand, Nerling et al. [9] used CORSIKA simulation code of EAS for estimating the energy of electrons and angular distributions of showers with high energies. The authors have developed an analytical representation of Cherenkov radiation emission in EAS, which provides the angular distribution and total number of photons. Parameterization has been used to estimate the contribution of Cherenkov radiation to shower profiles that was measured with air shower techniques. Mishev et al. [10] have presented simulations with CORSIKA code using different hadronic interaction models such as GHEISHA, FLUKA 2006, and QGSJET II. They calculated the primary energy of protons that initiated EAS. At the same time, Budnev et al. [11] have presented the main results of EAS Cherenkov array that covers the primary CR energy spectrum and chemical composition at the energy range of $3 \cdot 10^{15}$ to $3 \cdot 10^{16}$ eV. They developed a new model for Cherenkov light LDF for analyzing the depth of shower maximum distribution.

In this work, the Cherenkov light LDF simulations were performed for conditions and configurations of Yakutsk EAS array [12, 13] with the CORSIKA code [3, 4] using two models for simulation of hadronic processes (QGSJET [14] and GHEISHA [15]) and EGS4 model for electromagnetic cascade and Cherenkov light simulation. The approximation of a numerical simulation of Cherenkov light density (Q, m^{-2}) was performed on the basis of Breit–Wigner function which was proposed in [5, 6]. This function is used for describing and analyzing the Cherenkov light LDF form in EAS showers induced by primary particles. Its application may give the possibility for reconstruction of the EAS events (the type and the primary energy of EAS particles in addition to the angle and arrival time of particles at a fixed observation plane) registered on Yakutsk EAS array. The comparison of the approximated Cherenkov light LDF with the reconstructed EAS events registered with Yakutsk EAS Cherenkov array [12, 13] has shown an acceptable chance for identification of the primary particle and determination of its primary energy near the knee region.

1. CHERENKOV LIGHT LDF SIMULATION

The Cherenkov light LDF simulation of particles that initiated EAS was fulfilled using the Monte Carlo CORSIKA (COsmic Ray SImulations for KAscade) software package [3, 4] using two hadronic models: QGSJET (Quark Gluon String model with JETs) code [14] with energies larger than 80 GeV and GHEISHA (Gamma Hadron Electron Interaction SHower) code [15] with energies lower than 80 GeV. CORSIKA program simulates the interactions and decays of various nuclei, hadrons, muons, electrons, and photons in the atmosphere. In EAS, the primary particles are tracked through the atmosphere until they undergo interactions with an air nucleus or they decay in the case of unstable secondary particles [3]. The result of the simulations is detailed about the type, energy, momenta, location, and arrival time of the produced secondary particles at given selected altitude above the sea level. The Yakutsk EAS array consists of 48 Cherenkov light detectors (500 m spacing between detectors); 100 m of the observation level above the sea level (1020 g/cm^2); 0 to 360° of the azimuth angle, and 300 to 600 nm of wavelength range [13].

2. PARAMETERIZATION OF CHERENKOV LIGHT LDF

The lateral distribution function of Cherenkov radiation is a function to characterize the lateral variation of Cherenkov flux with the core distance that is widely used in event reconstruction, aimed at obtaining information about primary particles in EAS. Integration of LDF over the total range of core distance results in the shower size, i.e., the total number of particles. Estimation of core position and depth of shower maximum is also performed using the total number of photons (N_γ) which were radiated in EAS by electrons which is proportional directly to the energy of the primary particle E_0 [16]:

$$N_\gamma(E_0) \approx 3.7 \cdot 10^3 \, \frac{E_0(\text{eV})}{\xi_e} \approx 4.5 \cdot 10^{10} \frac{E_0(\text{eV})}{10^{15} \text{ eV}}, \tag{1}$$

where ξ_e is the critical energy of electrons that equals 81.4 MeV. The empirical measurements of this magnitude are actually hard, so for vertical EAS one can use the density of Cherenkov radiation — the photons number (ΔN_γ) per unit area of detector (ΔS), which appears as a function of the primary energy and distance from the core shower (R) [17]:

$$Q(E_0, R) = \frac{\Delta N_\gamma(E_0, R)}{\Delta S}. \tag{2}$$

Direct measurements of Cherenkov radiation have demonstrated that the LDF fluctuation in EAS is fundamentally less than the total number of photons N_γ. For parameterization of simulated LDF of Cherenkov radiation, we have used the proposed function, which depends on four parameters a, b, s, and r_0:

$$Q(E_0, R) = \frac{C s \exp\left[a - \dfrac{R}{b} + \dfrac{(R - r_0)}{b} + \left(\dfrac{R}{b}\right)^2 + \dfrac{(R - r_0)^2}{b^2}\right]}{b \left[\left(\dfrac{R}{b}\right)^2 + \dfrac{(R - r_0)^2}{b^2} + \dfrac{R s^2}{b}\right]}, \tag{3}$$

where C is the normalization constant [7]; a, b, s, and r_0 are parameters of Cherenkov light LDF which are parameterized as a function of the primary energy E_0. The estimation of Cherenkov light density (Q, m^{-2}) was performed in the energy range of 10^{13}–10^{16} eV for different primary particles and different zenith angles. Unlike [5], we found an energy dependence of the parameters a, b, s, and r_0 that allows us to estimate the LDF of Cherenkov radiation for any primary energy and fit the LDF which was simulated with CORSIKA program. The energy dependence of LDF parameters is parameterized with a 3rd order polynomial fit as

$$K(E_0) = c_0 + c_1 \log\left(E_0(\text{eV})\right) + c_2(\log E_0(\text{eV}))^2 + c_3(\log E_0(\text{eV}))^3, \qquad (4)$$

where $K(E_0)$ determines the four parameters of the function (3), namely: a, $\log(b/1\ \text{km})$, $\log s$, and $\log(r_0/1\ \text{km})$; c_0, c_1, c_2, and c_3 are coefficients that depend on the type of primary particles (see Tables 1 and 2).

Function (3) with its parameters in Eq. (4) represents the parameterization of Cherenkov light LDF model for EAS initiated by primary proton and iron nuclei. The obtained Cherenkov light LDFs in EAS due to two types of CR particles (p and Fe) are presented in Fig. 1 below and in the region of the "knee". It demonstrates the results of the simulated (solid lines) and parameterized LDF of Cherenkov radiation (dashed lines) for vertical showers for primary

Table 1. **Energy dependence (Eq. (4)) identified by coefficients c_i of the extrapolated parameters a, b, s, and r_0 for the primary particles (p and Fe) for vertical EAS of Yakutsk EAS array [12, 13]**

K	a	Error	b	Error	s	Error	r_0	Error
				p, $\theta = 0°$				
	$\chi^2 = 0.01245$		$\chi^2 = 0.01$		$\chi^2 = 0.0576$		$\chi^2 = 0.0003$	
c_0	$1.374 \cdot 10^{2}$	$0.884 \cdot 10^{2}$	$-2.826 \cdot 10^{1}$	$1.523 \cdot 10^{1}$	$8.126 \cdot 10^{1}$	$3.887 \cdot 10^{1}$	$-2.502 \cdot 10^{1}$	$3.303 \cdot 10^{1}$
c_1	$-3.309 \cdot 10^{1}$	$1.798 \cdot 10^{1}$	$6.008 \cdot 10^{0}$	$3.097 \cdot 10^{0}$	$-1.669 \cdot 10^{1}$	$7.903 \cdot 10^{0}$	$5.174 \cdot 10^{0}$	$6.717 \cdot 10^{0}$
c_2	$2.552 \cdot 10^{0}$	$1.214 \cdot 10^{0}$	$-4.152 \cdot 10^{-1}$	$2.092 \cdot 10^{-1}$	$1.132 \cdot 10^{0}$	$5.338 \cdot 10^{-1}$	$-3.58 \cdot 10^{-1}$	$4.537 \cdot 10^{-1}$
c_3	$-6.023 \cdot 10^{-2}$	$2.726 \cdot 10^{-2}$	$9.470 \cdot 10^{-3}$	$4.700 \cdot 10^{-3}$	$-2.548 \cdot 10^{-2}$	$1.198 \cdot 10^{-2}$	$8.070 \cdot 10^{-3}$	$1.018 \cdot 10^{-2}$
				Fe, $\theta = 0°$				
	$\chi^2 = 0.00262$		$\chi^2 = 0.19718$		$\chi^2 = 0.00684$		$\chi^2 = 0.00001524$	
c_0	$-3.127 \cdot 10^{2}$	$2.054 \cdot 10^{2}$	$-1.393 \cdot 10^{2}$	$9.325 \cdot 10^{1}$	$6.392 \cdot 10^{1}$	$1.429 \cdot 10^{2}$	$-2.276 \cdot 10^{1}$	$3.115 \cdot 10^{1}$
c_1	$5.605 \cdot 10^{1}$	$4.274 \cdot 10^{1}$	$2.983 \cdot 10^{1}$	$1.940 \cdot 10^{1}$	$-1.412 \cdot 10^{1}$	$2.974 \cdot 10^{1}$	$4.185 \cdot 10^{0}$	$6.482 \cdot 10^{0}$
c_2	$-3.361 \cdot 10^{0}$	$2.957 \cdot 10^{0}$	$-2.111 \cdot 10^{0}$	$1.342 \cdot 10^{0}$	$1.030 \cdot 10^{0}$	$2.057 \cdot 10^{0}$	$-2.519 \cdot 10^{-1}$	$4.484 \cdot 10^{-1}$
c_3	$7.018 \cdot 10^{-2}$	$6.803 \cdot 10^{-2}$	$4.960 \cdot 10^{-2}$	$3.088 \cdot 10^{-2}$	$-2.498 \cdot 10^{-2}$	$4.733 \cdot 10^{-2}$	$4.740 \cdot 10^{-3}$	$1.032 \cdot 10^{-2}$

Table 2. **Coefficients c_i which identify the dependence of primary energy (Eq. (4)) of the extrapolated parameters a, b, s, and r_0 for two particles (p and Fe) for inclined EAS of Yakutsk EAS array [12, 13]**

K	a	Error	b	Error	s	Error	r_0	Error
				p, $\theta = 30°$				
	$\chi^2 = 0.00325$		$\chi^2 = 0.00425$		$\chi^2 = 0.0000561252$		$\chi^2 = 0.000433775$	
c_0	$-7.715 \cdot 10^{1}$	$8.318 \cdot 10^{1}$	$-4.417 \cdot 10^{1}$	$1.531 \cdot 10^{1}$	$6.133 \cdot 10^{0}$	$1.382 \cdot 10^{1}$	$1.415 \cdot 10^{1}$	$6.818 \cdot 10^{1}$
c_1	$1.194 \cdot 10^{1}$	$1.730 \cdot 10^{1}$	$9.292 \cdot 10^{0}$	$3.187 \cdot 10^{0}$	$-1.353 \cdot 10^{0}$	$2.877 \cdot 10^{0}$	$-3.037 \cdot 10^{0}$	$1.418 \cdot 10^{1}$
c_2	$-6.242 \cdot 10^{-1}$	$1.197 \cdot 10^{0}$	$-6.40 \cdot 10^{-1}$	$2.205 \cdot 10^{-1}$	$9.383 \cdot 10^{-2}$	$1.99 \cdot 10^{-1}$	$2.137 \cdot 10^{-1}$	$9.814 \cdot 10^{-1}$
c_3	$1.389 \cdot 10^{-2}$	$2.754 \cdot 10^{-2}$	$1.458 \cdot 10^{-2}$	$5.070 \cdot 10^{-3}$	$-2.180 \cdot 10^{-3}$	$4.580 \cdot 10^{-3}$	$-5.16 \cdot 10^{-3}$	$2.258 \cdot 10^{-2}$
				Fe, $\theta = 10°$				
	$\chi^2 = 0.02903$		$\chi^2 = 0.00136$		$\chi^2 = 0.0000250436$		$\chi^2 = 0.0000249987$	
c_0	$-4.028 \cdot 10^{2}$	$1.254 \cdot 10^{2}$	$-7.279 \cdot 10^{0}$	$7.138 \cdot 10^{0}$	$2.138 \cdot 10^{1}$	$1.546 \cdot 10^{1}$	$-8.124 \cdot 10^{1}$	$3.377 \cdot 10^{1}$
c_1	$7.741 \cdot 10^{1}$	$2.610 \cdot 10^{1}$	$1.604 \cdot 10^{0}$	$1.485 \cdot 10^{0}$	$-4.105 \cdot 10^{-1}$	$3.217 \cdot 10^{0}$	$1.744 \cdot 10^{1}$	$7.027 \cdot 10^{0}$
c_2	$-5.011 \cdot 10^{0}$	$1.806 \cdot 10^{0}$	$-1.079 \cdot 10^{-1}$	$1.027 \cdot 10^{-1}$	$2.081 \cdot 10^{-2}$	$2.225 \cdot 10^{-1}$	$-1.243 \cdot 10^{0}$	$4.862 \cdot 10^{-1}$
c_3	$1.118 \cdot 10^{-1}$	$4.154 \cdot 10^{-2}$	$2.38 \cdot 10^{-3}$	$2.36 \cdot 10^{-3}$	$-3.301 \cdot 10^{-4}$	$5.12 \cdot 10^{-3}$	$2.919 \cdot 10^{-2}$	$1.118 \cdot 10^{-2}$

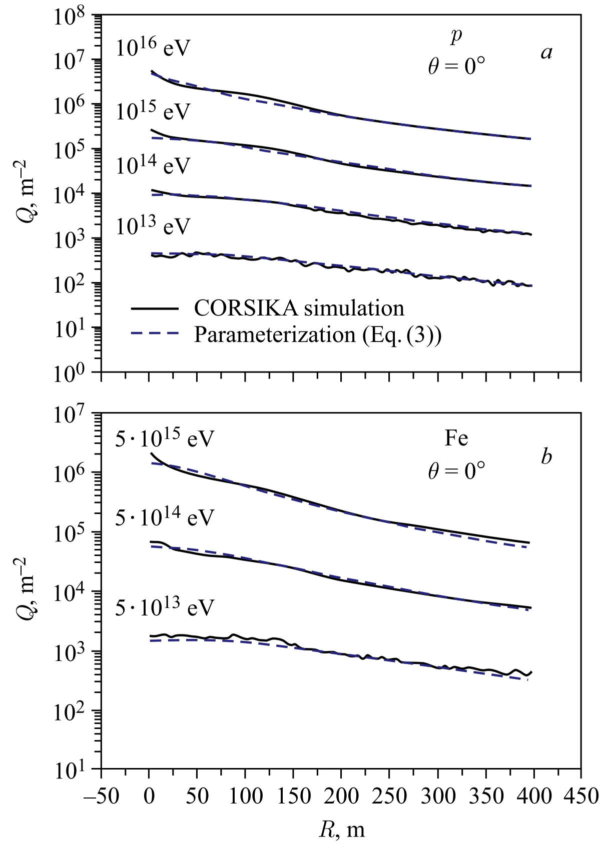

Fig. 1. Cherenkov light LDFs that were simulated using CORSIKA program (solid lines) and approximated (Eq. (3)) (dashed lines) for vertical EAS initiated by: *a*) proton at the energy range of 10^{13}–10^{16} eV; *b*) iron nuclei at the energies $5 \cdot 10^{13}$–$5 \cdot 10^{15}$ eV

proton and iron nuclei, respectively, at different primary energies. Monte Carlo statistics in Fig. 1 gives the possibilities for the type reconstruction of the primary particle that initiated the showers in EAS cascade. The approximation of parameter dependences on the primary energy is carried out using a 3rd order polynomial fit. These differences permit to distinguish the initiating primaries on the basis of different χ^2. For example, the χ^2 for protons is more times larger using the iron fit for the same parameters. The accuracy of the parameterization of Cherenkov light LDF with that simulated for proton is better than 15% at the distances of 10–150 m from the air shower core and about 5–15% for the other distances. The accuracy of iron nuclei was found close to 15% at the distances of 10–150 m from the shower core and about 5–10% at the other distances.

3. COMPARISON OF PARAMETERIZED LDF WITH YAKUTSK MEASUREMENTS

The wide-angle Yakutsk EAS array was designed for studying the chemical composition and the energy spectrum of CRs of extremely high energies (10^{15}–$5 \cdot 10^{19}$ eV), i.e., in the field of CR astrophysics. The construction of Yakutsk array depends on two main goals; the first one is the investigation of cascades of elementary particles in the atmosphere initiated by primary particles and the other is the reconstruction of the properties of the CR particle type, energy spectrum, and chemical composition [13]. The essential parameters of EAS measurements are zenith and azimuth angles, location of shower core, individual LDF, and the density of Cherenkov radiation $Q(R)$. The ability of reconstructing the particle type in EAS can be demonstrated in Figs. 2 and 3.

Figure 2, a demonstrates the comparison between the approximated LDF of Cherenkov radiation (dashed lines) and the measured LDF with Yakutsk EAS array [7, 8] (symbols) for primary proton and iron nuclei at the distance from 2.5 to 400 m from the shower core. Figure 2, b displays a good agreement between the approximated Cherenkov light LDF (dashed lines) and that measured with Yakutsk array [12, 13] (symbols) for two primary particles (p and Fe) at different energies and different zenith angles.

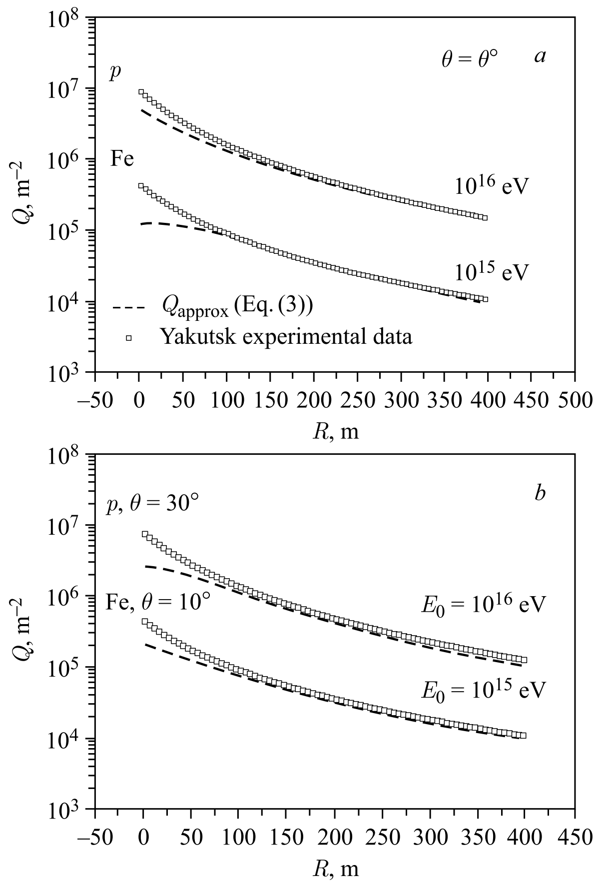

Fig. 2. Comparison between the parameterized LDF of Cherenkov radiation and the experimental data of LDF obtained by Yakutsk EAS array [12, 13] (symbols) for: *a*) iron nuclei and primary proton at the energies of 10^{15} and 10^{16} eV, respectively; *b*) two different primary particles (Fe and *p*), zenith angles (10 and 30°), and energies (10^{15} and 10^{16} eV)

The extrapolation of the Cherenkov light LDF parameterization for higher energies ($> 10^{16}$ eV) can be seen in Fig. 3, where Fig. 3, *a* displays the comparison between the approximated LDF of Cherenkov radiation which extrapolated to 20 and 30 PeV (dashed lines) and LDF measured with Yakutsk EAS array [12, 13] (symbols) for primary proton at $\theta = 30°$. Figure 3, *b* shows the comparison between the approximated LDF of Cherenkov radiation which extrapolated to 20 and 30 PeV (dashed lines) and LDF measured with Yakutsk EAS array [12, 13] (symbols) for iron nuclei for vertical showers. A good agreement between the model parameters of the extrapolated Cherenkov light LDF, which is a function of the energy of the primary particles, and that measured with Yakutsk array shows that this model (Eqs. (3) and (4)) is adequate and usable for different Cherenkov arrays.

The parameterized Cherenkov light LDF in Figs. 2 and 3 differs slightly from the measured LDF with Yakutsk EAS array; at 80–400 m from the shower core, the variation was close to 5–20% for proton and 5–13% for iron nuclei for vertical showers. For inclined showers, the distinction at the same distance interval is about 15–20% at $\theta = 30°$ for primary proton and about 8–20% at $\theta = 10°$ for iron nuclei.

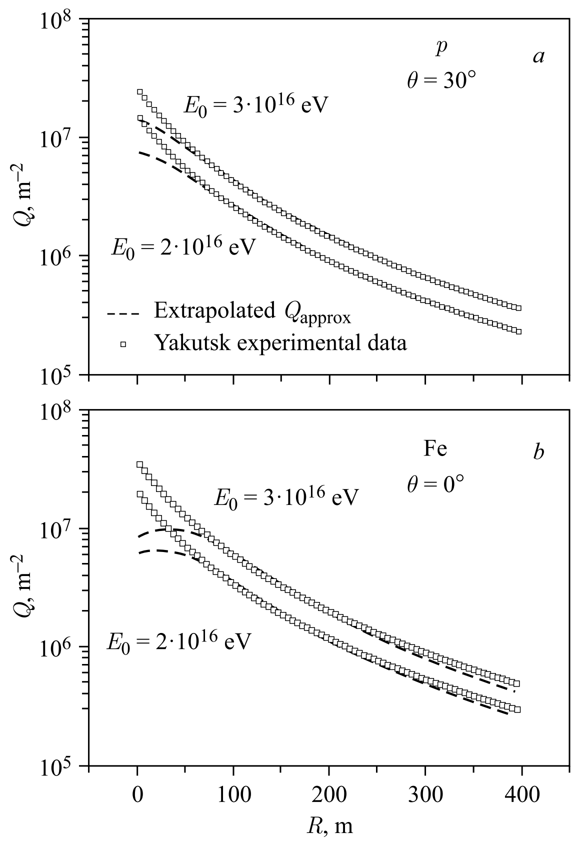

Fig. 3. Extrapolation of parameterized Cherenkov light LDF (Eq. (3)) (dashed lines) in comparison with experimental data obtained by Yakutsk EAS array (symbols) for the energies of $2 \cdot 10^{16}$ and $3 \cdot 10^{16}$ eV for: *a*) inclined showers ($\theta = 30°$) initiated by primary proton; *b*) vertical showers initiated by iron nuclei

CONCLUSIONS

The simulation of Cherenkov light lateral distribution function in extensive air showers induced by primary particles, like proton and iron nuclei, was performed at the energy range of 10^{13}–10^{16} eV using CORSIKA code. On the basis of this simulation and by depending on Breit–Wigner function, sets of approximating functions were constructed for two primary cosmic ray particles at different zenith angles. The comparison between the parameterized lateral distribution functions of Cherenkov radiation with Yakutsk EAS array has displayed the ability for determining the primary particles that initiate EAS showers and identifying their energy around and above the knee region of the CR spectrum. The extrapolation of the Cherenkov light lateral distribution function parameterization of the data with the CORSIKA program for the primary energies above 10^{16} eV is obtained. The fundamental feature of the presented approach consists in the ability of making a library of the Cherenkov light lateral distribution function models that could be used for the analysis of real events, that were detected by Cherenkov arrays, and reconstruction of the energy spectrum and chemical composition of the primary CR.

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