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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ ФИЗИКИ

ENTANGLED SOLITONS AND STOCHASTIC Q-BITS

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Stochastic realization of the wave function in quantum mechanics with the inclusion of soliton representation of extended particles is discussed. Two-solitons configurations are used for constructing entangled states in generalized quantum mechanics dealing with extended particles, endowed with nontrivial spin S. Entangled solitons construction being introduced in the nonlinear spinor field model, the Einstein–Podolsky–Rosen (EPR) correlation is calculated and shown to coincide with the quantum mechanical one for the $1/2$-spin particles. The concept of stochastic q-bits is used for quantum computing modelling.

Обсуждается стохастическая реализация волновой функции в квантовой механике на основе солитонного представления протяженных частиц. Для построения запутанных состояний в обобщенной квантовой механике протяженных частиц со спином S используется двухсолитонные конфигурации. Конструкция запутанных солитонов в модели нелинейного спинорного поля применяется для вычисления спиновой корреляции Эйнштейна–Подольского–Розена (ЭПР), и показано, что она совпадает с квантовой ЭПР-корреляцией для частиц спина $1/2$. Для моделирования квантовых вычислений используется концепция стохастических кубитов.

INTRODUCTION.
WAVE-PARTICLE DUALISM AND SOLITONS

As the first motivation for introducing stochastic representation of the wave function let us consider the de Broglie plane wave

$$\psi = A\,\mathrm{e}^{-ikx} = A\,\mathrm{e}^{-i\omega t + i(\mathbf{kr})}$$

for a free particle with the energy ω, momentum $\mathbf{k}$, and mass m, when the relativistic relation

$$k^2 = \omega^2 - \mathbf{k}^2 = m^2$$

holds (in natural units $\hbar = c = 1$).

Suppose, following L. de Broglie [1] and A. Einstein [2], that the structure of the particle is described by a regular bounded function $u(t, \mathbf{r})$, which is supposed to satisfy some nonlinear equation with the Klein–Gordon linear part. Let $\ell_0 = 1/m$ be the characteristic size of the soliton solution $u(t, \mathbf{r})$ moving with the velocity $\mathbf{v} = \mathbf{k}/\omega$.

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Now it is worthwhile to underline the remarkable fact behind this research [3], namely, the possibility to represent the de Broglie wave as the sum of solitons located at the nodes of a cubic lattice with the spacing $a \gg \ell_0$:

$$A\,\mathrm{e}^{-ikx} = \sum_{\mathbf{d}} u(t, \mathbf{r} + \mathbf{d}),$$

where $\mathbf{d}$ marks the positions of lattice nodes. To show this, one can take into account the asymptotic behavior of the soliton in its tail region:

$$u(x) = \int d^4k\,\mathrm{e}^{-ikx} g(k)\delta(k^2 - m^2)$$

and then use the well-known formula

$$\sum_{\mathbf{d}} \mathrm{e}^{i(\mathbf{k}\,\mathbf{d})} = \left(\frac{2\pi}{a}\right)^3 \delta(\mathbf{k}).$$

1. SOLITONIAN SCHEME IN QUANTUM MECHANICS

The main goal of this paper is to show that special «soliton» representation of quantum mechanics (QM) is possible, with the conservation of all QM principles in the point-like limit of particles, the spin-statistics correlation being included. The role of the one-particle wave function in this solitonian scheme is played by the linear combination of solitons which generalizes the aforementioned soliton representation of the de Broglie wave:

$$\Psi_N(t, \mathbf{r}) = (\hbar N)^{-1/2} \sum_{j=1}^{N} \varphi_j(t, \mathbf{r}). \tag{1}$$

Here the Einstein–de Broglie idea to represent particles by regular solutions $\phi(t, \mathbf{r})$ to some fundamental nonlinear equations is used. The complex function φ in (1) reads

$$\varphi(t, \mathbf{r}) = \frac{1}{\sqrt{2}}(\nu\phi + i\pi/\nu),$$

where

$$\pi(t, \mathbf{r}) = \partial\mathcal{L}/\partial\phi_t, \quad \phi_t = \partial\phi/\partial t,$$

is the conjugate momentum, and ν is the normalization constant, which is chosen from the condition

$$\hbar = \int d^3x\,|\varphi|^2,$$

with $\hbar$ being the Planck constant.

The index j in (1) runs over the set of independent (random) trials, the number N of which is supposed to be very large (the frequency hypothesis by Mises). Thus, the formula

(1) gives the stochastic realization of the wave function. To show that Ψ_N plays the role of the probability amplitude, it is sufficient to calculate the integral

$$\rho_N = \frac{1}{\triangle\vee} \int\limits_{\triangle\vee\subset\mathbb{R}^3} d^3x\, |\Psi_N|^2, \tag{2}$$

which is taken over the small volume $\triangle\vee \gg \vee_0$, where $\vee_0$ stands for the proper volume of the soliton. It can be easily shown that with the probability $P = 1 - \alpha \vee_0 / \triangle\vee$, $\alpha \sim 1$, the integral (2) equals to

$$\rho_N = \triangle N / N \triangle\vee,$$

where $\triangle N$ is the number of trials, for which the centers of solitons are contained in $\triangle\vee$. That is why ρ_N can be identified with the coordinate probability density.

Now let us consider the measuring procedure for some observable A corresponding, due to E. Noether's theorem, to the symmetry group generator $\hat{M}_A$. For example, the momentum $\mathbf{P}$ is related to the generator of space translation $\hat{M}_P = -i\,\bigtriangledown$, the angular momentum $\mathbf{L}$ is related to the generator of space rotation $\hat{M}_L = \mathbf{J}$ and so on. As a result, one can represent the classical observable A_j for the j-th trial in the form

$$A_j = \int d^3x\, \pi_j i \hat{M}_A \phi_j = \int d^3x\, \varphi_j^* \hat{M}_A \varphi_j.$$

The corresponding mean value is

$$\mathbb{E}(A) \equiv \frac{1}{N}\sum_{j=1}^{N} A_j = \frac{1}{N}\sum_{j=1}^{N} \int d^3x\, \varphi_j^* \hat{M}_A \varphi_j = \int d^3x\, \Psi_N^* \hat{A} \Psi_N + O\left(\frac{\vee_0}{\triangle\vee}\right), \tag{3}$$

where the Hermitian operator $\hat{A}$ reads $\hat{A} = \hbar \hat{M}_A$. Thus, up to the terms of the order $\vee_0/\triangle\vee \ll 1$, we obtain the standard quantum mechanical rule (3) for the calculation of mean values.

2. ENTANGLED SOLITONS

Now we consider two spinor solitons endowed with the spin $1/2$ and construct the entangled solitons configuration with the zero spin

$$\varphi_{12} = \frac{1}{\sqrt{2}}\left[\varphi_1^{\uparrow} \otimes \varphi_2^{\downarrow} - \varphi_1^{\downarrow} \otimes \varphi_2^{\uparrow}\right], \tag{4}$$

where $\varphi_1^{\uparrow}$ corresponds to the spin projection $+1/2$ and $\varphi_2^{\downarrow}$ corresponds to that $-1/2$. Finally, one can construct, on the base of (4), the stochastic wave function (1) and calculate the EPR-correlation of spins, which proves to coincide with the quantum one

$$P(\mathbf{a}, \mathbf{b}) = -(\mathbf{ab}). \tag{5}$$

By virtue of the orthogonality relation for the states with the opposite spin projections, one easily derives the following normalization condition for the entangled solitons configuration (4):

$$\int d^3x_1 \int d^3x_2\, \varphi_{12}^{+}\varphi_{12} = \hbar^2.$$

Now it is not difficult to find the expression for the stochastic wave function (1) for the singlet two-solitons state:

$$\Psi_N\left(t, \mathbf{r}_1, \mathbf{r}_2\right) = \left(\hbar^2 N\right)^{-1/2} \sum_{j=1}^{N} \varphi_{12}^{(j)}, \tag{6}$$

where $\varphi_{12}^{(j)}$ corresponds to the entangled solitons configuration in the j-th trial.

Our final step is the calculation of the spin correlation for the singlet two-solitons state. In the light of the fact that in the standard EPR-correlation the operator σ corresponds to the twice angular momentum operator, one should calculate the following expression:

$$P'(\mathbf{a}, \mathbf{b}) = \mathbb{M} \int d^3x_1 \int d^3x_2\, \Psi_N^{+} 2\left(\mathbf{J}_1\mathbf{a}\right) \otimes 2\left(\mathbf{J}_2\mathbf{b}\right) \Psi_N, \tag{7}$$

where $\mathbb{M}$ stands for the averaging over the random phases of the solitons. Inserting (6) into (7), using the independence of trials $j \neq j'$ and taking into account the relations:

$$\begin{aligned} &J_+\varphi^{\uparrow} = 0, \quad J_3\varphi^{\uparrow} = \frac{1}{2}\varphi^{\uparrow}, \qquad J_-\varphi^{\uparrow} = \varphi^{\downarrow}, \\ &J_-\varphi^{\downarrow} = 0, \quad J_3\varphi^{\downarrow} = -\frac{1}{2}\varphi^{\downarrow}, \quad J_+\varphi^{\downarrow} = \varphi^{\uparrow}, \end{aligned} \tag{8}$$

where $J_\pm = J_1 \pm iJ_2$, one easily finds that

$$P'(\mathbf{a}, \mathbf{b}) = -(\mathbf{ab}). \tag{9}$$

Comparing the correlations (9) and (5), one remarks their coincidence, that is, the solitonian model satisfies the EPR-correlation criterium. This fact permits one to use the entangled solitons configurations (4) in stochastic representation (6) for constructing stochastic q-bits.

The main results, obtained within the scope of the solitonian scheme, are represented in the papers [4–11].

3. STOCHASTIC Q-BITS AND SOLITONS

Now we intend to explain how stochastic q-bits (stobits) could be introduced into the solitonian scheme via random phase mechanism. To this end, one should define the random phase Φ_j for the j-th trial in our system of n solitons-particles. Let $\varphi^{(k)}(\mathbf{r})$ denote the standard (etalon) profile for the k-th soliton. The most probable position $\mathbf{d}_j^{(k)}(t)$ of the k-th soliton's center in j-th trial can be found from the following variational problem:

$$\left| \int d^3x\, \varphi_j^{*(k)}(t, \mathbf{r}) \varphi^{(k)}\left(\mathbf{r} - \mathbf{d}_j^{(k)}\right) \right| \to \max,$$

thus giving the random phase structure:

$$\Phi_j = \sum_{k=1}^{n} \arg \int d^3x\, \varphi_j^{*(k)}(t,\mathbf{r})\varphi^{(k)}\left(\mathbf{r} - \mathbf{d}_j^{(k)}\right). \tag{10}$$

The random phase (10) can be used for simulating quantum computing via generating the following K random dichotomic functions:

$$f_s(\theta_s) = \text{sign}\,[\cos(\Phi_j + \theta_s)]\,, \quad s = \overline{1, K}, \tag{11}$$

with θ_s being arbitrary fixed phases. Now recall that the quantum bit (q-bit) is identified with the state vector

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

corresponding to the superposition of two orthogonal states $|0\rangle$ and $|1\rangle$ as, for instance, two polarizations of the photon, or two possible $1/2$-spin states. Using the expression (5) for $1/2$-spin correlation in a singlet state of two particles, one can compare it with the random phases correlation for the case of $n = 2$ particles:

$$\mathbb{E}\,(f_1\, f_2) = 1 - \frac{2}{\pi}|\triangle\theta|, \tag{12}$$

where $\triangle\theta = \theta_1 - \theta_2$. The similarity of these two functions (5) and (12) of the angular variable seems to be a good motivation for the K q–bits simulation by the dichotomic random functions (11), popularized in the paper [12].

In conclusion, we express the hope that the random solitons phases realization could be effectively simulated by the generator of random numbers in classical computer. This very simple model will be called the stochastic q-bits simulation. We hope that it will be useful for the Shor's and Grover's quantum algorithms realization.

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